Lecture 3: Leverage cycle, Flight to Collateral and Contagion.

John Geanakoplos (Yale)

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Outline

1. Introduction
2. Model
3. The Leverage Cycle
4. Multiple Leverage Cycles
5. Agent-Based Model
6. Summary
Goal of Lecture 2

We will study the dynamic and cross sectional consequences of the theoretical results in Lecture 2.

Dynamic Properties: Leverage Cycle.


We will finish with a discussion of Agent-Based Models of Leverage Cycles.
We extend the model presented in Lecture 2 to a multiple-period model.
Finite tree $s \in S$ including a root $s = 0$, and terminal nodes $s \in S_T$.

We denote the time of $s$ as $t(s)$, so $t(0) = 0$.

$s \neq 0$ has a unique immediate predecessor $s^*$, and $s \in S \setminus S_T$ has a set $S(s)$ of immediate successors.

$L_s$ commodities in $s \in S$.

Let $p_s \in R_{L_s}^+$ the vector of commodity prices in each state $s \in S$. 

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Intended for a summary placeholder.
Agents

Each investor $h \in H$ is characterized by Bernoulli utilities, $u^h_s$, $s \in S$, a discount factor, $\beta_h$, and subjective probabilities, $\gamma^h_s$, $s \in S_T$.

$u^h_s : R^L_s \rightarrow R$, are differentiable, concave, and weakly monotonic (more of every good in any state strictly improves utility).

Expected utility to agent $h$ is

$$U^h = u^h(c_0) + \sum_{s \in S \setminus 0} \beta^t(s) \bar{\gamma}^h_s u^h(c_s).$$

where $\bar{\gamma}^h_s$ is the probability of reaching $s$ from 0.
Agent Endowments

Investor $h$’s endowment of the commodities is denoted by $e^h_s \in R^L_s$ in each state $s \in S$. 
For each $s \in S$ and $h \in H$, let $Z^h_s \subset \mathbb{R}^{L_s}$ denote the set of feasible intra-period production for agent $h$.

Commodities can enter as inputs and outputs of the intra-period production process; inputs appear as negative components $z_l < 0$ of $z \in Z^h_s$, and outputs as positive components $z_l > 0$ of $z \in Z^h_s$.

We assume that $Z^h_s$ is convex, compact and that $0 \in Z^h_s$. 
For each $h \in H, s \in S_T$, let $F^h_s : \mathbb{R}^{L_s^*}_+ \rightarrow \mathbb{R}^{L_s}_+$ be a linear inter-period production function connecting a vector of commodities $x_s^*$ at state $s^*$ with the vector of commodities $F^h_s(x_s^*)$ it becomes in each state $s$. 
Production

Production enables our model to include many different kinds of commodities. Commodities could either be:

- perishable consumption goods (like food),
- durable consumption goods (like houses),
- they could represent assets (like Lucas trees) that pay dividends.
Financial Contracts and Collateral

The heart of our analysis involves contracts and collateral.

In Arrow Debreu the question of why agents honor their promises is ignored.

We explicitly incorporate in our model repayment enforceability problems.

Collateral is the only enforcement mechanism: agents cannot be coerced into honoring their promises except by seizing collateral agreed upon by contract in advance.
But exclude *cash flow problems*.

But there is no doubt what the collateral will pay, conditional on the future state of nature.
Financial Contracts and Collateral

A financial contract is an ordered pair

\[ j = (j_s, c_{sj}) \]

**Promise:** \( j_s \in R^{L_s^+} \) commodities in each final state \( s \in S\backslash\{0\} \)

**Collateral:** backed by collateral \( c_{sj} \in R^{L_s^*}_{+} \). This allows for non-contingent promises of different sizes, as well as contingent promises.
We assume that all contracts are short term one period contracts.

Each contract $j$ has an issue state $s(j)$ and $j_{s'} \neq 0$ only if $s' \in S(s(j))$. 
Price of contract $j \in J$ is $\pi_{sj}$.

Let $\theta_{sj}^h$ be the number of contracts $j$ held by $h$ at time $s$. Let $\varphi_{sj}^h$ be the number of contracts $j$ issued by $h$ up to time $s$.

An investor can borrow $|\varphi_{sj}^h|\pi_{sj}$ today by selling $\varphi_{sj}^h$ and lend $\theta_{sj}^h\pi_{sj}$ by buying $\theta_{sj}^h$. 
No Cash Flow Problems

We wish to exclude cash flow problems (like adverse selection or moral hazard). For that:

- we restrict the sale of each contract $j$ to a set $H(j) \subset H$ of traders with the same durability functions, $F^h(c_{s^*j}) = F^{h'}(c_{s^*j})$ if $h, h' \in H(j)$.
- we suppose that if $c_{sj}$ is the collateral for some contract $j$, then there is a “large” contract $j'$ with $c_{sj'} = c_{sj}$ and $H(j') = H(j)$ and $j'_s \geq F^{H(j)}_s(c_{sj})$ for all $s$. 
Financial Contract Delivery

We assumed that the maximum borrowers can lose is their collateral if they do not honor their promise.

Actual delivery any one-period contract $j$ in states $s \in S(s(j))$:

$$\delta_s(j) = \min\{p_s \cdot j_s, p_s \cdot F^H_j(c_{s^*j})\}$$

The value of the collateral in each future state does not depend on the size of the promise, or on what other choices the seller $h \in H(j)$ makes, or on who owns the asset at the very end.
Budget Set

\[ B^h(p, \pi) = \{(z, x, \theta, \varphi) \in R^{S \times L} \times R^S_+ \times R^S_+ \times R^{SJ}_+ : \forall s \in S \}
\]

\[ p_s \cdot (x_s - e^h_s - z_s - F^h_s(x_s^*)) + \sum_{j \in J} (\theta_{sj} - \varphi_{sj}) \pi_{sj} \leq \]

\[ \leq \sum_{j \in J} (\theta^*_{s*j} - \varphi^*_{s*j}) \min \{p_s \cdot j_s, p_s \cdot F^{H(j)}_s(c^*_{s*j})\} \]

\[ z_s \in Z^h_s, \]

\[ \varphi_{sj} > 0 \text{ only if } h \in H(j) \]

\[ \sum_{j \in J} \varphi_{sj} c_{sj} \leq x_s \} . \]
Collateral Equilibrium

\[
((p, \pi), (z^h, x^h, \theta^h, \varphi^h)_{h \in H}) \in R^{S \times L} \times R^{SJ} \times (R^{S \times L} \times R^{S \times L} \times R^{SJ} \times R^{SJ})^H
\]
such that

\[
\sum_{h \in H}(x_s^h - e_s^h - z_s^h - F_s^h(x_{s^*}^h)) = 0, \forall s \in S.
\]

\[
\sum_{h \in H}(\theta_{sj}^h - \varphi_{sj}^h) = 0, \forall s \in S, j \in J.
\]

\[
(z^h, x^h, \theta^h, \varphi^h) \in B^h(p, \pi), \forall h
\]

\[
(z, x, \theta, \varphi) \in B^h(p, \pi) \Rightarrow U^h(x) \leq U^h(x^h), \forall h.
\]
We first describe the properties of the leverage cycle in an example that extends Example 2 in Lecture 2.
Time and Assets

\( T = 2, \) and \( S = \{0, U, D, UU, UD, DU, DD\} \).

There is one financial asset \( Y \) which pays dividends only in the final period:
\( d_{UU} = d_{UD} = 1 \) and \( d_{DU} = 1 \) and \( d_{DD} = .2 \).

Good news reduces uncertainty about the payoff value and bad news increases uncertainty (and down risk) about the payoff value of the asset.
Time and Assets

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Time and Assets

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Good news reduces uncertainty about the payoff value and bad news increases uncertainty (and down risk) about the payoff value of the asset.
Goods and Production

$L_s = \{c_s, Y\}, s = 0, U, D. \quad L_s = \{c_s\}, s = UU, UD, DU, DD.$

$F_s(c_0, Y) = d_s Y, s = UU, UD, DU, DD$
Agents

Two types of agents $H = \{O, P\}$ with logarithmic utilities who do not discount the future.

Agents differ in their beliefs and wealth:

- Beliefs: $\gamma^O_{sU} = .9$ and $\gamma^P_{sU} = .4$ for all $s \in S \setminus S_T$.

- Endowments: $e^h_{Y0} = 1$, $h = O, P,$ $e^O_{c0} = e^O_{cD} = 8.5$, $e^O_{cs} = 10$, $s \neq 0$, $D$ and $e^P_{cs} = 100$, $\forall s$. 
Agents

The diagram illustrates the agents in the context of the Leverage Cycle model, showing the probabilities of different outcomes. The model considers two primary states: Up (U) and Down (D), with transition probabilities as follows:

- From 0 to U, the probability is 0.4.
- From 0 to D, the probability is 0.6.
- From U to U, the probability is 0.9.
- From U to D, the probability is 0.1.
- From D to U, the probability is 0.1.
- From D to D, the probability is 0.6.

These probabilities are denoted as follows:
- $d_{UU} = 1$
- $d_{DU} = 1$
- $d_{DD} = 0.2$
Agents

\[ e_{c0} = 8.5 \]
\[ e_{Yo} = 1 \]

\[ e_{cD} = 8.5 \]
\[ e_{cD} = 100 \]

\[ e_{cDD} = 10 \]
\[ e_{cDD} = 100 \]

\[ (d_{DU} = 1) \]
\[ (d_{DD} = 0.2) \]

\[ e_{cUU} = 10 \]
\[ e_{cUU} = 100 \]

\[ (d_{UU} = 1) \]

\[ e_{cUD} = 10 \]
\[ e_{cUD} = 100 \]

\[ (d_{DU} = 1) \]
## Equilibrium

### Equilibrium Leverage Cycle.

<table>
<thead>
<tr>
<th>States</th>
<th>$s = 0$</th>
<th>$s = U$</th>
<th>$s = D$</th>
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<tr>
<td>$p$</td>
<td>0.909</td>
<td>0.982</td>
<td>0.670</td>
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<tr>
<td>$j^*$</td>
<td>0.670</td>
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<tr>
<td>$\pi j^*$</td>
<td>0.664</td>
<td>0.201</td>
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### Asset Holdings

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<tr>
<td>Pessimists</td>
<td>0</td>
<td>1.591</td>
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### Debt Contract Trades

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<th>Pessimists</th>
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</thead>
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<tr>
<td>Optimists</td>
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<td>2</td>
</tr>
<tr>
<td>Pessimists</td>
<td>−2</td>
<td>−2</td>
</tr>
</tbody>
</table>
Equilibrium

\[ p_U = 0.98 \]
\[ c^0_U = 10.22 \]
\[ c^p_U = 99.78 \]

\[ p_D = 0.67 \]
\[ c^0_D = 7.56 \]
\[ c^p_D = 100.94 \]

\[ c^0_0 = 8.92 \]
\[ c^p_0 = 99.58 \]

\[ c^0_D = 7.56 \]
\[ c^p_D = 100.94 \]
\[ c^0_U = 10.22 \]
\[ c^p_U = 101.59 \]
\[ c^0_{UU} = 10.41 \]
\[ c^p_{UU} = 101.59 \]
\[ c^0_{UD} = 10.41 \]
\[ c^p_{UD} = 101.59 \]
\[ c^0_{DU} = 11.60 \]
\[ c^p_{DU} = 100.40 \]
\[ c^0_{DD} = 10.00 \]
\[ c^p_{DD} = 100.40 \]

\[ d_{DU} = 1 \]
\[ d_{DD} = 0.2 \]
Ebullient Times

When volatility and down risk is low, as at $s = 0$, the existing scarce collateral can support large amounts of borrowing to buy assets that are acceptable collateral.

A bubble can emerge in which the prices of the assets that can be used as collateral rise to levels far above their “Arrow-Debreu” Pareto efficient levels, even though all agents are rational. In this example, leverage at time 0 is almost 4 to 1 ($LTV = .73$), and the asset price at time 0 is .91. In Arrow-Debreu equilibrium, the asset price would only be .71.
On top of all that, the optimists are willing to pay a collateral value of .06 above and beyond the asset payoff value of .85 to them, because holding it enables them to borrow more money.

The combination of high prices, low volatility and low down risk creates an illusion of prosperity. But in fact the seeds of collapse are growing as the assets get more and more concentrated in the hands of the most enthusiastic and leveraged buyers.
Leverage cycle crashes always occur because of a coincidence of three factors:

1) **The bad news** itself lowers the prices.

2) **Wealth Effect:** The reduction in wealth of the leveraged buyers.

3) **Leverage Effect:** If the bad news also creates more uncertainty and increases down-risk, then credit markets tighten and leverage will be reduced.
The Equations

\[
\frac{\gamma^O_U \frac{1}{\bar{x}_U} [p_{UY} - p_{DY}]}{p_0 Y - \pi_0 p_{DY}} = \frac{1}{x_0^O}
\]

\[
\frac{\gamma^P_U \frac{1}{\bar{x}_U} + \gamma^O_D \frac{1}{\bar{x}_D}}{\pi_0 p_{DY}} = \frac{1}{x_0^O}
\]

\[
x^O_U + 2p_0 Y = (8.5 + p_0 Y) + 2\pi_0 p_{DY}
\]
The price of the asset in our example goes down from .91 at 0 to .67 at $D$ after bad news, a drop of 24 points.

At both 0 and $D$, the optimists are the only agents holding the asset, and in their view the expected payoff of the asset drops only 7 points, from .99 to .92, after the bad news.

So there is something much more important than the bad news which explains the drop in asset price. This is the downward path of the leverage cycle.
First notice that the optimists, though still buying all the asset in the economy, lose wealth after bad news. They are forced to consume less.

The higher volatility and increased down-risk at $D$ reduces the amount they can leverage. Leverage plummets from 4 at 0 to 1.4 at $D$ (equivalently, the LTV goes from .73 to .29). Quantitatively the most important factor reducing price.
The Crash: Increased Liquidity Wedge

As a result their liquidity wedge, which is a measure of how much they are willing to pay above the riskless interest rate, increases dramatically, from 0.1 to 0.52.

By the Discount Theorem, they then discount all future cash flows at a much higher rate than the riskless rate: the payoff value of the assets sinks all the way to .60.

Of course there is still a collateral value of .07. But despite the high liquidity wedge, the collateral value of the assets is limited by the small amount of borrowing they support.
The most visible sign of the crash is the margin call.

After the bad news at $D$ starts to reduce asset prices, optimists who want to roll over their loans need to put up more money to maintain the same $LTV$ on their loans.

They do that by reducing their consumption.

They then effectively get a second margin call because the new $LTV$ is much lower than before, forcing them to reduce consumption further.
Volatility and Down-Risk

The signature of the leverage cycle is rising asset prices in tandem with rising leverage, followed by falling asset prices and leverage.

But the underlying cause of the change in leverage is a change in volatility, or more generally, in some kind of bad tail uncertainty.

In our example, the volatility of the asset’s value is .126 at time 0, when leverage is almost 4, and increases to .394 at $D$, when leverage plummets to 1.4.
Credit Cycle vs Leverage Cycle

The Leverage Cycle is not the same as a Credit Cycle.

A Leverage Cycle is a feedback between asset prices and leverage, whereas a Credit Cycle is a feedback between asset prices and borrowing.

Of course a leverage cycle always produces a credit cycle. But the opposite is not true.
Credit Cycle vs Leverage Cycle

Classical macroeconomic models of financial frictions such as Kiyotaki and Moore (1997) produce credit cycles but not leverage cycles. (counter-cyclical leverage despite the fact that borrowing goes down after bad news).

The reason for the discrepancy is that to generate leverage cycles, uncertainty is needed, and a particular type of uncertainty: one in which bad news is associated with an increase in future volatility and down-risk.
Credit Cycle vs Leverage Cycle

Classical macroeconomic models of financial frictions such as Kiyotaki and Moore (1997) ignore huge swings in asset prices that come from procyclical leverage.

They ignore importance of changes in volatility and down risk.

They take for granted that collateral constraints restrict borrowing and so reduce asset prices and investment. But they miss collateral value. Collateral constraints can actually raise asset prices and investments and cause bubbles and overinvestment.
Agent Heterogeneity

The leverage cycle relies crucially on agent heterogeneity. In the example, heterogeneity was created by differences in beliefs. But there are many other sources of heterogeneity.

It is very important to understand that the connection between leverage and asset prices does not rely on differences in beliefs.

We can change endowments of consumption goods and assume that both agents have identical beliefs and reproduce all the trades and prices in our example.
We now describe the properties of the leverage cycle in an example that extends Example 3 in Lecture 2. (Geanakoplos (2003)).

The only difference is in how we define the agents.
Agents

Continuum of investors $h \in H = [0, 1]$.

Risk neutral. No discounting.

Expected utility to agent $h$ is

$$U^h(c_0, c_U, c_D, c_{UU}, c_{UD}, c_{DU}, c_{DD}) =$$

$$\gamma^h_U c_U + \gamma^h_D c_D + \gamma^h_U \gamma^h_{UU} c_{UU} + \gamma^h_U \gamma^h_{UD} c_{UD} + \gamma^h_D \gamma^h_{DU} c_{DU} + \gamma^h_D \gamma^h_{DD} c_{DD} =$$

$$= hc_U + (1 - h)c_D + h^2 c_{UU} + h(1 - h)c_{UD} + (1 - h)hc_{DU} + (1 - h)^2 c_{DD}$$
Each agent $h \in H$ has an endowment of the consumption good of $e_{cO}^h = 1$, $e_{cs}^h = 0$, $\forall s \neq 0$, and of the asset of $e_{YO}^h = 1$, $e_{Ys}^h = 0$, $\forall s \neq 0$.

The only source of heterogeneity is in subjective probabilities, $\gamma_{sU}^h = h$. 
Agents

\[ U \quad D \]
\[ (d_{UU} = 1) \quad (d_{DD} = 0.2) \]

\[ h \quad (1-h) \]
\[ e^{h_{c}} = 1 \quad e^{h_{y}} = 1 \]

\[ e^{h} c_{0} = 1 \quad e^{h} Y_{0} = 1 \]
## Equilibrium Leverage Cycle

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<tr>
<td>$p$</td>
<td>0.95</td>
<td>1</td>
<td>0.69</td>
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<tr>
<td>$j^*$</td>
<td>0.69</td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>$\pi j^*$</td>
<td>0.69</td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>$LTV$</td>
<td>0.73</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>$h$</td>
<td>0.87</td>
<td></td>
<td>0.61</td>
</tr>
</tbody>
</table>
Equilibrium

\[ p_U = 1 \]
\[ p_D = 0.69 \]
\[ p_0 = 0.95 \]
\[ h_0 = 0.87 \]
\[ h_D = 0.61 \]
\[ d_{UU} = 1 \]
\[ d_{DD} = 0.2 \]
The Crash

Leverage cycle crashes always occur because of a coincidence of three factors:

1) The **bad news** itself lowers the prices.

2) **Wealth Effect:** Bankruptcy of optimist buyers. The new marginal buyer is someone more pessimistic.

3) **Leverage Effect:** If the bad news also creates more uncertainty and down risk, then credit markets tighten and leverage will be reduced.
The Crash: Bad News

The price of the asset in our example goes down from .95 at 0 to .69 at \(D\) after bad news, a drop of 26 points.

The marginal buyer of the asset at time 0, \(h = 87\), thinks there is only a 1.69% chance of ultimate default, and when he gets to \(D\) after the first piece of bad news, he thinks there is a 13% chance for ultimate default. So his valuation goes from 

\[
.986 = .9831(1) + .0169(.2)
\]


to

\[
.87(1) + .13(.2) = .896,
\]

a drop of only 9 points. Yet the price falls 26 points. In fact nobody thinks it should have fallen 26 points.

So there is something much more important than the bad news which explains the drop in asset price. This is the downward path of the leverage cycle.
The Crash: A New Marginal Buyer and Plummeting LTV

First notice that all original optimistic buyers go bankrupt. The new marginal buyer $h = .61$ is more pessimistic, and hence the price lowers.

The higher volatility and increased down-risk at $D$ reduces the amount they can leverage. Leverage plummets from 3.6 at 0 to 1.4 at $D$ (equivalently, the LTV goes from .73 to .3). Quantitatively the most important factor reducing price.
Wealth Effect

The main difference between the two examples I and II we just saw is in how the wealth effect works.

In the first example, the marginal buyer stays the same. What moves is consumption, marginal utility of consumption, and hence the liquidity wedge.

In the second example, the marginal buyer is the one that changes, and hence price through his valuation.
The marginal buyer .87 at 0 thinks Y is worth at least .98, yet he only spends .95. Why?.

Because he would rather wait for the crash.

But there are not too many of these guys, because they have to be optimistic enough at D but not too optimistic at 0.
The Equations

\[
\frac{\gamma_{h_0}^U 1 + \gamma_{h_0}^D p_{DY} \gamma_{h_0}^U}{p_0 Y} = \frac{\gamma_{h_0}^U 1 + \gamma_{h_0}^D \gamma_{h_0}^U}{\gamma_{h_1}^U} = 1
\]

\[(1 - h_0)(1 + p_0 Y) + p_{DY} = p_0 Y\]

\[
\frac{\gamma_{h_1}^U 1 + \gamma_{h_0}^D (.2)}{p_0 Y} = \frac{1}{1}
\]

\[(h_0 - h_1)(1 + p_0 Y) + .2 = p_{DY}\]
Long Loans

What if two period loans offered at 0? Sounds great for borrowers, because they wouldn’t be forced to sell at D.

But they wouldn’t take them out.

Because could only borrow .2.
Let us consider again our first example Leverage Cycle I presented above.

We now show that regulating leverage ex-ante can smooth the leverage cycle and be Pareto Improving.
Welfare I: Smoothing the Leverage Cycle

Asset prices are much too high at 0 (compared to Arrow Debreu first best prices) and then they crash at $D$, rising and falling in tandem with leverage.

But interest rates over the cycle in the Leverage Cycle example barely move.

The leverage cycle suggests that it might be more effective to stabilize leverage than to stabilize interest rates.
Welfare I: Smoothing the Leverage Cycle

Regulation limiting leverage at time 0 will lower asset price at time 0 and raise it at \( D \), smoothing the cycle. Goes up at \( D \) because Optimists have higher marginal propensity to buy the asset, and become richer when owe less debt.

Will not cause Pareto improvement because no trade in assets at \( D \): Optimists retain them all.

If modify example by giving Pessimists endowment of assets at \( U \) and \( D \), and letting discount of Pessimists be .95, then taxing leverage at time 0 and redistributing revenue to Pessimists will Pareto improve.
### Welfare I: Smoothing the Leverage Cycle

#### Smoothing the Leverage Cycle.

<table>
<thead>
<tr>
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<th>Unrest. Lev</th>
<th>Restr. Lev</th>
<th>Leverage Transfer</th>
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<td></td>
<td>$j = .58$</td>
<td>$j = .58, t = 0.0004$</td>
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<tr>
<td>Price at $s = 0$</td>
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<tr>
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<td>Utility Pessimists</td>
<td>1311.6860</td>
<td>1311.6858</td>
<td>1311.6862</td>
</tr>
</tbody>
</table>
Welfare I: Smoothing the Leverage Cycle

In the modified Leverage Cycle example, curtailing leverage at 0 not only raises the price of assets at $D$, but also raises the price of assets at $U$, because now optimists are more patient than pessimists and so will invest more of their extra money at $U$ into assets than pessimists withdraw when they receive smaller debt payments.

Since optimists are selling the asset at $U$, this price rise helps optimists and hurts pessimists. Since optimists care more about $U$ than pessimists do, this increases the sum of utilities (normalized so that the marginal utility of consumption at 0 is 1 for all agents). At $D$ the optimists are buying the extra endowment of assets, and so the price rise hurts optimists and helps the pessimist sellers. But pessimists care more about $D$ than optimists, and so the price change again raises total utility.
Welfare I: Smoothing the Leverage Cycle

But curtailing leverage has one more effect: it lowers the price of assets at 0, thereby helping optimists and hurting pessimists. In order to make all agents better off, the policy intervention should reduce leverage and transfer some consumption at time 0 from optimists to pessimists.

Both of these objectives could be achieved by taxing borrowing and then redistributing the revenue to all agents (and not back to those who paid the tax). Table 3 shows that such an intervention is indeed Pareto improving.
Welfare II: Smoothing the Leverage Cycle and Default/TBW Kubler

The most important benefit from curtailing leverage is not captured by the modified Leverage Example, because in binomial economies with financial assets there is no default.

Geanakoplos and Kubler (2013) constructs a multi state example with common beliefs in which there is heterogeneity because optimists get utility from housing. They are thus led to borrow so much on their mortgages that some of them will default in some of the states.
Curtailing leverage has the extra benefit that by raising the future price of houses, it reduces default, since whether a homeowner defaults depends on how far underwater he is.

Though the lender rationally anticipates that by curtailing the loan, he can reduce the chances of his own borrower defaulting, he does not take into account that by lending less he can help increase future housing prices and thus reduce other borrowers’ chances of defaulting.

If defaulting homeowners neglect repairs on their houses, curtailing leverage can lead to Pareto improvements.
Lessons from the Leverage Cycle

Increasing leverage on a broad scale can increase asset prices.

Leverage is endogenous and fluctuates with the fear of default.

Leverage is therefore related to the degree of uncertainty or volatility or down risk of asset markets.

The scarcity of collateral creates a collateral value that can lead to bubbles in which some asset prices are far above their efficient levels.

Booms and busts of the leverage cycle can be smoothed best not by controlling interest rates, but by regulating leverage.

The amplitude of the cycle depends on the heterogeneity of the valuations of the investors.
Outline

1. Introduction
2. Model
3. The Leverage Cycle
4. Multiple Leverage Cycles
5. Agent-Based Model
6. Summary
Multiple Leverage Cycles

Many kinds of collateral exist at the same time, hence there can be many simultaneous leverage cycles.

Collateral equilibrium theory not only explains how one leverage cycle might evolve over time, it also explains some commonly observed cross sectional differences and linkages between cycles in different asset classes.
Multiple co-existing leverage cycles can explain:

- Flight to collateral

- Contagion
Multiple Leverage Cycles: Flight to Collateral

When similar bad news hits two different asset classes, one asset class often preserves its value better than another.

This empirical observation is traditionally given the name Flight to Quality, because it is understood as a migration toward safer assets that have less volatile payoff values.

Fostel and Geanakoplos (2008) emphasized a new channel which they called Flight to Collateral: After volatile bad news, collateral values widen more than payoff values, thus giving a different explanation for the diverging prices.
Multiple Leverage Cycles: Flight to Collateral

Consider the same economy as in our first example Leverage Cycle I above except that now there are two financial assets.

Asset $Y$ pays $d_s^Y = 1$, $s = UU, UD, DU$ and $d_s^Y = .2$, $s = DD$.

Asset $Z$ is perfectly correlated with asset $Y$ and pays $d_s^Z = 1$, $s = UU, UD, DU$ and $d_s^Z = .1$, $s = DD$.

Agents start with asset endowments of .5 units of each asset, $e^{h}_{Y0} = .5$, $e^{h}_{Z0} = .5$, $h = O, P$ at the beginning.
Multiple Leverage Cycles: Flight to Collateral

\[
\begin{align*}
U &\quad (d_{UU}^Y=1, d_{UU}^Z=1) \\
UD &\quad (d_{UD}^Y=1, d_{UD}^Z=1) \\
DU &\quad (d_{DU}^Y=1, d_{DU}^Z=1) \\
UDU &\quad (d_{UDU}^Y=1, d_{UDU}^Z=1) \\
DUD &\quad (d_{DUD}^Y=1, d_{DUD}^Z=1) \\
DDU &\quad (d_{DDU}^Y=1, d_{DDU}^Z=1) \\
DDD &\quad (d_{DDD}^Y=.2, d_{DDD}^Z=.1)
\end{align*}
\]
Multiple Leverage Cycles: Flight to Collateral

<table>
<thead>
<tr>
<th></th>
<th>Asset Y</th>
<th></th>
<th>Asset Z</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$s = 0$</td>
<td>$p$</td>
<td>0.664</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$s = U$</td>
<td>$PV$</td>
<td>0.897</td>
<td>0.982</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$CV$</td>
<td>0.059</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>$s = D$</td>
<td>$\pi_j^*$</td>
<td>0.201</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$LTV$</td>
<td>0.686</td>
<td>0.162</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\omega$</td>
<td>0.105</td>
<td>0.541</td>
<td></td>
</tr>
</tbody>
</table>
Multiple Leverage Cycles: Flight to Collateral

$p_0^U = 0.906$
$p_0^Z = 0.897$

$p_U^U = 0.982$
$p_U^Z = 0.982$

$CV_0^U = 0.063$
$CV_0^Z = 0.059$

$p_D^U = 0.664$
$p_D^Z = 0.621$

$CV_D^U = 0.071$
$CV_D^Z = 0.035$

$CV^0_Y = 0.063$
$CV^0_Z = 0.059$

$CV_D^Y = 0.071$
$CV_D^Z = 0.035$

$(d_{UU}^Y, d_{UU}^Z) = 1$

$(d_{UD}^Y, d_{UD}^Z) = 1$

$(d_{DUU}^Y, d_{DUU}^Z) = 1$

$(d_{DUD}^Y, d_{DUD}^Z) = 1$

$(d_{DDU}^Y, d_{DDU}^Z) = 1$

$(d_{DDD}^Y, d_{DDD}^Z) = 1$
Each asset experiences a leverage cycle.

However, something interesting happens when we look at the cross section variation of all the variables. The gap between asset prices widens after bad news by more than the gap in expected payoffs.

The price of $Y$ falls from .906 at 0 to .664 at $D$, while the price of $Z$ falls from .897 to .621. After bad news, both payoff values go down, but gap increases from .009 to .043.

However, their collateral values move in opposite directions. The widening spread of .034 in prices is almost entirely explained by the widening of collateral values by .033.
Multiple Leverage Cycles: Flight to Collateral

Flight to collateral occurs when the liquidity wedge is high and the dispersion of $LTV$s is high.

During a flight to collateral, investors would rather buy those assets that enable them to borrow money more easily (higher $LTV$s).

The other side of the coin is that investors who need to raise cash get more by selling those assets on which they borrowed less money because the sales revenues net of loan repayments are higher.
Multiple Leverage Cycles: Contagion

Consider the same economy as before except that the asset payoffs are defined as follows
Multiple Leverage Cycles: Contagion

```
(d_{UU}^Y = 1, d_{UU}^Z = 1)
(d_{UD}^Y = 1, d_{UD}^Z = 1.1)
(d_{DUU}^Y = 1, d_{DUU}^Z = 1)
(d_{DUD}^Y = 1, d_{DUD}^Z = 1.1)
(d_{DDU}^Y = 2, d_{DDU}^Z = 1)
(d_{DDD}^Y = 2, d_{DDD}^Z = 1.1)
```
Bad news is about $Y$ only. So we should expect the price of $Y$ to go down after bad news due to a deterioration of its expected payoff value.

But we should not expect the price of asset $Z$ to go down after bad news about $Y$. 
## Multiple Leverage Cycles: Contagion

### Equilibrium Contagion

<table>
<thead>
<tr>
<th>States</th>
<th>$s = 0$</th>
<th>$s = U$</th>
<th>$s = D$</th>
<th>$s = 0$</th>
<th>$s = U$</th>
<th>$s = D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset $Y$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.925</td>
<td>0.991</td>
<td>0.667</td>
<td>0.789</td>
<td>0.827</td>
<td>0.624</td>
</tr>
<tr>
<td>$\pi_j^*$</td>
<td>0.660</td>
<td>0.201</td>
<td>0.617</td>
<td>0.099</td>
<td>0.100</td>
<td></td>
</tr>
<tr>
<td>$LTV$</td>
<td>0.721</td>
<td>0.299</td>
<td>0.792</td>
<td>0.119</td>
<td>0.160</td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.054</td>
<td>0.544</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset $Z$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table above illustrates the equilibrium contagion states for different assets under various conditions.
Multiple Leverage Cycles: Contagion

\[
p_{0U}^Y = 0.925 \\
p_{0U}^Z = 0.789 \\
p_{0D}^Y = 0.667 \\
p_{0D}^Z = 0.624 \\
p_{U}^Y = 0.991 \\
p_{U}^Z = 0.827 \\
p_{D}^Y = 0.667 \\
p_{D}^Z = 0.624 \\
\]

- \((d_{UU}^Y = 1, d_{UU}^Z = 1)\)
- \((d_{UD}^Y = 1, d_{UD}^Z = 0.1)\)
- \((d_{DUU}^Y = 1, d_{DUU}^Z = 1)\)
- \((d_{DUD}^Y = 1, d_{DUD}^Z = 0.2)\)
- \((d_{DDU}^Y = 0.2, d_{DDU}^Z = 1)\)
- \((d_{DDD}^Y = 0.2, d_{DDD}^Z = 0.1)\)
As expected, asset $Y$ experiences a leverage cycle: Its price rises from .925 to .991 after good news and crashes after bad news by more than its payoff value, going down from .925 to .667.

Surprisingly, the price of $Z$ also goes up from .789 to .827 after good news about $Y$, and goes down by more than 20% from .789 to .624 after bad news about $Y$.

The leverage cycle on $Y$ migrates to asset class $Z$, producing a leverage cycle on this market as well. In short, we see contagion in equilibrium.
Fostel-Geanakoplos (2008) showed that contagion is generated by a change in the liquidity wedge.

The $Y$ leverage cycle lowers the liquidity wedge at $U$ and sharply increases the liquidity wedge at $D$, as we have seen in our previous examples. A leverage cycle in one asset class alone can move the liquidity wedge. But, the liquidity wedge is a universal factor in valuing all assets.

Portfolio Effect, amplifying the movements of the liquidity wedge at $U$ and $D$. At $U$ optimists do not see any advantage in giving up consumption to invest in $Y$. At $D$ they buy all of $Y$, reducing their consumption, further increasing the liquidity wedge.
Agent Based Model/TBW
Summary/ TBW