A Dynamic Model of Vehicle Ownership, Type Choice, and Usage

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Introduction
How much is a Volvo in Denmark?
204.816 US Dollars!
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MSRP in US: $62,350
Danish car registration tax: 180%!
..... plus 25% VAT!!!
Annual Revenue

- 30–50 billion DKK
- $\approx 2–3$ pct. of GDP
- $\approx 4$–7 pct. of total tax revenue
- Most revenue originate from taxation of ownership and registration of new cars.

It is also widely understood that transport externalities are rarely appropriately priced (e.g., Parry and Small (2005)).

- Underpricing congestion.
- Incorrect taxation of gasoline.
Car taxes in Denmark

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• **Annual green ownership tax** was introduced in 1997 to create incentives for *owning* more energy efficient cars.
  - Reformed in 2007: even stronger incentives to choose smaller, more energy efficient cars including diesels.
  - Green owner tax now ranges from $3096 for cars getting less than 4.7 km/liter to only $94 for cars getting more than 20km/liter
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• **Vehicle registration tax** was designed to pay for roads and maintenance and to ration the number of cars in Denmark
  - Progressive tax with 180% marginal tax in top bracket (+25% VAT).
  - Reformed in 2007: Discounts registration fee by $605 per km over 16 km/L gas or 18km/L diesel and penalty of $152 for cars with efficiencies below these thresholds
  - Reformed in 2017: Marginal tax in top bracket reduced 150%
Trend in vehicle efficiency in Denmark

Fact Sheet: The Danish motor vehicle taxes

Purpose of the taxes
In Denmark, two different taxes must be paid; a registration tax and an annual green owner tax. The registration tax, which is a purchase tax, was introduced already in 1910. The main purpose was to ensure that vehicle owners contribute to the public expenditures of road maintenance and construction. Today the registration tax aims to reduce the number of vehicles in Denmark. In addition, a weight-based tax had to be paid from 1910 until 1997. In 1997, the green owner tax replaced the weight-based tax to provide an incentive to use vehicles with higher energy efficiency and thereby reducing environmental damage caused by cars.

How the taxes work today
The annual green owner tax is based on how energy efficient the motor vehicle is (the number of kilometres driven per litre fuel) and ranges from €2740 EUR (<4,7 km per litre) to €83 EUR (>20 km per litre). The registration tax is paid when the vehicle is purchased, it must be paid when vehicles are registered for the first time in Denmark, including motorcycles, taxis, and buses up to four tons. Both the registration tax and the green owner tax are low, when the car is small and highly energy efficient, while they are high, when the car is big and only drives a few kilometres per litre.

In 2014, the registration tax for passenger cars was 105% of the taxable value up to €10,951 EUR and 180% of the rest. In 2007, the tax was restructured, so that a discount is given on the registration fee of €536 EUR per km the car runs longer than 16 km per litres of petrol and 18 km per litre diesel, and a surcharge of €134 EUR per km the car runs below these limits. This regulation has led to a noticeable shift in car sales in the direction of small, more energy-efficient cars (see figure 1) and the average CO2 emission per kilometre from all the cars is still going down.

Source: The Danish Statistical Bureau

Figure 1: Newly registered passenger cars, 1997 - 2014

4,5-9,9 km per litre
10,0-13,2 km per litre
13,3-15,3 km per litre
15,4-19,9 km per litre
More than 20 km per litre

Figure 1: Newly registered passenger cars, 1997 - 2014

Source: The Danish Statistical Bureau
## Potential Policy Option

- Lower registration taxes, and
- higher usage taxes (road charging).
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Outcomes of interest

- Equilibrium dynamics of car ownership and type choice:
  - new car sales and trade in secondary markets
  - fleet age and scrappage,
  - value of the car stock.
- Driving, fuel demand, and emissions.
- Redistribution and welfare.
- Need to capture these effects simultaneously.... and account for macroeconomics shocks
Car sales over the business cycle 1992-2010

New Registrations of Private Cars

[Graph showing car sales and GDP growth over the years 1992 to 2010, with peaks and troughs indicating economic cycles.]
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
The 1995 Cohort of Cars
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The 1995 and 1986 Cohorts of Cars
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The 1995 and 1986 Cohorts of Cars
The 1995 and 1986 Cohorts of Cars
The 1995 and 1986 Cohorts of Cars
The 1995 and 1986 Cohorts of Cars
The 1995 and 1986 Cohorts of Cars
The 1995 and 1986 Cohorts of Cars
The 1995 and 1986 Cohorts of Cars
The Car Age Distribution Over Time
The Car Age Distribution Over Time
The Car Age Distribution Over Time
The Car Age Distribution Over Time
The Car Age Distribution Over Time

![Graph showing the car age distribution over time with data from 1998 to 2008.]
The Car Age Distribution Over Time
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Waves in car purchases 1995-2009
1. *Dynamics are crucial:* When deciding to keep your current car, trade for a new one, or get rid of the car you have, consumer take into account that investments in cars are partly irreversible due to transactions cost, technical depreciation and increasing maintenance cost.
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2. With dynamics come *expectations*: how much can I sell my current car for, and how much will it cost to buy another one?
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3. Macro shocks are evidently important: do I want to buy a new car now if the economy is going into recession and I might lose my job?
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3. Macro shocks are evidently important: do I want to buy a new car now if the economy is going into recession and I might lose my job?

4. There are many different makes/models and *ages* of cars, so finding an equilibrium is a *high dimensional* numerical problem.
**Goal:** Analyze the effects of a reform that changes the balance between taxes on registration, ownership, fuel and road use.

We develop a dynamic equilibrium model that simultaneously tracks the following mechanisms:

- Business cycle variation in new/used car purchases,
- Scrappage, replacement timing, new vs. used-car tradeoffs,
- Type choice, fuel-efficiency and driving,
- Trade/sorting of cars between heterogeneous consumers.

We structurally estimate this model using Danish register data: lots heterogeneity and rich dynamics. We simulate the counter-factual equilibrium and evaluate the effects of changing car taxation in Denmark.
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Previous work on equilibrium dynamics in auto markets
A brief history of auto models

- Manski (1982) and Berkovec (1985):
- Rust (1985): provided a dynamic framework for equilibrium prices and quantities, and showed that when transactions costs are zero, consumers trade every period for an optimal car.
- Konishi and Sandfort (2002): generalized Rust's analysis to allow for positive transaction costs and multiple makes/models of cars and proved the existence of a stationary equilibrium.
- Gavazza et al. (2014): numerically calculated equilibrium with one car type and discrete ages/qualities of cars and analyzed the impact of the secondary market with varying levels of transaction costs.
- All of the above: exogenous driving demand and no macro shocks.
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Supply side effect - new vehicles

Product differentiation and consumer choice of new vehicles (Bresnahan, 1981; BLP, 1995; Goldberg, 1995; Petrin, 2002).

- Model general patterns of substitution across differentiated products
- Static models of consumer demand and a Bertrand oligopoly model for automobile supply.
- Do not model secondary markets or the dynamics of the consumer decision process.

Environmental policies in vehicle market

- Jacobsen (2013): Effects of CAFE Standards in the US.
- Supply side effects less of a concern for the counterfactual we consider (most relevant for type substitution among new vehicles).
Data
## Data Sources

<table>
<thead>
<tr>
<th>Purpose</th>
<th>Source</th>
<th>Time</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ownership</td>
<td>Central motor register</td>
<td>Day</td>
<td>1990–2011</td>
</tr>
<tr>
<td>Usage</td>
<td>Ministry of Transportation tests</td>
<td>Day</td>
<td>1997–2007*</td>
</tr>
<tr>
<td></td>
<td>(occur approx. at car age 4, 6, 8, ...)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Car characteristics</td>
<td>Vehicle type approval documents</td>
<td>n.a.</td>
<td>1997–2011**</td>
</tr>
<tr>
<td>New car prices</td>
<td>Danish Automobile Dealer Association</td>
<td>Year</td>
<td>1997–2009</td>
</tr>
<tr>
<td>Demographics</td>
<td>Danish registers.</td>
<td>Year</td>
<td>1980–2012</td>
</tr>
<tr>
<td>Fuel prices</td>
<td>Danish Oil Industry Association</td>
<td>Day</td>
<td>1980–2014</td>
</tr>
</tbody>
</table>

* Driving periods for used cars observed until [2009; 2011]-periods.

** Early on, car characteristics are almost only available for new cars.
Cars in Denmark have many different owners

Figure 1: For all 15 y/o cars in 2009, the number of owners

Note: each bar shows the percentage of vehicles that have had the particular number of owners until they turned 15 years old in 2009. Only vehicles whose first owner is observed are used (61,919/92,021 of the vehicles). Truncated at 15 owners.
Younger cars are held longer
Vehicle ownership over the life cycle

Note: each bar shows the percentage of households within the given age category that own the particular number of cars in a particular year.
Gains from trade between rich and poor consumers

Rich mans Volvo

Poor mans Volvo
Holdings by income quintiles, Heavy Vehicle
Holdings by income quintiles, Light Vehicle
The rich keep more
Newer cars are driven more

Controls: none.
Selection: VKT within 1% and 99% percentiles, year in [1996;2009] and household age in [18;65].

By income  ▶  By HH age
Scrappage Decisions – by car age

Note: each bar shows the percentage of cars and vans that were scrapped within each car age.
Summing up

• Profound *waves* in the car age distribution over time,
  ...for both holdings, transactions and scrappage of cars.
Summing up

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  ...for both holdings, transactions and scrappage of cars.
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- Older and richer households tend to
  - keep more and purge less - but replace cars more often.
  - buy newer and own more cars high quality cars
    .... and sell them to poorer consumers.
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- Older cars are
  - Driven less intensively,
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  - Held shorter in durations.
- Driving intensity increases with income until the top decile, and then drops.
  ......older people drive their cars less intensively.
Model
Choice model + market equilibrium
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Model: Dynamic, finite-horizon, quasi-linear.
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States: 4 discrete, 2 continuous

- **Household**: Age, income, car (age, type),
- **Aggregate**: Fuel price, macro state (0/1).
Model

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Utility: Derived from

1. Driving,
2. Owning a car,
3. Outside consumption.
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Utility: Derived from
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2. Owning a car,
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Decisions: 
- Driving,
- Car ownership,
- Scrappage.
Decisions for households with no car

\[ d = (\tau', a') \]

- No existing car:
  \[ s = (\emptyset, \emptyset) \]
  \[ d = K \]

- Buying a car (new or used):
  \[ s' = (\tau', a') \]
  Cost: \[ P(s') + \text{Trans} \]

- Staying with no car:
  \[ s' = (\emptyset, \emptyset) \]
Decisions of car owners

- Car exists (car owner) $s = (\tau, a)$
  - Replacing the car $s' = (\tau', a')$
    - Cost: $P(s') + \text{Trans}$
  - Keeping existing car $s' = s = (\tau, a)$
  - Purging the car $s' = (\emptyset, \emptyset)$

- Sell Benefit: $P(s)$
- Scrap Benefit: $\underline{P}(s)$
<table>
<thead>
<tr>
<th>State</th>
<th>Type</th>
<th>Values</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_t$</td>
<td>Car type</td>
<td>discrete</td>
<td>${1, \ldots, \tau}$</td>
</tr>
<tr>
<td>$a_t$</td>
<td>Car age</td>
<td>discrete</td>
<td>${0, \ldots, 24}$</td>
</tr>
<tr>
<td>$y_t$</td>
<td>Household income</td>
<td>continuous</td>
<td>$[\underline{y}, \overline{y}]$</td>
</tr>
<tr>
<td>$p_t$</td>
<td>Fuel prices</td>
<td>continuous</td>
<td>$[\underline{p}, \overline{p}]$</td>
</tr>
<tr>
<td>$m_t$</td>
<td>Macro state</td>
<td>discrete</td>
<td>${0, 1}$</td>
</tr>
<tr>
<td>$t$</td>
<td>Household age</td>
<td>discrete</td>
<td>${20, \ldots, 85}$</td>
</tr>
</tbody>
</table>
The Choice Model

\[
\max_{\{d_t, d^s_t, vkt_t\}_{t=t_0}^T} \sum_{t=t_0}^{T} \beta^t E_t \left[ u_t(s_t, d_t, d^s_t, vkt_t, y_t, p_t, m_t) \right]
\]
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\max_{\{d_t, d_t^s, vkt_t\}_{t=t_0}^{T}} \sum_{t=t_0}^{T} \beta^t \mathbb{E}_t \left[ u_t(s_t, d_t, d_t^s, vkt_t, y_t, p_t, m_t) \right]
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\(d_t, d_t^s\) choice: holding/trading/scrappage decision
The Choice Model

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- $d_t, d_t^s$ choice: holding/trading/scrappage decision
- $vkt_t$ choice: driving decision (vehicle-kilometers-traveled)
The Choice Model

\[
\max_{\{d_t, d_t^s, vkt_t\}_{t=t_0}^{t=T}} \sum_{t=t_0}^{T} \beta^t \mathbb{E}_t \left[ u_t(s_t, d_t, d_t^s, vkt_t, y_t, p_t, m_t) \right]
\]

- \(d_t, d_t^s\) choice: holding/trading/scrappage decision
- \(vkt_t\) choice: driving decision (vehicle-kilometers-traveled)
- \(t_0, T\) household age (\(t_0 = 20, T = 85\))
The Choice Model

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\max_{\{d_t, d_t^s, vkt_t\}_{t=t_0}} \sum_{t=t_0}^T \beta^t \mathbb{E}_t \left[ u_t(s_t, d_t, d_t^s, vkt_t, y_t, p_t, m_t) \right]
\]

- \(d_t, d_t^s\) choice: holding/trading/scrappage decision
- \(vkt_t\) choice: driving decision (vehicle-kilometers-traveled)
- \(t_0, T\) household age \((t_0 = 20, T = 85)\)
- \(\beta\) discount factor \((\beta = 0.95)\)
The Choice Model

\[
\max \left\{ d_t, d_{t}^{s}, vkt_t \right\}_{t=t_0}^{T} \sum_{t=t_0}^{T} \beta^t E_t \left[ u_t(s_t, d_t, d_{t}^{s}, vkt_t, y_t, p_t, m_t) \right]
\]

\(d_t, d_{t}^{s}\) choice: holding/trading/scrappage decision

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\(u_t(\cdot)\) instantaneous utility function
The Choice Model

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\max_{\{d_t, d^s_t, vkt_t\}_{t=t_0}} \sum_{t=t_0}^{T} \beta^t \mathbb{E}_t \left[ u_t(s_t, d_t, d^s_t, vkt_t, y_t, p_t, m_t) \right]
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\(u_t(\cdot)\) instantaneous utility function

\(s_t = (\tau, a)\) the car existing in the beginning of the period \(t\)
The Choice Model

\[
\max_{\{d_t, d_t^s, vkt_t\}_{t=0}^T} \sum_{t=t_0}^T \beta^t \mathbb{E}_t \left[ u_t(s_t, d_t, d_t^s, vkt_t, y_t, p_t, m_t) \right]
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- \(d_t, d_t^s\) choice: holding/trading/scrappage decision
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- \(u_t(\cdot)\) instantaneous utility function
- \(s_t = (\tau, a)\) the car existing in the beginning of the period \(t\)
- \(s_t' = (\tau', a')\) car to be driven in period \(t\): \(s_t' = s_t'(d_t, s_t)\)
The Choice Model

\[
\max_{\{d_t, d_t^s, vkt_t\}_{t=t_0}} \sum_{t=t_0}^{T} \beta^t \mathbb{E}_t \left[ u_t(s_t, d_t, d_t^s, vkt_t, y_t, p_t, m_t) \right]
\]

\(d_t, d_t^s\) choice: holding/trading/scrappage decision

\(vkt_t\) choice: driving decision (vehicle-kilometers-traveled)

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\(s_t = (\tau, a)\) the car existing in the beginning of the period \(t\)

\(s_t' = (\tau', a')\) car to be driven in period \(t\): \(s_t' = s_t'(d_t, s_t)\)

\((y_t, p_t, m_t)\) household income and macro conditions
To simplify computations, we exploit:

Property: Continuation value depends only on the *chosen* car.
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Property: Continuation value depends only on the *chosen* car

Driving: No dynamic implications $\Rightarrow$ static choice.
To simplify computations, we exploit:

Property: Continuation value depends only on the chosen car.

Driving: No dynamic implications ⇒ static choice.

Scrappage: No dynamic implications ⇒ static choice.

(not the incoming car)
Utility specification

\[ u_t(s_t, s'_t, d^s_t, vkt_t, y_t, p_t, m_t) = \]

\[
\theta(y_t, m_t) \left[ y_t - p_{km}(\tau', a', p_t)vkt_t - c(a', \tau') - TC(s_t, s'_t, d^s_t, p_t, m_t) \right] + \phi vkt_t^2 + \gamma(a', m_t)vkt_t + \delta_\tau + \delta_n 1\{a' = 0\} - q(a')
\]

Consumption

Utility from car services
Utility specification

\[ u_t(s_t, s'_t, d^s_t, vkt_t, y_t, p_t, m_t) = \]

\[ \theta(y_t, m_t) \left[ y_t - p_{km}(\tau', a', p_t)vkt_t - c(a', \tau') - TC(s_t, s'_t, d^s_t, p_t, m_t) \right] \]

\[ + \phi vkt_t^2 + \gamma(a', m_t)vkt_t + \delta_{\tau} + \delta_n 1\{a' = 0\} - q(a') \]

\[ \theta(y_t, m_t) \text{ Constant marginal utility of money. Macro and income dependent. (no consumption/savings) } \]
Utility specification

\[ u_t(s_t, s'_t, d_t^s, vkt_t, y_t, p_t, m_t) = \]

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\[ + \phi vkt_t^2 + \gamma(a', m_t)vkt_t + \delta_\tau + \delta_n 1\{a'=0\} - q(a') \]

\[ \theta(y_t, m_t) \text{ Constant marginal utility of money. Macro and income dependent. (no consumption/savings)} \]

\[ p_{km}(\tau', a', p_t) \text{ per km cost of driving} \]
Utility specification

\[ u_t(s_t, s'_t, d^s_t, vkt_t, y_t, p_t, m_t) = \theta(y_t, m_t) \left[ y_t - p_{km}(\tau', a', p_t)vkt_t - c(a', \tau') - TC(s_t, s'_t, d^s_t, p_t, m_t) \right] \]

\[ \quad + \phi vkt^2_t + \gamma(a', m_t)vkt_t + \delta_\tau + \delta_n 1\{a' = 0\} - q(a') \]

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\[ \phi, \gamma(a', m_t) \text{ utility from driving} \]
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\( \delta_t \) type specific dummies
Utility specification

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**Consumption**

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\[ \delta_\tau \] type specific dummies

\[ \delta_n, q(a') \] the effect of new car and disutility of car aging

**Utility from car services**
Utility specification

\[ u_t(s_t, s'_t, d^s_t, vkt_t, y_t, p_t, m_t) = \theta(y_t, m_t) \left[ y_t - p_{km}(\tau', a', p_t)vkt_t - c(a', \tau') - TC(s_t, s'_t, d^s_t, p_t, m_t) \right] \]

\[ \text{consumption} \]

\[ + \phi vkt_t^2 + \gamma(a', m_t)vkt_t + \delta_\tau + \delta_n 1\{a' = 0\} - q(a') \]

\[ \text{utility from car services} \]

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\[ \delta_n, q(a') \] the effect of new car and disutility of car aging

\[ c(a', \tau') \] annual maintenance cost (type specific, increasing in age)
Utility specification

\[ u_t(s_t, s'_t, d^s_t, vkt_t, y_t, p_t, m_t) = \theta(y_t, m_t) \left[ y_t - p_{km}(\tau', a', p_t)vkt_t - c(a', \tau') - TC(s_t, s'_t, d^s_t, p_t, m_t) \right] \]

consumption

\[ + \phi vkt_t^2 + \gamma(a', m_t)vkt_t + \delta_\tau + \delta_n \mathbb{1}\{a' = 0\} - q(a') \]

utility from car services

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\[ \delta_n, q(a') \] the effect of new car and disutility of car aging

\[ c(a', \tau') \] annual maintenance cost (type specific, increasing in age)

\[ TC(\cdot) \] Includes transactions cost and selling price/scrap value.
Trade costs, $TC(s_t, s'_t, d^s_t, p_t, m_t)$

Purchase: Pay $P(\tau', a')$ to buy a $(\tau', a')$ car.
Trade costs, \(TC(s_t, s'_t, d^s_t, p_t, m_t)\)

Purchase: Pay \(P(\tau', a')\) to buy a \((\tau', a')\) car.

Transactions: The \textit{buyer} incurs transaction costs (to be estimated).
Trade costs, $TC(s_t, s'_t, d^s_t, p_t, m_t)$

Purchase: Pay $P(\tau', a')$ to buy a $(\tau', a')$ car.

Transactions: The buyer incurs transaction costs (to be estimated).

Selling: When getting rid of a $(\tau, a)$-car, either
- Sell at the market price $P(\tau, a)$, or
- Scrap and receive $\underline{P}(\tau) + \varepsilon$. 
Binary scrappage choice

- Decision arises when getting rid of a car.

Random utility model with fixed components:
- Scrap: Receive the fixed scrappage payment (about $250).
- Not scrap: Receive the market price.

This way, used-car prices affect scrappage. Can be handled as either nested logit or 2nd stage choice.
Binary scrappage choice

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Accidents

- **Problem:** 0.66% of 0–4 y/o cars are scrapped.
Accidents

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- **Accidents:** the involuntary scrappage of a car.
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• Accidents: the involuntary scrappage of a car.
• Data: “disappearance” event.
  • Current ownership spells end; no future spell observed.
Accidents

- **Problem:** 0.66% of 0–4 y/o cars are scrapped.
- **Accidents:** the involuntary scrappage of a car.
- **Data:** “disappearance” event.
  - Current ownership spells end; no future spell observed.
- **Model:** transition households to the no-car state.
  - Exogenous probability $\alpha$,
  - Occurs before choices are made,
  - Agent receives a fraction of the car’s market value (insurance).

$\text{Pr(accident)} \equiv \alpha := 0.0066 = \text{young scraps.}$

Graph of scrap rates
Accidents

- **Problem**: 0.66% of 0–4 y/o cars are scrapped.
- **Accidents**: the involuntary scrappage of a car.
- **Data**: “disappearance” event.
  - Current ownership spells end; no future spell observed.
- **Model**: transition households to the no-car state.
  - Exogenous probability $\alpha$,
  - Occurs before choices are made,
  - Agent receives a fraction of the car’s market value (insurance).
- **Identification**: $\Pr(\text{accident}) \equiv \alpha := 0.0066 = \text{young scraps}$. 

[Graph of scrap rates]
• Optimal driving found via FOC:

\[
\frac{\partial}{\partial x} \left[ \theta(y, m)(y - p_{km}(\tau, a, p_{fuel})x - c - TC) + \gamma(a, m)x + \phi x^2 + \ldots \right] = 0
\]

• Assume additive normally distributed measurement error.
• Optimal driving found via FOC:

\[
\frac{\partial}{\partial x} \left[ \theta(y, m)(y - p_{km}(\tau, a, p_{fuel})x - c - TC) + \gamma(a, m)x + \phi x^2 + \ldots \right] = 0
\]

• Results in linear form,

\[
x^* = \frac{1}{2\phi} \left[ \gamma(a, m) - \theta(y, m)p_{km}(\tau, a, p_{fuel}) \right].
\]
Driving

- Optimal driving found via FOC:

\[
\frac{\partial}{\partial x} \left[ \theta(y, m)(y-p_{km}(\tau, a, p_{fuel})x-c-TC) + \gamma(a, m)x + \phi x^2 + ... \right] = 0
\]

- Results in linear form,

\[
x^* = \frac{1}{2\phi} \left[ \gamma(a, m) - \theta(y, m)p_{km}(\tau, a, p_{fuel}) \right].
\]

- Assume additive normally distributed measurement error.
Bellman equation

\[ V_t(s_t, y_t, p_t, m_t) = \max_{d_t, d^s_t, vkt_t} \left\{ u_t(s_t, d_t, d^s_t, vkt_t, y_t, p_t, m_t) + \lambda \varepsilon(d_t, d^s_t) + \beta \mathbb{E}_t \left[ V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}) \right] \right\} \]

- \( \varepsilon(d_t, d^s_t) \) are EV i.i.d. \( \Rightarrow \mathbb{E}(\max\{\ldots\}) \) is Logsum
- CCPs are logit of discrete choice specific value functions

\[ v_t(s_t, y_t, p_t, m_t, d_t, d^s_t, vkt_t) = u_t(s_t, d_t, d^s_t, vkt_t, y_t, p_t, m_t) + \beta \mathbb{E}_t \left[ \max_{d_{t+1}, d^s_{t+1}, vkt_{t+1}} v_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}, d_{t+1}, d^s_{t+1}, vkt_{t+1}) \right] \]

- \( v_t(s_t, y_t, p_t, m_t, d_t, d^s_t, vkt_t) \) is very high-dimensional object
A: Separate static continuous choice

Next period value function $V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1})$ is independent of $v_{kt_t}$, therefore the Bellman equation simplifies to

$$V_t(\tau_t, a_t, y_t, p_t, m_t) =$$

$$\max_{d_t, d_t^s} \left\{ \max_{v_{kt_t}} \left[ u_t(s_t, s'_t, d_t^s, v_{kt_t}, y_t, p_t, m_t) \right] + \lambda \varepsilon(d_t, d_t^s) + \beta \mathbb{E}_t \left[ V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}) \right] \right\}$$

$$\frac{\partial u_t(\cdot)}{\partial v_{kt_t}} = \gamma(a', m_t) - \theta(y_t, m_t)p_{km}(\tau', p_t) + 2\phi v_{kt_t} = 0 \Rightarrow$$

$$v_{kt_t}^* = \frac{\theta(y_t, m_t)p_{km}(\tau', p_t) - \gamma(a', m_t)}{2\phi}$$

Pure discrete choice model conditional on optimal driving.
B: Separate static scrappage choice

Next period value function \( V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}) \) is independent of \( d_t^s \), therefore the Bellman equation further simplifies to

\[
V_t(\tau_t, a_t, y_t, p_t, m_t) = \max_{d_t} \left\{ \begin{array}{c}
\max \left\{ u_t(s_t, s'_t, -1, vkt^*_t, y_t, p_t, m_t), u_t(s_t, s'_t, +1, vkt^*_t, y_t, p_t, m_t) \right\} \\
\text{scrap} & \text{sell}
\end{array} \right\}
\]

\[+ \lambda \varepsilon(d_t) + \beta \mathbb{E}_t \left[ V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}) \right] \]

After adding extreme value shocks, the alternatives of the scrappage decision are characterized by the expected maximum utility (logsum) of the highlighted utilities of selling and scrapping

\( \lambda_s \) – scale parameter for the EV shocks on the scrappage decision
C: Reformulate in terms of expected value function

Next period value function $V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1})$ is only dependent of the car to drive $s'_t$ and not on the existing car $s_t$, therefore solving the Bellman equation in terms of expected value function has much lower dimensionality.

$$EV_t(s_{t+1}, y_t, p_t, m_t) = \mathbb{E}_t \left[ V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}) \right] =$$

$$= \sum_{m_{t+1}} \int_{y_{t+1}} \int_{p_{t+1}} \lambda \log \left( \sum_{d_{t+1}} \exp \frac{V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}, d_{t+1})}{\lambda} \right)$$

$$g_s(y_{t+1}|y_t, p_{t+1}, m_{t+1})h(p_{t+1}, m_{t+1}|p_t, m_t)dy_{t+1}dp_{t+1}dm_{t+1}$$

Assume the above dependence structure between $y_t$, $p_t$ and $m_t$. 
Choice specific value functions

Decision to keep existing car or no car

\[ v_s(\tau, a, y_t, p_t, m_t, d_t = K) = \]

\[ u_t(\tau, a, \tau, a, vkt_t^*, y_t, p_t, m_t) + \beta EV_t(\tau, a + 1, y_t, p_t, m_t) \]

Decision to purge existing car

\[ v_s(\tau, a, y_t, p_t, m_t, d_t = P) = \]

\[ u_t^{ls}(\tau, a, \emptyset, \emptyset, \pm 1, vkt_t^*, y_t, p_t, m_t) + \beta EV_t(\emptyset, \emptyset, y_t, p_t, m_t) \]

Decision to buy or replace the car

\[ v_s(\tau, a, y_t, p_t, m_t, d_t = (\tau', a')) = \]

\[ u_t^{ls}(\tau, a, \tau', a', \pm 1, vkt_t^*, y_t, p_t, m_t) + \beta EV_t(\tau', a' + 1, y_t, p_t, m_t) \]
Bellman equation in expected values

\[ EV_t(s', y, p, m) = \sum_{m'} \int_{y'} \int_{p'} \left[ \lambda \log \sum_{d'} \exp \left( \frac{u(s', s''(d'), y', p', m') + \beta EV_{t+1}(s''(d'), y', p', m')}{\lambda} \right) \right] g_s(y'|y_t, p', m') h(p', m'|p_t, m_t) dy' dp' dm' \]

Once \( EV_t(s', y, p, m) \) are computed, the choice probabilities are calculated with standard logit formula

\[ P_t(d) = \frac{\exp \left( \frac{1}{\lambda} u(s, d, y, p, m) + \frac{\beta}{\lambda} EV_{t+1}(s'(d), y, p, m) \right)}{\sum_{d'} \exp \left( \frac{1}{\lambda} u(s, d', y, p, m) + \frac{\beta}{\lambda} EV_{t+1}(s'(d'), y, p, m) \right)} \]
Computational details

1. Interpolation over \((y, p)\)
   - Chebyshev polynomials to approximate \(EV_t(s', y, p, m)\)
   - Expected value function is smooth, so this works very well
   - Only the Chebyshev coefficients are stored in memory to represent the whole function

2. Integrals over errors in AR(1) processes
   - Need to compute integrals over distributions of \((y, p)\)
   - Use two-dimensional Gaussian quadrature

3. Implementation: Matlab + C
• **Accidents** happen with a fixed probability $\alpha$ which is independent of car age, result in making “keep” decision $d = K$ infeasible, and incur a penalty to be paid

• Clunkers ($a = \bar{a}$) may be **allowed** or **not allowed** to be traded in the secondary market

• **Nested choice structure** to allow for “Not to keep” decisions to have correlated EV error terms
Alternative nested choice specification

Car exists (car owner)
\[ s = (\tau, a) \]

Keeping existing car
\[ d = \mathbb{K} \]
\[ s' = s = (\tau, a) \]

Not keeping existing car
\[ d = (\tau', a') \]

Replacing the car
\[ s' = (\tau', a') \]
Cost: \[ P(s') + \text{Trans} \]

Purging the car
\[ s' = (\emptyset, \emptyset) \]

Sell
Benefit: \[ P(s) \]

Scrap
Benefit: \[ P(s) \]

Sell
Benefit: \[ -P(s) \]

Scrap
Benefit: \[ -P(s) \]
Equilibrium
Choice model + market equilibrium
Overview

Choice model + market equilibrium
Idea: Choose $P(\tau, a)$ so # of buyers = # of sellers.
Equilibrium

Idea: Choose $P(\tau, a)$ so # of buyers = # of sellers.

Supply: Current owners contribute

$$\sum_{i: s_{it} = (\tau, a)} [1 - Pr(keep|\tau, a, P(\cdot)\ldots)]$$
Equilibrium

Idea: Choose $P(\tau, a)$ so # of buyers = # of sellers.

Supply: Current owners contribute

$$\sum_{i:s_{it}=(\tau, a)}[1 - \Pr(\text{keep}|\tau, a, P(\cdot)\ldots)]$$

Demand: Anyone contributes:

$$\sum_{i=1}^{N} \Pr \left[ d = (\tau, a)|s_{it}, P(\cdot)\ldots \right]$$
Idea: Choose $P(\tau, a)$ so $\#$ of buyers $= \#$ of sellers.

Supply: Current owners contribute

$$\sum_{i: s_{it} = (\tau, a)} [1 - \Pr(\text{keep}|\tau, a, P(\cdot)\ldots)]$$

Demand: Anyone contributes:

$$\sum_{i=1}^{N} \Pr[d = (\tau, a)|s_{it}, P(\cdot)\ldots]$$

Boundary:

- **New cars:** $P(\tau, 0) = \overline{P}(\tau)$.
- **Clunkers:** $P(\tau, \overline{a}) = \underline{P}(\tau)$. 

Challenges: expectations and estimation.
Equilibrium

Idea: Choose $P(\tau, a)$ so $\#$ of buyers $= \#$ of sellers.

Supply: Current owners contribute
$$\sum_{i:s_{it}=(\tau,a)} [1 - \Pr(\text{keep}|\tau, a, P(\cdot)\ldots)]$$

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Boundary:  
- **New cars:** $P(\tau, 0) = \overline{P}(\tau)$.  
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Challenges: expectations and estimation.
• **Problem:** to solve the backwards iteration, we need prices in the future.

Rational solution: the car stock becomes a state variable (Krusell and Smith, 1998).

Current Solution: agents assume the current temporal equilibrium to be permanent.
Price expectations

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  - computationally infeasible.
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- **Current Solution**: agents assume the current temporal equilibrium to be permanent.
Expected excess demand

For each car category \((\tau, a)\) define *Expected Excess Demand*

\[
EED(\tau, a, P(\cdot), \theta) = \sum_{i=1}^{N} \left( Pr[\text{buy}|a, \tau, P(\cdot)] - Pr[\text{sell}|a, \tau, P(\cdot)] \right)
\]
Expected excess demand

For each car category \((\tau, a)\) define *Expected Excess Demand*

\[
\text{EED}(\tau, a, P(\cdot), \theta) = \sum_{i=1}^{N} \left( \Pr \left[ \text{buy} | a, \tau, P(\cdot) \right] - \Pr \left[ \text{sell} | a, \tau, P(\cdot) \right] \right)
\]

- Minimize EED w.r.t. parameters in \(P_t(\tau, a)\)
For each car category \((\tau, a)\) define \textit{Expected Excess Demand}

\[
\text{EED}(\tau, a, P(\cdot), \theta) = \\
\sum_{i=1}^{N} \left( \Pr[\text{buy}|a, \tau, P(\cdot)] - \Pr[\text{sell}|a, \tau, P(\cdot)] \right)
\]

- Minimize EED w.r.t. parameters in \(P_t(\tau, a)\)
- Smoothness through the choice probabilities.
Expected excess demand

For each car category \((\tau, a)\) define *Expected Excess Demand* \(EED(\tau, a, P(\cdot), \theta) = \sum_{i=1}^{N} \left( \Pr \left[ \text{buy} \mid a, \tau, P(\cdot) \right] - \Pr \left[ \text{sell} \mid a, \tau, P(\cdot) \right] \right) \)

- Minimize EED w.r.t. parameters in \(P_t(\tau, a)\)
- Smoothness through the choice probabilities.
- Solve for a price for each \(P(\tau, a)\) via a system of equations.
Expected excess demand

For each car category \((\tau, a)\) define *Expected Excess Demand*

\[
EED(\tau, a, P(\cdot), \theta) = \sum_{i=1}^{N} \left( \Pr[buy|a, \tau, P(\cdot)] - \Pr[sell|a, \tau, P(\cdot)] \right)
\]

- Minimize EED w.r.t. parameters in \(P_t(\tau, a)\)
- Smoothness through the choice probabilities.
- Solve for a price for each \(P(\tau, a)\) via a system of equations.
- This objective function is never flat,
For each car category \((\tau, a)\) define *Expected Excess Demand* 

\[
EED(\tau, a, P(\cdot), \theta) = \sum_{i=1}^{N} \left( \Pr[\text{buy}\mid a, \tau, P(\cdot)] - \Pr[\text{sell}\mid a, \tau, P(\cdot)] \right)
\]

- Minimize EED w.r.t. parameters in \(P_t(\tau, a)\)
- Smoothness through the choice probabilities.
- Solve for a price for each \(P(\tau, a)\) via a system of equations.
- This objective function is never flat,
- Gradient based methods are applicable \(\Rightarrow\) faster
Searching for equilibrium: nonparametric prices

**Supply and demand, Car type 1**

- Demand
- Supply

**Supply and demand, Car type 2**

- Demand
- Supply

**Excess demand, Car type 1**

- Excess Demand

**Excess demand, Car type 2**

- Excess Demand

**Price functions**

- Type 1
- Type 2
Structural Estimation
Strategy:

1. Estimate utility parameters keeping prices fixed.
   - \( P_t(\tau, a) = \delta^a_{\tau,t} P_t(\tau, 0) \)
   - \( \delta_{\tau} \): from the Automobile Dealer Association.
Strategy:

1. Estimate utility parameters keeping prices fixed.
   - \( P_t(\tau, a) = \delta_{\tau, t}^a P_t(\tau, 0) \)
   - \( \delta_{\tau} \): from the Automobile Dealer Association.

2. Solve for equilibrium prices \( P_t^*(\tau, a) \) (fixing utility parameters).
Strategy:

1. Estimate utility parameters keeping prices fixed.
   - $P_t(\tau, a) = \delta_{\tau,t}^a P_t(\tau, 0)$
   - $\delta_{\tau}$: from the Automobile Dealer Association.
2. Solve for equilibrium prices $P_t^*(\tau, a)$ (fixing utility parameters).
3. Re-estimate utility parameters given $P_t^*(\tau, a)$
Strategy:

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   - \( P_t(\tau, a) = \delta_{\tau,t}^a P_t(\tau, 0) \)
   - \( \delta_{\tau} \): from the Automobile Dealer Association.

2. Solve for equilibrium prices \( P_t^*(\tau, a) \) (fixing utility parameters).

3. Re-estimate utility parameters given \( P_t^*(\tau, a) \)

4. If new prices do not constitute an equilibrium, go to 2
Structural estimation

Log likelihood

$$\ell_{it}(\theta) = \alpha \left[ \log P_{it}(d_{it}|s_{it}, y_{it}, p_t, m_t, \text{accident}) \right]$$

$$+ (1 - \alpha) \left[ \log P_{it}(d_{it}|s_{it}, y_{it}, p_t, m_t) + 1_{\text{gets rid of car}} \log \Pr(d_{it}^s|s_{it}) \right]$$

$$+ 1_{\text{owns car}} \log \phi \left[ \frac{vkt_{it} - vkt^*_{it}(d_{it}, s_{it}, y_{it}, p_t, m_t)}{\sigma_{vkt}} \right].$$
Structural estimation

Log likelihood

\[ \ell_{it}(\theta) = \alpha \left[ \log P_{it}(d_{it}|s_{it}, y_{it}, p_t, m_t, \text{accident}) \right] \\
+ (1 - \alpha) \left[ \log P_{it}(d_{it}|s_{it}, y_{it}, p_t, m_t) + 1_{\text{gets rid of car}} \log \Pr(d_{it}^s|s_{it}) \right] \\
+ 1_{\text{owns car}} \log \phi \left[ \frac{vkt_{it} - vkt_{it}^*(d_{it}, s_{it}, y_{it}, p_t, m_t)}{\sigma_{vkt}} \right]. \]

Contributions from

1. Car decision (keep, buy, ...),
2. Accidents: forced scrappage (i.e. keep is infeasible),
   - \( \Pr(\text{accident}) \equiv \alpha \) is identified from scrappage of young cars (0.66%).
3. Driving decision (if the household chooses to hold a car).
Results
## Structural Parameter Estimates

### Fixed parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. Hh. age</td>
<td>20</td>
</tr>
<tr>
<td>Max. Hh. age</td>
<td>85</td>
</tr>
<tr>
<td># of car ages</td>
<td>25</td>
</tr>
<tr>
<td># of car types</td>
<td>1</td>
</tr>
<tr>
<td>Clunkers in choiceset</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$ Discount factor</td>
<td>0.95</td>
</tr>
<tr>
<td>$\rho$ Inc. AR(1) term</td>
<td>0.86818</td>
</tr>
<tr>
<td>$\sigma_y$ Inc. s.d.</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_p$ Fuel price AR(1) term</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_y$ Fuel price s.d.</td>
<td>0</td>
</tr>
<tr>
<td>$\Pr(0</td>
<td>0)$ Macro transition</td>
</tr>
<tr>
<td>$\Pr(1</td>
<td>1)$ Macro transition</td>
</tr>
<tr>
<td>Accident prob.</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\lambda$ Logit error var.</td>
<td>1</td>
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</table>

### Monetary Utility

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std.err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_0$ Intercept</td>
<td>0.037476</td>
<td></td>
</tr>
<tr>
<td>$\theta_1$ Inc.</td>
<td>-1.2043e-05</td>
<td>2.54e-08</td>
</tr>
<tr>
<td>$\theta_2$ Inc. sq.</td>
<td>1.171e-08</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_3$ Macro</td>
<td>-0.00084546</td>
<td>9.604e-06</td>
</tr>
</tbody>
</table>

### Car Utility

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std.err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q(a)$ Car age, linear</td>
<td>0.12737***</td>
<td>0.0002645</td>
</tr>
<tr>
<td>$q(a)$ Car age, squared</td>
<td>-0.0010925***</td>
<td>1.157e-05</td>
</tr>
<tr>
<td>$\delta_1$ Car type dummy</td>
<td>1.1433***</td>
<td>0.004135</td>
</tr>
</tbody>
</table>

### Transaction costs

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std.err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed cost</td>
<td>138.8***</td>
<td>0.06326</td>
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<tr>
<td>Proportional cost</td>
<td>0</td>
<td></td>
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</tbody>
</table>

### Scrappage

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std.err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{scrap}}$ Scrappage error var.</td>
<td>1.2154***</td>
<td>0.002351</td>
</tr>
</tbody>
</table>

---

62
Model fit by state variables

CCPs for owners (year=2004)

CCPs for Non-owners (year=2004)

Demand for cars by age and type (year=2004)
Fit by car age: Keep probability

Prob. of Keep by carage (2004)
Fit by household income: Keep probability

Prob. of Keep by inc (2004)

Income in 1000 DKK

Prob. of Keep by hhage (2004)

Household age

Prob. of Keep by carage (2004)

carage

Prob. of Keep by fuelprice (2004)

Fuel price, DKK/l
Fit by household age: Keep probability

Prob. of Keep by hhage (2004)

- Predicted, owners
- Predicted, non-owners
- Data, owners
- Data, non-owners

Household age

Income in 1000 DKK

Fuel price, DKK/l
Non-equilibrium simulations: no clear waves
Equilibrium simulations: clear waves

Car age distribution

Car age

Calendar year

2000
2005
2010
2015
2020
2025
2030
0
500
1000
1500
2000

0
5
10
15
20
25
Car age

Purchases

Calendar time

2000
2005
2010
2015
2020
2025
0
500
1000
1500
2000

0
5
10
15
20
25
Car age

Car prices for type 1 (gasoline)

Calendar year

300
15
200
10
2005

300
15
200
10
2005

0
5
10
15
20
25
Car age
Equilibrium prices

Car prices in 1000 DKK
Scrappage: non-equilibrium simulation

Scrappage decisions

Calendar time
Car age

Car price
Car age
Car prices for type 1 (gasoline)

Purchases
Car age

Scrappage

Calendar year

1000
15 2010
20
1500
25 2015

2000
0
5
2005
10

70
Scrappage: equilibrium simulation

Scrappage decisions

- Car age distribution
- Calendar year
- Car prices for type 1 (gasoline)
- Purchases
- Calendar time

- Scrappage decisions
- Calendar year
- Car age
- Car price

- Purchases
- Calendar time
- Car age

- 2010
- 2015
- 2020
- 2005
- 2000

- 15
- 20
- 25
Counterfactual

- **Illustrative Counterfactual**: Lower new car prices by 10%.
  - Non-equilibrium: Lower new car prices and assume depreciation rates are unchanged.
  - Equilibrium: Change new-car prices and solve for the rest.
Counterfactual 10% Lower New Car Prices: New car sales

Non-equilibrium

Equilibrium
Simulated car stock: non-equilibrium

Car age distribution

- Car age distribution over time, showing the number of cars by age and calendar year from 2000 to 2050.

- The chart illustrates the evolution of the car stock over time, with peaks and troughs indicating changes in car ownership patterns.

- Key observations include:
  - A decrease in car ownership as time progresses, particularly after 2030.
  - Peaks in car ownership around 2010 and 2020, indicating increased purchases.
  - Scrappage decisions are reflected in the decay of older car age distributions over time.
Simulated car stock: equilibrium

Car age distribution

Car age distribution

Car age
Calendar year

Purchases
Car age
Calendar time
Simulated scrappage: non-equilibrium
Simulated scrappage: equilibrium

![Scrapage decisions](image-url)
Nonstationary temporary equilibria
1. Thus for each car type, $\tau$, the equilibrium condition is a system of $a - 1$ smooth nonlinear equations in $a - 1$ unknowns, $P(a, \tau)$, $a \in \{1, \ldots, a\}$ and $\tau \in \{1, \ldots, \tau\}$.
Finding an equilibrium following Berkovec’s approach

1. Thus for each car type, $\tau$, the equilibrium condition is a system of $\bar{a} - 1$ smooth nonlinear equations in $\bar{a} - 1$ unknowns, $P(a, \tau)$, $a \in \{1, \ldots, \bar{a}\}$ and $\tau \in \{1, \ldots, \bar{\tau}\}$

2. We solve for the price vector that sets expected excess demand in each of the $\bar{\tau} \times (a - 1)$ second hand markets to zero

$$EED(P) = 0$$
Finding an equilibrium following Berkovec’s approach

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2. We solve for the price vector that sets expected excess demand in each of the $\bar{\tau} \times (\bar{a} - 1)$ second hand markets to zero

$$EED(P) = 0$$

3. Since $EED$ is differentiable, we can use Newton’s method to find a solution

$$P_{t+1} = P_t - [\nabla EED(P_t)]^{-1} EED(P_t)$$

where $\nabla EED(P_t)$ is the $\bar{\tau}(\bar{a} - 1) \times \bar{\tau}(\bar{a} - 1)$ Jacobian matrix of $EED(P)$. 
Let $\Delta(P, \tau)\Omega$ be the ”demand transition matrix” matrix for type $\tau$

$$
\begin{bmatrix}
\Delta(\emptyset|\emptyset, P, \tau) & \Delta(1|\emptyset, P, \tau)\Omega_1 & \cdots & \Delta(J|\emptyset, P, \tau)\Omega_J \\
\Delta(\emptyset|1, P, \tau) & \Delta(1|1, P, \tau)\Omega_1 & \cdots & \Delta(J|1, P, \tau)\Omega_J \\
\vdots & \vdots & \ddots & \vdots \\
\Delta(\emptyset|J-1, P, \tau) & \Delta(1|J-1, P, \tau)\Omega_1 & \cdots & \Delta(J|J-1, P, \tau)\Omega_J \\
\Delta(\emptyset|J, P, \tau) & \Delta(1|J, P, \tau)\Omega_1 & \cdots & \Delta(J|J, P, \tau)\Omega_J
\end{bmatrix}
$$

where $\Omega_j$ is the $\bar{a}_j \times \bar{a}_j$ transition probability matrix reflecting accidents/scrappage for car type $j$ and $\Delta(j'|j, \tau, P)$ is a $\bar{a}_{j'} \times \bar{a}_j$ “demand transition matrix” for the probability that a consumer with a car of type $j$ chooses a car of type $j'$. 
Stationary equilibrium with finite car/consumer types

1. Let $q_\tau$ be the invariant distribution for consumers of type $\tau$

$$q_\tau = q_\tau \Delta(P, \tau) \Omega$$

2. Define $q$ and $\Delta(P)$ by

$$q = \sum_\tau q_\tau f(\tau)$$

$$\Delta(P) = \sum_\tau \Delta(P, \tau) f(\tau)$$

3. Then $P$ is a stationary equilibrium if and only if

$$q = q \Delta(P) \Omega.$$

4. Solving for equilibrium in the multiple car type model is similar to the single car type case, except for the determination of endogenous scrappage thresholds $(\bar{a}_1, \ldots, \bar{a}_J)$. 
1. Consider the following example economy with $n = 2$ two types of consumers with common discount factors $\beta = .95$ and extreme value scale parameters $\sigma = 1$. 

...
Simple computed examples of stationary equilibrium

1. Consider the following example economy with \( n = 2 \) two types of consumers with common discount factors \( \beta = .95 \) and extreme value scale parameters \( \sigma = 1 \).

2. “Rich consumers” have utility \( u(y, a, \tau_1) = 45 - 3.2a + y \) and “poor consumers” have utility \( u(y, a, \tau_2) = 55.1 - 2.7a + 1.5y \)
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3. Thus rich consumers have greater disutility of owning older cars but a smaller “marginal utility of income” \( y \) which is used to consume the outside good.
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3. Thus rich consumers have greater disutility of owning older cars but a smaller “marginal utility of income” \( y \) which is used to consume the outside good.

4. We assume \( f(\tau_1) = .2 \) and \( f(\tau_2) = .8 \), i.e. 20% of all consumers are rich and 80% are poor.
1. Consider the following example economy with $n = 2$ two types of consumers with common discount factors $\beta = .95$ and extreme value scale parameters $\sigma = 1$.

2. “Rich consumers” have utility $u(y, a, \tau_1) = 45 - 3.2a + y$ and “poor consumers” have utility $u(y, a, \tau_2) = 55.1 - 2.7a + 1.5y$.

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4. We assume $f(\tau_1) = .2$ and $f(\tau_2) = .8$, i.e. 20% of all consumers are rich and 80% are poor.

5. Let $T$ denote transactions costs. We assume $\overline{P} = 280$ and $\underline{P} = 1.5$. 
Heterogeneous agent stationary equilibrium: $T = 0$

Equilibrium Price Functions, $\sigma=1$, $T=0$, $\rho=0$

- Blue: Heterogeneous
- Red: Homogeneous type 1
- Green: Homogeneous type 2

Age of car vs. Equilibrium price of car
Heterogeneous agent stationary equilibrium: $T = 5$

Equilibrium Price Functions, $\sigma=1$, $T=5$, $\rho=0$
Heterogeneous agent stationary equilibrium: \( T = 0 \)

Holdings distributions (start of period, post-trade) \( \sigma = 1, T=0 \ \rho=0 \)

- **Aggregate holdings**
- **Rich consumers**
- **Poor consumers**

Fraction of car stock of each age

Car age:
- 0
- 2
- 4
- 6
- 8
- 10
- 12
- 14
- 16
- 18
- 20

Fraction of car stock of each age:
- 0.05
- 0.1
- 0.15
- 0.2
- 0.25
- 0.3
Heterogeneous agent stationary equilibrium: $T = 5$

Holdings distributions (start of period, post-trade) $\sigma=1$, $T=5$, $\rho=0$

![Graph showing holdings distributions for different consumer groups over the age of cars.](image_url)
1. IRUC = Intelligent Road User Charges
Analysis of IRUC Reform

1. IRUC = Intelligent Road User Charges

2. Idea: instead of the huge 180% new car tax, increase the gas tax or impose a tax on car mileage travelled
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2. Idea: instead of the huge 180% new car tax, increase the gas tax or impose a tax on car mileage travelled

3. Do analysis of a *revenue neutral tax reform* — i.e. need to raise gas tax so that Danish government gets approximately the same revenue from the gas tax as the new car tax
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2. Idea: instead of the huge 180% new car tax, increase the gas tax or impose a tax on car mileage travelled

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4. Compared to the example above, we extend the utility function to allow driving to do this. Get a discrete/continuous choice model
1. IRUC = Intelligent Road User Charges

2. Idea: instead of the huge 180% new car tax, increase the gas tax or impose a tax on car mileage travelled

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1. Using a calibrated model we solved the model under the assumption of a new car tax of 180\%
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2. Then we eliminated the new car tax and instituted a gas tax and iteratively solved the model until the aggregate revenues from the gas tax equals the revenues from the new car tax (7% of government revenues).
1. Using a calibrated model we solved the model under the assumption of a new car tax of 180%.

2. Then we eliminated the new car tax and instituted a gas tax and iteratively solved the model until the aggregate revenues from the gas tax equals the revenues from the new car tax (7% of government revenues).

3. We find a 30% fuel tax raises the same revenue as a 180% new car tax. This is due to a switch in the extensive margin (causing poor consumers who did not own cars to choose to buy them) that overwhelms the substitution in the intensive margin (people drive less due to the higher gross of tax price of driving).
Equilibrium prices with a 180% new car tax

Equilibrium Price Functions, $\sigma=1$, $T=1$, $\rho=0$

- Blue: Heterogeneous
- Red: Homogeneous type 1
- Green: Homogeneous type 2

<table>
<thead>
<tr>
<th>Age of car</th>
<th>Equilibrium price of car</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>250</td>
</tr>
<tr>
<td>2</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>
Prices with 0% new car tax but 30% fuel tax

Equilibrium Price Functions, $\sigma=1$, $T=1$, $\rho=0$

- Heterogeneous
- Homogeneous type 1
- Homogeneous type 2

Axes:
- Y-axis: Equilibrium price of car
- X-axis: Age of car
Equilibrium prices with a 180% new car tax

Holdings distributions (start of period, post-trade) $\sigma=1$, $T=1$ $\rho=0$

- Aggregate holdings
- Rich consumers
- Poor consumers

Car age:
- 0
- 2
- 4
- 6
- 8
- 10
- 12
- 14
- 16
- 18
- 20

Fraction of car stock of each age

- 0.05
- 0.1
- 0.15
- 0.2
- 0.25
- 0.3
- 0.35
- 0.4
- 0.45
- 0.5

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Holdings with 0% new car tax but 30% fuel tax

Holdings distributions (start of period, post-trade) $\sigma=1$, $T=1$, $\rho=0$
1. The huge reduction in prices of cars induces all consumers to own cars. Under the gas tax 8.2% of consumers buy new cars each year and cars are scrapped after 15 years instead of 21 years.
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2. Though the gas tax discourages driving per household (which drives 13% less per year), aggregate driving increases by 42% due to the huge substitution effect of all households owning cars.
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2. Though the gas tax discourages driving per household (which drives 13% less per year), aggregate driving increases by 42% due to the huge substitution effect of all households owning cars.

3. The tax change is a Pareto gain: rich households are better off by €26K and poor households are better off by €4K. This amounts to a welfare gain of €21.8 billion for the 2.66 million Danish households.
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3. The tax change is a Pareto gain: rich households are better off by €26K and poor households are better off by €4K. This amounts to a welfare gain of €21.8 billion for the 2.66 million Danish households.

4. Taxes increase by 353% to €7.5 billion from €2.1 billion.

5. However we did not account for road congestion externalities of so many more cars.
Nonstationary equilibrium in the auto market
1. The simulations above was based on a stationary equilibrium.
2. Why care about nonstationarity?
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2. Why care about nonstationarity? **Answer:** because there are constant shocks to actual markets that continually shift prices and quantities in the market.
1. The simulations above was based on a stationary equilibrium.

2. Why care about nonstationarity? **Answer:** because there are constant shocks to actual markets that continually shift prices and quantities in the market.

3. Rather that attempting to compute the nonstationary equilibrium implied by fully rational expectations, we will instead specify a temporary equilibrium relationship in which traders forecast the future at each date as a function of their information on current states of the economy.
1. The simulations above was based on a stationary equilibrium.
2. Why care about nonstationarity? **Answer:** because there are constant shocks to actual markets that continually shift prices and quantities in the market.
3. Rather that attempting to compute the nonstationary equilibrium implied by fully rational expectations, we will instead specify a temporary equilibrium relationship in which traders forecast the future at each date as a function of their information on current states of the economy.
4. We will operationalize this concept in the car market with a particularly simple/naive adaptive price forecasting function: consumers always believe today’s price structure $P$ will persist in the future.
1. Let \((P_t, q_t)\) be the temporary equilibrium prices and holdings of cars at time \(t\).
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2. In a stationary equilibrium these same prices and quantities will also be a temporary equilibrium in all future dates, and hence the temporary equilibrium is also a perfect foresight equilibrium and hence a rational expectations equilibrium as well.
Nonstationary Temporary equilibria

1. Let \((P_t, q_t)\) be the temporary equilibrium prices and holdings of cars at time \(t\).
2. In a stationary equilibrium these same prices and quantities will also be a temporary equilibrium in all future dates, and hence the temporary equilibrium is also a perfect foresight equilibrium and hence a rational expectations equilibrium as well.
3. However if the economy is not in stationary equilibrium at time \(t\) (or if a shock occurs that affects some aspect of consumer preferences, or income, or new car prices, etc.) then at time \(t + 1\) the original pair \((P_t, q_t)\) no longer constitutes a temporary equilibrium and some new temporary equilibrium \((P_{t+1}, q_{t+1})\) will arise at \(t + 1\) to clear the market for used cars.
Nonstationary Temporary equilibria

1. The result of this is a sequence (or stochastic process) of temporary equilibria \( \{(P_t, q_t)\} \) with the property that at each time \( t \) \( P_t \) clears the market for used cars, i.e. \( EED_t(P_t) = 0 \).
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2. Suppose that at \( t = 0 \) the economy is in a stationary equilibrium and there is a one time permanent shock to the economy at \( t = 1 \). Thus if \( (P_0, q_0) \) is the initial stationary equilibrium at \( t = 0 \), then due to the shock at \( t = 1 \), \( (P_0, q_0) \) will not be an equilibrium any longer at \( t = 1 \). Instead there will be a sequence of temporary equilibria \( \{(P_t, q_t)\} \) originating from this previous shock.
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   .. Instead there will be a sequence of temporary equilibria \( \{(P_t, q_t)\} \) originating from this previous shock.

3. Let \( (P_\infty, q_\infty) \) denote the new stationary equilibrium corresponding to the post-shock economy. Will it be the case that

   \[ \lim_{t \to \infty} (P_t, q_t) = (P_\infty, q_\infty) \]
1. We start by defining \((P_1, q_1)\) and then recursively define temporary equilibria \((P_t, q_t)\) for all \(t \geq 1\).
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2. At \(t = 1\) the market has (permanently shifted) so that \((P_0, q_0)\) is no longer a stationary equilibrium. We assume that prices immediately adjust at \(t = 1\) so that supply of cars \(a = 1, 2, \ldots, \bar{a} - 1\) in the stationary equilibrium at \(t = 0\) equals the (new) demand for these cars in the new regime in periods \(t \geq 1\). That is, we assume \(P_1\) is a solution to

\[
\sum_{\tau} q_{\tau,0}(a)f(\tau) = \sum_{\tau} q_{\tau,0}\Delta_n(P_1, \tau)(a)f(\tau), \quad a \in \{1, 2, \ldots, \bar{a} - 1\}
\]

\[2\]

where \(q_{\tau,0}\) is the stationary holdings distribution for a type \(\tau\) consumer at \(t = 0\).
1. Note that if $\bar{a}_0$ is the scrappage age in the initial stationary equilibrium $(P_0, q_0)$ there will be no car in the economy older than $\bar{a}_0$. However at the start of period $t = 1$ there will be cars of age $a = \bar{a}_0$ which would be scrapped if the equilibrium continued to remain at the stationary equilibrium $(P_0, q_0)$ but in a new equilibrium it is possible that consumers would decide to keep cars of age $\bar{a}_0$ instead.
Definition of Nonstationary Temporary equilibrium

1. Note that if \( \bar{a}_0 \) is the scrappage age in the initial stationary equilibrium \((P_0, q_0)\) there will be no car in the economy older than \( \bar{a}_0 \). However at the start of period \( t = 1 \) there will be cars of age \( a = \bar{a}_0 \) which would be scrapped if the equilibrium continued to remain at the stationary equilibrium \((P_0, q_0)\) but in a new equilibrium it is possible that consumers would decide to keep cars of age \( \bar{a}_0 \) instead.

2. In that case the largest possible scrap age would be \( \bar{a}_0 + 1 \) since the absence of any vehicles of this age in the previous equilibrium implies that there is a supply of used cars of age \( \bar{a}_0 \) but zero supply of cars of age \( \bar{a}_0 + 1 \).
1. It follows that there can only be trading in used cars up to age $\bar{a}_0$ at period $t = 1$, but there can be no trading in cars of age $\bar{a}_0 + 1$ or older due to the fact that there is no supply of cars of these older ages in the economy at this point.
Definition of Nonstationary Temporary equilibrium

1. It follows that there **can only be trading in used cars up to age \( \bar{a}_0 \) at period \( t = 1 \), but there can be no trading in cars of age \( \bar{a}_0 + 1 \) or older due to the fact that there is no supply of cars of these older ages in the economy at this point.

2. Call the scrappage age from the largest solution to (2) \( \bar{a}_1 \).
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2. Call the scrappage age from the largest solution to (2) $\bar{a}_1$.

3. Given $P_1$ define $q_{\tau,1}^0$ as follows: for $a \in \{1, 2, \ldots, \bar{a}_1 - 1\}$ we set $q_{\tau,1}^0 = q_{\tau,0}(a)$. We define $q_{\tau,1}^0(\emptyset)$, the fraction of type $\tau$ consumers who choose not to own a car by
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2. Call the scrappage age from the largest solution to (2) $\bar{a}_1$.

3. Given $P_1$ define $q^0_{\tau,1}$ as follows: for $a \in \{1, 2, \ldots, \bar{a}_1 - 1\}$ we set $q^0_{\tau,1} = q_{\tau,0}(a)$. We define $q^0_{\tau,1}(\emptyset)$, the fraction of type $\tau$ consumers who choose not to own a car by

$$q^0_{\tau,1}(\emptyset) = \prod(\emptyset|\emptyset, \tau, P_1)q_{\tau,0}(\emptyset) + \sum_{a=1}^{\bar{a}_1-1} \prod(\emptyset|a, \tau, P_1)q_{\tau,0}(a). \quad (3)$$
1. Finally, let the total quantity of new cars held by type $\tau$ consumers just after trading at the prices $P_1$ at the start of period 1 be given by

$$q_{\tau,1}^0(0) = \Pi(0|\emptyset, \tau, P_1)q_{\tau,0}(\emptyset) + \sum_{a=1}^{\overline{a}_1-1} \Pi(0|a, \tau, P_1)q_{\tau,0}(a). \quad (4)$$
1. Finally, let the total quantity of new cars held by type $\tau$ consumers just after trading at the prices $P_1$ at the start of period 1 be given by

$$q^{0}_{\tau,1}(0) = \Pi(0|\emptyset, \tau, P_1)q^{0}_{\tau,0}(\emptyset) + \sum_{a=1}^{\bar{a}_1-1} \Pi(0|a, \tau, P_1)q^{0}_{\tau,0}(a). \quad (4)$$

2. Now define $q_{\tau,1} = q^{0}_{\tau,1}\Omega_n$. This is the distribution of holdings of cars by type $\tau$ consumers at the end of period $t = 1$ (thus at the start of period 2, but before trading occurs at prices $P_2$). Finally let $q_1$ be the type-weighted average,

$$q_1 = \sum_{\tau} q_{\tau,1} f(\tau). \quad (5)$$
Definition of Nonstationary Temporary equilibrium

1. Thus, we have defined \((P_1, q_1)\), the temporary equilibrium prices and quantities in the first period after a permanent shock that disturbs the market from its initial stationary equilibrium \((P_0, q_0)\) at \(t = 0\).
Definition of Nonstationary Temporary equilibrium

1. Thus, we have defined \((P_1, q_1)\), the temporary equilibrium prices and quantities in the first period after a permanent shock that disturbs the market from its initial stationary equilibrium \((P_0, q_0)\) at \(t = 0\).

2. At time \(t = 2\) we can follow the same procedure: given the quantities \(q_{\tau,1}\) held by each consumer type \(\tau\), we use equation (2) with time indices shifted forward one period to \(t = 1\) to define the temporary equilibrium price vector \(P_2\) and corresponding scrappage age \(\bar{a}_2\), where \(\bar{a}_2\) is the largest scrappage age from the potentially multiple solutions to equation (2) that satisfy a) monotonicity, and b) \(P_2(a) \geq P_n\), \(a \in \{1, 2, \ldots, \bar{a}_2 - 1\}\) but subject to the constraint that \(\bar{a}_2 \leq \bar{a}_1 + 1\) where \(\bar{a}_1\) is the scrappage age in the temporary equilibrium \((P_1, q_1)\).
1. $\bar{a}_{t+1}$ can be at most 1 more the scrappage age in the previous TE, $\bar{a}_t$, since there is zero supply of cars that are older than $\bar{a}_t$ in the previous TE.
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2. Given $P_2$ we can define $q_{t,2}^0$, the post-trade age distribution of cars in period 2, using equations (3) and (4) but with time indices shifted one period ahead. Let $q_{t,2} = q_{t,2}^0 \Omega_n$ be the holdings of cars by type $\tau$ consumers at the end of period $t = 2$ (thus at the start of period 3, but before trading occurs at prices $P_3$), and $q_2$ be the type-weighted average,

$$q_2 = \sum_{\tau} q_{\tau,2} f(\tau). \quad (6)$$
1. $\bar{a}_{t+1}$ can be at most 1 more the scrappage age in the previous TE, $\bar{a}_t$, since there is zero supply of cars that are older than $\bar{a}_t$ in the previous TE.

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$$q_2 = \sum_{\tau} q_{\tau,2} f(\tau). \quad (6)$$

3. Thus, we have defined $(P_2, q_2)$, the temporary equilibrium prices and quantities in period $t = 2$, using as initial condition the temporary equilibrium $(P_1, q_1)$ at $t = 1$. 
Impact of one time rise in $\bar{P} = 240$ to $\bar{P} = 280$

Evolution of temporary equilibria after a one time rise in $\bar{P}$ from 240 to 280
Impact of one time rise in $\bar{P} = 240$ to $\bar{P} = 280$

Evolution of temporary equilibria after a one time rise in $\bar{P}$ from 240 to 280
Impact of one time fall in $\bar{P} = 240$ to $\bar{P} = 200$

Evolution of temporary equilibria after a one time fall in $\bar{P}$ from 240 to 200

![3D Graph showing the evolution of temporary equilibria after a one time fall in $\bar{P}$ from 240 to 200. The graph illustrates the relationship between Car age, a, Calender Time, t, and Price of car.](image)
Impact of one time rise in \( \bar{P} = 240 \) to \( \bar{P} = 200 \)

Evolution of temporary equilibria after a one time fall in \( \bar{P} \) from 240 to 200
Impact of repeated fluctuations in $\bar{P}$

Secondary market price waves caused by fluctuation in new car prices
Impact of repeated fluctuations in $P$

Waves in holding distributions for car caused by fluctuation in new car prices
Ride the wave

Evolution of temporary equilibria after a one time fall in $P$ from 240 to 200
Conclusion

- **Simulated waves:** Estimates reproduce waves in simulations, and it is essential to ensure that prices are in equilibrium.

- **Equilibrium model with heterogeneity:** Sorting of high and low quality cars on rich and poor consumers.

- **Proof of concept for estimation procedure:** Works well with a sub-sample of the data.

- **Future work:**
  - Add further heterogeneity (e.g., region of residence, commute distances, more vehicle classes).
  - Analyze quantitative effect of Road User Charging reform based on the empirical model.
  - Account for road congestion externalities and pollution in counterfactual.
Thank you for all your attention
Appendix: Data
## Summary Statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>$N$</th>
<th>Mean</th>
<th>sd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age of H.</td>
<td>22,041,601</td>
<td>38.93</td>
<td>11.66</td>
</tr>
<tr>
<td>Real income (2005 kr)</td>
<td>22,041,601</td>
<td>403,820.70</td>
<td>403,550.81</td>
</tr>
<tr>
<td>Urban resident</td>
<td>22,041,601</td>
<td>0.32</td>
<td>0.47</td>
</tr>
<tr>
<td>Work distance of H.</td>
<td>22,041,601</td>
<td>20.81</td>
<td>86.96</td>
</tr>
<tr>
<td>Unemployment for H.</td>
<td>16,242,835</td>
<td>0.08</td>
<td>0.28</td>
</tr>
<tr>
<td>Dummy for couple</td>
<td>22,041,601</td>
<td>0.45</td>
<td>0.50</td>
</tr>
<tr>
<td>Num of kids</td>
<td>22,041,601</td>
<td>0.61</td>
<td>0.97</td>
</tr>
<tr>
<td>Car age in years</td>
<td>7,085,310</td>
<td>7.27</td>
<td>4.88</td>
</tr>
<tr>
<td>Fuel price (period)</td>
<td>6,362,373</td>
<td>8.76</td>
<td>0.63</td>
</tr>
<tr>
<td>Fuel price (annual)</td>
<td>22,041,601</td>
<td>8.37</td>
<td>1.32</td>
</tr>
<tr>
<td>Dummy for diesel car</td>
<td>22,041,601</td>
<td>0.02</td>
<td>0.14</td>
</tr>
<tr>
<td>Total weight of car</td>
<td>7,085,310</td>
<td>1,576.66</td>
<td>217.83</td>
</tr>
<tr>
<td>Fuel efficiency (km/l)</td>
<td>3,547,818</td>
<td>14.21</td>
<td>2.49</td>
</tr>
<tr>
<td>VKT (km traveled/day)</td>
<td>7,085,310</td>
<td>46.52</td>
<td>24.22</td>
</tr>
<tr>
<td>Years to test</td>
<td>7,085,310</td>
<td>4.03</td>
<td>3.65</td>
</tr>
</tbody>
</table>

**Notes:** “H.” refers to the head of the household.

All Danish kroner (kr) in 2005 kroner.
Choiceset characteristics: diesel vs. gas
Older people buy newer cars

Age of purchased car by household age

Note: Local linear regression.
**Definition (Scrappage):** The last ownership spell ends for a given car.

Note: each bar shows the percentage of cars and vans that were scrapped or sold to another household at each age.
Car taxes 1980-2012

Car Taxation in Denmark, 1980-2012

Bill. DKK


Registration tax
Fuel taxes
Ownership taxes
Young households do not own cars

Note: each bar shows the share in each car state within each household age group.
Discrete Choices by Income

Note: each curve shows the percentage of households within the given income decile that is in the particular car state in a given year.
Vehicle Km Travelled (VKT) by household income

Controls: none.
Selection: VKT within 1% and 99% percentiles, year in [1996;2009] and household age in [18;65].
Vehicle Km Travelled (VKT) by household age

Controls: none.
Selection: VKT within 1% and 99% percentiles, year in [1996;2009] and household age in [18;65].
The poor purge more
Number of Cars Owned by Income

Note: each curve shows the percentage of households within the given income decile that own the particular number of cars in a given year.
Appendix: Model
Bellman equation

\[ V_t(\tau_t, a_t, y_t, p_t, m_t) = \]

\[ \max_{d_t, d^+_t, vkt_t} \left\{ u_t(s_t, s'_t, d^+_t, vkt_t, y_t, p_t, m_t) + \right. \]

\[ + \beta \mathbb{E}_t \left[ V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}) \mid y_t, p_t, m_t \right] \]
Bellman equation

$$V_t(\tau_t, a_t, y_t, p_t, m_t) =$$

$$\max_{d_t, d_t^s, vkt_t} \left\{ u_t(s_t, s_t', d_t^s, vkt_t, y_t, p_t, m_t) +$$

$$\beta E_t \left[ V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}) \bigg| y_t, p_t, m_t \right] \right\}$$

- **Driving**, $vkt_t$, is static,
Bellman equation

\[ V_t(\tau_t, a_t, y_t, p_t, m_t) = \]

\[
\max_{d_t, d_t^S, vkt_t} \left\{ u_t(s_t, s_t', d_t^S, vkt_t, y_t, p_t, m_t) + \right. \\
\left. + \beta E_t \left[ V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1} | y_t, p_t, m_t) \right] \right\}
\]

- **Driving**, \( vkt_t \), is static,
- **Scrappage**, \( d_t^S \), is static,
Bellman equation

\[ V_t(\tau_t, a_t, y_t, p_t, m_t) = \]

\[ \max_{d_t, d^s_t, vkt_t} \left\{ u_t(s_t, s'_t, d^s_t, vkt_t, y_t, p_t, m_t) + \right. \]

\[ + \beta E_t \left[ V_{t+1}(s_{t+1}, y_{t+1}, p_{t+1}, m_{t+1}) \left| y_t, p_t, m_t \right. \right] \right\} \]

- Driving, \( vkt_t \), is static,
- Scrappage, \( d^s_t \), is static,
- Continuation value depends only on the chosen car,

\[ s_{t+1} = s_{t+1}(s_t, d_t). \]
Appendix: Results
Some intuition on identification

- **Outside option**: Car type dummies shift the flow utility of having a car (absent usage).

- **Car type choice**: Car characteristics (fuel efficiency, weight, ...), (new) car prices and type dummies.

- **Heterogeneity**:
  - Income affects the utility of money.
  - Driving patterns change over the lifecycle (e.g. kids).

- **Scrappage**: Depends on the difference between market and scrap price.
  - The $\lambda$ parameter determines the sensitivity to this.

- **Waves**: The macro state shifts the utility of money
  - I.e. money becomes more dear in bad times.
  - Persistence due to transactions costs and price adjustments.
1st stage driving parameter estimates

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>std.err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_0$</td>
<td>Const</td>
<td>35.74***</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>Car age</td>
<td>-0.3444***</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>Car age sq.</td>
<td>0.002467**</td>
</tr>
<tr>
<td>$\kappa_3$</td>
<td>m</td>
<td>-19.63***</td>
</tr>
<tr>
<td>$\kappa_4$</td>
<td>Inc</td>
<td>-0.0004118***</td>
</tr>
<tr>
<td>$\kappa_5$</td>
<td>Inc sq.</td>
<td>3.826e-07***</td>
</tr>
<tr>
<td>$\kappa_6$</td>
<td>HH age</td>
<td>0.3178***</td>
</tr>
<tr>
<td>$\kappa_7$</td>
<td>HH age sq.</td>
<td>-0.004956***</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>std.err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa_8$</td>
<td>PPK</td>
<td>-26.46***</td>
</tr>
<tr>
<td>$\kappa_9$</td>
<td>PPK*inc</td>
<td>0.0007932***</td>
</tr>
<tr>
<td>$\kappa_{10}$</td>
<td>PPK*inc sq.</td>
<td>-5.81e-07***</td>
</tr>
<tr>
<td>$\kappa_{11}$</td>
<td>PPK*m</td>
<td>27.34***</td>
</tr>
</tbody>
</table>

Avg. PPK-elasticity | -0.6652 |
$R^2$        | 0.1030 |
$N$        | 111231 |

Data for all years 1996–2009 is used.

PPK: Price Per Kilometer (fuel price / efficiency).
Lower transaction costs

Motivation: Estimated transaction costs are unrealistically high.

Solution: Fix at lower value.

Fixed part: 20,000 DKK (about 5%-10% of new car price),
Proportional: 20% of the car’s price.

Who? Levied both on the seller and buyer.

Result: Vast underprediction of keep probability.
Lower transaction costs: model fit

Prob. of Keep by inc (2002)

Prob. of Keep by hhage (2002)

Prob. of Keep by carage (2002)

Prob. of Keep by fuelprice (2002)
Lower transaction costs: simulation forward in time

Simulation of the car stock

Car age distribution

Calendar time

New cars

Clunkers
Searching for equilibrium: nonparametric prices

Supply and demand, Car type 1

Supply and demand, Car type 2

Excess demand, Car type 1

Excess demand, Car type 2

Price functions

Type 1

Type 2
Searching for equilibrium: nonparametric prices

Supply and demand, Car type 1

Excess demand, Car type 1

Supply and demand, Car type 2

Excess demand, Car type 2

Price functions
Searching for equilibrium: nonparametric prices

Supply and demand, Car type 1
- Demand
- Supply

Supply and demand, Car type 2
- Demand
- Supply

Excess demand, Car type 1
- Excess Demand

Excess demand, Car type 2
- Excess Demand

Price functions
- Type 1
- Type 2
Searching for equilibrium: nonparametric prices

[Graphs showing supply and demand for Car types 1 and 2, as well as excess demand for each type, alongside price functions for both types.]
Searching for equilibrium: nonparametric prices

- Supply and demand, Car type 1
- Supply and demand, Car type 2
- Excess demand, Car type 1
- Excess demand, Car type 2
- Price functions

- Type 1
- Type 2
Searching for equilibrium: nonparametric prices

Supply and demand, Car type 1

Supply and demand, Car type 2

Excess demand, Car type 1

Excess demand, Car type 2

Price functions
Searching for equilibrium: nonparametric prices

Supply and demand, Car type 1

Supply and demand, Car type 2

Excess demand, Car type 1

Excess demand, Car type 2

Price functions

Return
Searching for equilibrium: nonparametric prices

**Supply and demand, Car type 1**
- Demand
- Supply

**Excess demand, Car type 1**
- Excess Demand

**Supply and demand, Car type 2**
- Demand
- Supply

**Excess demand, Car type 2**
- Excess Demand

**Price functions**
- Type 1
- Type 2
Searching for equilibrium: nonparametric prices
Searching for equilibrium: nonparametric prices

Supply and demand, Car type 1
- Demand
- Supply

Excess demand, Car type 1

Supply and demand, Car type 2
- Demand
- Supply

Excess demand, Car type 2

Price functions
- Type 1
- Type 2
Searching for equilibrium: nonparametric prices

Supply and demand, Car type 1
- Demand
- Supply

Excess demand, Car type 1
- Excess Demand

Supply and demand, Car type 2
- Demand
- Supply

Excess demand, Car type 2
- Excess Demand

Price functions
- Type 1
- Type 2