Formula Sheet for Econ 367 Midterm 1

- **APR** = \( n \times \left\{ [1 + EAR]^{1/n} - 1 \right\} \)

- **Compounding.** \( V(0) \) today is worth \( V(n) = V(0) \times \exp(n) \) in \( n \) periods. Hence \( r = \frac{\ln(V(n)) - \ln(V(0))}{n} \)

- **Present Value:** \( P = \frac{C(1)}{1+r} + \frac{C(2)\times (1+r)}{(1+r)^2} \ldots + \frac{C(T)\times (1+r)^T}{(1+r)^T} \)

- Suppose that \( X_1, X_2, \ldots X_n \) are random variables and \( k_1, k_2, \ldots k_n \) are constants. Then \( E(\Sigma k_jX_j) = \Sigma k_jE(X_j) \) and \( Var(\Sigma k_jX_j) = \Sigma \Sigma k_jk_j \text{Cov}(X_i, X_j) \)

- **Minimum Variance Portfolio weights (2 assets):** \( w_D = \frac{\sigma_E^2 - \text{Cov}(R_D, R_E)}{\sigma_E^2 + \sigma_D^2 - 2\text{Cov}(R_D, R_E)} \) and \( w_E = 1 - w_D \)

- **Maximum Sharpe Ratio Portfolio weights (2 assets):** \( w_D = \frac{E(R_D)\sigma_E^2 - E(R_E)\text{Cov}(R_D, R_E)}{E(R_D)^2 + E(R_E)^2 - [E(R_D) + E(R_E)]\text{Cov}(R_D, R_E)} \)

and \( w_E = 1 - w_D \) where \( R_D \) and \( R_E \) are excess returns over the riskfree rate

- **Optimal complete portfolio.** If an investor has a utility function \( E(r) - \frac{A}{2\sigma^2} \) then the weight in the tangent portfolio will be \( \frac{E(r_p) - r_f}{A\sigma_p^2} \) where \( r_p \) is the return on the tangent portfolio and \( \sigma_p^2 \) is its variance.

- **Zero beta CAPM.** In the zero-beta CAPM (no risk-free asset), with two possible assets, if the market portfolio has a weight of \( w \) in asset 1, then the zero-beta portfolio will have a weight of \( v \) in asset 1 where:

\[
v = \frac{(w-1)\text{Var}(r_2) - w\text{Cov}(r_1, r_2)}{w\text{Var}(r_1) - (1-w)\text{Var}(r_2) + (1-2w)\text{Cov}(r_1, r_2)}
\]

- **Factor Model**

\[
r_i = E(r_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \ldots \beta_{ik}F_k + e_i
\]

\[
E(r_i) = r_f + \lambda_i\beta_{i1} + \lambda_2\beta_{i2} + \ldots + \lambda_k\beta_{ik}
\]