A q-Ramsey Model

Consider a Ramsey economy in which the capital stock cannot be freely adjusted; instead, as in the q model of investment, capital is subject to quadratic costs of adjustment. The dynamic budget constraint is

$$\begin{aligned} F(k_t) &= k_t + k_t^{\alpha} \\ a_t &= F(k_t) - c_t - j_t \\ k_{t+1} &= a_t \end{aligned} \tag{1}$$

where for analytical simplicitly we neglect capital depreciation (though the illustrative figures below show results of a model that properly includes depreciation) and the cost-of-adjustment function takes the form

$$\mathrm{j}(i, k) = (k/2)(i/k)^2\omega \tag{2}$$

for some constant ω, so that the cost of adjustment incurred in period t is given by

$$j_t = \mathrm{j}(\overbrace{a_t - k_t}^{\equiv i_t}, k_t). \tag{3}$$

A social planner is assumed to maximize the discounted sum of utility from consumption c_t, where the utility function is CRRA, $\mathrm{u}(c) = c^{1-\rho}/(1-\rho)$. The social planner's problem can be rewritten in the form of a Bellman equation,

$$\begin{aligned} \mathrm{v}(k_t) &= \max_{c_t} \ \mathrm{u}(c_t) + \beta \mathrm{v}(k_{t+1}) \\ &\text{s.t.} \\ k_{t+1} &= F_t - c_t - j_t. \end{aligned}$$

Because (given k_t), choosing a_t is equivalent to choosing c_t,[1] we can rewrite the problem as:

$$\mathrm{v}(k_t) = \max_{a_t} \ \mathrm{u}(F(k_t) - a_t - \mathrm{j}(a_t - k_t, k_t)) + \beta \mathrm{v}(a_t). \tag{4}$$

The first order condition is found by setting the derivative w.r.t. a_t to zero:

$$(1 + j_t^i)\mathrm{u}'(c_t) = \beta \mathrm{v}^k(\underbrace{a_t}_{=k_{t+1}}). \tag{5}$$

The envelope theorem tells us that the marginal value of capital does not depend on its effect on the investment policy function $\mathrm{a}(k)$:

$$\begin{aligned} \mathrm{v}^k(k_t) &= (F'(k_t) + \mathrm{j}^i(a_t - k_t, k_t) - \mathrm{j}^k(a_t - k_t, k_t))\mathrm{u}'(c_t) \\ &\equiv \lambda_t, \end{aligned} \tag{6}$$

(where notice that λ_t does *not* have the simple interpretation of a share price as in `qModel` because here it involves $\mathrm{u}'(c_t)$).

[1] Solving (1) for c_t and substituting this expression in the place of c_t in the Bellman equation, and then substituting a_t for k_{t+1}.

Substituting period $t+1$'s version of (6) into (5) allows us to rewrite the Euler equation in the form:

$$
\begin{aligned}
\mathrm{u}'(c_t)(1+j^i_t) &= \beta(\mathcal{F}'(k_{t+1}) + j^i_{t+1} - j^k_{t+1})\mathrm{u}'(c_{t+1}) \\
\mathrm{u}'(c_t) &= \beta \underbrace{(\mathcal{F}'(k_{t+1}) + j^i_{t+1} - j^k_{t+1})/(1+j^i_t)}_{\mathsf{R}_{t+1}} \mathrm{u}'(c_{t+1}).
\end{aligned} \tag{7}
$$

This economy reduces to a standard Ramsey model when the cost of adjustment parameter is set to $\omega = 0$, because all the j terms disappear so that the interest factor becomes the usual $\mathsf{R}_{t+1} = \mathcal{F}'(k_{t+1}) = 1+\mathrm{f}'(k_{t+1})$. The presence of adjustment costs does not change the steady state of the model (because in steady state, adjustment costs are zero), but reduces the speed of convergence toward that steady state. This can be seen by considering the policy functions plotted in figure 1, where the solid lines reflect the solution to a model with $\omega = 0$ (the standard Ramsey model) while the dashed lines reflect a model with a high cost of adjustment (the 'q-Ramsey' model).

The differences between the solid and the dashed loci indicate that a faster rate of convergence to the steady state requires a high level of i below the steady state at which $k = \check{k}$ and low level of i when k is above $\check{k}$. Higher adjustment costs work against fast convergence, since, when k is below the steady state (and positive investment is needed to increase k toward $\check{k}$, adjustment costs reduce investment, while they increase investment (making it less negative) when k is above the equilibrium. In both cases, the difference occurs because because adjusting capital involves convex costs, and thus it is optimal to proceed slowly in moving the capital stock to minimize those costs. Interestingly, even though the optimal choices of investment and consumption change quite substantially in the model with a larger adjustment cost parameter, the actual size of costs of adjustment borne is quite modest (the dashing line for j_t is barely distinguishable from the horizontal axis except very far from the steady state). This tells us that even if the observed costs paid are not very large, those costs can have a large effect in changing behavior away from the frictionless optimum.

Increasing the desired degree of consumption smoothing, captured by the coefficient of relative risk aversion ρ, leads to similar implications. Figure 2 shows that a higher ρ (the dashed loci), implies again lower investment below the steady state and higher above it. This is now caused by a low intertemporal elasticity of substitution: if the economy falls below steady state, a larger ρ implies that the representative agent is less willing to cut consumption in order to boost investment and quickly return to the steady state. Similarly, the increase in consumption above the steady state is more moderate, thus leading to a smaller reduction in investment and a more gradual return to equilibrium.

By comparing the policy functions, we have thus seen that either an intensified consumption smoothing motive (higher ρ) and or a stronger investment smoothing motive (higher ω) have similar implications: they restrain sharp adjustments to consumption and investment, thus slowing down the speed of convergence to the steady state.

We now consider the responses of the model to several shocks, starting from steady state. Figure 3 shows the economy's dynamics following the destruction of part of the capital stock. In the standard Ramsey $\omega = 0$ model (black), this leads to an increase

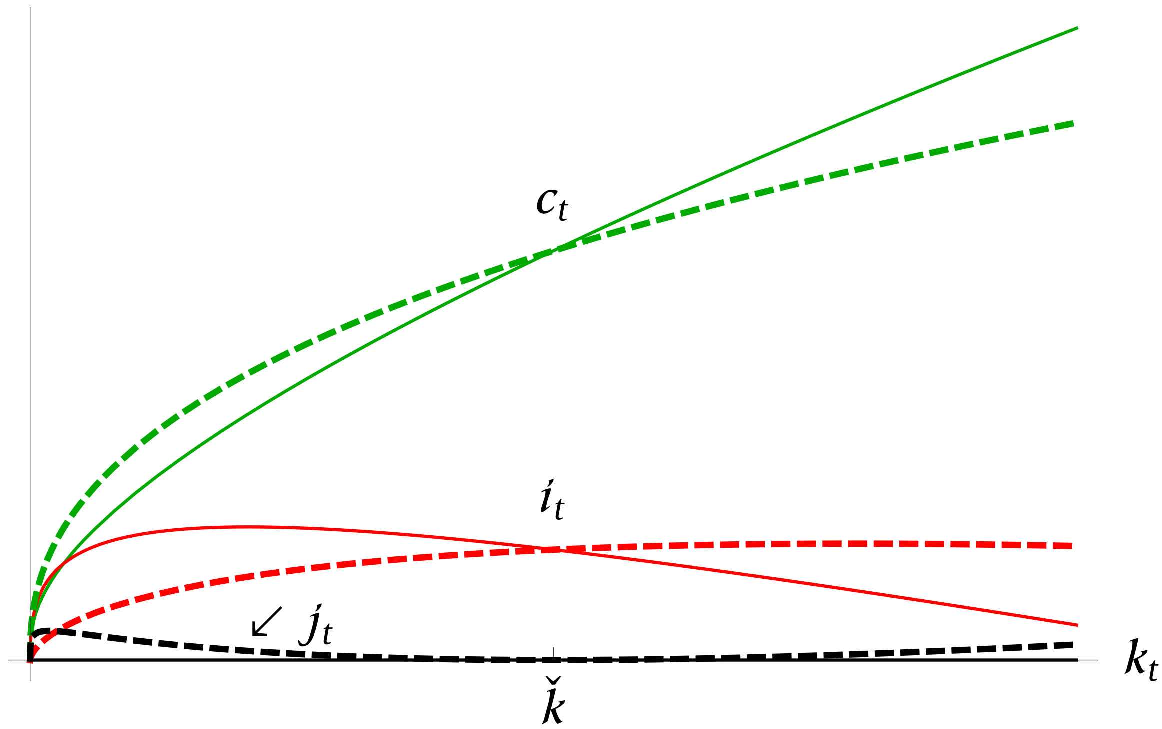


Figure 1 Solid Loci: Standard Ramsey Model; Dashed: With Costs of Adjustment

in the marginal productivity of capital which boosts investment. In the $\omega > 0$ model with adjustment costs (red), the level of investment actually falls. This is because costs of adjustment are assumed to be *relative* to the size of the capital stock, and with a shrunken capital stock the original level of investment would incur very large costs of adjustment. Investment therefore drops to a level that is large relative to the (shrunken) capital stock but nevertheless smaller than its initial level. Even this lower investment level, though, is large relative to the new lower level of the capital stock, and so the capital stock rises back toward the original equilibrium – just more slowly than in the frictionless model.

Consumption drops due to the negative wealth effect and the need to finance investment. But since investment is lower initially in the model with investment costs, consumption can be higher initially (the first red consumption dot is above the first black one, post-shock).

Figure 4 shows the dynamics triggered by an increase in patience, captured by a permanent rise in β. The most striking difference is in the interest factor R. In the Ramsey model with no investment costs, the interest rate is simply the marginal product of capital. Here, it must also take account of costs of adjustment. Since costs of adjustment are high when the economy is trying to change the size of the capital stock, the interest rate is lower. This result is interesting because one problem with using the Ramsey model for studying business cycle dynamics is that the aggregate capital stock barely moves at all over such a short time period as a business cycle, so the non-q

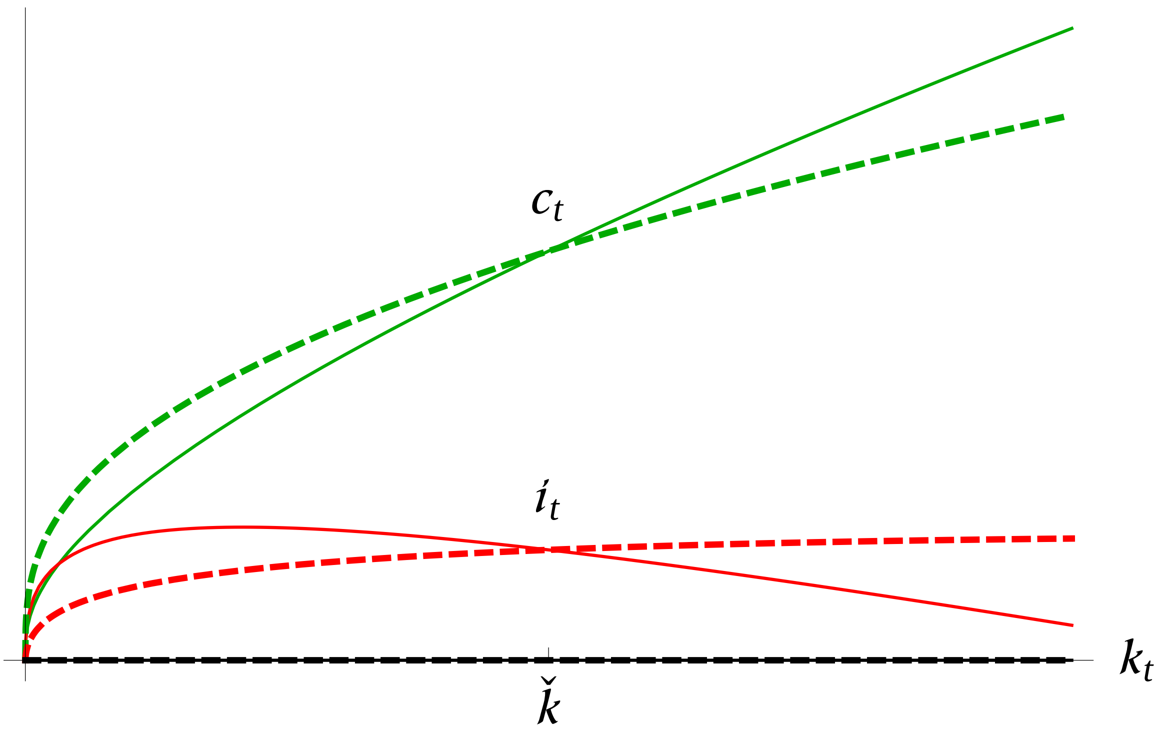


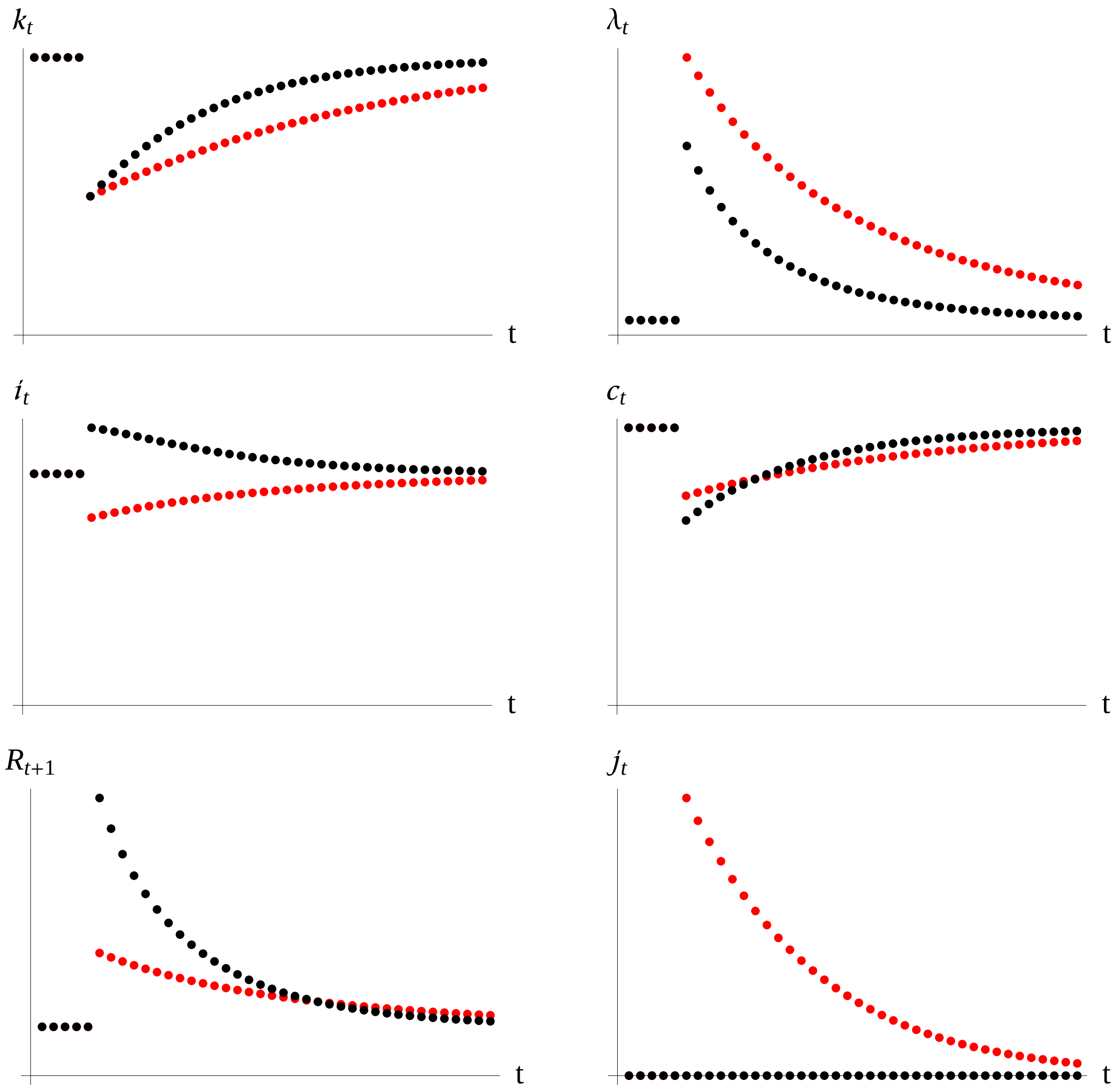
Figure 2 Higher ρ (Dashed Loci) Has a Similar Effect to Adjustment Costs

Ramsey model has no hope of matching empirical interest rate fluctuations. Adding costs of adjustment allows much bigger movements in R and thus gives the model a fighting chance.

Given the lower interest rate (and its implications through the consumption Euler equation), the growth rate of consumption after the increase in patience will be less than in the standard Ramsey model.

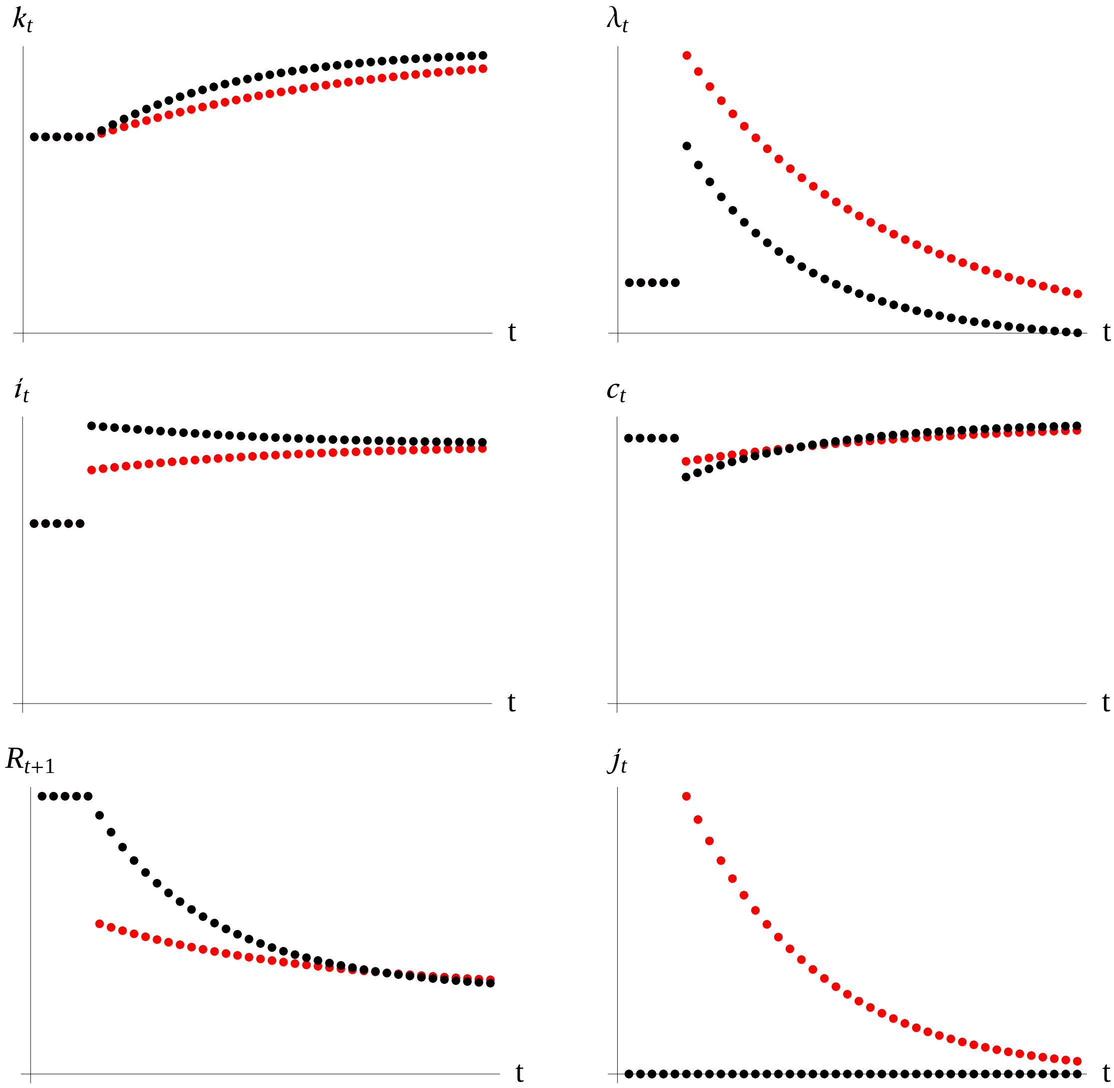
Even though consumption drops less, λ_t rises more. Recall that λ_t is a composition of the marginal utility of consumption and the "share price" of ownership of a unit of capital. The extra rise in λ reflects the fact that the existing capital is more valuable in a period when the rate of investment will be high (going forward), so the market value of a unit of "installed" capital rises to above the purchase price of a unit of capital (which is always 1). This can be interpreted as a boom in asset prices.

Figure 3 Impulse response functions to 50% destruction of the capital stock



$\omega = 0$ (standard Ramsey) in black; $\omega > 0$ (q-Ramsey) in red

Figure 4 Impulse responses to an increase in patience (higher β)



Black: Ramsey; Red: q-Ramsey