Generational Accounts and the Government

1 The Government Budget Constraint

Consider a government that raises taxes T_t, makes expenditures X_t, and has an outstanding stock of debt D_t at the beginning of period t, on which it must pay interest at rate r_t. The government can run a deficit only by raising funds via the issuing of new bonds.

The government's Dynamic Budget Constraint (DBC) is given by

$$
\overbrace{D_{t+1} - D_t}^{\text{Deficit}} = \overbrace{(X_t + \mathsf{r}_t D_t)}^{\text{Outlays}} - T_t
$$
$$
D_{t+1} = X_t + \mathsf{R}_t D_t - T_t \tag{1}
$$
$$
D_t = \left(\frac{D_{t+1} + T_t - X_t}{\mathsf{R}_t} \right).
$$

But we can obtain a similar formula for D_{t+1} in terms of D_{t+2}, and substitute it into (1). Continued substitution gives

$$
D_t = \overbrace{(T_t - X_t)}^{\equiv Z_t} / \mathsf{R}_t + (T_{t+1} - X_{t+1}) / \mathsf{R}_t \mathsf{R}_{t+1} + \dots
$$
$$
= Z_t / \mathsf{R}_t + Z_{t+1} / \mathsf{R}_t \mathsf{R}_{t+1} + \dots \tag{2}
$$
$$
\mathsf{R}_t D_t = \mathbb{P}_t(Z)
$$
$$
= \mathbb{P}(\text{Govt Primary Surpluses})
$$

where $\mathbb{P}$ denotes the present discounted value; this can be rewritten

$$
\mathbb{P}_t(X) = \mathbb{P}_t(T) - \mathsf{R}_t D_t. \tag{3}
$$

Equation (3) should look familiar: recall that in the consumption problem we had an Intertemporal Budget Constraint that said

$$
\mathbb{P}_t(C) = \mathbb{P}_t(Y) + \mathsf{R}_t K_t \tag{4}
$$

where K_t is the beginning-of-period level of capital wealth (before interest has been earned).

In each case, the PDV of expenditures must be equal to the PDV of income plus current wealth. Thus, equations (2) and (3) are different ways to express the Government Intertemporal Budget Constraint (GIBC).[1]

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[1]The R_t is present in the government's problem because for the consumer we were thinking about the situation after any interest income was received; if we were to think of the consumer's beginning-of-period capital as K_t then we would have $B_t = \mathsf{R}_t K_t$; and note that the sign difference reflects the fact that D is debt while B is balances.

Now let's suppose that the only kind of expenditures the government engages in are transfers, so that X_t simply reflects money handed out to some members of the population in period t. Then Z_t will be equal to total net transfers among the members of the population at period t. Note that there is nothing that says that Z_t must be positive or negative in any particular period. The GIBC only places restrictions on the present discounted value of net transfers.

The fact that government only has to satisfy the GIBC means that the government can potentially treat different generations very differently from each other. It is therefore useful to have a mechanism to keep track of how different generations are treated. The standard way of doing this is to construct a set of 'generational accounts,' as initially proposed by Auerbach, Kotlikoff, and Gokhale (1991).

If we assume that consumers live two-period lives, the generational account for the generation born at time t is:

$$\bar{Z}_t = Z_{1,t} + Z_{2,t+1}/\mathsf{R}_{t+1} \tag{5}$$
$$= \text{PDV of lifetime taxes net of transfer payments.}$$

In the US and most other countries, working-age people pay more in taxes than they receive in transfers, so $Z_{1,t}$ is positive, while old people receive more in transfers than they pay in taxes, so $Z_{2,t}$ is negative.

Note now that the aggregate total of net transfers can be subdivided into the net transfers of the two age groups in the population,

$$Z_t = Z_{1,t} + Z_{2,t}. \tag{6}$$

Now write out the GIBC (2) explicitly:

$$\begin{aligned}
\mathbb{P}_t(Z) = \ & Z_t + Z_{t+1}/\mathsf{R}_{t+1} + \ldots \\
= \ & Z_{1,t} + Z_{1,t+1}/\mathsf{R}_{t+1} + Z_{1,t+2}/\mathsf{R}_{t+1}\mathsf{R}_{t+2} + \ldots \\
& + Z_{2,t} + Z_{2,t+1}/\mathsf{R}_{t+1} + Z_{2,t+2}/\mathsf{R}_{t+1}\mathsf{R}_{t+2} + \ldots \\
= \ & Z_{2,t} + [Z_{1,t} + Z_{2,t+1}/\mathsf{R}_{t+1}] + [Z_{1,t+1} + Z_{2,t+2}/\mathsf{R}_{t+2}]/\mathsf{R}_{t+1} + \ldots \\
= \ & Z_{2,t} + \bar{Z}_t + \bar{Z}_{t+1}/\mathsf{R}_{t+1} + \bar{Z}_{t+2}/\mathsf{R}_{t+1}\mathsf{R}_{t+2} + \ldots
\end{aligned}$$

which again shows that the GIBC is consistent with any treatment of any particular generation; any pattern of generational accounts that satisfies the GIBC is feasible.

2 Social Security and Generational Accounts

Consider an economy that initially has no government so that $Z_{1,t} = Z_{2,t} = Z_{2,t-1} = 0$. Now consider introducing a Pay As You Go (PAYG) Social Security system at date s, which is to remain of constant size forever after introduction,

$$\begin{aligned}
Z_{2,t} &= -Z_{1,t} \neq 0 \ \ \forall \, t \ \geq \ s \\
Z_{1,t+1} &= Z_{1,t}.
\end{aligned} \tag{7}$$

Consider the generation born at time $s - 1$. It paid nothing into the Social Security

system when young, yet gets $Z_{2,s}$ out when old. Its generational account is therefore

$$
\begin{aligned}
\bar{Z}_{s-1} &= Z_{1,s-1} + Z_{2,s}/\mathsf{R}_s \\
&= 0 + Z_{2,s}/\mathsf{R}_s
\end{aligned}
\tag{8}
$$

so this generation benefits from the introduction of SS because it paid no taxes yet receives benefits.

The GA's for succeeding generations are

$$
\begin{aligned}
\bar{Z}_t &= Z_{1,t} + Z_{2,t+1}/\mathsf{R}_{t+1} \\
&= Z_{1,t}(1 - 1/\mathsf{R}_{t+1}) \\
&= \mathsf{r}_{t+1} Z_{1,t}/\mathsf{R}_{t+1}
\end{aligned}
\tag{9}
$$

so future generations are worse off by this amount.

The reason the introduction of Social Security makes future generations worse off is that without SS they could have invested the amount $Z_{1,t}$ and earned interest on it of $\mathsf{r}_{t+1}Z_{1,t}$ in period 2. Now the money is taken away from them when young and returned *without interest* when old. Thus, the loss is precisely the loss in interest income on $Z_{1,t}$ in period $t+1$, discounted back to the present.

Note that if there is zero population growth, the foregoing analysis all holds in per-capita terms as well, so that the per-capita change in generational accounts from introducing Social Security is

$$
\bar{z}_t = \mathsf{r}_{t+1} z_{1,t}/\mathsf{R}_{t+1}
\tag{10}
$$

2.1 Effects of Population Growth

If there is perpetual population growth, it is possible to finance a positive rate of return on Social Security contributions. Define

$$
z_{1,t} = Z_{1,t}/L_t
\tag{11}
$$

and assume there is constant population growth, $\Xi = L_{t+1}/L_t$. If we assume that Social Security taxes per capita are constant, then we can achieve a positive rate of return on Social Security contributions equal to the growth rate of population:

$$
\begin{aligned}
z_{2,t+1} &= Z_{2,t+1}/L_t \\
&= -Z_{1,t+1}/L_t \\
&= -\left(\frac{Z_{1,t+1}}{L_{t+1}}\right)\left(\frac{L_{t+1}}{L_t}\right) \\
&= -z_{1,t+1}\Xi = -z_{1,t}\Xi.
\end{aligned}
\tag{12}
$$

Not only does this prove that it is *possible* for the Social Security system to pay a rate of return equal to the rate of population growth - it proves that the *only* rate of return that is consistent with constant per-capita taxes on the young is a rate of return of Ξ.

2.2 Effects of Productivity Growth and Population Growth

Suppose there is wage growth G betwen t and $t+1$, and suppose that workers contribute a constant *percentage* of their incomes to the Social Security system, $z_{1,t} = \zeta \mathsf{W}_{1,t}$. In this case it is possible to earn a rate of return on SS contributions equal to the product of the growth factor for wages and the growth factor for population:

$$
\begin{aligned}
z_{1,t} &= \zeta \mathsf{W}_{1,t} \\
\mathsf{W}_{1,t+1} &= \mathsf{G}\mathsf{W}_{1,t} \\
z_{2,t+1} &= -Z_{1,t+1}/L_t \\
&= -(Z_{1,t+1}/L_{t+1})(L_{t+1}/L_t) \\
&= -\zeta \underbrace{\mathsf{W}_{1,t+1}}_{=\mathsf{G}\mathsf{W}_{1,t}} \Xi \\
&= -\zeta \mathsf{W}_{1,t}\mathsf{G}\Xi \\
&= -z_{1,t}\mathsf{G}\Xi
\end{aligned}
\tag{13}
$$

so viewed from the perspective of the young generation in period t, their Social Security contributions are returned to them larger by a factor of $\mathsf{G}\Xi$ than what they paid in; the effective rate of return is therefore $\mathsf{G}\Xi$.

2.3 Generational Accounts in a Growing Economy

Now consider the per-capita generational accounts in an economy with constant population growth and constant wage growth and a Social Security system that imposes a constant tax of ζ on the wages of the young:

$$
\begin{aligned}
\bar{z}_t &= z_{1,t} + z_{2,t+1}/\mathsf{R}_{t+1} \\
&= \zeta \mathsf{W}_{1,t} - \mathsf{G}\Xi\zeta\mathsf{W}_{1,t}/\mathsf{R}_{t+1} \\
&= \zeta \mathsf{W}_{1,t} \left(1 - \mathsf{G}\Xi/\mathsf{R}_{t+1} \right) \\
&= \zeta \mathsf{W}_{1,t} \left(\frac{\mathsf{R}_{t+1} - \mathsf{G}\Xi}{\mathsf{R}_{t+1}} \right).
\end{aligned}
\tag{14}
$$

Note that this expression will be *negative* if $\mathsf{G}\Xi > \mathsf{R}_{t+1}$, meaning that the introduction of a Social Security system with a positive tax rate ζ actually *improves* the lifetime budget constraint! This is another way of seeing that an economy is *dynamically inefficient* if the return factor for capital R is less than the product of the population growth and productivity growth factors. (Or, using approximations, the rate of return is less than the sum of the population growth rate and the productivity growth rate).

References

Auerbach, Alan J., Laurence J. Kotlikoff, and Jagadeesh Gokhale (1991): "Generational Accounting: A Meaningful Alternative to Deficit Accounting," in *Tax Policy and the Economy*, ed. by David Bradford, vol. 5. MIT Press, Cambridge, MA.