Gross Saving and Growth in the RCK Model

In the neoclassical growth model with labor-augmenting technological progress at rate γ, utility function $u(c) = c^{1-\rho}/(1-\rho)$, time preference rate ϑ and depreciation rate δ the steady-state will be at the point where the growth rate of consumption is equal to the growth rate of labor-augmenting technological progress, γ,

$$\begin{aligned} \frac{\dot{C}}{C} &= \rho^{-1}(f'(k) - \delta - \vartheta) \\ &= \gamma \end{aligned} \tag{1}$$

which implies that

$$\begin{aligned} f'(k) &= \rho\gamma + \vartheta + \delta \\ \alpha k^{\alpha-1} &= \rho\gamma + \vartheta + \delta \\ k &= \left[\frac{\rho\gamma + \vartheta + \delta}{\alpha}\right]^{\frac{1}{\alpha-1}}. \end{aligned} \tag{2}$$

The aggregate gross saving rate is defined as

$$\begin{aligned} s &= \frac{y - c}{y} \\ &= \frac{k^{\alpha} - c}{k^{\alpha}} \\ &= 1 - c/k^{\alpha}. \end{aligned} \tag{3}$$

In steady-state by definition

$$\dot{k} = 0 \tag{4}$$

but from the capital accumulation equation we know that

$$\dot{k} = k^{\alpha} - (\delta + \gamma)k - c \tag{5}$$

so in steady-state

$$c = k^{\alpha} - (\delta + \gamma)k. \tag{6}$$

This can be substituted into (3) to obtain

$$\begin{aligned} s &= 1 - \frac{k^{\alpha} - (\delta + \gamma)k}{k^{\alpha}} \\ &= (\delta + \gamma)k^{1-\alpha} \end{aligned} \tag{7}$$

and the expression for the steady-state level of capital per capita can be substituted in to yield

$$\begin{aligned} s &= (\delta + \gamma) \left(\frac{\rho\gamma + \vartheta + \delta}{\alpha} \right)^{-1} \\ &= \left(\frac{\alpha(\gamma + \delta)}{\delta + \vartheta + \rho\gamma} \right) \end{aligned} \tag{8}$$

The derivative of this expression with respect to γ is

$$\begin{aligned} \frac{ds}{d\gamma} &= \left(\frac{\alpha(\delta + \vartheta + \rho\gamma) - \alpha(\gamma + \delta)\rho}{(\delta + \vartheta + \rho\gamma)^2} \right) \\ &= \left(\frac{\alpha(\delta(1-\rho) + \vartheta)}{(\delta + \vartheta + \rho\gamma)^2} \right). \end{aligned} \tag{9}$$

This will be positive if its numerator is positive, i.e. if

$$\begin{aligned} \rho\delta &< \vartheta + \delta \\ \rho &< 1 + \vartheta/\delta. \end{aligned} \tag{10}$$

A typical assumption is $\vartheta = .04$ and $\delta = .08$, implying that the steady-state relationship between saving and growth in the neoclassical model is positive only if the coefficient of relative risk aversion ρ is less than 1.5. Typically we assume values of ρ in the range from 2 to 5, so the model leads us to expect a negative relationship between saving and growth.