Econ-367: Logistics

- Professor: Jonathan Wright
- Office Hour: Thursdays 3-4
- Email: wrightj@jhu.edu

<table>
<thead>
<tr>
<th>TAs</th>
<th>Email</th>
<th>Office Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kevin Yuan</td>
<td><a href="mailto:kyuan1@jhu.edu">kyuan1@jhu.edu</a></td>
<td>Wednesdays 9:30-10:30. Greenhouse.</td>
</tr>
</tbody>
</table>
Econ-367: Logistics

- **Requirements**
  - About 6 Homeworks 20%
  - Midterm 1 20%
  - Midterm 2 20%
  - Final 40%

- **Exams**
  - Midterm 1 October 2, in class
  - Midterm 2 October 30, in class
  - Final December 18, 9-12.
Econ-367: Logistics

- Slides for projection
  
  http://www.econ.jhu.edu/courses/367/index.html

- The book

  “Investments” by Bodie, Kane and Marcus
Econ-367: Prerequisites

- Micro Theory
- Statistics (111 or 112)
Minor in Financial Economics

- Four required courses:
  - Elements of Macroeconomics
  - Elements of Microeconomics
  - Corporate Finance
  - Investments and Portfolio Management
- Two elective courses
- Plus the prerequisites for these courses
- Check CFE website for detailed rules on the minor

http://cfe.econ.jhu.edu/
Benefits of this Course

- Required for the financial economics minor
- Topics that are important in economics/finance
- Skills that are useful in interviews/jobs
Important Notes

- There is no senior option.
- Anything covered in class can be on the exam. Attendance in class is highly recommended.
- Grades depend on exams/homework alone.
- All regrade requests must be submitted in writing within 2 weeks of the homework/exam being returned.
Major Classes of Financial Assets or Securities

- Money market
- Bond market
- Equity Securities
- Indexes
- Derivative markets
Debt and Equity (Relative to GDP)
Federal Government Debt (Relative to GDP)
Household debt (relative to GDP)
Nonfinancial business debt (relative to GDP)
Financial business debt (relative to GDP)
The Money Market

- Treasury bills
- Certificates of Deposit
- Interbank Loans
  - Eurodollars
  - Federal Funds
- Commercial Paper
- Repurchase Agreements (RPs)
Bank Discount Rate (T-Bills)

\[
r_{BD} = \frac{10,000 - P}{10,000} \cdot \frac{360}{n}
\]

- \( r_{BD} \) = bank discount rate
- \( P \) = market price of the T-bill
- \( n \) = number of days to maturity

Example: 90-day Tbill, \( P=9,875 \)

\[
r_{BD} = \frac{10,000 - 9,875}{10,000} \cdot \frac{360}{90} = 5\%
\]
The Spread between 3-month CD and Treasury Bill Rates
**Repo Agreement**

- Selling an asset with an explicit agreement to repurchase the asset after a set period of time
  1. Bank A sells a treasury security to Bank B at $P_0$
  2. Bank A agrees to buy the treasury back at a higher price $P_f > P_0$
  3. Bank B earns a rate of return implied by the difference in prices

\[ i_{RA} = \frac{P_f - P_0}{P_0} \times \frac{360}{\text{days}} \]
## Major Components of the Money Market

<table>
<thead>
<tr>
<th>Component</th>
<th>$ Billion</th>
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<tbody>
<tr>
<td>Repurchase agreements</td>
<td>$1,141</td>
</tr>
<tr>
<td>Small-denomination time deposits and savings deposits*</td>
<td>7,202</td>
</tr>
<tr>
<td>Large-denomination time deposits*</td>
<td>1,603</td>
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<tr>
<td>Treasury bills</td>
<td>1,478</td>
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<td>Commercial paper</td>
<td>1,445</td>
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<tr>
<td>Money market mutual funds</td>
<td>2,645</td>
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</table>

*Small denominations are less than $100,000.

Sources:  
The Bond Market

- Treasury Notes and Bonds
- Inflation-Protected Treasury Bonds
- Federal Agency Debt
- International Bonds
- Municipal Bonds
- Corporate Bonds
- Mortgages and Mortgage-Backed Securities
Treasury Notes and Bonds

- Maturities
  - Notes – maturities up to 10 years
  - Bonds – maturities in excess of 10 years
  - 30-year bond
- Par Value - $1,000
- Quotes – percentage of par
# Listing of Treasury Issues

## U.S. Treasury Quotes

### TREASURY NOTES & BONDS

**GO TO:** Bills

**Monday, August 29, 2016**

Treasury note and bond data are representative over-the-counter quotations as of 3pm Eastern time. For notes and bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues below par.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Coupon</th>
<th>Bid</th>
<th>Asked</th>
<th>Chg</th>
<th>Asked yield</th>
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<tr>
<td>8/31/2016</td>
<td>0.500</td>
<td>99.9688</td>
<td>99.9844</td>
<td>-0.0391</td>
<td>6.333</td>
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<tr>
<td>8/31/2016</td>
<td>1.000</td>
<td>99.9844</td>
<td>100.0000</td>
<td>unch.</td>
<td>0.998</td>
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<td>8/31/2016</td>
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<td>99.9844</td>
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<td>8.810</td>
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<td>9/15/2016</td>
<td>0.875</td>
<td>99.9922</td>
<td>100.0078</td>
<td>-0.0156</td>
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<td>100.0313</td>
<td>100.0469</td>
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<td>100.1953</td>
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Government Sponsored Enterprises Debt

- Major issuers
  - Federal Home Loan Bank
  - Federal National Mortgage Association
  - Government National Mortgage Association
  - Federal Home Loan Mortgage Corporation
Municipal Bonds

- Issued by state and local governments

- Types
  - General obligation bonds
  - Revenue bonds
    - Industrial revenue bonds

- Maturities – range up to 30 years
Municipal Bond Yields

- Interest income on municipal bonds is not subject to federal and sometimes not to state and local tax.

- To compare yields on taxable securities a Taxable Equivalent Yield is constructed.
Equivalent Taxable Yields Corresponding to Various Tax-Exempt Yields

<table>
<thead>
<tr>
<th>Marginal Tax Rate</th>
<th>1%</th>
<th>2%</th>
<th>3%</th>
<th>4%</th>
<th>5%</th>
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<tr>
<td>20%</td>
<td>1.25%</td>
<td>2.50%</td>
<td>3.75%</td>
<td>5.00%</td>
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<tr>
<td>30</td>
<td>1.43</td>
<td>2.86</td>
<td>4.29</td>
<td>5.71</td>
<td>7.14</td>
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<tr>
<td>40</td>
<td>1.67</td>
<td>3.33</td>
<td>5.00</td>
<td>6.67</td>
<td>8.33</td>
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<tr>
<td>50</td>
<td>2.00</td>
<td>4.00</td>
<td>6.00</td>
<td>8.00</td>
<td>10.00</td>
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</table>
Size of the municipal bond market

- Industrial revenue bonds
- General obligation
Ratio of Yields on Muni to Corporate Bonds
Corporate Bonds

- Issued by private firms
- Semi-annual interest payments
- Subject to larger default risk than government securities
- Options in corporate bonds
  - Callable
  - Convertible
Mortgages and Mortgage-Backed Securities

- Developed in the 1970s to help liquidity of financial institutions
- Proportional ownership of a pool or a specified obligation secured by a pool
Major Components of the Bond Market

- Treasury Debt: $10,827.5
- Federal Agency and Gov’t Sponsored Enterprise: $2,953.1
- Corporate Bonds: $1,049.3
- Tax-Exempt*: $3,428.0
- Mortgage-Backed Securities: $5,192.5
- Other Asset-Backed Securities: $6,202.0
Equity Securities

- Common stock
- Residual claim
- Limited liability
### Stocks Traded on the NYSE

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<tr>
<th>NAME</th>
<th>SYMBOL</th>
<th>CLOSE</th>
<th>NET CHG</th>
<th>VOLUME</th>
<th>52 WK HIGH</th>
<th>52 WK LOW</th>
<th>DIV</th>
<th>YIELD</th>
<th>P/E</th>
<th>YTD% CHG</th>
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<td>Gencorp</td>
<td>GY</td>
<td>13.59</td>
<td>-0.29</td>
<td>491,300</td>
<td>20.75</td>
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<td>dd</td>
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<td>Genentech</td>
<td>DNA</td>
<td>83.68</td>
<td>-0.35</td>
<td>3,986,300</td>
<td>94.46</td>
<td>75.58</td>
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<td></td>
<td>49</td>
<td>3.1</td>
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<td>General Cable</td>
<td>BGC</td>
<td>42.67</td>
<td>-1.11</td>
<td>679,700</td>
<td>45.41</td>
<td>20.3</td>
<td></td>
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<td>23</td>
<td>-2.4</td>
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<td>General Dynamics</td>
<td>GD</td>
<td>74.59</td>
<td>0.17</td>
<td>1,497,300</td>
<td>77.98</td>
<td>56.68</td>
<td>0.92</td>
<td>1.2</td>
<td>16</td>
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<td>General Electric</td>
<td>GE</td>
<td>37.56</td>
<td>-0.19</td>
<td>26,907,700</td>
<td>38.49</td>
<td>32.06</td>
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<td>General Gwth Prop</td>
<td>GGP</td>
<td>51.51</td>
<td>-0.8</td>
<td>1,308,200</td>
<td>56.14</td>
<td>41.92</td>
<td>1.8</td>
<td>3.5</td>
<td>215</td>
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<td>General Maritime</td>
<td>GMR</td>
<td>34.56</td>
<td>-0.83</td>
<td>597,400</td>
<td>40.64</td>
<td>30.34</td>
<td>4.8</td>
<td>13.9</td>
<td>5</td>
<td>-1.8</td>
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<tr>
<td>General Mills</td>
<td>GIS</td>
<td>56.97</td>
<td>-0.42</td>
<td>1,355,600</td>
<td>59.23</td>
<td>47.05</td>
<td>1.48</td>
<td>2.6</td>
<td>18</td>
<td>-1.1</td>
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<tr>
<td>General Motors</td>
<td>GM</td>
<td>30.24</td>
<td>0.6</td>
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<td>Genesco Inc</td>
<td>GCO</td>
<td>36.75</td>
<td>-0.9</td>
<td>127,900</td>
<td>43.72</td>
<td>25.5</td>
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<td>-1.5</td>
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<td>Genesee &amp; Wyoming</td>
<td>GWR</td>
<td>25.86</td>
<td>-0.5</td>
<td>364,500</td>
<td>36.75</td>
<td>21</td>
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<td>GPC</td>
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<td>1.35</td>
<td>2.9</td>
<td>17</td>
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<td>-0.32</td>
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<td>36.47</td>
<td>31</td>
<td>0.36</td>
<td>1.1</td>
<td>13</td>
<td>-1.2</td>
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<td>Geo Group Inc</td>
<td>GEO</td>
<td>37.57</td>
<td>-1.53</td>
<td>157,500</td>
<td>40.3</td>
<td>14.69</td>
<td></td>
<td></td>
<td></td>
<td>0.1</td>
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<tr>
<td>Georgia Gulf</td>
<td>GGC</td>
<td>18.69</td>
<td>-0.38</td>
<td>479,000</td>
<td>34.65</td>
<td>18.36</td>
<td>0.32</td>
<td>1.7</td>
<td>6</td>
<td>-3.2</td>
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<td>Gerber Scientific</td>
<td>GRB</td>
<td>12.32</td>
<td>-0.07</td>
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<td>9</td>
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<td>0.08</td>
<td>0.9</td>
<td>7</td>
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<td>Gerdau S.A. Ads</td>
<td>GGB</td>
<td>15.57</td>
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<td>1,729,100</td>
<td>18.16</td>
<td>11.27</td>
<td>0.58</td>
<td>3.7</td>
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<td>-2.7</td>
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</tbody>
</table>
Stock Market Indexes

- There are several broadly based indexes computed and published daily
Dow Jones Industrial Average

- Includes 30 large blue-chip corporations
- Computed since 1896
- Price-weighted average
Standard & Poor’s Indexes

• Broadly based index of 500 firms
• Market-value-weighted index
• Index funds
• Exchange Traded Funds (ETFs)
Other U.S. Market-Value Indexes

- NASDAQ Composite
- NYSE Composite
- Wilshire 5000
Investing in an index

- Investor may buy stocks
- Or an index tracking mutual fund
- Or an Exchange-traded fund (ETF) designed to match the index
  - Spiders (S&P Depository Receipt) tracks S&P 500
  - Cube track Nasdaq 100
- ETFs consist of a share in a basket of securities corresponding to the relevant index
- There is a redemption procedure
Derivatives Markets

Options
- Basic Positions
  - Call (Buy)
  - Put (Sell)
- Terms
  - Exercise Price
  - Expiration Date
  - Assets

Futures
- Basic Positions
  - Long (Buy)
  - Short (Sell)
- Terms
  - Delivery Date
  - Assets
Trading Data on IBM Options

PRICES AT CLOSE, July 17, 2012

<table>
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<tr>
<th>Expiration</th>
<th>Strike</th>
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<tr>
<td></td>
<td></td>
<td>Last</td>
<td>Volume</td>
</tr>
<tr>
<td>Jul</td>
<td>180.00</td>
<td>5.50</td>
<td>620</td>
</tr>
<tr>
<td>Aug</td>
<td>180.00</td>
<td>6.85</td>
<td>406</td>
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<tr>
<td>Oct</td>
<td>180.00</td>
<td>9.70</td>
<td>184</td>
</tr>
<tr>
<td>Jan</td>
<td>180.00</td>
<td>12.58</td>
<td>52</td>
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<tr>
<td>Jul</td>
<td>185.00</td>
<td>2.80</td>
<td>2231</td>
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<td>Aug</td>
<td>185.00</td>
<td>4.10</td>
<td>656</td>
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<td>Oct</td>
<td>185.00</td>
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<td>843</td>
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<td>Jan</td>
<td>185.00</td>
<td>9.75</td>
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Underlying stock price: 183.65
### Agriculture Futures

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<tr>
<th></th>
<th>OPEN</th>
<th>HIGH</th>
<th>LOW</th>
<th>SETTLE</th>
<th>CHG</th>
<th>OPEN INT</th>
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<tbody>
<tr>
<td><strong>Corn (CBT)</strong> - 5,000 bu.; cents per bu.</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>March</td>
<td>371.00</td>
<td>372.50</td>
<td>360.50</td>
<td>362.25</td>
<td>-8.25</td>
<td>591,430</td>
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<td>Dec</td>
<td>361.75</td>
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<td>357.00</td>
<td>359.00</td>
<td>-3.00</td>
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<tr>
<td>March</td>
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<td>265.75</td>
<td>258.25</td>
<td>261.25</td>
<td>-.75</td>
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<td>Dec</td>
<td>233.00</td>
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<td>232.50</td>
<td>233.75</td>
<td>.75</td>
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<td><strong>Soybeans (CBT)</strong> - 5,000 bu.; cents per bu.</td>
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<tr>
<td>Jan</td>
<td>667.00</td>
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<td>659.75</td>
<td>662.75</td>
<td>-6.50</td>
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<td>687.75</td>
<td>672.50</td>
<td>675.50</td>
<td>-6.50</td>
<td>220,362</td>
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### Currency Futures

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<th>SETTLE</th>
<th>CHG</th>
<th>OPEN INT</th>
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<tbody>
<tr>
<td><strong>Japanese Yen (CME)</strong> - ¥12,500,000; $ per 100¥</td>
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<tr>
<td>March</td>
<td>.8456</td>
<td>.8485</td>
<td>.8447</td>
<td>.8479</td>
<td>.0016</td>
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<td>June</td>
<td>.8561</td>
<td>.8579</td>
<td>.8545</td>
<td>.8577</td>
<td>.0016</td>
<td>5,119</td>
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<td><strong>British Pound (CME)</strong> - £62,500; $ per £</td>
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<td>March</td>
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<td>1.9403</td>
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<td>1.9531</td>
<td>1.9402</td>
<td>1.9443</td>
<td>-.0063</td>
<td>191</td>
</tr>
</tbody>
</table>

### Index Futures

<table>
<thead>
<tr>
<th></th>
<th>OPEN</th>
<th>HIGH</th>
<th>LOW</th>
<th>SETTLE</th>
<th>CHG</th>
<th>OPEN INT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DJ Industrial Average (CBT)</strong> - $10 x index</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>March</td>
<td>12543</td>
<td>12575</td>
<td>12470</td>
<td>12549</td>
<td>19</td>
<td>64,555</td>
</tr>
<tr>
<td>June</td>
<td>12629</td>
<td>12647</td>
<td>12601</td>
<td>12647</td>
<td>18</td>
<td>44</td>
</tr>
<tr>
<td><strong>S&amp;P 500 Index (CME)</strong> - $250 x index</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>March</td>
<td>1425.20</td>
<td>1431.50</td>
<td>1417.00</td>
<td>1427.50</td>
<td>2.70</td>
<td>601,655</td>
</tr>
<tr>
<td>June</td>
<td>1432.00</td>
<td>1444.50</td>
<td>1430.50</td>
<td>1440.10</td>
<td>2.60</td>
<td>13,287</td>
</tr>
</tbody>
</table>
Suppose that interest of 1 percent is paid every month.

The “APR” with monthly compounding will be 12 percent.

\[
\text{APR} = \frac{\text{Annual Percentage Rate}}{(\text{periods in year})} \times (\text{rate for period})
\]
Quoting Conventions

**EAR**

Suppose again that interest of 1 percent is paid every month.

The “EAR” will be how much interest is earned on $1 after 1 year.

**EAR = Effective Annual Rate**

\[
(1 + \text{rate for period})^{\text{Periods per yr}} - 1
\]

In this case \( EAR = (1.01)^{12} - 1 = 12.68\% \)

After \( n \) years $1 turns becomes \( $(1 + EAR)^n \)
Quoting Conventions
Relationship between APR and EAR

For annual compounding they are the same thing

For compounding n times a year

\[ EAR = \left(1 + \frac{APR}{n}\right)^n - 1 \]

\[ APR = n \times \{[1 + EAR]^{1/n} - 1\} \]
Quoting Conventions
Continuous Compounding

Suppose that I compound every second

Let $r$ be the APR with continuous compounding

$$EAR = \lim_{n \to \infty} \left(1 + \frac{r}{n}\right)^n - 1 = \exp(r) - 1$$

After 1 year, the value of $1$ is $\$\exp(r)$

After $n$ years, the value of $1$ is $\$\exp(r)^n = \$\exp(rn)$
Quoting Conventions
Continuous Compounding

Suppose that an asset is worth $V(0)$ today and $V(n)$ in years

\[ V(n) = V(0) \exp(rn) \]

Inverting this yields

\[ r = \frac{\ln(V(n)) - \ln(V(0))}{n} \]
Quoting Conventions: Example

Say we want to get an EAR of 5.8%.

What APR do we need at different compounding frequencies?

\[ APR = n \times \left(1 + \frac{EAR}{n}\right)^{\frac{1}{n}} - 1 \]

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Periods per year</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual</td>
<td>1</td>
<td>5.80</td>
</tr>
<tr>
<td>Quarterly</td>
<td>4</td>
<td>5.68</td>
</tr>
<tr>
<td>Monthly</td>
<td>12</td>
<td>5.65</td>
</tr>
<tr>
<td>Daily</td>
<td>365</td>
<td>5.64</td>
</tr>
</tbody>
</table>
Continuous Compounding: Example

- Start with $1 and after 1 year have $1.058

- APR with continuous compounding is

\[
\frac{\ln(1.058) - \ln(1)}{1} = 0.0564
\]
A stock is worth $10 in year 1, $20 in year 2 and $10 in year 3.

Q1. What is the return from year 1 to year 2?
   - $\ln(20) - \ln(10) = 0.69$

Q2. What is the return from year 2 to year 3?
   - $\ln(10) - \ln(20) = -0.69$

Q3. What is the return from year 1 to year 3?
   - $\frac{\ln(10) - \ln(10)}{2} = 0$

Continuous compounded returns have the useful feature that they “add up”
Present value

- Suppose that we have a stream of cash flows and let \( r \) be the constant EAR

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( C(1) )</td>
</tr>
<tr>
<td>2</td>
<td>( C(2) )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>( T )</td>
<td>( C(T) )</td>
</tr>
</tbody>
</table>

- Present value:

\[
PV = \frac{C(1)}{1+r} + \frac{C(2)}{(1+r)^2} + \cdots + \frac{C(T)}{(1+r)^T}
\]

- NPV function in Excel
Present value example

Let the EAR be 5% and consider the following cash flow:

<table>
<thead>
<tr>
<th>Time (years)</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$75</td>
</tr>
<tr>
<td>2</td>
<td>$50</td>
</tr>
<tr>
<td>3</td>
<td>$100</td>
</tr>
</tbody>
</table>

Present value:

\[
PV = \frac{75}{1.05} + \frac{50}{1.05^2} + \frac{100}{1.05^3} = 203.16
\]
Another Present value example

- Consider the following two projects

<table>
<thead>
<tr>
<th></th>
<th>t=0</th>
<th>t=1</th>
<th>t=2</th>
<th>t=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project A</td>
<td>-200</td>
<td>50</td>
<td>50</td>
<td>120</td>
</tr>
<tr>
<td>Project B</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>-220</td>
</tr>
</tbody>
</table>

- What are their present values at 1% and 10% interest rates?

<table>
<thead>
<tr>
<th></th>
<th>1%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV of A</td>
<td>15</td>
<td>-23</td>
</tr>
<tr>
<td>PV of B</td>
<td>-15</td>
<td>21</td>
</tr>
</tbody>
</table>
Suppose that we have a stream of cash flows for which an investor is willing to pay $P$ today. The value of $r$ that solves the equation

$$
P = \frac{C(1)}{1 + r} + \frac{C(2)}{(1 + r)^2} \ldots + \frac{C(T)}{(1 + r)^T}$$

is called the *internal rate of return*. 
### History of T-bill Rates, Inflation and Real Rates, 1926-2012

<table>
<thead>
<tr>
<th></th>
<th>T-Bills</th>
<th>Inflation</th>
<th>Real T-Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>All months</td>
<td>3.55</td>
<td>3.04</td>
<td>0.52</td>
</tr>
<tr>
<td>First half</td>
<td>1.79</td>
<td>1.74</td>
<td>0.10</td>
</tr>
<tr>
<td>Recent half</td>
<td>5.35</td>
<td>4.36</td>
<td>0.95</td>
</tr>
</tbody>
</table>
Interest Rates and Inflation, 1926-2012
Risk and Risk Premiums

Rates of Return: Single Period

\[ HPR = \frac{P_1 - P_0 + D_1}{P_0} \]

\( HPR \) = Holding Period Return

\( P_0 \) = Beginning price

\( P_1 \) = Ending price

\( D_1 \) = Dividend during period one
Rates of Return: Single Period Example

Ending Price = 48
Beginning Price = 40
Dividend = 2

\[ \text{HPR} = \frac{(48 - 40 + 2)}{40} = 25\% \]
Expected returns

\[ E(r) = \sum_s p(s)r(s) \]

- \( p(s) \) = probability of a state
- \( r(s) \) = return if a state occurs
- \( s \) = state
## Scenario Returns: Example

<table>
<thead>
<tr>
<th>State</th>
<th>Prob. of State</th>
<th>$r$ in State</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.1</td>
<td>-.05</td>
</tr>
<tr>
<td>2</td>
<td>.2</td>
<td>.05</td>
</tr>
<tr>
<td>3</td>
<td>.4</td>
<td>.15</td>
</tr>
<tr>
<td>4</td>
<td>.2</td>
<td>.25</td>
</tr>
<tr>
<td>5</td>
<td>.1</td>
<td>.35</td>
</tr>
</tbody>
</table>

\[
E(r) = (.1)(-.05) + (.2)(.05) \ldots + (.1)(.35)
\]

\[
E(r) = .15
\]
Variance or Dispersion of Returns

Variance:

\[ Var = \sum_{s} p(s)[r(s) - E(r)]^2 \]

Standard deviation = \([\text{variance}]^{1/2}\)

Using Our Example:

\[ \text{Var} = [(0.1)(-0.05 - 0.15)^2 + (0.2)(0.05 - 0.15)^2 \ldots + 0.1(0.35 - 0.15)^2] \]

\[ \text{Var} = 0.012 \]

\[ \text{S.D.} = [0.012]^{1/2} = 0.1095 \]
Covariance and correlation

- Let \( r(1) \) and \( r(2) \) be two random variables.
- The covariance between them is

\[
E((r(1) - E(r(1)))(r(2) - E(r(2))))
\]

- Need the joint probabilities of \( r(1) \) and \( r(2) \) to work this out.
Covariance Example

- Let $r(1)$ and $r(2)$ be two returns with the following joint distribution

<table>
<thead>
<tr>
<th></th>
<th>$r(1)=0$</th>
<th>$r(1)=0.1$</th>
<th>$r(1)=0.3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r(2)=0$</td>
<td>0.05</td>
<td>0.3</td>
<td>0.15</td>
</tr>
<tr>
<td>$r(2)=0.2$</td>
<td>0.35</td>
<td>0.1</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Covariance Example

- First need the means

<table>
<thead>
<tr>
<th></th>
<th>( r(1)=0 )</th>
<th>( r(1)=0.1 )</th>
<th>( r(1)=0.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(2)=0 )</td>
<td>0.05</td>
<td>0.3</td>
<td>0.15</td>
</tr>
<tr>
<td>( r(2)=0.2 )</td>
<td>0.35</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

- \( E(r(1)) = (0.4 \times 0) + (0.4 \times 0.1) + (0.2 \times 0.3) = 0.1 \)

- \( E(r(2)) = (0.5 \times 0) + (0.5 \times 0.2) = 0.1 \)
Covariance Example

• Possible outcomes for

\[ (r(1) - E(r(1))) \times (r(2) - E(r(2))) \]

• Weight each of them by their probability

• Add up

• Covariance is -0.005
Covariance and correlation

- Covariance is measured in units that are hard to interpret.

- Correlation avoids this problem. The correlation between $r(1)$ and $r(2)$ is

\[
\frac{\text{Cov}(r(1), r(2))}{\sqrt{\text{Var}(r(1))\text{Var}(r(2))}}
\]

- It must be between -1 and +1.
The Normal Distribution

- 68.26% within 1σ
- 95.44% within 2σ
- 99.74% within 3σ
A Normal and Skewed Distributions
(mean = 6% SD = 17%)
Properties of stock returns

- Excess return on S&P is about 0.07
- Standard deviation of S&P is about 0.17
- Negative skewness
- Fat tails
The Reward-to-Volatility (Sharpe) Ratio

Sharpe Ratio for Portfolios = \frac{\text{Risk Premium}}{\text{SD of Excess Return}}

Risk Premium: Average Excess Return
For stocks, Sharpe Ratio is about 0.4
How Securities are Traded

- **Primary Market**
  - Market for newly-issued securities
  - Firms issue new securities through underwriter (investment banker) to public

- **Secondary Market**
  - Investors trade previously issued securities among themselves
How Firms Issue Securities

• Raise capital from a wider range of investors through *initial public offering, IPO*
  • *Seasoned equity offering*: The sale of additional shares in firms that already are publicly traded
• Public offerings are marketed by investment bankers or *underwriters*
• Registration must be filed with the SEC
How Firms Issue Securities

- Initial Public Offerings
  - Underwriter may bear price risk associated with placement of securities (bought deals v. best effort deals)
    - IPOs are commonly underpriced compared to the price they could be marketed
    - Some IPOs, however, are overpriced (ex.: Facebook); others cannot even fully be sold
Bid and Asked Prices

- Bid Price is price at which dealer will buy
- Ask Price is price at which dealer will sell
- Ask>Bid
Trading Mechanisms

• Dealer markets
• Electronic communication networks (ECNs)
  • True trading systems that can automatically execute orders
• Specialists markets
  • Maintain a “fair and orderly market”
  • Have been largely replaced by ECNs
Types of Orders

- Market Order:
  - Executed immediately
  - Trader receives current market price

- Price-Contingent Order:
  - Traders specify buying or selling price
  - A large order may be filled at multiple prices
Trading Costs

- **Brokerage Commission**: Fee paid to broker for making the transaction
  - Explicit cost of trading
  - Full service vs. discount brokerage
- **Spread**: Difference between the bid and asked prices
  - Implicit cost of trading
  - Reflects order processing costs and asymmetric information
Buying on Margin

- Borrowing part of the total purchase price of a position using a loan from a broker
- Investor contributes the remaining portion
- Margin refers to the percentage or amount contributed by the investor
- You profit when the stock rises
Buying on Margin

- Initial margin is set by the Federal Reserve
  - Currently 50%
  - Regulation T
- Maintenance margin
  - Margin call if value of securities falls too much
  - At least 25% by market convention
Short Sales

- **Purpose**
  - To profit from a decline in the price of a stock or security

- **Mechanics**
  - Borrow stock through a dealer
  - Sell it and deposit proceeds and margin in an account
  - Regulation T applies
  - Closing out the position: Buy the stock and return to the party from which it was borrowed
**Example: Short Sale: Initial Conditions**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dot Bomb</td>
<td>1000 Shares</td>
</tr>
<tr>
<td>50% Initial Margin</td>
<td></td>
</tr>
<tr>
<td>30% Maintenance Margin</td>
<td></td>
</tr>
<tr>
<td>$100 Initial Price</td>
<td></td>
</tr>
<tr>
<td>Sale Proceeds</td>
<td>$100,000</td>
</tr>
<tr>
<td>Margin</td>
<td>$50,000</td>
</tr>
<tr>
<td>Stock Owed</td>
<td>1000 shares</td>
</tr>
</tbody>
</table>
Example
Short Sale: Dot Bomb falls to $70 per share

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$100,000 (sale proceeds)</td>
<td>$70,000 (buy shares)</td>
</tr>
<tr>
<td>$50,000 (initial margin)</td>
<td></td>
</tr>
</tbody>
</table>

**Equity**

$80,000

**Profit = Ending equity – Beginning equity**

= $80,000 - $50,000 = $30,000

= Decline in share price x Number of shares sold short
Example

Short Sale: Margin Call

How much can the stock price rise before a margin call?

\[
\frac{($150,000^* - 1000P)}{(1000P)} = 30\%
\]

\[
P = $115.38
\]

* Initial margin plus sale proceeds
Downside of HFT

- Volatility may be heightened at times of stress
- Several possible examples of this
Decision Making Under Uncertainty: Utility and Risk Aversion

- A utility function measures happiness as a function of consumption/wealth.
- Investors care about expected utility, not expected wealth per se.
- Generally think that utility function is concave.
  - Risk Aversion
A Concave Utility Function
Risk averse investors and risk premia

- Have diminishing marginal utility of wealth
- Reject investment portfolios that are fair games or worse
- Consider only risk-free or speculative prospects with positive risk premiums
  - Expected return has to be bigger than risk-free rate
  - This is the risk premium
Certainty Equivalent Value

- The certainty equivalent value is the sum of money for which an individual would be indifferent between receiving that sum and taking the gamble.
- The certainty equivalent value of a gamble is less than the expected value of a gamble for someone who is risk averse.
Certainty Equivalence Example

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Value</td>
<td>40</td>
</tr>
<tr>
<td>Certainty Equivalent</td>
<td>30</td>
</tr>
<tr>
<td>Risk Premium</td>
<td>10</td>
</tr>
</tbody>
</table>

The graph shows the relationship between utility and wealth, with the certainty equivalent and risk premium indicated.

Utility

Wealth
Risk Neutrality

\[ U(M_0 + B) \]

\[ EU_G = U(M_0) \]

\[ U(M_0 - B) \]

\[ U = U(M) \]

\[ M_0 - B \quad M_0 \quad M_0 + B \]
Risk Seeking
Expected Utility

- Basic premise of standard decision making under uncertainty is that agents maximize expected utility.
- Say the utility function is $W^{1/2}$
- Wealth is 1 or 9 each w.p. $1/2$
- Expected utility is $0.5*1+0.5*3=2$
Example

- An investor’s utility function is $5w^{1/2}$ where $w$ is wealth in dollars. She currently has $100 but can purchase any number $x$ (including fractional amounts) of an asset for $20 apiece that pays off the following amount in the three possible states of the world.

<table>
<thead>
<tr>
<th>State of World</th>
<th>Payoff</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$60</td>
<td>40%</td>
</tr>
<tr>
<td>B</td>
<td>$0</td>
<td>20%</td>
</tr>
<tr>
<td>C</td>
<td>$20</td>
<td>40%</td>
</tr>
</tbody>
</table>
Example continued

• Q. What is expected holding period return on the asset?
  • A. \((2 \times 0.4) + ((-1) \times 0.2) + (0 \times 0.4) = 0.6\)

• Q. What is the standard deviation of the return?
  • A. Variance is \((1.4 \times 1.4 \times 0.4) + (1.6 \times 1.6 \times 0.2) + (0.6 \times 0.6 \times 0.4) = 1.44\) so standard deviation is 1.2
Q. How many units should investor buy to maximize expected utility?

<table>
<thead>
<tr>
<th>State of the world</th>
<th>Payoff</th>
<th>Probability</th>
<th>Wealth</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$60</td>
<td>40%</td>
<td>100 + 40x</td>
<td>5(100 + 40x)(^{1/2})</td>
</tr>
<tr>
<td>B</td>
<td>$0</td>
<td>20%</td>
<td>100 - 20x</td>
<td>5(100 - 20x)(^{1/2})</td>
</tr>
<tr>
<td>C</td>
<td>$20</td>
<td>40%</td>
<td>100</td>
<td>5(100)(^{1/2})</td>
</tr>
</tbody>
</table>

Expected utility is: \[2\sqrt{100 + 40x} + \sqrt{100 - 20x} + 20\]

First Order Condition: \[2 \times \frac{40}{\sqrt{100 + 40x}} - \frac{20}{\sqrt{100 - 20x}} = 0\]

\[x = \frac{25}{6}\]
Coefficient of Risk Aversion

- Coefficient of Absolute Risk Aversion is \( -\frac{u''(W)}{u'(W)} \)

- Consider the utility function \( u(W) = -\frac{\exp(-AW)}{A} \)

- This has a coefficient of risk aversion of A

- Constant Absolute Risk Aversion (CARA)
Uncertainty about fixed dollar amount

- Consider asset that pays $x$ or $x+1$ when investor has CARA utility.

- Certainty Equivalent solves

  \[
  -\frac{\exp(-AC)}{A} = -0.5\frac{\exp(-Ax)}{A} - 0.5\frac{\exp(-A(x+1))}{A}
  \]

- After a little algebra

  \[
  C = x + \frac{1}{A} \ln\left(\frac{2}{1 + \exp(-A)}\right)
  \]
Coefficient of Relative Risk Aversion

- Coefficient of Relative Risk Aversion is $-\frac{Wu''(W)}{u'(W)}$

- Consider the utility functions $u(W) = \ln(W)$
  
  
  $u(W) = \frac{W^{1-\rho}}{1-\rho}$
What is the certainty equivalent of
- $100,000 \text{ wp 0.5}
- $50,000 \text{ wp 0.5}
What is the certainty equivalent of:
- $100,000 wp 0.5
- $50,000 wp 0.5

<table>
<thead>
<tr>
<th>Rel. Risk Aversion</th>
<th>Certainty Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70,711</td>
</tr>
<tr>
<td>2</td>
<td>66,667</td>
</tr>
<tr>
<td>5</td>
<td>58,566</td>
</tr>
<tr>
<td>10</td>
<td>53.991</td>
</tr>
<tr>
<td>30</td>
<td>51,210</td>
</tr>
</tbody>
</table>
Utility Function

\[ U = E(r) - \frac{1}{2} A\sigma^2 \]

Where

- \( U \) = utility
- \( E(r) \) = expected return on the asset or portfolio
- \( A \) = coefficient of risk aversion
- \( \sigma^2 \) = variance of returns
Three possible portfolios

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(r)</td>
<td>0.07</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>σ(r)</td>
<td>0.05</td>
<td>0.10</td>
<td>0.20</td>
</tr>
</tbody>
</table>
Utility of three possible portfolios

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>M</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(r)</td>
<td>0.07</td>
<td>0.09</td>
<td>0.13</td>
</tr>
<tr>
<td>σ(r)</td>
<td>0.05</td>
<td>0.10</td>
<td>0.20</td>
</tr>
<tr>
<td>A=2</td>
<td>0.0675</td>
<td>0.08</td>
<td>0.09</td>
</tr>
<tr>
<td>A=5</td>
<td>0.6375</td>
<td>0.065</td>
<td>0.03</td>
</tr>
<tr>
<td>A=8</td>
<td>0.06</td>
<td>0.05</td>
<td>-0.03</td>
</tr>
</tbody>
</table>
The Trade-off Between Risk and Returns of a Potential Investment Portfolio, $P$
The Indifference Curve
Indifference Curves with $A = 2$ and $A = 4$
Basic Results from Statistics

• Suppose $X_1$ and $X_2$ are random variables

\[ E(k_1 X_1 + k_2 X_2) = k_1 E(X_1) + k_2 E(X_2) \]

\[ Var(k_1 X_1 + k_2 X_2) = k_1^2 Var(X_1) + k_2^2 Var(X_2) + 2k_1 k_2 Cov(X_1, X_2) \]

• If $X_1, X_2, \ldots, X_n$ are random variables

\[ E(\sum k_j X_j) = \sum k_j E(X_j) \]

\[ Var(\sum k_j X_j) = \sum_{i=1}^{n} \sum_{j=1}^{n} k_i k_j Cov(X_i, X_j) \]
Mean, Variance and Covariance of Portfolios

- Suppose there are assets with returns $r(1), r(2), \ldots r(n)$
- Suppose I form a portfolio with weights $w(1), w(2), \ldots w(n)$
- The properties of the portfolio are

\[
\begin{align*}
  r(p) &= \sum_{i=1}^{n} w(i) r(i) \\
  E(r(p)) &= \sum_{i=1}^{n} w_i E(r(i)) \\
  Var(r(p)) &= \sum_{i=1}^{n} \sum_{j=1}^{n} w(i) w(j) Cov(r(i), r(j))
\end{align*}
\]
It’s possible to split investment funds between safe and risky assets.

- Risk free asset: proxy; T-bills
- Risky asset: stock (or a portfolio)
Properties of combination of risk-free and risky asset

- \( r_c = \) combined portfolio putting weight \( y \) in the risky asset and \( 1-y \) on the risk-free asset

- \( E(r_c) = yE(r_p) + (1-y) r_f \)

- \( \sigma_c = |y| \sigma_p \)
Example

- $r_f = 7\%$, $\sigma_f = 0$
- $E r_p = 15\%$, $\sigma_p = 22\%$

- $y = 0.75$
  - $E(r_c) = 0.75 \times 0.15 + 0.25 \times 0.07 = 0.13$
  - $\sigma_c = 0.75 \times 0.22 = 0.165$

- $y = 0$
  - $E(r_c) = 0.07$
  - $\sigma_c = 0$
Example

- $r_f = 7\%$, $\sigma_f = 0$
- $E r_p = 15\%$, $\sigma_p = 22\%$

- $y = 1.5$
  - $E(r_c) = 1.5 \times 0.15 - 0.5 \times 0.07 = 0.19$
  - $\sigma_c = 1.5 \times 0.22 = 0.33$
Capital Allocation Line

$E(r_p) = 15\%$

$r_f = 7\%$

$S = 8/22$

$\sigma_p = 22\%$

$E(r_p) - r_f = 8\%$
Capital Allocation Line with Differential Borrowing and Lending Rates
Optimal Portfolio Using Indifference Curves

- $E(r_p) = 0.15$
- $E(r_d) = 0.1028$
- $r_f = 0.07$
- $\sigma_p = 0.0902$
- $\sigma_d = 0.22$
- $U = 0.094$
- $U = 0.08653$
- $U = 0.078$
- $U = 0.07$

Calhoun Line (CAL)
Utility as a Function of Allocation to the Risky Asset, $y$
Risk Tolerance and Asset Allocation

- The investor must choose one optimal portfolio, $y^*$, from the set of feasible choices

\[
\max E(r_c) - \frac{A\sigma_c^2}{2} = \max yE(r_p) + (1 - y)r_f - \frac{Ay^2\sigma_p^2}{2}
\]

\[\Rightarrow y^* = \frac{E(r_p) - r_f}{A\sigma_p^2}\]
Example

- \( r_f = 7\% \), \( \sigma_f = 0 \)
- \( Er_p = 15\% \), \( \sigma_p = 22\% \)
- \( A = 4 \rightarrow y^* = (0.15 - 0.07)/(4*0.22^2) = 0.41 \)
An Optimal Portfolio of Risky Assets

- Before we considered choice between one risky asset and a risk-free asset.

- But there are in fact many risky assets.

- Suppose we are now picking an optimal portfolio of risky assets (no risk-free asset).
Covariance

$$\text{Cov}(r_D, r_E) = \rho_{DE} \sigma_D \sigma_E$$

$$\rho_{D,E} = \text{Correlation coefficient of returns}$$

$$\sigma_D = \text{Standard deviation of returns for Security D}$$

$$\sigma_E = \text{Standard deviation of returns for Security E}$$
Correlation Coefficients: Possible Values

Range of values for correlation coefficient

\[ +1.0 \geq \rho \geq -1.0 \]

If \( \rho = 1.0 \), the securities would be perfectly positively correlated

If \( \rho = -1.0 \), the securities would be perfectly negatively correlated
Two assets (debt and equity) in a portfolio

- \( r_P = y r_D + (1-y) r_E \)

\[
E(r_p) = y E(r_D) + (1 - y) E(r_E)
\]

\[
Var(r_p) = y^2 Var(r_D) + (1 - y)^2 Var(r_E) + 2y(1 - y) \text{Cov}(r_D, r_E)
\]
Two assets (debt and equity) in a portfolio

- Suppose that $\rho = -1$

\[ \sigma_P^2 = y^2 \sigma_D^2 + (1 - y)^2 \sigma_E^2 - 2y(1 - y)\sigma_D \sigma_E \]
\[ = \left\{ y\sigma_D - (1 - y)\sigma_E \right\}^2 \]

- Now let $y = \frac{\sigma_E}{\sigma_E + \sigma_D}$. Then $\sigma_P^2 = 0$
Two assets (debt and equity) in a portfolio

- Suppose that $\rho = +1$

$$\sigma_P^2 = y^2 \sigma_D^2 + (1 - y)^2 \sigma_E^2 + 2y(1 - y)\sigma_D \sigma_E$$

$$= \{y\sigma_D + (1 - y)\sigma_E\}^2$$

$$\geq \min(\sigma_D, \sigma_E)^2$$

- No risk reduction possible.
Correlation Effects

- The relationship depends on the correlation coefficient.
- $-1.0 \leq \rho \leq +1.0$
- The smaller the correlation, the greater the risk reduction potential.
- If $\rho = +1.0$, no risk reduction is possible.
Example

<table>
<thead>
<tr>
<th></th>
<th>Debt</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Return</td>
<td>0.08</td>
<td>0.13</td>
</tr>
<tr>
<td>St Dev</td>
<td>0.12</td>
<td>0.20</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>Covariance</td>
<td>0.0072</td>
<td></td>
</tr>
</tbody>
</table>
Portfolio Expected Return as a Function of Investment Proportions

\[ w(\text{bonds}) = 1 - w(\text{stocks}) \]
Portfolio Standard Deviation as a Function of Investment Proportions
Portfolio Expected Return as a Function of Standard Deviation
Formula for Minimum Variance Portfolio

\[ w_D = \frac{\sigma_E^2 - \text{Cov}(R_D, R_E)}{\sigma_E^2 + \sigma_D^2 - 2\text{Cov}(R_D, R_E)} \]

\[ w_E = 1 - w_D \]

where \( R_D \) and \( R_E \) are the returns.
The Opportunity Set of the Debt and Equity Funds

- Expected Return (%)
- Standard Deviation (%)
Minimum-Variance Frontier of Risky Assets
The Opportunity Set of the Debt and Equity Funds

- But we don’t just have the two risky assets.

- There’s also a risk-free asset

- Combination of the risk-free asset and any point on the opportunity set of the debt and equity funds is the capital allocation line
The Opportunity Set of the Debt and Equity Funds and Two Feasible CALs
The Opportunity Set of the Debt and Equity Funds with the Optimal CAL and the Optimal Risky Portfolio
Determination of the Optimal Overall Portfolio
The Sharpe Ratio

- Tangency portfolio maximizes the slope of the CAL for any possible portfolio, \( p \)
- The objective function is the slope:

\[
S_p = \frac{E(r_p) - r_f}{\sigma_p}
\]
Formula for Optimal Risky Portfolio

\[ w_D = \frac{E(R_D)\sigma_E^2 - E(R_E)\text{Cov}(R_D, R_E)}{E(R_D)\sigma_E^2 + E(R_E)\sigma_D^2 - [E(R_D) + E(R_E)]\text{Cov}(R_D, R_E)} \]

\[ w_E = 1 - w_D \]

where \( R_D \) and \( R_E \) are the excess returns over the riskfree rate.
### Example

<table>
<thead>
<tr>
<th></th>
<th>Debt</th>
<th>Equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected Return</td>
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<td></td>
</tr>
<tr>
<td>Covariance</td>
<td>0.0072</td>
<td></td>
</tr>
</tbody>
</table>

**T-Bill yield: 5 percent**

**Optimal Portfolio**

\[
\begin{align*}
    w_D &= \frac{0.03 \times 0.04 - 0.08 \times 0.0072}{0.03 \times 0.04 + 0.08 \times 0.0144 - (0.03 + 0.08) \times 0.0072} = 0.4 \\
    w_E &= 0.6
\end{align*}
\]
Example (continued)

\[ E(r) = (0.4 \times 0.08) + (0.6 \times 0.13) = 11\% \]

\[ \sigma^2(r) = (0.4^2 \times 0.0144) + (0.6^2 \times 0.04) + (2 \times 0.4 \times 0.6 \times 0.0072) \]

\[ = 0.020164 \]

\[ \sigma(r) = 14.2\% \]

CAL of optimal portfolio has slope of

\[ \frac{11 - 5}{14.2} = 0.42 \]
Optimal Complete Portfolio

- Consider investor with utility function
  \[ U = E(r) - \frac{A}{2} \sigma^2 \]

- Position in portfolio will be
  \[ y = \frac{E(r_p) - r_f}{A \sigma_p^2} \]

- For example, if \( A = 4 \),
  \[ y = \frac{0.11 - 0.05}{4 \times 0.142^2} = 0.7439 \]
Optimal Complete Portfolio in Example

- Portfolio $P$: 74.39%
- Bonds: 29.76%
- Stocks: 44.63%
- T-bills: 25.61%
Markowitz Portfolio Selection Model

- Security Selection
  - First step is to determine the risk-return opportunities available
  - All risky portfolios that lie on the minimum-variance frontier from the global minimum-variance portfolio and upward provide the best risk-return combinations
Capital Allocation and the Separation Property

- **Separation property**: The property that portfolio choice problem is separated into two independent parts:
  - (1) determination of the optimal risky portfolio, and
  - (2) the personal choice of the best mix of the risky portfolio and risk-free asset.

- Implication of the separation property: the optimal risky portfolio $P$ is the same for all clients of a fund manager.
The Power of Diversification

- We have \( \sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \text{Cov}(r_i, r_j) \)

- Suppose that all securities in the portfolio have the same weight, have variance \( \sigma^2 \) and covariance \( \text{Cov} \)

- We can then express portfolio variance as:

\[
\sigma_p^2 = \frac{1}{n} \sigma^2 + \frac{n-1}{n} \text{Cov}
\]
### Risk Reduction of Equally Weighted Portfolios

<table>
<thead>
<tr>
<th>Number of Stocks</th>
<th>$\rho=0$</th>
<th>$\rho=0.4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5000</td>
<td>0.5000</td>
</tr>
<tr>
<td>2</td>
<td>0.3536</td>
<td>0.4183</td>
</tr>
<tr>
<td>10</td>
<td>0.1581</td>
<td>0.3391</td>
</tr>
<tr>
<td>100</td>
<td>0.0500</td>
<td>0.3186</td>
</tr>
<tr>
<td>Infinite</td>
<td>0.0000</td>
<td>0.3162</td>
</tr>
</tbody>
</table>
Portfolio Risk as a Function of the Number of Stocks in the Portfolio

Market risk = \( \lim_{n \to \infty} \sigma_p^2 = Cov \)
An Example

- Suppose there are many stocks
  - All have mean 15%
  - Standard deviation 60%
  - Correlation 0.5

1. What is the expected return and standard deviation of a portfolio of 25 stocks?

2. How many stocks are needed to get the standard deviation down to 43%
An Example

3. What is the systematic (market) risk in this universe?

4. If T bills are available and yield 10% what is the slope of the CAL?
Where next?

- Analysis using optimal portfolio choice requires a lot of parameters to be estimated.
- Hard to use in practice.
- So there are some models that are better at handling many assets:
  - Single Index Model
  - CAPM
  - APT
Excess Returns on HP and S&P 500 April 2001 – March 2006
Single-Index Model

\[ R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t) \]

\( \beta_i \) = index of a securities’ particular return to the market
The alphas and betas of stocks

- Alphas is a measure of which stocks are under and overpriced

- Beta is a measure of market sensitivity
  - Which stocks have high beta?
  - Which stocks have low beta?
Single Index Model Decomposition

\[ R_i(t) = \alpha_i + \beta_i R_m(t) + e_i(t) \]

- \( \alpha_i \): is stock market mispricing
- \( \beta_i R_M(t) \): is component of return due to systematic risk
- \( e_i(t) \): is component of return due to idiosyncratic risk (or firm-specific risk)
How to estimate betas

- Compute the excess returns of the stock
- Compute the excess returns of the market portfolio (proxied by S&P 500 Index)
- Regress excess returns of the stock on the excess returns of the S&P 500 index
- Slope coefficient is beta
- Also gives alpha and error estimates
Single-Index Model Input List

- Risk premium on the S&P 500 portfolio
- Estimate of the SD of the S&P 500 portfolio
- $n$ sets of estimates of
  - Alpha values
  - Beta coefficient
  - Stock residual variances
- Fewer inputs than with a general covariance matrix
Security Characteristic Line

- Plots excess returns of a single asset against excess market returns for different time periods
Portfolios With the Single-Index Model

- For a portfolio \( R_P(t) = \alpha_P + \beta_P R_M(t) + e_P(t) \)

- Alpha and beta of a portfolio:

\[
\alpha_p = \sum_{i=1}^{n} x_i \alpha_i \quad \text{and} \quad \beta_p = \sum_{i=1}^{n} x_i \beta_i
\]

- Variance of a portfolio:

\[
\sigma_p^2 = \beta_p^2 \sigma_m^2 + \sigma_{ep}^2
\]

- When \( n \) gets large, \( \sigma_{ep}^2 \) becomes negligible
The Variance of an Equally Weighted Portfolio with Risk Coefficient $\beta_p$ in the Single-Factor Economy

\[ \sigma^2_p = \frac{\sigma^2(e)}{n} \]

- Diversifiable Risk
- Systematic Risk
Optimal Risky Portfolio of the Single-Index Model

- Mean, variance and covariances of returns
  \[ E(R_i) = \alpha_i + \beta_i E(r_m) \]
  \[ Var(R_i) = \beta_i^2 \sigma_m^2 + \sigma_{e_i}^2 \]
  \[ Cov(R_i, R_j) = \beta_i \beta_j \sigma_m^2 \]

- Can then work out efficient frontier as before.
Efficient Frontier with full covariance matrix and the single index model
Question

- Why do some assets have higher returns than others?

- Capital Asset Pricing Model provides an answer
Capital Asset Pricing Model (CAPM)

- It is the equilibrium model that underlies all modern financial theory
Assumptions

- Investors are price takers
- Single-period investment horizon
- Same expectations
- “Perfect” markets
  - No transactions costs or taxes
  - Short selling and borrowing/lending at risk-free rate are allowed
- Investors have preferences on the mean and variance of returns
Expected return is a function of
- Beta
- Market return
- Risk-free return
The CAPM Pricing Equation

\[ E(r_i) - r_f = \beta_i [E(r_m) - r_f] \]

where

\[ \beta_i = \frac{Cov(r_i - r_f, r_m - r_f)}{Var(r_m - r_f)} \]

• This is also known as the security market line (SML), or the “Expected Return-Beta” relationship
CAPM Implications

- If $\beta_i = 1$, $E(r_i) = E(r_m)$
- If $\beta_i > 1$, $E(r_i) > E(r_m)$
- If $\beta_i < 1$, $E(r_i) < E(r_m)$
- If $\beta_i = 0$, $E(r_i) = r_f$
Using the CAPM: An Example

Suppose you have the following info:
- Risk free rate: 3.5%
- Expected market return: 8.5%
- Beta of IBM is 0.75

Q. What is the expected return on IBM stock?
A. 0.035 + 0.75 * [0.085 - 0.035] = 7.25%
The Security Market Line

\[ E(r) \]

\[ E(r_M) - r_f = \text{Slope of SML} \]

\[ \beta_M = 1.0 \]
Expected Return-Beta Relationship

- CAPM holds for any portfolio because:

\[ E(r_P) = \sum_k w_k E(r_k) \] and

\[ \beta_P = \sum_k w_k \beta_k \]

\[ E(r_P) - r_f = \beta_P [E(r_m) - r_f] \]

- This also holds for the market portfolio:

\[ E(r_M) - r_f = \beta_M \left[ E(r_M) - r_f \right] \]

\[ \beta_M = 1 \]
Different Agents

- CAPM does not assume that all agents are the same
- They can have different risk aversion
- This will affect only the mix of the market and riskfree asset that they hold.
Market Risk Premium

The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the average investor:

\[ E(r_M) - r_f = \bar{A} \sigma_M^2 \]

where \( \sigma_M^2 \) is the variance of the market portfolio and \( \bar{A} \) is the average degree of risk aversion across investors.
CAPM: Variance Decomposition

- CAPM implies that variance of returns on an asset are

\[ \beta^2 \sigma_m^2 + \sigma_e^2 \]
CAPM Intuition: Systematic and Unsystematic Risk

- Unsystematic risk can be diversified and is irrelevant
- Systematic risk cannot be diversified and is relevant
  - Measured by beta
  - Beta determines the level of expected return
- The risk-to-reward ratio should be the same across assets
CAPM versus the Index model

- The index model
  \[ R_i(t) - R_f(t) = \alpha_i + \beta_i[R_m(t) - R_f(t)] + e_i(t) \]
- The CAPM index model beta coefficient turns out to be the same beta as that of the CAPM expected return-beta relationship
  \[ E(R_i(t)) - R_f(t) = \beta_i[E(R_m(t)) - R_f(t)] \]
- Indeed, if you take the index model, take expectations and let \( \alpha_i = 0 \) you get the CAPM
CAPM versus the Index model

- CAPM is an equilibrium model
  - Derived assuming investors maximize utility
  - Index model is not

- The CAPM implies no intercept

- Index model says nothing about expected market return
Exercise: Can the following be consistent with CAPM?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>13.4%</td>
<td>40%</td>
<td>1.2</td>
</tr>
<tr>
<td>B</td>
<td>15.5%</td>
<td>50%</td>
<td>1.5</td>
</tr>
<tr>
<td>Market</td>
<td>12.0%</td>
<td>34%</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Exercise: Can the following be consistent with CAPM?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6%</td>
<td>20%</td>
<td>0.7</td>
</tr>
<tr>
<td>B</td>
<td>10%</td>
<td>20%</td>
<td>1.0</td>
</tr>
<tr>
<td>C</td>
<td>12%</td>
<td>45%</td>
<td>1.5</td>
</tr>
</tbody>
</table>
Exercise: Can the following be consistent with CAPM?

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Expected Return</th>
<th>Standard Deviation</th>
<th>Residual Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12%</td>
<td>30%</td>
<td>6%</td>
</tr>
<tr>
<td>Market</td>
<td>10%</td>
<td>20%</td>
<td>0%</td>
</tr>
</tbody>
</table>
Exercise: CAPM Example

<table>
<thead>
<tr>
<th>Asset</th>
<th>Total Variance</th>
<th>Residual Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock A</td>
<td>30%</td>
<td>17.2%</td>
</tr>
<tr>
<td>Stock B</td>
<td>15%</td>
<td>7.8%</td>
</tr>
<tr>
<td>Market</td>
<td>5%</td>
<td>0%</td>
</tr>
</tbody>
</table>

- What is the beta of stock A and stock B?
- If the expected market return is 10% and the riskfree rate is 5%, what are the expected returns on stock A and stock B?
The CAPM and Reality

- Testing the CAPM
  - Proxies must be used for the market portfolio
  - And for the risk-free rate
Does Beta Explain the Cross-section of Returns?

- Early tests found strong evidence for an expected return-beta relationship
  - Black, Jensen and Scholes (1972)

- More recent results have been less positive.
Tests of the CAPM: Fama MacBeth Procedure

\[ E(r_i) = r_f + \beta_i \left[ E(r_M) - r_f \right] \]

Tests of the expected return beta relationship:

- **First Pass Regression**
  - Estimate betas (and maybe variance of residuals)
- **Second Pass**: Using estimates from the first pass to determine if model is supported by the data

\[ R_i = \gamma_0 + \gamma_1 \beta_i + \varepsilon_i \]

\[ R_i = \gamma_0 + \gamma_1 \beta_i + \gamma_2 Var(e_{it}) + \varepsilon_i \]
Classic Test of the CAPM
(Black, Jensen and Scholes, 1972)
Single Factor Test Results

Return %

Predicted

Actual

Beta
Does Beta Explain Everything?

- Does it explain anything?
- Fama and French (1992) results
  - Cross sectional regression
    \[ r_i = \gamma_0 + \gamma_1 (B / M)_i + \gamma_2 \log(ME_i) + \gamma_3 \beta_i \]
  - Book to market and size matter
  - Beta explains almost nothing!
Average Return as a Function of Book-To-Market Ratio, 1926–2011
Average Annual Return for 10 Size-Based Portfolios, 1926 – 2011
The Relationship Between Illiquidity and Average Returns

Average Monthly Return (% per month)

Bid–Asked Spread (%)
The CAPM and Reality

- There are problems with the CAPM
- Other factors are important in explaining returns
  - Firm size effect
  - Ratio of book value to market value
  - Liquidity
- Still the benchmark asset pricing model
The CAPM without a risk-free asset

- Suppose there is no risk-free asset (e.g. due to inflation uncertainty)
  \[ E(R_i) = E(R_z) + \beta_i (E(R_m) - E(R_z)) \]
  where \( z \) is the return on any zero-beta portfolio

- This is called the “zero-beta” CAPM (Black (1972))

- Q. How do we find a zero-beta portfolio?
Finding a zero-beta portfolio

- Suppose that there are two assets and the market portfolio has a weight $w$ in the first and $1-w$ in the second
- Let $v$ and $1-v$ be portfolio weights for zero beta portfolio

$Cov(wr_1 + (1-w)r_2, vr_1 + (1-v)r_2) = 0$

$w v Var(r_1) + (1-w)(1-v)Var(r_2) + [(1-w)v + (1-v)w]Cov(r_1, r_2) = 0$

$w v Var(r_1) + (1-w)Var(r_2) - v(1-w)Var(r_2) + vCov(r_1, r_2) + wCov(r_1, r_2) - 2 w v Cov(r_1, r_2) = 0$

$v = \frac{(w-1)Var(r_2) - wCov(r_1, r_2)}{w Var(r_1) - (1-w)Var(r_2) + (1-2w)Cov(r_1, r_2)}$

- Same idea works with many assets in market portfolio
Factor Models

- Returns on a security come from two sources
  - Common factor
  - Firm specific events

- Possible common factors
  - Market excess return
  - Gross Domestic Product Growth
  - Interest Rates
Single Factor Model Equation

\[ r_i = E(r_i) + \beta_i F + e_i \]

\( r_i \) = Return for security \( i \)
\( \beta_i \) = Factor sensitivity or factor loading or factor beta
Estimate by regression
\( F \) = \textbf{Surprise} in macro-economic factor \((E(F)=0)\)
  \((F \) could be positive, negative or zero\)
\( e_i \) = Firm specific events
APT & Well-Diversified Portfolios

\[ r_P = E(r_p) + \beta_p F + e_P \]
\[ F = \text{some factor} \]

- For a well-diversified portfolio: \( e_P \) approaches zero
Returns as a Function of the Systematic Factor

**Well Diversified Portfolios**

Return (%)

- 10
- 0

**Individual Stocks**

Return (%)

- 10
- 0
Arbitrage Pricing Theory (APT)

Arbitrage - arises if an investor can construct a zero investment portfolio with a sure profit

• Since no investment is required, an investor can create large positions to secure large levels of profit

• In efficient markets, profitable arbitrage opportunities will quickly disappear
Pricing

- Implication of the factor structure for a well-diversified portfolio:

  \[ E(r_i) = r_f + \lambda \beta_i \]

- If not, there’s an arbitrage (show next slide)
- Expected returns proportional to factor beta
- Intuition: only systematic risk should command higher expected returns
APT Pricing Derivation

- Suppose there are 2 well-diversified portfolios
  \[ r_1 = E(r_1) + \beta_1 f \]
  \[ r_2 = E(r_2) + \beta_2 f \]
- Suppose I invest \( w \) in 1 and \( 1-w \) in 2
  \[ r_p = wE(r_1) + (1-w)E(r_2) + [w\beta_1 + (1-w)\beta_2]f \]
- Let \( w = \beta_2 / (\beta_2 - \beta_1) \). Then
  \[ r_p = \frac{\beta_2}{\beta_2 - \beta_1} E(r_1) - \frac{\beta_1}{\beta_2 - \beta_1} E(r_2) = r_f \]

----or else there is an arbitrage opportunity

\[ r_p = \frac{\beta_2}{\beta_2 - \beta_1} E(r_1) - \frac{\beta_1}{\beta_2 - \beta_1} E(r_1) + \frac{\beta_2}{\beta_2 - \beta_1} E(r_2) - \frac{\beta_1}{\beta_2 - \beta_1} E(r_2) = r_f \]

\[ r_p = \frac{\beta_2}{\beta_2 - \beta_1} E(r_1) - \frac{\beta_1}{\beta_2 - \beta_1} E(r_1) + \frac{\beta_1}{\beta_2 - \beta_1} E(r_1) - \frac{\beta_1}{\beta_2 - \beta_1} E(r_2) = r_f \]
APT Pricing Derivation

- So \( \frac{\beta_2[E(r_1) - E(r_2)]}{\beta_2 - \beta_1} + E(r_2) = r_f \Rightarrow \frac{E(r_2) - r_f}{\beta_2} = \frac{E(r_1) - E(r_2)}{\beta_1 - \beta_2} \)

- And \( \frac{\beta_1[E(r_1) - E(r_2)]}{\beta_2 - \beta_1} + E(r_1) = r_f \Rightarrow \frac{E(r_1) - r_f}{\beta_1} = \frac{E(r_1) - E(r_2)}{\beta_1 - \beta_2} \)

- Therefore \( \frac{E(r_1) - r_f}{\beta_1} = \frac{E(r_2) - r_f}{\beta_2} = \lambda \)

- And so \( E(r_1) = r_f + \lambda \beta_1 \)
  \( E(r_2) = r_f + \lambda \beta_2 \)
APT and CAPM

• Suppose that $F = r_m - E(r_m)$

• Then $r_i = E(r_i) + \beta_i [r_m - E(r_m)] + e_i$ and for any well-diversified portfolio

  $E(r_i) = r_f + \beta_i [E(r_m) - r_f]$

where $\beta_i = \frac{Cov(r_i - r_f, r_m - r_f)}{Var(r_f)}$

• This is the CAPM, except that it applies only to well diversified portfolios
APT and CAPM Compared

- APT applies to well diversified portfolios and not necessarily to individual stocks
  - Possible for some individual stocks to be mispriced
    - not lie on the SML
- APT doesn’t need investors to have mean-variance utility (which CAPM does)
- APT doesn’t necessarily require us to measure market portfolio
- APT can have more than one factor.
Factor model

\[ r_i = E(r_i) + \beta_i F + e_i \]

- Variance of returns is

\[ \beta_i^2 \text{Var}(F) + \text{Var}(e_i) \]
Factor model: Example

- The risk-free rate is 4 percent and X and Y are two well-diversified portfolios:

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Beta</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Y</td>
<td>1/2</td>
<td>?</td>
</tr>
</tbody>
</table>

- Q. What is the expected return on Y?
- A. \( 10 = 4 + \lambda \) and so \( \lambda = 6 \). The expected return on Y is \( 4 + \lambda \times 0.5 = 4 + 3 = 7 \)
Factor model: An arbitrage

• Suppose instead that the expected return on Y were 9%

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Beta</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>Y</td>
<td>1/2</td>
<td>9</td>
</tr>
</tbody>
</table>

Q. What can an investor do?

1. Invest $100 in Y.
   • Payoff: $100*[1+0.09+0.5F]= 109+50F

2. Invest -$50 in the riskfree asset:
   • Payoff: -50*1.04=-52

3. Invest -$50 in X.
   • Payoff: -50*[1+0.10+F]= -55-50F

• Total payoff: $2: This is an arbitrage.
Factor model: creating an arbitrage

- **General rule**
  - Go long the stock with higher return
  - Go short the stock with lower return
  - Portfolio weights are ratios of the betas
  - Invest whatever is left in risk-free asset
Multifactor Models

- Use more than one factor
  - Examples include gross domestic product, expected inflation, interest rates etc.
  - Estimate a beta or “factor loading” for each factor using multiple regression.
Multifactor Model Equation

\[ r_i = E(r_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + e_i \]

- \( r_i \) = Return for security \( i \)

- \( \beta_{i1} \) = Factor sensitivity for first factor (e.g. GDP)

- \( \beta_{i2} \) = Factor sensitivity for second factor (e.g. interest rate)

- \( e_i \) = Firm specific events
Pricing with Multiple Factors

\[ E(r_i) = r_f + \beta_{1i} \lambda_1 + \beta_{2i} \lambda_2 \]

e.g. \[ E(r_i) = r_f + \beta_{i,GDP} \lambda_{GDP} + \beta_{i,IR} \lambda_{IR} \]

\( \beta_{i,GDP} \) = Factor sensitivity for GDP

\( \lambda_{GDP} \) = Risk premium for GDP

\( \beta_{i,IR} \) = Factor sensitivity for Interest Rate

\( \lambda_{IR} \) = Risk premium for Interest Rate
There are 3 factors

The betas are 1.1, 0.5 and 2

The lambdas are 10%, 10% and 1%

The riskfree rate is 5%

Q: What is the expected return on stock i?

A: $0.05 + (1.1 \times 0.1) + (0.5 \times 0.1) + (2 \times 0.01) = 23\%$
Consider an APT model for a stock

Q If T Bills yield 6 percent, what is the expected return on this stock?

A: \[0.06 + (1.2 \times 0.06) + (0.5 \times 0.08) + (0.3 \times 0.03) = 18.1\%\]
Here are the actual and expected values for the factors a stock

<table>
<thead>
<tr>
<th>Factor</th>
<th>Expected</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
<td>5%</td>
<td>4%</td>
</tr>
<tr>
<td>IP Growth</td>
<td>3%</td>
<td>6%</td>
</tr>
<tr>
<td>Oil Prices</td>
<td>2%</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Q** What is the revised expectation of the stock return once the factors are known?

**A**: \[0.181 + 1.2*(-0.01) + 0.5*0.03 + 0.3*(-0.02) = 17.8\%\]
What Factors?

- Fama-French Three Factor Model
- Factors that are important to performance of the general economy (Chen, Roll and Ross)
Fama French Three Factor Model

- Factors seem to predict average returns well
  \[ r_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + e_{it} \]
  where:
  - SMB = Small Minus Big, i.e., the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks
  - HML = High Minus Low, i.e., the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of stocks with a low book-to-market ratio
Empirical Methodology

- Method: Two-stage regression
  - Stage 1: Estimate the betas
  - Stage 2: Run a cross-sectional regression
Fama French Second Stage Estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>T-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>-0.21</td>
<td>-1.25</td>
</tr>
<tr>
<td>SMB</td>
<td>0.25</td>
<td>8.38</td>
</tr>
<tr>
<td>HML</td>
<td>0.35</td>
<td>10.22</td>
</tr>
</tbody>
</table>
Stock Prices as a Random Walk

- Kendall (1953) found that log stock prices were approximately a random walk
  \[ p_t = p_{t-1} + \epsilon_t \]
- Erratic market behavior, or
- A well functioning market (where prices reflect all available information)?
Efficient Market Hypothesis (EMH)

- EMH says stock prices already reflect all available information.
- A forecast about favorable future performance leads to favorable current performance, as market participants rush to trade on new information.
Versions of the EMH

- Weak (Past trading data)
- Semi-strong (All public info)
- Strong (All info)
Cumulative Abnormal Returns Before Takeover Attempts: Target Companies
Stock Price Reaction to CNBC Reports

The graph shows the cumulative return (%) of stock prices over time relative to mentions in CNBC Reports. The graph compares midday-positive and midday-negative reports. The x-axis represents minutes relative to the mention moment, while the y-axis shows the cumulative return in percentage. The graph indicates a positive reaction to midday-positive reports and a negative reaction to midday-negative reports.
Cumulative Abnormal Returns in Response to Earnings Announcements
Dividend yield and stock returns
Dividend yields since 1900
Mutual Fund Performance

- Some evidence of persistent positive and negative performance ("hot hands")
- Mutual fund fees
Mutual funds in ranking & post ranking quarter

![Graph showing quarterly return (%) vs. performance decile in ranking quarter. The graph compares the performance of mutual funds in the ranking quarter (solid line) and the post-ranking quarter (dashed line). The x-axis represents the performance decile in the ranking quarter, ranging from 1 to 10, and the y-axis represents the quarterly return, ranging from -6% to 6%. The graph indicates a decline in quarterly return as the performance decile increases in both quarters.]
Hedge Funds

- Non-traditional mutual fund with 4 key characteristics
  - Attempts to earn returns in rising or falling markets
  - Subject to lighter regulation than mutual funds
  - Not open to the general public
  - Charges higher fees (“2 and 20” is typical)
Hedge Funds vs. Mutual Funds

**Hedge Fund**
- Transparency: Limited Liability Partnerships that provide only minimal disclosure of strategy and portfolio composition
- No more than 100 “sophisticated”, wealthy investors

**Mutual Fund**
- Transparency: Regulations require public disclosure of strategy and portfolio composition
- Number of investors is not limited
Hedge Funds vs. Mutual Funds

**Hedge Fund**
- Investment strategy: Very flexible, funds can act opportunistically and make a wide range of investments
- Often use shorting, leverage, options
- Liquidity: Often have lock-up periods, require advance redemption notices

**Mutual Fund**
- Investment strategy: Predictable, stable strategies, stated in prospectus
- Limited use of shorting, leverage, options
- Liquidity: Can often move more easily into and out of a mutual fund
### Hedge Funds vs. Mutual Funds

<table>
<thead>
<tr>
<th>Hedge Fund</th>
<th>Mutual Fund</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Compensation structure:</td>
<td>• Compensation structure:</td>
</tr>
<tr>
<td>Typically charge a</td>
<td>Fees are usually a fixed</td>
</tr>
<tr>
<td>management fee of 1-2% of</td>
<td>percentage of assets,</td>
</tr>
<tr>
<td>assets and an incentive fee</td>
<td>typically 0.5% to 1.5%</td>
</tr>
<tr>
<td>of 20% of profits</td>
<td></td>
</tr>
</tbody>
</table>
Hedge Fund Strategies

- Directional
- Non-directional
  - Exploit temporary misalignments in relative valuation across sectors
  - Buy one type of security and sell another
  - Strives to be market neutral
Statistical Arbitrage

- Uses quantitative systems that seek out many temporary and modest misalignments in prices
- Involves trading in hundreds of securities a day with short holding periods
- Pairs trading: Pair up similar companies whose returns are highly correlated but where one is priced more aggressively
Weak or Semistrong Tests: Anomalies

- Momentum
- Long-term reversals
- Size and book-to-market effects
- January effect
- Post-Earnings Announcement Price Drift
- Persistence in mutual fund performance
- Relations between dividend yields & future returns
Are Markets Efficient?

- Magnitude Issue
  - Difficulty in detecting a small gain in performance
- Selection Bias Issue
  - Results of tests of market efficiency are only reported if there is evidence against efficiency
- Lucky Event Issue
  - Some method will appear to work well ex post by chance alone
Equity Premium Puzzle

- The average equity premium is about 7%
- The volatility (standard deviation) is about 16%
- Why?
  - Investors very risk averse
  - Irrational fear of stocks
  - The history in the U.S. in the last 100 years was a “fluke”
  - Survivor bias
Cross-Country Real Equity Returns
Behavioral Finance

- Investors Do Not Always Process Information Correctly
- Investors Often Make Inconsistent or Systematically Suboptimal Decisions
- Asset Price Bubbles
Information Processing Critique

• Forecasting Errors
  • Too much weight on recent experience
  • Example: Malmendier and Nagel (2011) find that stock market participation depends on returns experienced in an investor’s lifetime.

• Overconfidence
  • 98% confidence intervals include actual outcome 60% of the time (Alpert and Raiffa (1982))
Behavioral Biases

- Loss Aversion
- Ambiguity Aversion
Loss aversion

- People are more motivated by avoiding a loss than acquiring a similar gain.
  - Once I own something, not having it becomes more painful, because it is a loss.
  - If I don’t yet own it, then acquiring it is less important, because it is a gain.
Loss Aversion

- Coval and Shumway (2005) investigate morning and afternoon trades of 426 traders of CBOT
- Assume significantly more risk in the afternoon trading following morning losses than gains
Loss Aversion

- Kahneman and Tversky considered the following choices:
  - A: Gain $24,000
  - B: Gain $100,000 wp 0.25 and nothing otherwise
  - C: Lose $76,000
  - D: Lose $100,000 wp 0.75 and nothing otherwise
- Most people choose A over B, but choose D over C
- Risk-averse about gains but risk-seeking over losses
Prospect Theory

A: Conventional Utility Function

B: Utility Function under Prospect Theory
Ambiguity Aversion

- You are to draw a ball at random from a bag containing 100 balls. You win a prize if the ball drawn is red. One bag contains 50/50 blue/red balls. The distribution of balls in the other bag is unknown. Which bag would you prefer to draw the ball from?
Asset Price Bubbles

- A bubble is an increase in asset prices justified by future price appreciation alone.

- Bubbles exist in tulip bulbs, stocks, houses etc.
A bubble
Or not
Limits to Arbitrage

- Fundamental Risk
  - Things may get worse before they get better
- Implementation Costs
  - Costly to implement strategy
- Model Risk
  - Maybe it wasn’t really an arbitrage
Limits to Arbitrage

- Siamese Twin Companies
- Equity Carve-outs
- Closed-End Funds
- Index Inclusions
Pricing of Royal Dutch Relative to Shell (Deviation from Parity)
3Com and Palm

• In March 2000 3Com announced that at the end of the year it would give their shareholders 1.5 shares in Palm for each 3Com share they owned.

• Afterwards:
  • Price of Palm: $95.06
  • Price of 3 Com: $81.11
  • Value of “Non-Palm” 3 Com: -$61
Closed-End Funds

- Closed End Funds buy assets, but shareholder cannot redeem the underlying stocks
- Could trade at a discount (or premium)
- Can trace in Bloomberg
Index Inclusions

- When a stock is added to the S&P 500 it jumps 3.5% on average
- But there are bound to be close substitutes
- Not a perfect arbitrage
On-the-run/Off-the-run spread
Dividend Discount Model and Required Return

- CAPM gave us required return:

\[ k = r_f + \beta \left[ E(r_M) - r_f \right] \]

- If the stock is priced correctly

\[ E\left( \frac{P_1 + D_1 - P_0}{P_0} \right) = k \Rightarrow P_0 = \frac{E(P_1) + D_1}{1 + k} \]
Specified Holding Period

\[ P_0 = \frac{D_1}{(1+k)} + \frac{D_2}{(1+k)^2} \ldots + \frac{D_H + E(P_H)}{(1+k)^H} \]
Dividend Discount Models: General Model

\[ P_0 = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} \]

\( D_t = \) Dividend

\( k = \) required return
No Growth Model

\[ P_o = \frac{D}{k} \]

- Stocks that have earnings and dividends that are expected to remain constant
- Preferred stock (assuming divs get paid)
  - Do not share in the earnings or earnings growth
  - Do not have the security of bonds
  - Higher yield than bonds
  - TARP investments in banks are in preferred stock
No Growth Model: Example

\[ P_0 = \frac{D}{k} \]

\[ D_1 = $5.00 \]
\[ k = .15 \]
\[ P_0 = \frac{$5.00}{.15} = $33.33 \]
Estimating Dividend Growth Rates

\[ g \equiv ROE \times b \]

- \( g \) = growth rate in dividends
- \( ROE \) = Return on Equity for the firm
- \( b \) = plowback or retention percentage rate
  (1- dividend payout percentage rate)
Dividend Growth for Two Earnings Reinvestment Policies
Constant Growth Model

\[ P_o = \frac{D_0 (1 + g)}{1 + k} + \frac{D_0 (1 + g)^2}{(1 + k)^2} = D_0 \star \frac{1}{1 - \frac{1 + g}{1 + k}} - D_0 \]

\[ = D_0 \star \frac{1 + k}{k - g} - D_0 = D_0 \frac{1 + g}{k - g} = \frac{D_1}{k - g} \]

\[ g = \text{constant perpetual growth rate} \]
Constant Growth Model: Example

\[ P_0 = \frac{D_0(1+g)}{k-g} = \frac{D_1}{k-g} \]

\[ E_1 = 5.00 \quad b = 40\% \quad k = 15\% \]
\[ (1-b) = 60\% \quad D_1 = 3.00 \quad g = 8\% \]
\[ P_0 = \frac{3.00}{(.15 - .08)} = 42.86 \]
Bond Characteristics

- Face or par value
- Coupon rate
  - Zero coupon bond
Different Issuers of Bonds

- U.S. Treasury
  - Notes and Bonds
- Corporations
- Municipalities
- International Governments and Corporations
- Innovative Bonds
  - Floaters and Inverse Floaters
  - Asset-Backed
  - Indexe-Linked Bonds
  - Catastrophe Bonds
## Listing of Treasury Issues

### U.S. Government Bonds and Notes

Representative Over-the-Counter quotation based on transactions of $1 million or more.
Treasury bond, note and bill quotes are from midafternoon. Colons in bond and note bid-and-asked quotes represent 32nds; 101:01 means 101 1/32. Net change in 32nds. n- Treasury Note. inflation-indexed issue. Treasury bill quotes in hundredths, quoted in terms of a rate of discount. Days to maturity calculated from settlement date. All yields are to maturity and based on the asked quote. For bonds callable prior to maturity, yields are computed to the earliest call date for issues quoted above par and to the maturity date for issues quoted below par.

*When issued. Daily change expressed in basis points.

### Treasury Bills

<table>
<thead>
<tr>
<th>MATURITY</th>
<th>BID</th>
<th>ASKED</th>
<th>CHG</th>
<th>ASK</th>
<th>YLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.375</td>
<td>Apr 11</td>
<td>99:11</td>
<td>99:12</td>
<td>+2</td>
<td>2.53</td>
</tr>
<tr>
<td>4.875</td>
<td>Apr 11</td>
<td>100:16</td>
<td>100:18</td>
<td>+3</td>
<td>4.73</td>
</tr>
<tr>
<td>4.375</td>
<td>Jan 12</td>
<td>100:04</td>
<td>100:02</td>
<td>+3</td>
<td>4.70</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>MATURITY</th>
<th>BID</th>
<th>ASKED</th>
<th>CHG</th>
<th>ASK</th>
<th>YLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.125</td>
<td>Jun 11</td>
<td>101:17</td>
<td>101:18</td>
<td>+4</td>
<td>4.73</td>
</tr>
<tr>
<td>4.875</td>
<td>Jul 11</td>
<td>100:18</td>
<td>100:19</td>
<td>+4</td>
<td>4.73</td>
</tr>
<tr>
<td>3.375</td>
<td>Jan 12</td>
<td>100:04</td>
<td>100:02</td>
<td>+3</td>
<td>4.70</td>
</tr>
</tbody>
</table>

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<table>
<thead>
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<th>CHG</th>
<th>ASK</th>
<th>YLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.250</td>
<td>Nov 28</td>
<td>104:12</td>
<td>104:13</td>
<td>+8</td>
<td>4.92</td>
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</tbody>
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<table>
<thead>
<tr>
<th>MATURITY</th>
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<th>ASKED</th>
<th>CHG</th>
<th>ASK</th>
<th>YLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.875</td>
<td>Apr 29</td>
<td>124:17</td>
<td>124:18</td>
<td>+16</td>
<td>2.44</td>
</tr>
<tr>
<td>6.250</td>
<td>May 30</td>
<td>118:19</td>
<td>118:20</td>
<td>+10</td>
<td>4.90</td>
</tr>
<tr>
<td>5.375</td>
<td>Feb 31</td>
<td>106:20</td>
<td>106:21</td>
<td>+8</td>
<td>4.90</td>
</tr>
<tr>
<td>3.375</td>
<td>Apr 31</td>
<td>119:09</td>
<td>119:10</td>
<td>+16</td>
<td>2.35</td>
</tr>
<tr>
<td>4.500</td>
<td>Feb 36</td>
<td>94:19</td>
<td>94:20</td>
<td>+9</td>
<td>4.84</td>
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<th>ASKED</th>
<th>CHG</th>
<th>ASK</th>
<th>YLD</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.250</td>
<td>Feb 29</td>
<td>104:14</td>
<td>104:15</td>
<td>+9</td>
<td>4.92</td>
</tr>
<tr>
<td>3.875</td>
<td>Apr 29</td>
<td>124:17</td>
<td>124:18</td>
<td>+16</td>
<td>2.44</td>
</tr>
<tr>
<td>6.250</td>
<td>May 30</td>
<td>118:19</td>
<td>118:20</td>
<td>+10</td>
<td>4.90</td>
</tr>
<tr>
<td>5.375</td>
<td>Feb 31</td>
<td>106:20</td>
<td>106:21</td>
<td>+8</td>
<td>4.90</td>
</tr>
<tr>
<td>3.375</td>
<td>Apr 31</td>
<td>119:09</td>
<td>119:10</td>
<td>+16</td>
<td>2.35</td>
</tr>
<tr>
<td>4.500</td>
<td>Feb 36</td>
<td>94:19</td>
<td>94:20</td>
<td>+9</td>
<td>4.84</td>
</tr>
</tbody>
</table>
## Listing of Corporate Bonds

<table>
<thead>
<tr>
<th>ISSUER NAME</th>
<th>SYMBOL</th>
<th>COUPON</th>
<th>MATURITY</th>
<th>FITCH</th>
<th>HIGH</th>
<th>LOW</th>
<th>LAST</th>
<th>CHANGE</th>
<th>YIELD %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gatx</td>
<td>GMT.IK</td>
<td>8.875%</td>
<td>Jun 2009</td>
<td>Baa1/BBB/Baa3/-</td>
<td>107.545</td>
<td>107.538</td>
<td>107.545</td>
<td>-0.100</td>
<td>5.433</td>
</tr>
<tr>
<td>Marshall &amp; Ilsley</td>
<td>MI.YL</td>
<td>3.800%</td>
<td>Feb 2008</td>
<td>Aa3/A+/A+</td>
<td>98.514</td>
<td>98.470</td>
<td>98.514</td>
<td>0.064</td>
<td>5.263</td>
</tr>
<tr>
<td>Capital One</td>
<td>COF.HK</td>
<td>7.686%</td>
<td>Aug 2036</td>
<td>Baa2/BBB/Baa2/-</td>
<td>113.895</td>
<td>113.390</td>
<td>113.733</td>
<td>0.257</td>
<td>6.621</td>
</tr>
<tr>
<td>Entergy Gulf States</td>
<td>ETR.KC</td>
<td>6.180%</td>
<td>Mar 2035</td>
<td>Baa3/BBB+/Baa3+</td>
<td>99.950</td>
<td>94.616</td>
<td>99.469</td>
<td>0.219</td>
<td>6.220</td>
</tr>
<tr>
<td>AOL Time Warner</td>
<td>AOL.HG</td>
<td>6.875%</td>
<td>May 2012</td>
<td>Baa2/BBB+/Baa2+</td>
<td>107.205</td>
<td>105.402</td>
<td>106.565</td>
<td>0.720</td>
<td>5.427</td>
</tr>
<tr>
<td>Household Intl</td>
<td>HI.HJG</td>
<td>8.875%</td>
<td>Feb 2008</td>
<td>Aa3/AA-/AA-</td>
<td>100.504</td>
<td>100.504</td>
<td>100.504</td>
<td>-0.109</td>
<td>5.348</td>
</tr>
<tr>
<td>SBC Comm</td>
<td>SBC.IF</td>
<td>5.875%</td>
<td>Feb 2012</td>
<td>A2/A/A</td>
<td>102.116</td>
<td>102.001</td>
<td>102.001</td>
<td>-0.156</td>
<td>5.415</td>
</tr>
<tr>
<td>American General Finance</td>
<td>AIG.GOU</td>
<td>5.750%</td>
<td>Sep 2016</td>
<td>A1/A+/A+</td>
<td>101.229</td>
<td>101.135</td>
<td>101.135</td>
<td>-0.530</td>
<td>5.595</td>
</tr>
</tbody>
</table>
Provisions of Bonds

- Secured or unsecured
- Call provision
- Convertible provision
- Floating rate bonds
- Preferred Stock
Preferred Stock

- Shares characteristics of equity & fixed income
  - Dividends are paid in perpetuity
  - Nonpayment of dividends does not mean bankruptcy
  - Preferred dividends are paid before common
Bond Pricing: Present value calculation

\[ P_B = \sum_{t=1}^{T} \frac{C}{(1 + r)^t} + \frac{ParValue}{(1 + r)^T} \]

\( P_B \) = Price of the bond  
\( C_t \) = interest or coupon payments  
\( T \) = number of periods to maturity  
\( r \) = semi-annual discount rate or the semi-annual yield to maturity
Price: 10-yr, 8% Coupon, Face = $1,000

\[ P = \sum_{t=1}^{20} \frac{40}{1.03^t} + \frac{1000}{1.03^{20}} \]

\[ P = 1,148.77 \]

\[ C_t = 40 \text{ (Semi Annual)} \]
\[ P = 1000 \]
\[ T = 20 \text{ periods} \]
\[ r = 3\% \text{ (Semi Annual)} \]

Can also be calculated using the Excel PRICE function
Yield to Maturity

- Interest rate that makes the present value of the bond’s payments equal to its price

Solve the bond formula for \( r \)

\[
P_B = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{ParValue}{(1+r)^T}
\]
Yield to Maturity Example

\[ 950 = \sum_{t=1}^{20} \frac{35}{(1 + r)^t} + \frac{1000}{(1 + r)^{20}} \]

10 yr Maturity    Coupon Rate = 7%
Price = $950
Solve for \( r \) = semiannual rate    \( r = 3.8635\% \)
Prices and Yields (required rates of return) have an inverse relationship.

When yields get very high the value of the bond will be very low.

When yields approach zero, the value of the bond approaches the sum of the cash flows.
The Inverse Relationship Between Bond Prices and Yields

![Graph showing the inverse relationship between bond prices and interest rates. The graph plots bond price on the y-axis and interest rate on the x-axis. As the interest rate increases, the bond price decreases.]
## Bond Prices at Different Interest Rates (8% Coupon Bond)

<table>
<thead>
<tr>
<th>Time to Maturity</th>
<th>4%</th>
<th>6%</th>
<th>8%</th>
<th>10%</th>
<th>12%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>1038.83</td>
<td>1029.13</td>
<td>1000.00</td>
<td>981.41</td>
<td>963.33</td>
</tr>
<tr>
<td>10 years</td>
<td>1327.03</td>
<td>1148.77</td>
<td>1000.00</td>
<td>875.35</td>
<td>770.60</td>
</tr>
<tr>
<td>20 years</td>
<td>1547.11</td>
<td>1231.15</td>
<td>1000.00</td>
<td>828.41</td>
<td>699.07</td>
</tr>
<tr>
<td>30 years</td>
<td>1695.22</td>
<td>1276.76</td>
<td>1000.00</td>
<td>810.71</td>
<td>676.77</td>
</tr>
</tbody>
</table>
Yield Measures

Bond Equivalent Yield
7.72% = 3.86% x 2

Effective Annual Yield
(1.0386)^2 - 1 = 7.88%

Current Yield
Annual Interest / Market Price
$70 / $950 = 7.37%
Accrued Interest

- Buyer of bond pays accrued interest
  \[
  \text{Days since last coupon} / \text{Days between coupons} \times \text{Coupon}
  \]

- Quoted price doesn’t include this

- Dirty Price = Clean Price + Accrued Interest
Callable bonds are always worth less than regular debt.
Prices over Time of 30-Year Maturity, 6.5% Coupon Bonds

Price path for premium bond selling for more than face value (at yield to maturity = 4%)

Price path for discount bond (selling at yield to maturity = 11.5%)

Today

Maturity Date
Special Bonds

- Perpetuity (Coupons, but never redeemed)
  \[ P = \sum_{t=1}^{\infty} \frac{C}{(1+r)^t} = C\sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^t = \frac{C}{r} \]

- Zero coupon bonds (no coupons)
  \[ P = \frac{Par}{(1+r)^T} \]

- Treasury STRIPS...effectively zero-coupon bonds
Yields and Prices to Maturities on Zero-Coupon Bonds ($1,000 Face Value)

<table>
<thead>
<tr>
<th>Maturity (years)</th>
<th>Yield to Maturity</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5%</td>
<td>952.38 = 1000/1.05</td>
</tr>
<tr>
<td>1</td>
<td>7.01%</td>
<td>934.50</td>
</tr>
<tr>
<td>2</td>
<td>6%</td>
<td>890.00</td>
</tr>
<tr>
<td>3</td>
<td>7%</td>
<td>816.30</td>
</tr>
<tr>
<td>4</td>
<td>8%</td>
<td>735.03</td>
</tr>
</tbody>
</table>
The Price of a 30-Year Zero-Coupon Bond over Time at a Yield to Maturity of 10%
Rates on TIPS relative to nominal Treasury bonds are called “breakeven” rates and contain information about inflation expectations.
Holding-Period Return: Single Period

\[ HPR = \frac{[I + (P_1 - P_0)]}{P_0} \]

where

\( I \) = interest payment

\( P_1 \) = price in one period

\( P_0 \) = purchase price
Holding-Period Return Example

\[ CR = 8\% \quad YTM = 8\% \quad N=10 \text{ years} \]

Semiannual Compounding \quad \[ P_0 = $1000 \]

In six months the rate falls to 7\%

\[ P_1 = $1068.55 \]

\[ HPR = \frac{40 + (1068.55 - 1000)}{1000} \]

\[ HPR = 10.85\% \text{ (semiannual)} \]

Note that HPR and yield are two completely different things
Default Risk and Ratings

- Rating companies
  - Moody’s Investor Service
  - Standard & Poor’s
  - Fitch

- Rating Categories
  - Investment grade
    - BBB or higher for S&P
    - BAA or higher for Moody’s and Fitch
  - Speculative grade/Junk Bonds
Factors Used to assess financial stability

- Coverage ratios
- Leverage ratios
  \[
  \frac{\text{Debt}}{\text{Equity}} \quad \text{or} \quad \frac{\text{Assets}}{\text{Equity}} = \frac{\text{Debt} + \text{Equity}}{\text{Equity}}
  \]
- Liquidity ratios
- Profitability ratios
- Cash flow to debt
# Financial Ratios by S&P Ratings Class

<table>
<thead>
<tr>
<th>Ratio Type</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT interest coverage multiple</td>
<td>23.8</td>
<td>19.5</td>
<td>8.0</td>
<td>4.7</td>
<td>2.5</td>
<td>1.2</td>
<td>0.4</td>
</tr>
<tr>
<td>EBITDA interest coverage multiple</td>
<td>25.5</td>
<td>24.6</td>
<td>10.2</td>
<td>6.5</td>
<td>3.5</td>
<td>1.9</td>
<td>0.9</td>
</tr>
<tr>
<td>Funds from operations/total debt (%)</td>
<td>203.3</td>
<td>79.9</td>
<td>48.0</td>
<td>35.9</td>
<td>22.4</td>
<td>11.5</td>
<td>5.0</td>
</tr>
<tr>
<td>Free operating cash flow/total debt (%)</td>
<td>127.6</td>
<td>44.5</td>
<td>25.0</td>
<td>17.3</td>
<td>8.3</td>
<td>2.8</td>
<td>(2.1)</td>
</tr>
<tr>
<td>Total debt/EBITDA multiple</td>
<td>0.4</td>
<td>0.9</td>
<td>1.6</td>
<td>2.2</td>
<td>3.5</td>
<td>5.3</td>
<td>7.9</td>
</tr>
<tr>
<td>Return on capital (%)</td>
<td>27.6</td>
<td>27.0</td>
<td>17.5</td>
<td>13.4</td>
<td>11.3</td>
<td>8.7</td>
<td>3.2</td>
</tr>
<tr>
<td>Total debt/total debt + equity (%)</td>
<td>12.4</td>
<td>28.3</td>
<td>37.5</td>
<td>42.5</td>
<td>53.7</td>
<td>75.9</td>
<td>113.5</td>
</tr>
</tbody>
</table>
## Default Rates

### Default Rate by S&P Bond Rating (15 Years)

<table>
<thead>
<tr>
<th>S&amp;P Bond Rating</th>
<th>Default Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.52%</td>
</tr>
<tr>
<td>AA</td>
<td>1.31%</td>
</tr>
<tr>
<td>A</td>
<td>2.32%</td>
</tr>
<tr>
<td>BBB</td>
<td>6.64%</td>
</tr>
<tr>
<td>BB</td>
<td>19.52%</td>
</tr>
<tr>
<td>B</td>
<td>35.76%</td>
</tr>
<tr>
<td>CCC</td>
<td>54.38%</td>
</tr>
</tbody>
</table>

Yield Spreads between long-term corporates and Treasuries
Components of a corporate risk spread

- Expected default
- Recovery rate
- Risk premium
Treasury yields

Graph showing historical interest rates for 10-Year Treasury and 90-Day T-Bills, with a line for the difference between the two.
Overview of Term Structure

- Information on expected future short term rates can be implied from the yield curve.

- The yield curve is a graph that displays the relationship between yield and maturity.
Yield Curve Under Certainty

- Say we knew next year’s interest rate
- An upward sloping yield curve implies that short-term rates are going to be higher next year
Two 2-Year Investment Programs

Alternative 1: Buy and hold 2-year zero

\[ \text{2-Year Investment} \]

\[ \$890 \rightarrow \$890 \times 1.06^2 = \$1000 \]

Alternative 2: Buy a 1-year zero, and reinvest proceeds in another 1-year zero

\[ \text{1-Year Investment} \rightarrow \text{1-Year Investment} \]

\[ \$890 \rightarrow \$890 \times 1.05 = \$934.50 \rightarrow \$934.50(1 + r_2) \]
Yield Curve Under Certainty

- The two programs must have the same yield

\[(1 + y_2)^2 = (1 + r_1) \cdot (1 + r_2)\]

\[1 + y_2 = \left[ (1 + r_1) \cdot (1 + r_2) \right]^{1/2}\]

- So \( y_2 > y_1 \) if and only if \( r_2 > r_1 \)
# Short Rates versus Spot Rates

<table>
<thead>
<tr>
<th>Year</th>
<th>Short Rate in Each Year</th>
<th>Current Spot Rates (Yields to Maturity) for Various Maturities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$r_1 = 5%$</td>
<td>1-Year Investment</td>
</tr>
<tr>
<td>2</td>
<td>$r_2 = 7.01%$</td>
<td>2-Year Investment</td>
</tr>
<tr>
<td>3</td>
<td>$r_3 = 9.025%$</td>
<td>3-Year Investment</td>
</tr>
<tr>
<td>4</td>
<td>$r_4 = 11.06%$</td>
<td>4-Year Investment</td>
</tr>
<tr>
<td></td>
<td>$y_1 = 5%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_2 = 6%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_3 = 7%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_4 = 8%$</td>
<td></td>
</tr>
</tbody>
</table>
Yield Curve Under Certainty

\[(1 + y_n)^n = (1 + r_1)(1 + r_2) \cdots (1 + r_n)\]

\[1 + y_n = \left[(1 + r_1)(1 + r_2) \cdots (1 + r_n)\right]^{\frac{1}{n}}\]
Interest Rate Uncertainty

- What can we say when future interest rates are not known today?

- Suppose that today’s rate is 5% and the expected short rate for the following year is $E(r_2) = 6\%$
Theories of Term Structure

- Expectations Hypothesis
  - Treat expected future rates as though they were known
  \[
  (1 + y_2)^2 = (1 + r_1) \times [1 + E(r_2)] = 1.05 \times 1.06
  \]
  \[\Rightarrow y_2 = 5.5\%
  \]

- Liquidity Preference
  - Upward bias over expectations
  - Investors require extra yield to compensate them for the risk of capital loss if interest rates go up
Expectations Hypothesis

- Observed long-term rate is a function of today’s short-term rate and expected future short-term rates

\[
(1 + y_n)^n = (1 + r_1) \times (1 + E(r_2)) \times \ldots \times (1 + E(r_n))
\]

\[
y_n \approx n^{-1} (r_1 + E(r_2) + \ldots + E(r_n))
\]
Liquidity Premium Theory

- Long-term bonds are more risky

- Investors will demand a premium for the risk associated with long-term bonds

- The yield curve has an upward bias built into the long-term rates because of the risk premium (or term premium)

- Risk premium may change over time
Engineering a Synthetic Forward Loan

- The one-year rate next year is not known
- But the one- and two-year yields today are
- Can
  - Sell a two-year zero coupon bond for $\frac{1000}{(1 + y_2)^2}$
  - Buy $\frac{1 + y_1}{(1 + y_2)^2}$ one-year zero coupon bonds for $\frac{1000}{(1 + y_2)^2}$

- Cash flows:
  - Today: Nothing
  - In one year receive $1000 \ast \frac{1 + y_1}{(1 + y_2)^2}$
  - In two years pay $1000$
Engineering a Synthetic Forward Loan

- In this way, I can lock in a rate to borrow at some point in the future.
- In one year, I receive $1000 \times \frac{(1 + y_1)}{(1 + y_2)^2}$ and in two years I pay $1000.
- So the interest rate that I am locking in is
  \[ f = \frac{(1 + y_2)^2}{1 + y_1} - 1 \]

and this is called a forward rate.
Forward Rates from Observed Rates

\[ f_n = \frac{(1 + y_n)^n}{(1 + y_{n-1})^{n-1}} - 1 \]

- \( f_n \) = one-year forward rate for period \( n \)
- \( y_n \) = yield for a security with a maturity of \( n \)
Example: Forward Rates

\[ 4 \text{ yr} = 8.00\% \quad 3\text{yr} = 7.00\% \quad fn = ? \]

\[ f_n = \frac{(1.08)^4}{(1.07)^3} - 1 \]

\[ f_n = .1106 \text{ or } 11.06\% \]
# Downward Sloping Spot Yield Curve Example

<table>
<thead>
<tr>
<th>Zero-Coupon Rates</th>
<th>Bond Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td>12%</td>
<td>1</td>
</tr>
<tr>
<td>11.75%</td>
<td>2</td>
</tr>
<tr>
<td>11.25%</td>
<td>3</td>
</tr>
<tr>
<td>10.00%</td>
<td>4</td>
</tr>
<tr>
<td>9.25%</td>
<td>5</td>
</tr>
</tbody>
</table>
Forward Rates for Downward Sloping Y C Example

1yr Forward Rates

1yr \[ \frac{(1.1175)^2}{1.12} - 1 \] = 0.115006

2yrs \[ \frac{(1.1125)^3}{(1.1175)^2} - 1 \] = 0.102567

3yrs \[ \frac{(1.1)^4}{(1.1125)^3} - 1 \] = 0.063336

4yrs \[ \frac{(1.0925)^5}{(1.1)^4} - 1 \] = 0.063008
Forward rates and future interest rates

- If the path of future interest rates is known for certain, then forward rates are equal to future interest rates.

- With interest rate uncertainty,
  - Under the expectations hypothesis, forward rates are equal to expected future short-term interest rates.
  - Under liquidity preference theory, forward rates are greater than expected future short-term interest rates.
Yield curve slope

- Typically yield curve slopes up
- Consistent with liquidity preference
- Slope of yield curve related to bank profitability
  - Banks lend long-term and finance themselves short-term
Prices of 8% Coupon Bond (Coupons Paid Semiannually)

<table>
<thead>
<tr>
<th>Yield to Maturity</th>
<th>1 year</th>
<th>10 year</th>
<th>20 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>8%</td>
<td>1000.00</td>
<td>1000.00</td>
<td>1000.00</td>
</tr>
<tr>
<td>9%</td>
<td>990.64</td>
<td>934.96</td>
<td>907.99</td>
</tr>
<tr>
<td>Percent Fall in Price</td>
<td>0.94%</td>
<td>6.50%</td>
<td>9.20%</td>
</tr>
</tbody>
</table>
## Prices of Zero-Coupon Bond

<table>
<thead>
<tr>
<th>Yield to Maturity</th>
<th>1 year</th>
<th>10 year</th>
<th>20 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>8%</td>
<td>924.56</td>
<td>456.39</td>
<td>208.29</td>
</tr>
<tr>
<td>9%</td>
<td>915.73</td>
<td>414.64</td>
<td>171.93</td>
</tr>
<tr>
<td>Percent Fall in Price</td>
<td>0.96%</td>
<td>9.15%</td>
<td>17.46%</td>
</tr>
</tbody>
</table>
Bond Pricing Relationships

- Price is more sensitive to change in yield for:
  - Longer maturity bonds
  - Lower coupon bonds
  - Lower initial yield to maturity bonds
Change in Bond Price as a Function of Change in Yield to Maturity

<table>
<thead>
<tr>
<th>Bond</th>
<th>Coupon</th>
<th>Maturity</th>
<th>Initial YTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12%</td>
<td>5 years</td>
<td>10%</td>
</tr>
<tr>
<td>B</td>
<td>12%</td>
<td>30 years</td>
<td>10%</td>
</tr>
<tr>
<td>C</td>
<td>3%</td>
<td>30 years</td>
<td>10%</td>
</tr>
<tr>
<td>D</td>
<td>3%</td>
<td>30 years</td>
<td>6%</td>
</tr>
</tbody>
</table>

![Graph showing the relationship between change in bond price and change in yield to maturity.](image)
Illustration of Duration
Duration

- A measure of the effective maturity of a bond
- The weighted average of the times until each payment is received, with the weights proportional to the present value of the payment
- Duration is shorter than maturity for all bonds except zero coupon bonds
- Duration is equal to maturity for zero coupon bonds
Duration: Calculation

\[ w_t = \frac{CF_t}{(1 + y)^t \text{ Price}} \]

\[ D = \sum_{t=1}^{T} tw_t \]
### Spreadsheet

**Calculating the Duration of Two Bonds**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Time</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>until</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Payment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>Period</td>
<td>0.5</td>
<td>40</td>
<td>38.095</td>
<td>0.0395</td>
<td>0.0197</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Years)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>A. 8%</td>
<td>2</td>
<td>1.0</td>
<td>40</td>
<td>36.281</td>
<td>0.0376</td>
<td>0.0376</td>
</tr>
<tr>
<td></td>
<td>coupon</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>bond</td>
<td>3</td>
<td>1.5</td>
<td>40</td>
<td>34.554</td>
<td>0.0358</td>
<td>0.0537</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td>2.0</td>
<td>1040</td>
<td>855.611</td>
<td>0.8871</td>
<td>1.7741</td>
</tr>
<tr>
<td>8</td>
<td>Sum:</td>
<td></td>
<td></td>
<td></td>
<td>964.540</td>
<td>1.0000</td>
<td>1.8852</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>B. Zero-</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0.000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td>coupon</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>1.0</td>
<td>0</td>
<td>0.000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>1.5</td>
<td>0</td>
<td>0.000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>2.0</td>
<td>1000</td>
<td>822.702</td>
<td>1.0000</td>
<td>2.0000</td>
</tr>
<tr>
<td>14</td>
<td>Sum:</td>
<td></td>
<td></td>
<td></td>
<td>822.702</td>
<td>1.0000</td>
<td>2.0000</td>
</tr>
<tr>
<td>16</td>
<td>Semiannual int rate:</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td><em>Weight = Present value of each payment (column E) divided by the bond price.</em></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Suppose that there is a level change in interest rates

\[
\frac{\Delta P}{P} \approx -D \left[ \frac{\Delta y}{1 + y} \right]
\]

\[D_{MOD} = D / (1 + y) \text{ modified duration} \]

with semiannual compounding y is yield per 6 months

\[
\frac{\Delta P}{P} \approx -D_{MOD} \Delta y
\]
Rules for Duration

Rule 1  The duration of a zero-coupon bond equals its time to maturity

Rule 2  Holding maturity constant, a bond’s duration is higher when the coupon rate is lower

Rule 3  Holding the coupon rate constant, a bond’s duration generally increases with its time to maturity

Rule 4  Holding other factors constant, the duration of a coupon bond is higher when the bond’s yield to maturity is lower

Rule 5  The duration of a perpetuity is equal to: 
\[
\frac{(1+y)}{y}
\]

Rule 6  The duration of a portfolio with weight \( w_k \) on a bond with duration \( D_k \) is 
\[
\sum_k w_k D_k
\]
Bond Duration versus Bond Maturity

![Graph showing bond duration versus maturity](image)
Bond Durations (Yield to Maturity = 8% APR; Semiannual Coupons)

<table>
<thead>
<tr>
<th>Years to Maturity</th>
<th>6%</th>
<th>8%</th>
<th>10%</th>
<th>12%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.985</td>
<td>0.981</td>
<td>0.976</td>
<td>0.972</td>
</tr>
<tr>
<td>5</td>
<td>4.361</td>
<td>4.218</td>
<td>4.095</td>
<td>3.990</td>
</tr>
<tr>
<td>10</td>
<td>7.454</td>
<td>7.067</td>
<td>6.772</td>
<td>6.541</td>
</tr>
</tbody>
</table>

Perpetuity: 13 years
Duration, Modified Duration and DV01

- DV01 is the “dollar value of a basis point”
  - How much the price rises when the interest rate falls by one hundredth of one percentage point
- Relations:

\[
DV01 \approx D_{MOD} \times P \times 0.0001
\]

\[
D_{MOD} = \frac{D}{1 + y}
\]
Consider a two-year 8% bond with a 10% yield.

- The duration is 1.8852 (calculated earlier).
- The modified duration is \( \frac{1.8852}{1.05} = 1.7954 \).
- The price of the bond is $96.45405.
- If the interest rate were 10.01%, the price would be $96.43673.
- The DV01 is 1.732 cents.
- And \( D_{\text{MOD}} \times 0.0001 \times P = 1.732 \) cents.
Convexity

- The relationship between bond prices and yields is not linear
- Duration rule is a good approximation for only small changes in bond yields
Bond Price Convexity: 30-Year Maturity, 8% Coupon; Initial Yield to Maturity = 8%
Convexity of Two Bonds

The graph illustrates the percentage change in bond price for two different bonds, A and B, as the change in yield to maturity (in percentage) increases. The curve for Bond A is steeper than that for Bond B, indicating that Bond A is more convex than Bond B.
Correction for Convexity

\[ w_t = \left[ \frac{CF_t}{(1 + y)^t} \right] / \text{Price} \]

\[
\text{Convexity} = \frac{1}{(1 + y)^2} \sum_{t=1}^{n} w_t (t + t^2)
\]

Correction for Convexity:

\[
\frac{\Delta P}{P} \approx -D_{MOD} \Delta y + \frac{1}{2} \text{Convexity}(\Delta y)^2
\]
Convexity Example

- Suppose that a bond trades at par and pays an 8% coupon once a year with 10 years to maturity
- What is its duration, modified duration, DV01 and convexity?
- From EXCEL, the answers are

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration</td>
<td>7.25</td>
</tr>
<tr>
<td>Modified Duration</td>
<td>6.71</td>
</tr>
<tr>
<td>DV01</td>
<td>0.0671</td>
</tr>
<tr>
<td>Convexity</td>
<td>60.53</td>
</tr>
</tbody>
</table>
Effects of convexity:
Typical Treasury ZC Yield Curve
Callable Bonds

- As rates fall, there is a ceiling on possible prices
- The bond cannot be worth more than its call price
- Negative convexity
Price – Yield Curve for a Callable Bond

- Region of Negative Convexity
  (Price-yield curve is below its tangency line.)

- Region of Positive Convexity
Mortgage backed securities

- Prepayment risk is important for MBS
- Homeowners will refinance if interest rates fall
- Implies negative convexity
- Because homeowners are sometimes slow to refinance, the value can exceed the principal balance
Price - Yield Curve for a Mortgage-Backed Security

Bond Price

Principal Balance

Interest Rate
Suppose an insurance company issues a guaranteed investment contract for $10,000 paying 8% over 5 years.

In 5 years, they need $10,000 \times (1.08)^5 = $14,693.28.

Suppose the insurance company buys an 8% 5 year bond for $10,000 to hedge this risk.
### Alternative Scenarios

#### Interest Rates Rise to 9%

<table>
<thead>
<tr>
<th>Year</th>
<th>Calculation</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>$800 \times (1.09)^4$</td>
<td>$1129.27$</td>
</tr>
<tr>
<td>Year 2</td>
<td>$800 \times (1.09)^3$</td>
<td>$1036.02$</td>
</tr>
<tr>
<td>Year 3</td>
<td>$800 \times (1.09)^2$</td>
<td>$950.48$</td>
</tr>
<tr>
<td>Year 4</td>
<td>$800 \times (1.09)^1$</td>
<td>$872.00$</td>
</tr>
<tr>
<td>Year 5</td>
<td></td>
<td>$10800$</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>$14787.77$</td>
</tr>
<tr>
<td>Needed</td>
<td></td>
<td>$14693.28$</td>
</tr>
<tr>
<td>PROFIT</td>
<td></td>
<td>$94.99$</td>
</tr>
</tbody>
</table>
## Alternative Scenarios

### Interest Rates Fall to 7%

<table>
<thead>
<tr>
<th>Year</th>
<th>Calculation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>$800 \times (1.07)^4$ =</td>
<td>$1048.64</td>
</tr>
<tr>
<td>Year 2</td>
<td>$800 \times (1.07)^3$ =</td>
<td>$980.03</td>
</tr>
<tr>
<td>Year 3</td>
<td>$800 \times (1.07)^2$ =</td>
<td>$915.92</td>
</tr>
<tr>
<td>Year 4</td>
<td>$800 \times (1.07)^1$ =</td>
<td>$856.00</td>
</tr>
<tr>
<td>Year 5</td>
<td>$10800</td>
<td>$10800</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>$14600.59</td>
</tr>
<tr>
<td>Needed</td>
<td></td>
<td>$14.693.28</td>
</tr>
<tr>
<td>PROFIT</td>
<td></td>
<td>($92.69)</td>
</tr>
</tbody>
</table>
Immunization of Interest Rate Risk

- The hedge did not work
- Intuitively the bond front loaded the coupons too much opening the strategy to reinvestment risk

- But now suppose the insurance company buys an 8% 6 year bond for $10,000 to hedge this risk
  - This bond has a duration of about 5 years
## Alternative Scenarios

### Interest Rates Rise to 9%

<table>
<thead>
<tr>
<th>Year</th>
<th>Calculation</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>$800 \times (1.09)^4$</td>
<td>$1129.27</td>
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<td>$1036.02</td>
</tr>
<tr>
<td>Year 3</td>
<td>$800 \times (1.09)^2$</td>
<td>$950.48</td>
</tr>
<tr>
<td>Year 4</td>
<td>$800 \times (1.09)^1$</td>
<td>$872.00</td>
</tr>
<tr>
<td>Year 5 Coupon</td>
<td>$800</td>
<td>$800</td>
</tr>
<tr>
<td>Year 5 Sell Bond</td>
<td>$10800 / 1.09</td>
<td>$9,908.26</td>
</tr>
</tbody>
</table>

**TOTAL** $14,696.03

**Needed** $14,693.28

**PROFIT** $2.75
## Alternative Scenarios

### Interest Rates Fall to 7%

<table>
<thead>
<tr>
<th>Year</th>
<th>Calculation</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>$800 \times (1.07)^4$</td>
<td>$1048.64</td>
</tr>
<tr>
<td>Year 2</td>
<td>$800 \times (1.07)^3$</td>
<td>$980.03</td>
</tr>
<tr>
<td>Year 3</td>
<td>$800 \times (1.07)^2$</td>
<td>$915.92</td>
</tr>
<tr>
<td>Year 4</td>
<td>$800 \times (1.07)^1$</td>
<td>$856.00</td>
</tr>
<tr>
<td>Year 5 Coupon</td>
<td>$800</td>
<td>$800</td>
</tr>
<tr>
<td>Year 5 Sell Bond</td>
<td>$10800 / 1.07$</td>
<td>$10,093.46</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td>$14,694.05</td>
</tr>
<tr>
<td>Needed</td>
<td></td>
<td>$14,693.28</td>
</tr>
<tr>
<td>PROFIT</td>
<td></td>
<td>$0.77</td>
</tr>
</tbody>
</table>
Immunization of Interest Rate Risk

- General principle:
  Immunize a liability with an asset of the same duration and the same present value
- A liability in 5 years should be matched with an asset of 5 years duration.
The interest rate is 5%. An insurance company must pay $1 million \times 1.05^3 = $1.158 million in 3 years.

- Immunize this with
  - A one year bill (weight $x$) and
  - A 10 year strip (weight $1-x$)

- Duration of liability: 3 years
- Duration of immunizing portfolio: 3 years
- $3 = x + 10(1-x)$: Solving this yields $x = \frac{7}{9}$
- The immunizing portfolio is $\$778k$ in bills and $\$222k$ in strips.
Constructing an immunized portfolio: Example

- The interest rate is 10%. An insurance company must pay $10,000 \times 1.1^7 = $19,487 in 7 years
- Immunize this with
  - A perpetuity and
  - A three-year zero coupon bond
- Duration of liability: 7 years
- Duration of immunizing portfolio: 7 years
Constructing an immunized portfolio: Example….continued

- The duration of the perpetuity is $1.10/0.1=11$
- The duration of the zero coupon bond is 3 years
- The duration of the immunizing portfolio is
  \[ 3\omega + 11(1 - \omega) \]
- Setting this to 7 implies that $\omega = 1/2$
- The immunizing portfolio is
  - $5000$ in zero coupon bonds
  - $5000$ in perpetuities
Rebalancing the immunized portfolio: Example

- 1 year has passed. The interest rate is 10%. What change should be made to the portfolio?
- Both the present value of the liability and the asset are $11,000 and are equal.
- The duration of the liability is now 6 years. So the portfolio weights solve

\[ 2\omega + 11(1 - \omega) = 6 \implies \omega = \frac{5}{9} \]

- So now the immunizing portfolio is
  - $6,111.11 in two-year zero coupon bonds
  - $4,888.89 in perpetuities
Swaps

- Interest rate swap
- Inflation swaps
- Variance swaps
- Credit default swaps
Interest rate swap

- Agreement between two parties to exchange a fixed rate a notional underlying principal for a floating rate

- For example:
  - Pay LIBOR times $100 million
  - Receive fixed rate times $100 million
  - Only net amount changes hands
Interest rate swap: example

- Suppose that the 3 month LIBOR rate is 4% and the fixed rate is 7%

- Party paying fixed gives the counterparty
  \[0.25 \times (7\% \times \$100 \text{ million} - 4\% \times \$100 \text{million}) = \$750,000\]
The Swap Dealer

- Dealer enters a swap with Company A
  - Pays fixed rate and receives LIBOR
- Dealer enters another swap with Company B
  - Pays LIBOR and receives a fixed rate
- When two swaps are combined, dealer’s position is effectively neutral on interest rates
Interest Rate Swap

Company A

6.95%  
LIBOR

Swap Dealer

7.05%  
LIBOR

Company B
Uses of Interest Rate Swaps

- Converting liabilities or assets from fixed to floating
- Banks have short-term liabilities and long-term assets
  - Banks generally pay fixed, receive floating
- Betting on interest rate movements
- Managerial or investor myopia
Example of Interest Rate Swap

- Companies X and Y have been offered the following rates per annum on a $20 million 5-year loan:

<table>
<thead>
<tr>
<th></th>
<th>Fixed</th>
<th>Floating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Company X</td>
<td>12%</td>
<td>LIBOR+0.1%</td>
</tr>
<tr>
<td>Company Y</td>
<td>13.4%</td>
<td>LIBOR+0.6%</td>
</tr>
</tbody>
</table>

- Company X wants a floating rate loan
- Company Y wants a fixed rate loan
- Swap dealer charges 10 bp spread
Example of Interest Rate Swap

- Company X borrows at 12%
- Enters swap to pay LIBOR and get 12.3%
- Effectively X pays LIBOR - 0.3%

- Company Y borrows at LIBOR + 0.6%
- Enters swap to receive LIBOR and pay 12.4%
- Effectively Y pays 13%
Case Study: Harvard and Use of Swaps

- In 2004 Harvard planned big expansion
- Planned to borrow $2.3 billion starting in 2008
- Entered into *forward* swap agreements to pay fixed and receive floating
  - No upfront cost, unlike a forward rate agreement
Case Study: Harvard and Use of Swaps

- In 2008 Harvard didn’t want to borrow
- But they still had swaps
- With short rates falling, pay fixed receive floating is costly
- Had to post margin
- Paid nearly $1 billion to exit swap contracts at worst possible time
Treasury and Swap Rates

![Graph showing Treasury and Swap Rates from 2005 to 2014](image)
Swap Spreads

![Graph of Swap Spreads showing 10 Year and 30 Year trends from 2005 to 2014.]
Swap Bond Arbitrage

- If swap rate is above bond rate
  - Short the bond
  - Invest the proceeds in floating
  - Receive fixed in a swap contract
- Limitation is default risk
- If swap rate is below bond rate
  - Borrow at the floating rate
  - Pay fixed
  - Go long the bond
- Why isn’t this being done (more)?
3 month LIBOR-OIS spreads
Variance Swaps

- Payment is based on the volatility of an asset over a period of time (say month).
- If the volatility turns out to be above a threshold, the seller of volatility protection has to pay buyer of volatility protection.
- Variance swaps are *on average* favorable to the seller of volatility protection.
Inflation Swaps

- Buyer of inflation protection pays a fixed rate on a notional principle
- Seller of inflation protection pays whatever CPI inflation turns out to be over next n years
- Pension funds might want to buy inflation protection
Credit Default Swaps

- Payment on a CDS is tied to the financial status of a bond
- The buyer of credit protection pays a fixed rate
- If the bond defaults, the issuer of the protection has to buy the bond at par value
  - Settlement can be *physical* or *cash*
Credit Default Swaps: Example

- Investor buys protection for $10 million of Risky Bank Debt
- CDS spread is 30 bps
- Each year, buyer of protection pays seller of protection $30,000
- On default, the seller pays the buyer $10 million less the value of the defaulted bonds
Credit Default Swaps: Pricing Example

- Suppose investors are risk neutral
- Risky Bank has a 20% chance of failing in 5 years
- If it fails, recovery will be 60%
- Q. What should the CDS spread be?

- A. Cost = Expected Benefit
  \[5 \times S = 0.2 \times 0.4\]
  \[S = 0.016 \text{ (160 basis points)}\]
Bank CDS rates in 2008
CDS Bond Arbitrage

• In theory

Corporate Bond + CDS Protection = Risk-free Bond

Corporate Bond Spread = CDS Rate
CDS Bond Arbitrage

- CDS are more liquid
- CDS are joint bets on the issuer and the counterparty
- The definition of what constitutes default can be manipulated
Quanto CDS

- Sovereign CDS generally don’t pay off in currency of issuer
- US Treasury CDS pay off in euros
- European CDS pay off in dollars
- But you can go long Italian CDS in dollars and short Italian CDS in euros
- Taking a bet on the currency in the event of default
Option Terminology

- Buy - Long
- Sell - Short
- Call
- Put

- Key Elements
  - Exercise or Strike Price
  - Premium or Price
  - Maturity or Expiration
Market and Exercise Price Relationships

**In the Money** - exercise of the option would be profitable
- Call: market price > exercise price
- Put: exercise price > market price

**Out of the Money** - exercise of the option would not be profitable
- Call: market price < exercise price
- Put: exercise price < market price

**At the Money** - exercise price and asset price are equal
American vs. European Options

American - the option can be exercised at any time before expiration or maturity

European - the option can only be exercised on the expiration or maturity date
Different Types of Options

- Stock Options
- Index Options
- Futures Options
- Foreign Currency Options
- Interest Rate Options
Payoffs and Profits at Expiration - Calls

Notation

Stock Price = $S_T$  Exercise Price = $X$

Payoff to Call Holder

$(S_T - X)$ if $S_T > X$

0 if $S_T \leq X$

Profit to Call Holder

Payoff - Purchase Price
Payoffs and Profits at Expiration - Calls

**Payoff to Call Writer**

- \(- (S_T - X)\) if \(S_T > X\)
- \(0\) if \(S_T \leq X\)

**Profit to Call Writer**

Payoff + Premium
Payoff and Profit to Call Option at Expiration
Payoff and Profit to Call Writers at Expiration
Payoffs and Profits at Expiration - Puts

Payoffs to Put Holder

\[
\begin{align*}
0 & \quad \text{if } S_T \geq X \\
(X - S_T) & \quad \text{if } S_T < X
\end{align*}
\]

Profit to Put Holder

Payoff - Premium
Payoffs and Profits at Expiration – Puts Continued

Payoffs to Put Writer

\[
\begin{align*}
0 & \quad \text{if } S_T \geq X \\
-(X - S_T) & \quad \text{if } S_T < X
\end{align*}
\]

Profits to Put Writer

Payoff + Premium
Payoff and Profit to Put Option at Expiration

Profit = Value of Put at Expiration

Price of Put

$100

$0
Value of Protective Put Portfolio at Option Expiration

<table>
<thead>
<tr>
<th></th>
<th>$S_T \leq X$</th>
<th>$S_T &gt; X$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stock</strong></td>
<td>$S_T$</td>
<td>$S_T$</td>
</tr>
<tr>
<td>$+ \text{ Put}$</td>
<td>$X - S_T$</td>
<td>$0$</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>$X$</td>
<td>$S_T$</td>
</tr>
</tbody>
</table>
Value of a Protective Put Position at Option Expiration
Protective Put versus Stock Investment (at-the-money option)
Option Strategies

Straddle (Same Exercise Price)

   Long Call and Long Put

Spreads - A combination of two or more call options or put options on the same asset with differing exercise prices or times to expiration.

Vertical or money spread:

   Same maturity
   Different exercise price

Horizontal or time spread:

   Different maturity dates
Value of a Straddle Position at Option Expiration

<table>
<thead>
<tr>
<th></th>
<th>$S_T &lt; X$</th>
<th>$S_T \geq X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payoff of call</td>
<td>0</td>
<td>$S_T - X$</td>
</tr>
<tr>
<td>Payoff of put</td>
<td>$X - S_T$</td>
<td>0</td>
</tr>
</tbody>
</table>

= TOTAL

$X - S_T$  $S_T - X$
Value of a Straddle at Expiration

**A:** Call

Payoff of Call

Payoff Profit

0

X

S_t

C

**B:** Put

Payoff of Put

Payoff Profit

0

X

S_t

P

**C:** Straddle

Payoff of Straddle

Payoff Profit

0

X

S_t

X - P - C

X - P

P + C

- (P + C)
Value of a Bullish Spread Position at Expiration

<table>
<thead>
<tr>
<th></th>
<th>$S_T \leq X_1$</th>
<th>$X_1 &lt; S_T \leq X_2$</th>
<th>$S_T \geq X_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payoff of purchased call, exercise price = $X_1$</td>
<td>0</td>
<td>$S_T - X_1$</td>
<td>$S_T - X_1$</td>
</tr>
<tr>
<td>Payoff of written call, exercise price = $X_2$</td>
<td>$-0$</td>
<td>$-0$</td>
<td>$-(S_T - X_2)$</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td>0</td>
<td>$S_T - X_1$</td>
<td>$X_2 - X_1$</td>
</tr>
</tbody>
</table>
Value of a Bullish Spread Position at Expiration

A: Call Held (Call 1)

B: Call Written (Call 2)

C: Bullish Spread $X_2 - X_1$
Put Call Parity

Violations of relationship mean arbitrage will be possible

\[ C + \frac{X}{(1+r_f)^T} = S_0 + P \]

- C is the price of a call option at strike X
- P is the price of a put option at strike X
- T is the maturity time of the options
- \( r_f \) is the riskfree rate
- \( S_0 \) is the stock price today
Put Call Parity - Disequilibrium Example

Stock Price = 110   Call Price = 17
Put Price = 5      Risk Free = 5%
Maturity = 1 yr    X = 105

\[ C + \frac{X}{(1 + r_f)^T} = S_0 + P \]

117 > 115

Implies an arbitrage opportunity
Arbitrage Strategy

<table>
<thead>
<tr>
<th>Position</th>
<th>Immediate Cash Flow</th>
<th>Cash Flow in 1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$S_T &lt; 105$</td>
</tr>
<tr>
<td>Buy stock</td>
<td>$-110$</td>
<td>$S_T$</td>
</tr>
<tr>
<td>Borrow $105/1.05 = $100</td>
<td>$+100$</td>
<td>$-105$</td>
</tr>
<tr>
<td>Sell call</td>
<td>$+17$</td>
<td>$0$</td>
</tr>
<tr>
<td>Buy put</td>
<td>$-5$</td>
<td>$105 - S_T$</td>
</tr>
<tr>
<td>TOTAL</td>
<td>$2$</td>
<td>$0$</td>
</tr>
</tbody>
</table>
Put Call Parity - Example

Stock Price = 80  Call Price for a strike at 90= 10
Risk Free = 2%    Maturity = 1 yr

What is the put price for a strike of 90 (1 year maturity)

\[ C + \frac{X}{(1 + r_f)^T} = S_0 + P \]

\[ \therefore P = C + \frac{X}{(1 + r_f)^T} - S_0 = 10 + \frac{90}{1.02} - 80 = 18.24 \]
Suppose that there are $K$ possible outcomes
  - Called “states of the world”

An “Arrow Debreu security” pays off $1$ in one particular state of the world and $0$ in all others

Let $v(k)$ denote the price of the Arrow Debreu security that pays $1$ in the $k$th state of the world
  - Probability of payoff under risk neutrality
Now suppose that there are $K$ different options.

Suppose that the $i$th option has a payoff $p(i,k)$ in the $k$th state of the world.

Price of the option

$$\sum_{k=1}^{K} p(i,k) v(k)$$
Risk-Neutral Implied Densities

- Given the prices of K-1 options can reverse-engineer the probabilities of the different states.

- Example: The price of oil will be $50, $60 or $70
  - Price of an option to buy at $60 is $4
  - Price of an option to buy at $50 is $12

\[
4 = P_{70} \times 10 \\
12 = (P_{70} \times 20) + (P_{60} \times 10) \\
1 = P_{50} + P_{60} + P_{70}
\]

Implies \( P_{70} = 0.4; \quad P_{60} = 0.4; \quad P_{50} = 0.2 \)
Arbitrage Restrictions on Option Values

- Values cannot be negative (call or put)
- American options are always at least as valuable as European options
Arbitrage Restrictions on Option Value: Call
Stock Pays no Dividends

\[ C \geq 0 \]
\[ C \leq S_0 \]
\[ C \geq S_0 - \frac{X}{(1 + r_f)^T} \]
\[ C \geq S_0 - X \text{ for an American option only} \]
Arbitrage Restrictions on Option Value: Put Stock Pays no Dividends

\[ P \geq 0 \]
\[ P \leq X \]
\[ P \geq \frac{X}{(1 + r_f)^T} - S_0 \]

\[ P \geq X - S_0 \] for an American option only
Early Exercising of an American Option

• It may be rational to exercise an American Option early

• Exception: It is never rational to exercise an American Call option early on a non-dividend paying stock
Binomial Option Pricing

- Binomial option pricing approximates stock movements by two possible outcomes up or down.
- Creates a portfolio of a bond and the stock which has the same payoff as the option.
- And so should have the same price.
- Called a replication pricing method.
Binomial Option Pricing: Example

Stock Price

100

120

90

Call Option Value

$X = 110$

C

10

0
Alternative Portfolio
Buy 1 share of stock at $100
Borrow $81.82 (10% Rate)
Net outlay $18.18

Payoff
Value of Stock  90  120
Repay loan       -90 -90
Net Payoff       0   30

Payoff Structure is exactly 3 times the Call
Binomial Option Pricing: Text Example Continued

\[ 3C = \$18.18 \]
\[ C = \$6.06 \]
Generalizing the Two-State Approach

Assume that we can break the year into two six-month segments.

In each six-month segment the stock could increase by 10% or decrease by 5%.

Assume the stock is initially selling at 100.

Possible outcomes:
- Increase by 10% twice
- Decrease by 5% twice
- Increase once and decrease once (2 paths)
Generalizing the Two-State Approach Continued
Generalizing the Two-State Approach Continued

Work backwards from the last node

A call option with a strike of $110 has a payoff of $11 or $0.
Alternative portfolio: Buy 1 share for $110 and borrow $99.52

<table>
<thead>
<tr>
<th></th>
<th>Top Branch</th>
<th>Bottom Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>121</td>
<td>104.5</td>
</tr>
<tr>
<td>Repay Loan</td>
<td>104.5</td>
<td>104.5</td>
</tr>
<tr>
<td>Total</td>
<td>16.5</td>
<td>0</td>
</tr>
</tbody>
</table>
Generalizing the Two-State Approach Continued

Call costs C and has a payoff of $11 or $0.
Alternative portfolio costs $10.48 and has a payoff of $16.50 or $0
Hence C=6.99
Generalizing the Two-State Approach Continued

At the other end node, the stock price is

\[
\begin{array}{c}
95 \\
90.25 \\
104.50
\end{array}
\]

Call option is worthless
Generalizing the Two-State Approach Continued

At the first node

Call option payoff

100

\[ \begin{aligned}
\text{110} \\
\text{95} \\
\text{6.99} \\
\text{0}
\end{aligned} \]
The call option costs $C$ and has a payoff of $6.99$ or $0$. Alternative portfolio: Buy 1 share for $100$ and borrow $90.47$.

<table>
<thead>
<tr>
<th></th>
<th>Top Branch</th>
<th>Bottom Branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>110</td>
<td>95</td>
</tr>
<tr>
<td>Repay Loan</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>Total</td>
<td>15</td>
<td>0</td>
</tr>
</tbody>
</table>

Alternative portfolio costs $9.53$ and has a payoff of $15$ or $0$. So the call option costs $4.44$. 
Example A

- The price of a stock today is $50.
- In one year’s time, it will either go up to $55 or down to $45.
- The risk-free interest rate is 5 percent.
- Find the price of a put option with a strike price of $50.
Example A

- Price the call option
  - Call option worth $5 if price rises
  - Buy 1 share and borrow $45/1.05=$42.86
  - Price of the call option is $3.57

- By put-call parity
  \[
  3.57 + \frac{50}{1.05} = 50 + P
  \]

- Put option is worth $1.19
A general formula

- The price of a stock today is $P(0)$.
- In one year’s time, it will either go up to $P(u)$ or down to $P(l)$.
- The risk-free interest rate is $r$.
- Find the price of a call option with a strike price of $X$ where $P(l) \leq X \leq P(u)$.
General Formula: Continued

\[ P_0 - \frac{P(l)}{1 + r_f} \]

\[ P(u) - P(l) \]

\[ P(u) - X \]

\[ C = \left[ P_0 - \frac{P(l)}{1 + r_f} \right] \frac{P(u) - X}{P(u) - P(l)} \]
A general formula

- Same setup as before except that $P(l) > X$.
- Option expires in the money on both branches.
- Turns out to be simpler.
General Formula: Continued

\[ P_o - \frac{X}{1 + r_f} \]

\[ P(l) - X \]

\[ P(u) - X \]

\[ P(l) - X \]

\[ C = \left[ P_o - \frac{X}{1 + r_f} \right] \]
The price of a stock today is $50. The risk-free rate is 0. One of the following scenarios occurs:

<table>
<thead>
<tr>
<th></th>
<th>Price in 6 months</th>
<th>Price in 1 year</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>52</td>
<td>55</td>
</tr>
<tr>
<td>B</td>
<td>52</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>48</td>
<td>50</td>
</tr>
<tr>
<td>D</td>
<td>48</td>
<td>46</td>
</tr>
</tbody>
</table>

Price a call option at $52 in 1 Year
Assume that we can break the year into three intervals

For each interval the stock could increase by 5% or decrease by 3%

Assume the stock is initially selling at 100
Expanding to Consider Three Intervals Continued
Possible Outcomes with Three Intervals

<table>
<thead>
<tr>
<th>Event</th>
<th>Probability</th>
<th>Final Stock Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 up</td>
<td>1/8</td>
<td>$100 \times (1.05)^3$ = 115.76</td>
</tr>
<tr>
<td>2 up 1 down</td>
<td>3/8</td>
<td>$100 \times (1.05)^2 \times (0.97)$ = 106.94</td>
</tr>
<tr>
<td>1 up 2 down</td>
<td>3/8</td>
<td>$100 \times (1.05) \times (0.97)^2$ = 98.79</td>
</tr>
<tr>
<td>3 down</td>
<td>1/8</td>
<td>$100 \times (0.97)^3$ = 91.27</td>
</tr>
</tbody>
</table>
Binomial Option Pricing

- Note that binomial option pricing makes no assumption on preferences
  - Agents can be risk-averse or risk-neutral
Black-Scholes Option Valuation

\[ C_o = S_o N(d_1) - X e^{-rT} N(d_2) \]

\[ d_1 = \left[ \ln \left( \frac{S_o}{X} \right) + (r + \frac{\sigma^2}{2})T \right] / (\sigma T^{1/2}) \]

\[ d_2 = d_1 - (\sigma T^{1/2}) \]

where

\( C_o \) = Current call option value

\( S_o \) = Current stock price

\( N(d) \) = probability that a random draw from a standard normal distribution will be less than \( d \)
$X =$ Exercise price  
$e = 2.71828$, the base of the natural log  
$r =$ Risk-free interest rate (annualizes continuously compounded with the same maturity as the option)  
$T =$ time to maturity of the option in years  
$ln =$ Natural log function  
$\sigma =$ Standard deviation of annualized cont. compounded rate of return on the stock
Call Option Example

\[ S_o = 100 \quad X = 95 \]
\[ r = .10 \quad T = .25 \text{ (quarter)} \]
\[ \sigma = .50 \]
\[ d_1 = \left[ \ln\left( \frac{100}{95} \right) + \left( .10 + (.5^{2/2}) \right) \times 0.25 \right] / (.5 \times .25^{1/2}) \]
\[ = .43 \]
\[ d_2 = .43 - ((.5)( .25^{1/2})) \]
\[ = .18 \]
\[ N(0.43) = 0.6664 \]
\[ N(0.18) = 0.5714 \]
Call Option Value

\[ C_o = S_o N(d_1) - X e^{-rT} N(d_2) \]
\[ C_o = 100 \times 0.6664 - 95 \times e^{-0.10 \times 0.25} \times 0.5714 \]
\[ C_o = 13.70 \]

**Implied Volatility**

Using Black-Scholes and the actual price of the option, solve for volatility.

Is the implied volatility consistent with the stock volatility?
Implied Volatility of the S&P 500

[Diagram showing implied volatility from Jan-90 to Jan-08 with significant events marked: Gulf War, LTCM, 9/11, Iraq, Subprime Mortgages]
Implied Volatility of the S&P 500 Index as a Function of Exercise Price
Put Value Using Black-Scholes

\[ P = Xe^{-rT}[1-N(d_2)] - S_0[1-N(d_1)] \]

Using the sample call data
\[ S = 100 \quad r = .10 \quad X = 95 \quad g = .5 \quad T = .25 \]

\[ 95e^{-10 \times .25}(1-.5714)-100(1-.6664) = 6.35 \]
Put Option Valuation: Using Put-Call Parity

\[
C + Xe^{-rT} = S_o + P
\]

Using the example data
\[
C = 13.70 \quad X = 95 \quad P = 6.35 \quad S = 100
\]
\[
r = .10 \quad T = .25
\]
\[
C + Xe^{-rT} = 13.70 + 95 e^{-0.10 \times 0.25} = 106.35
\]
\[
S + P = 106.35
\]
Call Option Example – Varying stock price

\( S_o = \text{various} \quad X = 95 \)

\( r = .10 \quad T = .25 \) (quarter)

\( \sigma = .50 \)

\[ d_1 = \frac{\ln(100/95) + (.10 + (.5^{2/2})) \times 0.25}{(.5^{1/2})} \]

\[ d_1 = .43 \]

\[ d_2 = .43 - ((.5) \times (.25^{1/2})) \]

\[ d_2 = .18 \]

\( N(0.43) = 0.6664 \)

\( N(0.18) = 0.5714 \)
Call Option Example – Varying stock price

<table>
<thead>
<tr>
<th>Stock Price</th>
<th>Call Option Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>98</td>
<td>12.3900</td>
</tr>
<tr>
<td>99</td>
<td>13.0400</td>
</tr>
<tr>
<td>100</td>
<td>13.7000</td>
</tr>
<tr>
<td>101</td>
<td>14.3700</td>
</tr>
<tr>
<td>102</td>
<td>15.0600</td>
</tr>
</tbody>
</table>

Slope: 0.67
Call Option Value and Hedge Ratio

Slope is 0.67
Can Hedge Option with 0.67 shares
Hedging: Hedge ratio or delta

The number of stocks required to hedge against the price risk of holding one option

Call = \( N (d_1) \)

Put = \( N (d_1) - 1 \)

Option Elasticity

Percentage change in the option’s value given a 1% change in the value of the underlying stock
Black-Scholes Greeks

Delta: Sensitivity of call option price to stock price
Vega: Sensitivity of call option price to volatility
Theta: Minus sensitivity of call option price to time
Rho: Sensitivity of call option price to risk-free rate
Dividends and Black Scholes

- Original version of Black Scholes assumes no dividends
- Q. What if the dividend yield is d?
- A. Both call and put formulas go through with

\[
C_0 = S_0 e^{-dT} N(d_1) - X e^{-rT} N(d_2)
\]
\[
P_0 = X e^{-rT} [1 - N(d_2)] - S_0 e^{-dT} [1 - N(d_1)]
\]
\[
d_1 = \left[ \ln\left(\frac{S_0}{X}\right) + (r - d + \sigma^2 / 2)T \right] / (\sigma T^{1/2})
\]
\[
d_2 = d_1 - (\sigma T^{1/2})
\]
Black Scholes and American Options

- Black Scholes applies to European Options
- Since right of early exercise is disposable, it gives a *lower bound* on the price of American options.
- More precise adjustments of Black Scholes for the possibility of early exercise are available
  - Barone-Adesi and Whaley (Journal of Finance; 1987)
- Note: Options traded on exchanges are mainly American; OTC options are mainly European.
Futures and Forwards

- Forward - an agreement calling for a future delivery of an asset at an agreed-upon price
- Futures - similar to forward
- How futures are different from forwards: futures are
  - Standardized and traded on exchanges
  - Secondary trading - liquidity
  - Clearinghouse warrants performance
Key Terms for Futures Contracts

- Futures price - agreed-upon price at maturity
- Long position - agree to purchase
- Short position - agree to sell
- Profits on positions at maturity
  Long = spot minus original futures price
  Short = original futures price minus spot
Profits to Buyers and Sellers of Futures and Option Contracts

A. Long futures profit = $P_T - F_0$

B. Short futures profit = $F_0 - P_T$

C. Buy a call option
Sample of Future Contracts

<table>
<thead>
<tr>
<th>Foreign Currencies</th>
<th>Agricultural and Energy</th>
<th>Metals and Energy</th>
<th>Interest Rate Futures</th>
<th>Equity Indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro</td>
<td>Soybean</td>
<td>Copper</td>
<td>Eurodollars</td>
<td>S&amp;P 500</td>
</tr>
<tr>
<td>Pound</td>
<td>Corn</td>
<td>Gold</td>
<td>Federal Funds</td>
<td>Nasdaq</td>
</tr>
<tr>
<td>Yen</td>
<td>Coffee</td>
<td>Oil</td>
<td>Treasury Bonds</td>
<td>FTSE Index</td>
</tr>
</tbody>
</table>
Trading Mechanics

- Clearinghouse - acts as a party to all buyers and sellers
  - Obligated to deliver or supply delivery
- Closing out positions
  - Reversing the trade
  - Take or make delivery
  - Most trades are reversed and do not involve actual delivery
- Open Interest
Panel A, Trading without a Clearinghouse.
Panel B, Trading with a Clearinghouse
Margin and Trading Arrangements

Initial Margin - funds deposited to provide capital to absorb losses

Marking to Market - each day the profits or losses from the new futures price are reflected in the account

Maintenance or variation margin - an established value below which a trader’s margin may not fall
Margin and Trading Arrangements
Continued

Margin call - when the maintenance margin is reached, broker will ask for additional margin funds

Convergence of Price - as maturity approaches the spot and futures price converge

Delivery - Actual commodity of a certain grade with a delivery location or for some contracts cash settlement

Cash Settlement – some contracts are settled in cash rather than delivery of the underlying assets
Trading Strategies

- Speculation -
  - short - believe price will fall
  - long - believe price will rise

- Hedging -
  - long hedge - protecting against a rise in price
  - short hedge - protecting against a fall in price
Hedging Revenues Using Futures,  
(Futures Price = $97.15)

Hedged revenues are constant at $97.15 per barrel, equal to the futures price.

Sales revenue increases with oil price.

Profit on short futures position falls with oil price.

Sales Revenue per Barrel, Futures Profits per Barrel, Total Proceeds.
Basis

- Basis - the difference between the futures price and the spot price (spot=price today)
- over time the basis converges
Convergence of cash and futures price
Spot-futures parity theorem - two ways to acquire an asset for some date in the future

- Purchase it now and store it: assume this is free
- Take a long position in futures
- These two strategies must be equivalent
Example

- S&P 500 fund that has a current value equal to the index of $1,500
- Assume dividends of $25 will be paid on the index at the end of the year
- Interest rate is 5%
- If I borrow to buy the stock today, my cost in one year is $1,500*1.05-$25=$1,550
- Therefore the futures price should be $1,550
General Spot-Futures Parity

\[ F_0 = S_0 (1 + rf)^T - D \]

- If there are no dividends, then the futures price should lie above the spot price
General Spot-Futures Parity

\[ F_1 = F_0 (1 + r_f)^T - D \]

- \( F_1 \) and \( F_0 \) are prices on two futures contracts maturing \( T \) periods apart
Arbitrage Possibilities

- If spot-futures parity is not observed, then arbitrage is possible
- If the futures price is too high, short the futures and acquire the stock by borrowing the money at the risk-free rate
- If the futures price is too low, go long futures, short the stock and invest the proceeds at the risk-free rate
Gold Futures Prices

![Graph showing the price of gold futures in January 2008 for different delivery dates: February, June, and December. The graph indicates a trend of increasing prices over the month, with the December delivery showing the highest price.](image-url)
Storage costs

- Relationship between futures, spot price and interest rates applies in the absence of storage costs
- Storing oil is possible, but expensive
- If the futures curve slopes down, this is called normal Backwardation
- If it slopes up, it is called Contango
What determines the slope of the futures curve?

- Expectations
- Storage Costs
- Hedging demand
  - Oldest story: Producers sell into futures markets to lock in prices creating normal backwardation
- Portfolio theory (e.g. correlation with market returns)
Interest Rate Futures

- Two main interest rate futures
  - Treasury bond
  - Eurodollar
Eurodollar Futures

- The Eurodollar futures contract is the most widely traded short-term interest rate futures.
- It is based upon a 90-day $1 million Eurodollar time deposit.
- It is settled in cash.
- Contracts mature in March, June, Sep, December
- At expiration, the settlement price is 100-LIBOR
- At expiration if the settlement price is S and the futures price is F, in basis points, then the short side pays the long side $25*(S-F) per contract.
Eurodollar Futures Example

- In February you buy a March Eurodollar futures contract. The quoted futures price at the time you enter into the contract is 94.86.

- Q. If the 90-day LIBOR rate at the end of March turns out to be 4.14% p.a., what is the payoff on your futures contract?

- A. Settlement price: 95.86. So you receive a payoff of $25*100=$2,500.
Hedging with a Eurodollar Futures Contract

• Suppose a firm knows that it must borrow $1 million for three months in the future.
• **Sell** a Eurodollar futures contract.
• Suppose the current futures rate is 94.86. This implies a LIBOR rate of 5.14%.
• Now consider three scenarios:

<table>
<thead>
<tr>
<th>Borrowing Rate</th>
<th>5.14</th>
<th>6.14</th>
<th>4.14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eurodollar Payoff</td>
<td>0</td>
<td>$2,500</td>
<td>-$2,500</td>
</tr>
<tr>
<td>Borrowing Cost</td>
<td>$12,850</td>
<td>$15,350</td>
<td>$10,350</td>
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<tr>
<td>Total Expense</td>
<td>$12,850</td>
<td>$12,850</td>
<td>$12,850</td>
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</table>
Foreign Exchange

• 1 Unit of currency A = X units of currency B
• Quoting conventions differ by currency pair
• Euro-Dollar: 1 Euro = X Dollars
• Dollar- Yen: 1 Dollar = X Yen
• Enormous liquid market
Foreign Currency Futures and Forward Rates

- Currency futures are an agreement to buy a foreign currency at a point in the future
  - Currency forwards are more common
- Covered Interest Parity

\[
S_t (1 + R_t)^T = F_t (1 + R_t^*)^T
\]

where

\( S_t \): Exchange Rate (US$/Foreign Currency)
\( F_t \): Forward Exchange Rate
\( R_t \): $ Interest Rate
\( R_t^* \): Foreign Interest Rate
Covered Interest Parity Example 1

- Suppose that the exchange rate today is 1 Euro=$1.50
- Interest rate is 0.25% in the US
- Interest rate is 1% in the euro zone
- Covered interest parity says that the one year forward rate must satisfy
  \[ 1.5 \times 1.0025 = F_t \times 1.01 \]
- The forward rate is 1 Euro=$1.489
Covered Interest Parity Example 1

- In the example on the last slide, suppose instead that the forward rate was 1 Euro=$1.50.
- What could I do?
  Borrow $1, Convert it to Euro.
  Receive: Euro 0.666.
  Invest it at 1% interest. In one year I have Euro 0.6733
  Convert this back to dollars to receive $1.01
  Pay off the loan for $1.0025
  Profit of 0.75 cents
Covered Interest Parity Example 2

- Suppose that the spot exchange rate in 2005 was $1=100 Yen.
- Interest rate was 0 in Japan
- One year forward rate was $1=95 Yen.
- Q. What must US$ interest rate have been?

Spot: 1 Yen=$0.01. Forward 1 Yen=$0.0105

$$0.01 \times (1+R_t) = 0.0105$$

- US interest rate is 5 percent.
Foreign Currency Swaps

- Foreign Exchange Swaps
  - Spot sales of a currency combined with a forward repurchase of the currency.
  - Make up sizeable share of forex trading
Covered Interest Parity

- A violation of covered interest parity is an arbitrage opportunity
- Violations used to be rare and small
- Since 2008, there have been violations
- Banks need to use swaps market to borrow dollars
  - Cross-currency basis: negative means more expensive to borrow dollars via swaps than to do so directly
Hedging with Foreign Currency Futures

- A firm's profits go down by $200,000 for every 5 cent rise in the dollar/pound exchange rate.
- Each pound contract calls for the delivery of 62,500 pounds.
- Q. How should this firm hedge exchange rate risk?
- A. Enters a long position on pound futures.
  - 1 contract goes up $0.05 \times 62,500$ when the pound appreciates by 5 cents.
  - The firm needs $\frac{200,000}{0.05 \times 62,500} = 64$ contracts.
Uncovered interest parity

• Suppose I borrow dollars today, invest it in a foreign currency and plan to buy it back tomorrow.

• No forward/futures contract

• Profit will be

\[ S_{t+T} (1 + R^*_t)^T - S_t (1 + R_t)^T \]

• But there is no arbitrage strategy
Uncovered interest parity

- Uncovered interest parity says that

\[ E_t(S_{t+T})(1 + R^*_t)^T = S_t(1 + R_t)^T \]

- For example, if exchange rate today is AUD 1=$0.80
- One year interest rate is 0.25 percent in US
- One year interest rate is 4.25 percent in Australia
- Expected exchange rate in one year is AUD=$0.769
- Currency with the higher interest rate is expected to depreciate
Uncovered interest parity
Empirical Evidence

- Evidence *for* covered interest parity fairly strong
- Evidence goes *against* uncovered interest parity
- If anything, the currency with the *higher* interest rate tends to *appreciate*
- Motivates the “carry trade”
  - Borrow in currency with low interest rate (*funding* currency)
  - Convert to the high interest currency and invest
  - Do NOT hedge the exchange rate risk
  - Convert back
- From 1990 till crisis, yen was the natural funding currency
  - From 2008 till now dollar also a natural funding currency
JPY-AUD
Skewness and interest differentials
Performance Attribution

- A common attribution system decomposes performance into components:
  1. Allocation choices *across* broad asset classes.
  2. Choices *within* each asset class.
Set up a ‘Benchmark’ portfolio:
- Select a benchmark index portfolio for each asset class.
Performance Attribution

- Superior performance is achieved by:
  - Picking right asset classes
  - Selection within asset classes
Formulas for Attribution

\[ r_B = \sum_{i=1}^{n} w_{Bi} r_{Bi} \quad \& \quad r_{p} = \sum_{i=1}^{n} w_{pi} r_{pi} \]

\[ r_{p} - r_{B} = \sum_{i=1}^{n} w_{pi} r_{pi} - \sum_{i=1}^{n} w_{Bi} r_{Bi} = \]

\[ \sum_{i=1}^{n} (w_{pi} r_{pi} - w_{Bi} r_{Bi}) = \]

\[ \sum_{i=1}^{n} w_{pi} (r_{pi} - r_{Bi}) + \sum_{i=1}^{n} (w_{pi} - w_{Bi}) r_{Bi} \]

where B is the benchmark portfolio and p is the managed portfolio.
Performance Attribution

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<thead>
<tr>
<th></th>
<th>Portfolio</th>
<th>Benchmark</th>
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<tr>
<td>TOTAL</td>
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<td>Equity</td>
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<tr>
<td>Cash</td>
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Excess Return is 1.37
### Contribution of Asset Allocation

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## Contribution of Selection within Markets

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<tr>
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<th>Portfolio Weight</th>
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<td>0</td>
<td>0.23</td>
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<tr>
<td>TOTAL</td>
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