The Prescott Real Business Cycle Model

This handout presents the elements of the original Real Business Cycle model of aggregate fluctuations, as laid out by Prescott (1986), along with a few critiques articulated by Summers (1986) and others.

Consider a representative household whose goal is to maximize

$$\mathbb{E}_t\left[\sum_{t=0}^{\infty}\beta^t \mathrm{u}(c_t, z_t)\right] \tag{1}$$

where z_t is the fraction of time the representative agent spends at leisure (not working); the alternative to leisure is the number of hours you work, which will be designated ℓ_t, and the time endowment is normalized to 1, so that

$$\ell_t + z_t = 1. \tag{2}$$

Assume that the structure of the utility function is

$$\begin{aligned} \mathrm{u}(c, z) &= \left(\frac{(c^{1-\zeta}z^{\zeta})^{1-\rho}}{1-\rho}\right) \\ \mathrm{u}^c &= (c^{1-\zeta}z^{\zeta})^{-\rho}c^{-\zeta}z^{\zeta}(1-\zeta) \\ \mathrm{u}^z &= (c^{1-\zeta}z^{\zeta})^{-\rho}c^{1-\zeta}z^{\zeta-1}\zeta. \end{aligned} \tag{3}$$

Think about the maximum amount of income that could be gained if the representative agent worked every waking hour:

$$y_t = \mathsf{W}_t. \tag{4}$$

The representative agent can then think of deciding to 'purchase' two things with this endowment of income: leisure z_t whose price is W_t, or consumption, whose price is normalized to one.

Over the past century in the U.S., wages have risen very substantially, but hours worked have not declined much if at all. (Ramey and Francis (2006)). What kind of utility function implies that the budget share of a good (leisure) remains constant even as the price of the good changes sharply? A Cobb-Douglas utility function. Hence the assumption that utility is obtained from a Cobb-Douglas aggregate of consumption and leisure is consistent with the lack of a strong trend in hours worked per worker.

Since workers are choosing how many hours to work as well how much to consume, a first order condition will characterize the optimal choice *between* consumption and leisure *within* a period. In particular, the price of leisure is W_t and the price of consumption is 1, so the ratio of the marginal utility of leisure to the marginal utility of consumption should be

$$\left(\frac{\mathsf{W}_t}{1}\right) = \left(\frac{\mathrm{u}^z_t}{\mathrm{u}^c_t}\right). \tag{5}$$

To see this, note that the consumer's goal is to

$$\max_{\{c_t, z_t\}} \mathrm{u}(c_t, z_t). \tag{6}$$

Suppose the consumer has decided to spend a given amount χ_t in period t on a combination of consumption and leisure,

$$c_t + \mathsf{W}_t z_t = \chi_t \tag{7}$$

Then (6) becomes

$$\max_{\{z_t\}} \mathrm{u}(\chi_t - \mathsf{W}_t z_t, z_t) \tag{8}$$

for which the FOC is

$$\begin{aligned} -\mathrm{u}_t^c \mathsf{W}_t + \mathrm{u}_t^z &= 0 \\ \mathsf{W}_t &= (\mathrm{u}_t^z / \mathrm{u}_t^c). \end{aligned} \tag{9}$$

Returning to (5)

$$\begin{aligned} \left(\frac{\mathsf{W}_t}{1}\right) &= \left(\frac{\mathrm{u}_t^z}{\mathrm{u}_t^c}\right) \\ \mathsf{W}_t &= \left(\frac{c_t^{1-\zeta} z_t^{\zeta-1}}{c_t^{-\zeta} z_t^{\zeta}}\right) \left(\frac{\zeta}{1-\zeta}\right) \\ &= \left(\frac{c_t}{z_t}\right) \left(\frac{\zeta}{1-\zeta}\right) \end{aligned} \tag{10}$$

or

$$\mathsf{W}_t z_t / c_t = \left(\frac{\zeta}{1-\zeta}\right). \tag{11}$$

Since we know that z has been roughly constant over long periods of time, this implies that as wages rise, consumption rises by roughly the same amount.

One of the original proimises of the DSGE literature was to calibrate its business-cycle models based on either long-run facts (like the lack of a trend in z) or on micro data (like intertemporal elasticities estimated using household data). So how is ζ calibrated?

If wages are defined as per unit of labor, then if on average consumption roughly equals labor income $c = (1 - z)w$ we have

$$\begin{aligned} \left(\frac{\mathsf{w} z}{\mathsf{w}(1-z)}\right) &= \left(\frac{\zeta}{1-\zeta}\right) \\ \left(\frac{1-z}{z}\right) &= \left(\frac{1-\zeta}{\zeta}\right) \\ 1/z - 1 &= 1/\zeta - 1 \\ z &= \zeta. \end{aligned} \tag{12}$$

So ζ should be calibrated to be equal to the proportion of their available (i.e. non-sleep) time people spend not working. A 40-hour work week (along with 8 hours of sleep a day) would yield $\zeta = 2/3$. Among other taste parameters, Prescott chooses log utility ($\lim_{\rho \to 1} \mathrm{u}(c)$) and $\beta = 0.96$.

The aggregate production function is assumed to be Cobb-Douglas,

$$\begin{aligned} K_{t+1} &= \overbrace{\daleth}^{1-\delta} K_t + Y_t - C_t \\ Y_t &= A_t K_t^{1-\nu} L_t^{\nu} \end{aligned} \tag{13}$$

where $\ell_t = (1 - z_t)\mathfrak{H}_t = \ell_t \mathfrak{H}_t$ where $\mathfrak{H}_t = \sum_i \mathfrak{h}_i$ is the aggregate amount of Hours available to members of the working population. Constant income shares and perfect competition imply

$$\begin{aligned} \mathrm{F}_L L/Y &= \nu A_t K_t^{1-\nu} L^{\nu-1} L_t/Y_t \\ &= \nu \end{aligned} \tag{14}$$

so that labor's share of GDP is roughly constant. Prescott sets labor's share to a constant 64 percent, and chooses a depreciation rate of $\delta = 0.10$.

The crucial assumption, however, is about the productivity process, since 'technology shocks' are assumed to drive business cycles.

Prescott defines the 'hat' operator $\hat{}$ as:

$$\hat{X}_t = \left(\frac{X_t - X_{t-1}}{X_{t-1}} \right) \tag{15}$$

which implies from the production function that

$$\hat{A}_t \approx \hat{Y}_t - \nu \hat{L}_t - (1 - \nu)\hat{K}_t. \tag{16}$$

(this is just the Solow residual).

Prescott 'estimates' a productivity process that takes the form

$$\hat{A}_t = \phi_t + \epsilon_t \tag{17}$$

with a standard deviation of $\sigma_\epsilon = 0.76$ per quarter.

Prescott makes sufficient assumptions (perfect competition, etc.) so that the social planner's problem is the same as the decentralized solution. With log utility, the social planner's problem is

$$\max \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t \left((1 - \zeta) \log c_t + \zeta \log(1 - \ell_t) \right) \right] \tag{18}$$

subject to

$$\begin{aligned} k_{t+1} &= \daleth k_t + A_t k_t^{1-\nu} \ell^{\nu} - c_t \\ \hat{A}_t &= \phi + \epsilon_t. \end{aligned} \tag{19}$$

Prescott argues that the way to judge the model is by whether it produces plausible statistics for standard deviations of the key variables. He produces a table that argues it does:

Since the first column is calibrated, it isn't a test of the model. The second column comes out of the model, and isn't too bad a fit. However, the third column is a terrible

	σ_z	σ_y	σ_n
US Data	0.76	1.76	1.67
Model	0.76	1.48	0.76

fit. What it says is that labor input is much more variable over the course of the business cycle than this model would suggest.

What's going on? To understand the answer, we need to understand why hours fluctuate in this model at all. Recall that we deliberately constructed the model (by choosing a utility function that was Cobb-Douglas in consumption and leisure) in a way designed to *prohibit* any *long-run* response of hours worked to wages. Since hours worked are being chosen freely on a day-by-day basis by workers in this model, there must be some incentive that causes them to be willing to put up with *short-term* variation in hours (over the business cycle).

The answer is that transitory productivity shocks provide an incentive to work harder some times than others. In particular, if there is a temporary positive productivity shock you will be willing to work longer hours than usual, while if there is a negative productivity shock everybody wants to take a vacation.

To see this formally, consider again the first order conditions from the maximization problem. We showed in (11) that

$$\begin{aligned} \mathsf{W}_t z_t/c_t &= \left(\frac{\zeta}{1-\zeta}\right) \\ &= \mathsf{W}_{t+1}z_{t+1}/c_{t+1}. \end{aligned} \tag{20}$$

Now note that since (18) is separable in consumption and leisure the intertemporal FOC will imply that

$$\begin{aligned} 1/c_t &= \mathsf{R}_{t+1}\beta/c_{t+1} \\ c_{t+1} &= \mathsf{R}_{t+1}\beta c_t. \end{aligned} \tag{21}$$

Combining this with (20) gives

$$\begin{aligned} \mathsf{W}_t z_t &= \mathsf{W}_{t+1}z_{t+1}/\mathsf{R}_{t+1}\beta \\ z_{t+1}/z_t &= \left(\frac{\mathsf{R}_{t+1}\beta}{\mathsf{W}_{t+1}/\mathsf{W}_t}\right) \\ \hat{z}_{t+1} &\approx -\hat{\mathsf{w}}_{t+1} + (\mathsf{r}_{t+1}-\theta) \end{aligned} \tag{22}$$

What this equation tells us is that there are two ways to make z_t (and therefore ℓ_t) fluctuate over the business cycle:

1. Make transitory movements in real wages induce a strong labor supply response
   - Problem: A large number of microeconomic studies have estimated the elasticity of labor supply with respect to wages to be much too small to explain the observed fluctuations in hours over the business cycle. Indeed, there is even controversy about whether real wages rise or fall over the business cycle. But even if we accept that wages fall during recessions, the evidence at the

micro level does not seem to suggest that people are willing to dramatically change their work hours in response to transitory fluctuations in wages

2. Make movements in interest rates induce a strong labor supply response (the idea is that you will work hard during the period of high interest rates in order to earn more cash which can be invested to take advantage of the high interest rate - in thinking about this, consider that a high value of R_{t+1} will induce high leisure growth between t and $t+1$ by resulting in low leisure in period t)

   - Problem: If this is the mechanism, there should at the same time be a strong consumption response:

$$\begin{aligned} \mathsf{W}_t z_t / c_t &= \mathsf{W}_{t+1} z_{t+1} / c_{t+1} \\ c_{t+1}/c_t &= \mathsf{W}_{t+1} z_{t+1} / \mathsf{W}_t z_t \end{aligned} \tag{23}$$

     so if the labor supply response is not being driven by wage differences, there should be a one-for-one comovement of consumption with leisure - i.e. recessions should be periods of high consumption and booms should be periods of low consumption!

This latter is a quite general problem with the DSGE framework in which fluctuations in employment over the business cycle are driven by voluntary changes in hours worked.

References (click to download `.bib` file)

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