1. (15 points) 28 years ago, Joseph purchased a bond with 10% per annum coupons, 30 years to maturity and a face value of $100. Assume that the yield at the time of purchase was 10% and the yield today is 3% (with annual compounding). Also assume that coupon payments occur only once a year. So the bond pays a total of 30 coupons of $10 each.
   (a) At what price did Joseph purchase the bond 28 years ago?
   (b) What is the price of the bond today, assuming that the 28th coupon has just been paid?

2. (10 points) Suppose that the one year yield is 5% and that investors know that the one-year yield will be 7% in one years’ time (with annual compounding). What is the price of a 2-year maturity bond with a 10% coupon rate paid annually? (Face value = $100)

3. (15 points)
   (a) There is some evidence that the default rates on mortgages issued to homeowners with FICO scores just above 620 is higher than the default rates on mortgages issued to homeowners with FICO scores just below 620. What explanation has been proposed for this phenomenon?
   (b) Which of the following is the closest approximation to the current size of the Federal Reserve Balance Sheet?
      A. $4.5 billion.
      B. $900 billion
      C. $4.5 trillion
      D. $90 trillion.
   (c) Name a major central bank that has purchased Exchange Traded Funds as part of a quantitative easing program.

4. (10 points) Mary has entered into a plain vanilla ten-year interest rate swap contract where she pays fixed and receive floating (three-month LIBOR). The notional underlying is $10 million and there are four payment dates per year. Today is a payment date and the LIBOR fixing is 35 basis points. The fixed rate is 1.95 percent. Does Mary make or receive a payment, and what is the dollar payment that she make or receive?

5. (10 points) What is the duration of a bond that sells at par, has a coupon rate of 8% (paid annually; with annual compounding) and a remaining time to maturity of 3 years.

6. (20 points) Consider a bond selling at par with modified duration of 10.6 years and convexity of 210. Yields decrease by 2%.
   (a) Using the simple duration approximation, what should the percentage price change be?
   (b) Using the duration-with-convexity approximation, what should the percentage price change be?
7. (20 points). You observe the following prices for European options on a non-dividend paying stock which expire on December 31, 2014.

<table>
<thead>
<tr>
<th>Strike</th>
<th>Call</th>
<th>Put</th>
</tr>
</thead>
<tbody>
<tr>
<td>$60</td>
<td>$9.50</td>
<td>$7.50</td>
</tr>
<tr>
<td>$65</td>
<td></td>
<td>$8.50</td>
</tr>
<tr>
<td>$70</td>
<td></td>
<td>$11</td>
</tr>
<tr>
<td>$75</td>
<td>$1</td>
<td>$14</td>
</tr>
</tbody>
</table>

Assume that interest rates are zero.
(a) Using put-call parity, what must the price of the stock be today?
(b) Using put-call parity, find the prices of the call options at strikes of $65 and $70.
(c) Assume that the price of the stock on December 31, 2014 is $60, $65, $70, $75 or $80 and that options traders are risk neutral. Find the probabilities of these five possible outcomes.
Solutions

1. (a) Since the yield is equal to the coupon, no calculations are needed. Joseph bought the bond for $100.

\[
\frac{10}{1.03} + \frac{110}{1.03^2} = 113.39
\]

(b) The two-year yield will be \((1.05*1.07)^{1/2} - 1\) which is 6%. The price of the two-year coupon bearing bond is

\[
\frac{10}{1.05} + \frac{110}{1.06^2} = 107.42.
\]

2. (a) The fact that it was much easier to securitize mortgages with a FICO score just above 620. This made originators more careless, because they knew that they were not going to have to hold onto the mortgages.

(b) C.

(c) Bank of Japan.

4. Mary pays \(0.25*(1.95 - 0.35)*10,000,000 = 40,000\).

5. The price of the bond is $100 and the yield is 8%. The duration is

\[
\left(\frac{1}{100^{1.08}}*1\right) + \left(\frac{1}{100^{1.08^2}}*2\right) + \left(\frac{1}{100^{1.08^3}}*3\right) = 2.78 \text{ years.}
\]

6. (a) The price should rise by \(10.6*0.02=0.212\) or 21.2%.

(b) The price should rise by \(10.6*0.02+(\frac{1}{2}*210*0.02*0.02)=0.254\) or 25.4%.

7. (a) Taking the options at a strike price of $60, by put-call parity, \(9.50 + 60 = 7.50 + S_0\), so the stock price \(S_0\) is $62. You would get the same answer from the options with a strike price of $75.

(b) By put-call parity, the call option with a strike of $65 should satisfy \(C + 65 = 8.5 + 62\), so the call option price \(C\) is $5.50. Similarly the price of the call option with a strike of $70 should be $3.

(c) Let \(P60, P65, P70, P75\) and \(P80\) denote the 5 probabilities. We know that \(5P80 = 1\) and so \(P80 = 0.2\).

From a sequence of bullish spread positions, we know that \(5(P75 + P80) = 3 - 1 \Rightarrow P75 + P80 = 0.4\) and so \(P75 = 0.2\) and that \(5(P70 + P75 + P80) = 5.50 - 3 \Rightarrow P70 + P75 + P80 = 0.5\) and so \(P70 = 0.1\) and that \(5(P65 + P70 + P75 + P80) = 9.50 - 5.50 \Rightarrow P65 + P70 + P75 + P80 = 0.8\) and so \(P65 = 0.3\) and \(P60 = 0.2\).
Grading Rubric.

1. (a) 5 points and (b) 10 points. No partial credit.

2. 5 points for getting that the two-year yield is 6 percent. No points off for getting this as \((5+7)/2\), as opposed to the more correct geometric average. 10 points for getting the price of the coupon bond (given whatever two-year yield was calculated). 3 points off for an arithmetic error at this stage.

3. 5 points per sub-question. No partial credit. On part (a) must make reference to securitization, mortgage backed securities, or securities that are originated to distribute.

4. 10 points for correct answer. 3 points off for arithmetic error. 5 points for 
\[(1.95 - 0.35) \times 10,000,000 = 160,000\]
which fails to recognize that it is a quarterly payment. 5 points for getting the direction of payment wrong.

5. No points if fail to realize that par means that price is $100. 10 points for correct answer. 3 points off for arithmetic error.

6. (a) 10 points. 5 points for saying 0.212%. Full credit for 0.212. 3 points off for arithmetic error. 
(b) 10 points. 5 points for saying 0.254%. Full credit for 0.254. 3 points off for arithmetic error.

7. (a) 5 points. 2 points off for correct formula with arithmetic error. 
(b) 2.5 points per options price. No partial credit. 
(c) 10 points for all probabilities correct. 7 points for right formula but with arithmetic error. 6 points for being able to use butterflies to price the probabilities of $65 and $70 but not knowing how to handle the tails.