Data Sources and Solution Methods for Empirical and Theoretical Results in 'Unemployment Expectations, Jumping (S,s) Triggers, and Household Balance Sheets'

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The paper itself, the set of RATS programs that generate all of our empirical results, and the set of Mathematica programs that generate all of our theoretical results are available at Carroll's home page, http://www.econ.jhu.edu/ccarroll.

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1 Introduction

This document provides background information about empirical and theoretical results published in the paper 'Unemployment Expectations, Jumping (S,s) Triggers, and Household Balance Sheets,' published in the *NBER Macroeconomics Annual, 1997.* Specifically, we describe the aggregate time series data used to generate the empirical results, an explanation of the general methods of solving and then simulating the theoretical model, and an overview of the structure and flow of the Mathematica code which solves the model.

2 Empirical Data Sources

The bulk of the aggregate time series data used to generate the results in the paper are contained in the *Citibase* database (CITIBASE). Other data series were obtained from the Federal Reserve Board (FRB), the Flow of Funds tables (FOF), Fannie Mae (FNMA), or the National Bureau of Economic Research (NBER). The table below lists the name of each variable used in the RATS programs along with its definition and source. The names of variables obtained from *Citibase* also correspond to the names in the database itself.

All variables are seasonally adjusted except for MT, LB, AS, and CIC, which we seasonally adjust ourselves in the RATS programs. The flow of funds variables are from the outstanding levels table L.100 for households and nonprofit organizations, which can be found at the Board's public website (http://www.federalreserve.gov). Household net worth is total assets (both financial and tangible) minus total liabilities; annual data on net worth can be found in the flow of funds table B.100, but these data are not released publically on a quarterly basis. Consumer installment credit is total consumer credit outstanding minus non-installment credit.[1]

[1] Prior to June 1996, the Federal Reserve Board's Statistical Release G.19 included only installment credit in its measure of total consumer credit outstanding. However, beginning with the June 11, 1996 release the measure of consumer credit was broadened to include non-installment consumer credit, which in 1996 amounted to about $65 billion, or 6 percent of the total of installment plus non-installment credit. Consumer credit is now reported as three separate components: automobile credit, revolving credit, and 'other' credit. All non-installment credit is included in the 'other' category.

Variable Name	Source	Definition
GYD	CITIBASE	Disposable personal income (DPI)
GW	CITIBASE	Wage and salary disbursements
GPT	CITIBASE	Transfer payments
HNS	CITIBASE	New single family home sales
HESHS	CITIBASE	Existing single family home sales
POP	CITIBASE	Population
FYGM3	CITIBASE	Interest rate: 3-month Treasury bill
HHSNTR	CITIBASE	University of Michigan Index of Consumer Sentiment
HHSNTN	CITIBASE	University of Michigan Index of Consumer Expectations
GCN	CITIBASE	Personal consumption expenditures (PCE): nondurables
GCD	CITIBASE	PCE: durables
GCDA	CITIBASE	PCE: motor vehicles and parts
GCS	CITIBASE	PCE: services
GDC	CITIBASE	Implicit price deflator (PCE)
LHUR	CITIBASE	Unemployment rate
LHEL	CITIBASE	Help wanted advertising index
FSPCOM	CITIBASE	S&P's common stock price index
FYPR	CITIBASE	Prime rate
FYMCLE	CITIBASE	Interest rate: conventional home mortgage loans closed
RECSNS	NBER	NBER designated business cycles
ORIGVA	FNMA	VA home mortgage originations
ORIGA	FNMA	Total home mortgage originations
MT	FOF	Household home mortgages (FL153165105)
LB	FOF	Household total liabilities (FL194190005)
AS	FOF	Household total financial assets (FL194090005)
WEALTH	FRB	Household net worth (FL152090005)
CIC	FRB	Consumer installment credit (FL153166000)
DSB	FRB	Debt service burden
MQ	FRB	Univ. of Michigan Unemployment Expectations - More
LQ	FRB	Univ. of Michigan Unemployment Expectations - Less
XGAP	FRB	Gap between actual and trend GDP
YDGAP	FRB	Gap between actual and trend ratios of DPI to GDP
PICNIA	FRB	Inflation rate
RFFE	FRB	Interest rate: federal funds (effective)

The debt service burden is a measure of the fraction of disposable personal income taken up by service payments on outstanding debt. It is based on an estimate of total debt service payments using contemporaneous interest rates and the flow of funds measure of outstanding debt in the household sector. The debt service burden data were created by the Federal Reserve Board but are not publically released. The University of Michigan Survey Research

Center's unemployment expectations variables are based on the monthly surveys of consumers in which respondents are asked about their expectations about future unemployment. MQ is the percent of respondents who expect unemployment to rise, and LQ is the percent of respondents who expect unemployment to fall. Our index of unemployment expectations is simply (MQ-LQ)/10,000.

The remaining FRB variables, XGAP, YDGAP, PICNIA and RFFE are explained in the next section.

3 Methods of Constructing Annuity Income

Below are more detailed descriptions of the two methods for constructing annuity income mentioned in the paper.

3.1 FRB-US Method

The FRB-US formula for annuity income (A_t^{FRB-US}) is based on a log-linear approximation:

$$A_t^{FRB-US} = YOVERGDP_t * \overline{GDP}_t * exp(PDVXGAP_t/100 + PDVYGAP_t/100) \quad (1)$$

where $YOVERGDP_t$ is a nine-year centered moving average of the ratio of labor income (wages plus transfers) to GDP, $\overline{GDP}_t$ is trend real GDP (created at the Fed by the FRB-US model staff), $PDVXGAP_t$ is a weighted average of expected future GDP gaps, and $PDVYGAP_t$ is a weighted average of expected future gaps between the actual ratio of labor income to GDP and the trend ratio. $PDVYGAP$ and $PDVXGAP$ are calculated as follows:

$$PDVXGAP_t = \frac{R-1}{R}\sum_{j=0}^{\infty} R^{-j} E_t XGAP_{t+j} \quad (2)$$

$$PDVYGAP_t = \frac{R-1}{R}\sum_{j=0}^{\infty} R^{-j} E_t YGAP_{t+j} \quad (3)$$

where the interest rate R is chosen such that the annual interest rate is 25 percent.

A VAR forecasting system is used to estimate the expected output gap and the expected gap in the ratio of income to GDP. The system includes four equations for inflation (PICNIA), the Fed funds rate (RFFE), the output gap (XGAP), and the gap between the actual ratio of disposable income to GDP and the trend ratio of income to GDP (YDGAP). We augmented this specification with four lags of income growth and the unemployment expectations index. The coefficients of the system are restricted so that the forecasts of $XGAP_{t+j}$ and $YGAP_{t+j}$ made at time t converge to zero as j goes to infinity, and forecasts of inflation and interest rates as j goes to infinity converge to survey-based time series data on inflation and interest rates expected to prevail in the long run.

A more detailed overview of the design of the VAR forecasting system in the FRB/US model can be found on-line under the FEDS paper series on the Board's public web site at http://www.federalreserve.gov.

3.2 Carroll-Dunn Method

We begin by computing the actual presented discounted value of the sum of the next two years of labor income (N_t):

$$N_t \quad = \quad \sum_{j=1}^{8} Y_{t+j}/R^j \tag{4}$$

where R is defined as in the FRB-US method. We compute a similar sum over the previous two years of labor income (L_t):

$$L_t \quad = \quad \sum_{j=1}^{8} Y_{t-j}/R^j \tag{5}$$

and take the log difference of the two to get m_t:

$$m_t \quad = \quad \log N_t - \log L_t \tag{6}$$

We regressed m_t on m_{t-9}, the set of instruments from Carroll, Fuhrer and Wilcox (1994), the unemployment expectations index, the change in the CPI, the log difference between nondurables consumption and labor income, and the level of the help wanted index.[2]

The expectation of the present discounted value of the next two years of labor income is then calculated by taking the inverse log of the sum of the fitted values from the above regression and the log of L_t:

$$\hat{N}_t = \exp(\hat{m}_t + \log L_t) \tag{7}$$

We assume that beyond the two year forecast, income is expected to grow at a constant rate equal to the average growth rate ($G = 1 + g$) over the entire sample period. Using this growth rate, we calculate the present discounted value of expected labor income from two years to infinity and add this to the forecasted PDV of the next two years of labor income, and multiply by $(R-1)/R$ to get the annuity value of the PDV:

$$A_t^{Ours} = \frac{R-1}{R}\left(\hat{N}_t + \frac{(G/R)^8}{1-(G/R)}\right). \tag{8}$$

4 Numerical Solution of the Model

4.1 Solving The Model

The consumer's objective is to maximize the expected discounted value of lifetime utility from a nondurable good (C) and the flow of services from a durable (Z). The period utility function takes the standard form with constant relative risk aversion (CRRA) and Cobb-Douglas aggregation of goods and services:

$$\max_{\{C_t, Z_t\}} \sum_{t=0}^{\infty} \beta^t E_t \left(\frac{(C_t^{1-\alpha} Z_t^{\alpha})^{1-\rho}}{(1-\rho)}\right) \tag{9}$$

[2]The variables we added to the Carroll, Fuhrer, Wilcox instrument set were chosen because Stock and Watson (1997) showed they had good predictive power for GDP.

The consumer's choice of C_t and Z_t in each quarter is constrained by the values of five state variables: (1) the level of 'cash on hand' (X_t), which equals the sum of wealth and current labor income (Y_t); (2) the stock of the durable owned by the consumer at the beginning of the quarter (H_t^b); (3) the level of permanent labor income (P_t); (4) the aggregate state of the economy (I_t); and (5) the consumer's current employment status (J_t). The evolution of each of these is described in detail in the paper. I_t, J_t, and P_t are all assumed to evolve exogenously, with permanent labor income following a first order Markov process with drift.

Consumers begin every quarter by making a decision regarding homeownership (H_t^e) given the stock of owned housing with which they began the period (H_t^b). Households who begin the period with no house ($H_t^b = 0$) have two choices: they can continue renting at cost $q\lambda$ where $q = 1.5$ and λ is the flow cost of homeownership, or they can buy a house whose value is equal to $\phi = 3$ times their permanent income. At the time of purchase, buyers must put up a down payment of amount $d = .20$ proportional to the value of the house, and pay fees and taxes in amount $b = .03$ (note that similar fees are also paid by the seller, so that each home sale incurs fees totalling 6 percent of home value). Households who begin the period owning a home have three choices: they can sell the house and rent, sell the house and buy another, or continue living in the same house. For homeowners, the flow of housing services is equal to the size of the house ($Z_t = H_t^e$), and the cost of servicing debt in each period is equal to a fixed mortgage rate $m = \delta + r = .04$, where r is the after-tax real rate of interest and δ is the depreciation rate of the house. The universe of possible homeownership-related actions that the consumer can take is summarized in the following table, where S_t denotes the level of liquid assets that the consumer holds at the end of the period:

Initial Status	Period t Action(s)	S_t	H_t^e	Z_t
$H_t^b = 0$	Keep Renting	$X_t - C_t - q\lambda Z_t$	0	Optimal
$H_t^b = 0$	Buy	$X_t - C_t - (d+b)H_t^e$ $-[m(1-d)+n]H_t^e$	ϕP_t	H_t^e
$H_t^b > 0$	Sell and Rent	$X_t - C_t + (d-b)H_t^b - q\lambda Z_t$	0	Optimal
$H_t^b > 0$	Hold	$X_t - C_t - [m(1-d)+n]H_t^e$	H_t^b	H_t^e
$H_t^b > 0$	Sell and Buy	$X_t - C_t + (d-b)H_t^b - (d+b)H_t^e$ $-[m(1-d)+n]H_t^e$	ϕP_t	H_t^e

For a fuller description of the model, see Carroll and Dunn (1997). The model is solved by numerical iteration on the value function. If a value function for period $t+1$ is defined, then the value function for period t is given by:

$$v_t(x_t, h_t^b, I_t, J_t) = \max_{\{c_t, z_t, h_t^e\}} u(c_t, z_t) + \beta E_t (G_{t+1}\Pi_{t+1})^{1-\rho} v_{t+1}(x_{t+1}, h_{t+1}^b, I_{t+1}, J_{t+1}).$$

Assuming we have a well-defined value function for period $t+1$ (we discuss below our method for obtaining a startup v_{t+1}), the procedure for calculating the value function at time t is as follows. As is usual in problems of this type, the first step is to discretize the two continuous state variables x and h into uniform grids over the range of values of x and h that we expect actually to arise among consumers behaving according to the model. Denoting the number of grid points for h and x as n_h and n_x, and similarly denoting the number of possible values for I and J as n_I and n_J, the total number of possible combinations of the state variables is $n_h n_x n_I n_J$. Call this set of points P.

Conceptually, the next step is, for each of these possible locations in state space, to determine the feasible course of action which yields the greatest value to the consumer. The discreteness of the homeownership choice, however, makes determining optimal behavior somewhat complicated, because the set of possible actions open to the consumer (particularly whether to sell a house) depend partly on the values of the state variables. For example, consumers who do not own a house do not have the option of selling their house.

The procedure, then, is to cycle through all possible configurations of the state variables, calculating for each configuration the value of pursuing each *possible* course of action. This will generate what might be referred to as 'contingent value functions.' For example, for a homeowner, we calculate the value function *contingent* on the assumption that the consumer sells his house and rents, and the value function *contingent* on the decision to sell the existing home and buy a new house. Thus, each configuration of state variables will be associated with several contingent value functions; obviously, the consumer would then choose the course of action which is associated with the highest contingent value function. This leads to a unique value which is associated with each possible configuration of state variables. Our approximated continuous value function is then generated by a multidimensional linear interpolation between the numerical values calculated at the sample of points contained in the set P defined above.

Assuming that the problem defines a contraction mapping, we can start off with any arbitrary value function v_{T+1} and iterate the procedure outlined above to generate v_T, v_{T-1}, $\ldots v_{T-n}$, and as $n \to \infty$, v_{T-n} will approach the true infinite-horizon value function associated with the infinite-horizon problem. In practice, we begin with the value function $v_{T+1}(x_{T+1}, h^b_{T+1}, I_{T+1}, J_{T+1}) = 0$ which, in addition to possessing the virtue of simplicity, also possesses the virtue that the value function v_{T-n} can be interpreted as the value function that would be associated with a finite horizon version of the problem n periods before the end of life.

We have not proven that this problem defines a contraction mapping. However, it seems likely that the conditions under which this problem defines a contraction mapping will be the same as those under which the simpler problems analyzed by Deaton (1991) and Carroll (1996) define contraction mappings. The central theoretical requirement in those models was $R\beta E_t(G\tilde{N}_{t+1})^{-\rho} < 1$, a condition that is satisfied by all of the parametric combinations we explore. As expected, the actual value functions do appear to converge numerically to an arbitrary degree of precision if we let the procedure iterate long enough.

In practice, we have found it useful to vary the fineness of the grids for the continuous varibles, beginning with a very coarse grid and iterating for 60 periods, then increasing the density of the grid and iterating another 40 periods, then increasing the density yet more and iterating for a final 4 periods with a very fine grid. In a final step, a procedure is run to fill in additional grid points near changes in the optimal homeownership decision (e.g. from renting to owning, from keeping a house to selling it and buying a new one, etc.). The decision rules are then rederived one last time using the supplemented grid. The final set of decision rules for optimal consumption and homeownership represents our solution to the model.

The foregoing description is actually slightly simpler than the problem we actually solve, because we must add two more state variables to address the effects of financial liberalization. If the required down payment ratio on the home is lowered, for some time after the reduction some consumers will be living in a home for which they paid a pre-liberalization down payment, while other consumers will be living in a home for which they paid a post-liberalization down payment. As a result, both the current debt ratio (one minus the down payment ratio) and the debt ratio that prevailed at the time of the consumer's last home purchase must also be kept track of and treated as additional state variables. The extension to the procedures outlined above is exactly parallel to the treatment of the discrete variables I and J.

4.2 Running Simulations

The procedure for generating simulated data from the model is as follows. We construct a model economy populated by 20,000 consumers who behave according to the optimal decision rules that solve the maximization problem. Both the state of financial market liberalization (before or after liberalization) and the aggregate state (recession, recovery, or expansion) are specified by predetermined values contained in an input file (see below for an explanation of the evolution of the aggregate state); the debt ratio remains constant unless we are studying the effects of a relaxation in the down payment constraint. Our set of model consumers begin 'life' at an essentially arbitrary point, with ratios of housing wealth to permanent income evenly distributed over the range of zero to three, and the ratio of cash-on-hand to permanent income set to the same initial value for every consumer. In the initial period the nondurable and durable expenditure shares are set to their Cobb-Douglas shares, permanent and transitory income shocks are set to zero, and everyone is employed.

In the second and subsequent periods, random permanent and transitory income shocks are drawn from their appropriate respective distributions (which differ depending on the aggregate state) for each consumer. Individual employment states are also randomly drawn from the three possible employment states using the steady-state employment probability distribution. Given the realizations of employment and income, the current level of cash-on-hand (x_t) can be calculated. The beginning-of-period housing stock (h_t) is equal to the value of the housing stock in the previous period minus depreciation. Once the second period state variables (x_t, h_t^b, I_t and J_t) are fully defined, the converged decision rules are used to determine housing and non-housing consumption expenditures (z_t and c_t) and the homeownership decision. Calculations are then performed to determine income, saving, the value of the housing stock at the end of the period, etc. This exercise is then repeated for the next period, and so on.

The economy is initially simulated for 400 quarters of expansion, by which time it has settled into a stochastic steady-state with a reasonably settled distribution of consumers across the state space. Then for quarters 401 through 539 (corresponding to 1962Q2 through 1995Q4), the aggregate state of the simulated economy is set to the aggregate state of the corresponding quarter for the US economy as indicated by the official NBER chronology. (We arbitrarily assume that every recession is followed by a recovery period that is four quarters long, which is the expected duration implied by the transition matrix.)

5 Computational Requirements

If you have gotten this far, now would be a good time to mention the computer resources required to solve and simulate the model. On a Sun SPARC Ultra 140 workstation, the original model took approximately five days to solve and one day to simulate. Informal testing on a 300 megahertz PC with 128 megabytes (meg) of RAM suggested that the process would run approximately 20 percent faster than on the Sun. Solving the model, however, requires a large amount of hard disk space (at least by today's standards), both for virtual memory while the programs are running (at least 200 meg) and for storing the resulting decision rules for optimal consumption (20 meg).

It should also be noted that, computationally, solving a finite horizon model of this type (see Dunn(1998)) is significantly more difficult because rather than saving a single set of decision rules, there are different rules for each period of the consumer's life; for example, in a quarterly model with a 40 year pre-retirement lifespan, there will be 40 years * 4 quarters * 20 meg = 3.2 gigabytes of storage needed for the solution to the model. This is somewhat intractable given the current state of technology. The virtual memory required to read and write files of this size in Mathematica is also enormous. In practice, the best workaround is to save only one set of decision rules for every one (or two) years, and use one rule across the entire year as an approximation.

6 Program Flow and Documentation

To help clarify the actual Mathematica code used to generate the results in the paper, the following table lists the individual files containing the code used to solve and simulate the model, along with a brief explanation of each file's purpose, any files that it calls, and what file(s) call it.[3] The file 'DoAll' is the top-level program that, at least theoretically, will solve the model, run the simulations, and generate the results. In practice, however, each of these steps is quite time-intensive, and therefore must be run one step at a time.

We suggest two final hints for deciphering the code: In solving the model, the main loop is the function 'KeepSolvingPeriods', defined in the 'iteration_routines.m' file. This calculates the expected value of next period's value function, and then finds the consumption rules for the given period. In the simulations, the main loop is the function 'KeepSimulatingFromTo', defined in 'simulation_routines.m'. This loops over each simulation period and, within each period, loops over the people in the simulated economy. Users attempting to understand the overall structure of the programs should begin with these files.

[3] The zip file also contains code to generate the regression results and figures in the paper; however, explanations of these files are left up to the individual reader.

No.	Program	Purpose	Called By	Calls
1	clear vars	set up empty lists for decision rules	15	none
2	clearextradata	clear decision rules for period $t+2$ to save on storage	8	none
3	construct grids	construct the grid points for discretized variables	15, 21	24
4	DoAll	(theoretical) top-level program to solve & simulate the model	none	15, 21, 20, 17, 18, 19, 26
5	find crossings	fill in extra grid points near changes in the homeownership decision	21	6
6	FindBest	define functions that return the value of pursuing each homeownership choice	9, 22, 5	none
7	function definitions	define the functions EtVtp1Raw, EtVtp1, Vt, Choicet, CtOpt, ZtOpt, htOpt, stOp	15	none
8	iteration routines	define the functions SolvePeriods and KeepSolvingPeriods	15	9, 10, 22, 2
9	last period	find the value of the optimal decision in each possible state in the last period	8	6, 11
10	make EtVtp1	construct the expectation of next period's Vt	8	none
11	make interpolating	construct consumption and other functions via linear interpolation	9, 22	none
12	params	define all parameter values	15	none
13	prepare simulated data	create simulated aggregate time series data comparable to aggregate US variables	26	14

No.	Program	Purpose	Called By	Calls
14	read post60 recessions	read in the indicators of NBER boom/recession quarters	17. 18	none
15	setup everything	set up complete environment for solving or simulating	4, 26	16, 12, 1, 25, 24, 3, 8, 7
16	setup workspace	initialize variables, define basic functions, and load necessary Mathematica packages	15	none
17	simulate baseline	run simulations with baseline parameter values (down payment ratio = .3)	4	none
18	simulate highdebt	run simulations with high debt parameter values (down payment ratio = .2)	4	none
19	simulate switch	run simulations switching from baseline to high debt parameter value	4	20
20	simulation routines	define the major functions used in simulations	4, 19	none
21	solve model	solve the model	4	3, 5, 22
22	solveperiod	find the value of the optimal decision for each possible state in a given period	8, 21	6, 11
23	tabulate grids	create all of the combinations of state variables	3, 5	none
24	transitions	create aggregate state transition matrices and calculate statistics	15	none
25	unemployment	create employment state transition matrices and calculate statistics	15	none
26	write datasets	load a set of simulations and use it to generate a time series data file	4	15, 13

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