GAMING PERFORMANCE FEES BY PORTFOLIO MANAGERS*

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We show that it is very difficult to devise performance-based compensation contracts that reward portfolio managers who generate excess returns while screening out managers who cannot generate such returns. Theoretical bounds are derived on the amount of fee manipulation that is possible under various performance contracts. We show that recent proposals to reform compensation practices, such as postponing bonuses and instituting clawback provisions, will not eliminate opportunities to game the system unless accompanied by transparency in managers' positions and strategies. Indeed there exists no compensation mechanism that separates skilled from unskilled managers solely on the basis of their returns histories.

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I. Background

Incentives for financial managers are coming under increased scrutiny because of their tendency to encourage excessive risk-taking. In particular, the asymmetric treatment of gains and losses gives managers an incentive to increase leverage and take on other forms of risk without necessarily increasing expected returns for investors. Various changes to the incentive structure have been proposed to deal with this problem, including postponing bonus payments, clawing back bonus payments if later performance is poor, requiring managers to hold an equity stake in the funds that they manage, and so forth. These apply both to managers of financial institutions, such as banks, and also to managers of private investment pools, such as hedge funds.[1]

The purpose of this paper is to show that, while these and related reforms may *moderate* the incentives to game the system, gaming cannot be eliminated. The problem is especially acute when there is no transparency, so investors cannot see the trading strategies that are producing the returns for which managers are being rewarded. In this setting, where managerial compensation is based solely

[1] For a general discussion of managerial incentives in the financial sector see Bebchuk and Fried (2004) and Bebchuk and Spamann (2009). The literature on incentives and risk-taking by portfolio managers will be discussed in greater detail in section 2.

on historical performance, we establish two main results. First, if a performance-based compensation contract does not levy *out-of-pocket penalties for underperformance*, then managers with no superior investment skill can capture a sizable amount of the fees that are intended to reward superior managers by mimicking the latter's performance. The potential amount of fee capture has a concise analytical expression. Second, if a compensation contract imposes penalties that are sufficiently harsh to deter risk-neutral mimics, then it will also deter managers of arbitrarily high skill levels. In other words, there exist no performance-based compensation schemes that screen out risk-neutral mimics while rewarding managers who generate excess returns. This contrasts with statistical measures of performance, some of which can discriminate in the long run between "expert" and "non-expert" managers.[2]

[2] There is a substantial literature on statistical tests that discriminate between true experts and those who merely pretend to be experts. In finance, Goetzmann, Ingersoll, Spiegel, and Welch (2007) propose a class of measures of investment performance that we discuss in greater detail in section 6. Somewhat more distantly related is the literature on how to distinguish between experts who can predict the probability of future events and imposters who manipulate their predictions in order to look good (Lehrer, 2001; Sandroni, Smorodinsky, and Vohra, 2003; Sandroni, 2003; Olszewski and Sandroni, 2008, forthcoming). Another paper that is thematically related is Spiegler (2006), who shows how 'quacks' can survive in a market due to the difficulty that customers have in distinguishing them from the real thing.

Our results are proved using a combination of game theory, probability theory, and elementary principles of mechanism design. One of the novel theoretical elements is the concept of *performance mimicry*. This is analogous to a common biological strategy known as "mimicry" in which one species sends a signal, such as a simulated mating call, in order to lure potential mates, who are then devoured. An example is the firefly *Photuris versicolor*, whose predaceous females imitate the mating signals of females from other species in order to attract passing males, some of whom respond and are promptly eaten (Lloyd, 1974). Of course, the imitation may be imperfect and the targets are not fooled all of the time, but they are fooled often enough for the strategy to confer a benefit on the mimic.[3]

In this paper we shall apply a variant of this idea to modeling the competition for customers in financial markets. We show that portfolio managers with no private information or special investment skills can generate returns over an extended period of time that look just like the returns that would be generated by highly skilled managers; moreover, they can do so without any knowledge of

[3] Biologists have documented a wide range of mimicking repertoires, including males mimicking females, and harmless species mimicking harmful ones in order to deter predators (Alcock, 2005).

how the skilled managers actually produce such returns.[4]

Of course, a mimic cannot reproduce a skilled manager's record forever; instead he reproduces it with a certain probability and pays for it by taking on a small probability of a large loss. In practice, however, this probability is sufficiently small that the mimic can get away with the imitation for many years (in expectation) without being discovered. Our framework allows us to derive precise analytical expressions for: i) the probability with which an unskilled manager can mimic a skilled one over any specified length of time; and ii) the minimum amount the mimic can expect to earn in fees as a function of the compensation structure.

The paper is structured as follows. In the next section we review the prior theoretical and empirical literature on performance manipulation. In section 3 we introduce the model, which allows us to evaluate a very wide range of

[4] It should be emphasized that mimicry is not the same as *cloning* or *replication (*Kat and Palaro, 2005; Hasanhodzic and Lo, 2007). These strategies seek to reproduce the statistical properties of a given fund or class of funds, whereas mimicry seeks to fool investors into thinking that returns are being generated by one type of distribution when in fact they are being generated by a different (and less desirable) distribution. Mimicry is also distinct from strategy stealing, which is a game-theoretic concept that involves one player copying the entire strategy of another (Gale, 1974). In our setting the performance mimic cannot steal the skilled manager's investment strategy because if he knew the strategy then he too would be skilled.

compensation contracts and different ways of manipulating them. Section 4 shows how much fee capture is possible under any compensation arrangement that does not assess personal financial penalties on the manager. In section 5 we explore the implications of this result through a series of concrete examples. Section 6 discusses manipulation-proof performance measures and why they do not solve the problem of designing manipulation-proof compensation schemes. In section 7 we derive an impossibility theorem, which shows that there is essentially no compensation scheme that is able to reward skilled managers and screen out unskilled managers based solely on their 'track records'. Section 8 shows how to extend these results to allow for the inflow and outflow of money based on prior performance. Section 9 concludes.

II. Related literature

The fact that standard compensation contracts give managers an incentive to manipulate returns is not a new observation; indeed there is a substantial prior literature on this issue. In particular, the two-part fee structure that is common in the hedge fund industry has two perverse features: the fees are convex in the level of performance, and gains and losses are treated asymmetrically. These features create incentives to take on increased risk, a point that has been discussed in both the empirical and theoretical finance literature (Starks, 1987; Carpenter, 2000; Lo, 2001; Hodder and Jackwerth, 2007).

The approach taken here builds on this work by considering a much more general class of compensation contracts and by deriving theoretical bounds on how much manipulation is possible. Of the prior work on this topic, Lo (2001) is the closest to ours because he focuses explicitly on the question of how much money a strategic actor can make by deliberately manipulating the returns distribution using options trading strategies. Lo examines a hypothetical situation in which a manager takes short positions in S&P 500 put options that mature in 1-3 months, and shows that such an approach would have generated very sizable excess returns relative to the market in the 1990s. (Of course this strategy could have lost a large amount of money if the market had gone down sufficiently.) The present paper builds on Lo's approach by examining how far this type of manipulation can be taken and how much fee capture is theoretically possible. We do this by explicitly defining the strategy space that is available to potential entrants, and how they can use it to mimic high-performance managers.

A related strand of the literature is concerned with the potential manipulation of standard performance measures, such as the Sharpe ratio, the appraisal ratio, and Jensen's alpha. It is well-known that these and other measures can be 'gamed' by manipulating the returns distribution without generating excess returns in expectation (Ferson and Siegel, 2001; Lhabitant, 2000). It is also known, however, that one can design performance measures that are immune to many forms of manipulation. These take the form of a constant relative risk aversion utility function averaged over the returns history (Goetzmann, Ingersoll, Spiegel, and Welch, 2007). We shall discuss these connections further in section 6. Our main conclusion, however, is that a similar possibility theorem does *not* hold for compensation mechanisms. At first this may seem surprising: for example, why would it not suffice to pay fund managers according to a linear increasing function of one of the manipulation-proof measures mentioned above? The difficulty is that a compensation mechanism must not only reward managers according to their actual ability, it must also *screen out* managers who have no ability. In other words, the mechanism must create incentives for skilled managers to participate and for unskilled managers *not* to participate. This turns out to be considerably more demanding because managers of different skill levels have different opportunity costs and therefore different incentive-compatibility constraints.

III. The model

Performance-based compensation contracts rely on two types of inputs: the returns generated by the fund manager and the returns generated by a benchmark portfolio that serves as a comparator. Consider first a benchmark portfolio that generates a sequence of returns in each of T periods. Throughout we shall assume that returns are reported at discrete intervals, say at the end of each month or each quarter (though the value of the asset may evolve in continuous time). Let r_{ft} be the risk-free rate in period *t* and let X_t be the total return of the benchmark portfolio in period *t*, where X_t is a nonnegative random variable whose distribution may depend on the prior realizations $x_1, x_2, ..., x_{t-1}$. A fund that has initial value $s_0 > 0$ and is passively invested in the benchmark will therefore have value $s_0 \prod_{1 \le t \le T} X_t$ by the end of the T^{th} period. If the benchmark asset is risk-free then $X_t = 1 + r_{ft}$. Alternatively, X_t may represent the return on a broad market index such as the S&P 500, in which case it is stochastic, though we do not assume stationarity.

Let the random variables $Y_t \ge 0$ denote the period-by-period returns generated by a particular managed portfolio, $1 \le t \le T$. A compensation contract is typically based on a comparison between the returns Y_t and the returns X_t generated by a suitably chosen benchmark. It will be mathematically convenient to express the returns of the managed portfolio as a *multiple* of the returns generated by the benchmark asset. Specifically, let us assume that $X_t > 0$ in each period *t*, and consider the random variable $M_t \geq 0$ such that

(1) $$Y_t = M_t X_t \,.$$

A *compensation contract* over *T* periods is a vector-valued function $\phi : R_+^{2T} \to R^{T+1}$ that specifies the payment to the manager in each period $t = 0,1,2,...,,T$ as a function of the amount of money invested and the realized sequences $\vec{x} = (x_1, x_2, ..., x_T)$ and $\vec{m} = (m_1, m_2, ..., m_T)$. We shall assume that the payment in period *t* depends only on the realizations $x_1, ..., x_t$ and $m_1, ..., m_t$. We shall also assume that the payment is made at the *end* of the period, and cannot exceed the funds available at that point in time. (Payments due at the start of a period can always be taken out at the end of the *preceding* period, so this involves no real loss of generality. The payment in period zero, if any, corresponds to an upfront management fee.)

This formulation is very general, and includes standard incentive schemes as well as commonly-proposed reforms, such as 'postponement' and 'clawback' arrangements, in which bonuses earned in prior periods can be offset by maluses in later periods. These and a host of other variations are embedded in the assumption that the payment in period *t*, $\phi_t(\vec{m},\vec{x})$, can depend on the *entire sequence* of returns through period *t*.

Let us consider a concrete example. Suppose that the contract calls for a 2% management fee that is taken out at the *end* of each year plus a 20% performance bonus on the return generated during the year in excess of the risk-free rate. Let the initial size of the fund be s_0. Given a pair of realizations $(\vec{m},\vec{x})$, let $s_t = s_t(\vec{m},\vec{x})$ be the size of the fund at the *start* of year t after any upfront fees have been deducted. Then the management fee at the end of the first year will be $0.02m_1x_1s_1$ and the bonus will be $0.2(m_tx_t - 1 - r_{ft})_+ s_1$. Hence

(2) $$\phi_1 = [.02m_1x_1 + .2(m_1x_1 - 1 - r_{f1})_+]s_1 .$$

Letting $s_2 = s_1 - \phi_1$ and continuing recursively we find that in each year *t*,

(3) $$\phi_t = [.02 m_t x_t + .2(m_t x_t - 1 - r_{ft})_+] s_t .$$

Alternatively, suppose that the contract specifies a 2% management fee at the end of each year plus a *one-time* 20% performance bonus that is paid only at the end of T years. In this case the size of the fund at the start of year t is $s_t(\vec{m}, \vec{x}) = s_0 (.98)^{t-1} \prod_{1 \le s \le t-1} m_s x_s$. The management fee in the t^{th} year equals

(4) $$\phi_t(\vec{m}, \vec{x}) = .02 s_t(\vec{m}, \vec{x}) m_t x_t = [.02(.98)^{t-1} \prod_{1 \le s \le t} m_s x_s] s_0 .$$

The final performance bonus equals 20% of the *cumulative* excess return relative to the risk-free rate, which comes to $.2[\prod_{1 \le t \le T} m_t x_t - \prod_{1 \le t \le T} (1 + r_{ft})]_+ s_0$.

IV. Performance mimicry

We shall say that a manager has *superior skill* if, in expectation, he delivers excess returns relative to a benchmark portfolio (such as a broad-based market index), either through private information, superior predictive powers, or access to payoffs outside the benchmark payoff space. A manager has *no skill* if he cannot deliver excess returns relative to the benchmark portfolio. Investors should not be willing to pay managers with no skill, because the investors can obtain the same expected returns by investing passively in the benchmark. We claim, however, that under any performance-based compensation contract, either the unskilled managers can capture some of the fees intended for the skilled managers, or else the contract is sufficiently unattractive that both the skilled and unskilled managers will not wish to participate.

We begin by examining the case where the contract calls only for nonnegative payments, that is, $\phi_t(\vec{m},\vec{x}) \geq 0$ for all $t,\vec{m},\vec{x}$. (In section 7 we shall consider the situation where $\phi_t(\vec{m},\vec{x}) < 0$ for some realizations $\vec{m}$ and $\vec{x}$.) Note that nonnegative payments are perfectly consistent with clawback provisions, which reduce prior bonuses but do not normally lead to net assessments against the manager's personal assets.

Given realized sequences $\vec{m}$ and $\vec{x}$, define the manager's *cut* in period *t* to be the fraction of the available funds at the end of the period that the manager takes in fees, namely,

(5) $$c_t(\vec{m},\vec{x}) = \phi_t(\vec{m},\vec{x}) / m_t x_t s_t(\vec{m},\vec{x}) .$$

By assumption the fees are nonnegative and cannot exceed the funds available, hence $0 \le c_t(\vec{m},\vec{x}) \le 1$ for all $\vec{m},\vec{x}$. (If $m_t x_t s_t(\vec{m},\vec{x}) = 0$ we let $c_t(\vec{m},\vec{x}) = 1$ and assume that the fund closes down.) The *cut function* is the vector-valued function $c : R_+^{2T} \to [0,1]^{T+1}$ such that $c(\vec{m},\vec{x}) = (c_0(\vec{m},\vec{x}), c_1(\vec{m},\vec{x}), ..., c_T(\vec{m},\vec{x}))$ for each pair $(\vec{m},\vec{x})$.

In our earlier example with a 2% end-of-period management fee and a 20% annual bonus, the cut function is

(6) $$c_0(\vec{m},\vec{x}) = 0 \text{ and } c_t(\vec{m},\vec{x}) = .02 + .2[1 - \frac{1+r_{ft}}{m_t x_t}]_+ \text{ for } 1 \le t \le T .$$

Proposition 1. *Let ϕ be a nonnegative compensation contract over T periods that is benchmarked against a portfolio generating returns $\vec{X} = (X_1, X_2, ..., X_T) > \vec{0}$, and let c be the associated cut function. Given any target sequence of excess returns $\vec{m} \ge \vec{1}$ there exists a mimicking strategy $\vec{M}^0(\vec{m})$ that delivers zero expected excess returns in every period ($E[M_t^0] = 1$), such that for every realization $\vec{X} = \vec{x}$ of the benchmark asset, the mimic's expected fees in period t (conditional on $\vec{x}$) are at least*

(7) $$c_t(\vec{m},\vec{x})[(1 - c_0(\vec{m},\vec{x})) \cdots (1 - c_{t-1}(\vec{m},\vec{x}))][x_1 \cdots x_t] s_0 .$$

Note that, in this expression, the factor $[(1-c_0(\vec{m},\vec{x}))\cdots(1-c_{t-1}(\vec{m},\vec{x}))]$ is the fraction left over after the manager has taken out his cut in previous periods. Hence the proposition says that in expectation the mimic's cut in period t, $c_t(\vec{m},\vec{x})$, is the *same* as the cut of a skilled manager who generates the excess returns sequence $\vec{m}$ with certainty. The difference is that the mimic's cut is assessed on a fund that is compounding at the rate of the benchmark asset ($\prod_{1\le s\le t} x_s$), whereas the skilled manager's cut is based on a portfolio compounding at the higher rate $\prod_{1\le s\le t} m_s x_s$. It follows that the skilled manager will earn more than the mimic in expectation. The key point, however, is not that skilled managers earn more than mimics, but that mimics can earn a great deal compared to the alternative, which is not to enter the market at all.

To understand the implications of this result, let us work through a simple example. Suppose that the benchmark asset consists of risk-free government bonds growing at a fixed rate of 4% per year. Consider a skilled manager who can deliver 10% over and above this every year, and is paid according to the standard two and twenty contract: a bonus equal to 20% of the excess return plus a management fee of 2%.[5] In this case the excess annual return is $(1.10)(1.04)-1.04=0.104$, so the performance bonus is $.20(0.104)=0.0208$ per dollar in the fund at the start of the period. This comes to about 0.0208/[(1.10)(1.04)] = 0.0182 per dollar at the end of the period. By assumption the management fee is .02 per dollar at the end of the period. Therefore the cut, which is the total fee per dollar at the end of the period, is 0.0382 or 3.82%.

Proposition 1 says that a manager with no skill has a mimicking strategy that *in expectation* earns at least 3.82% *per year* of a fund that is compounding at 4% per year before fees, and 0.027% after fees ($1.04(1-0.0382)=1.00027$). As t becomes large the probability goes to one that the fund will go bankrupt before then. However, the mimic's *expected* earnings in any given year t are actually increasing with t, because in expectation the fund is compounding at a faster rate (4%) than the manager is taking out the fees (3.82%). The key to proving proposition 1 is the following result.

[5] Of course it is unlikely that anyone would generate the same return year after year but this assumption keeps the computations simple.

Lemma. *Consider any target sequence of excess returns* $\vec{m} = (m_1, ..., m_T) \geq (1,1,...,1)$. *A mimic has a strategy* $\vec{M}^0(\vec{m})$ *that, for every realized sequence of returns* $\vec{x}$ *of the benchmark portfolio, generates the returns sequence* $(m_1 x_1, ..., m_T x_T)$ *with probability at least* $1/\prod_{1\leq t\leq T} m_t$.

We shall sketch the idea of the proof here; in the Appendix we show to execute the strategy using puts and calls on standard market indexes with Black-Scholes pricing.

Proof sketch. Choose a target excess returns sequence $m_1, m_2, \cdots, m_T \geq 1$. At the start of period 1 the mimic has capital equal to s_0. Assume that he invests it entirely in the benchmark asset. He then uses the capital as collateral to take a position in the options market. The options position amounts to placing a fair bet that bankrupts the fund with low probability $(1-1/m_1)$ and inflates it by the factor m_1 with high probability $(1/m_1)$ by the end of the period. If the high-probability outcome occurs, the mimic has end-of-period capital equal to $m_1 x_1 s_0$, while if the low probability outcome occurs the fund goes bankrupt.

The mimic repeats this construction in each successive period t using the corresponding value m_t as the target. By the end of the T^{th} period the strategy will have generated the returns sequence $(m_1 x_1, ..., m_T x_T)$ with probability $1/\prod_{1\le t\le T} m_t$, and this holds for every *realization* $\vec{x}$ of the benchmark portfolio. This concludes the outline of the proof of Lemma 1.

Proposition 1 is now proved as follows. Choose a particular sequence of excess returns $\vec{m} \ge \vec{1}$. Under the mimicking strategy defined in the Lemma, for every realization $\vec{x}$ and every period t, the mimic generates excess returns $m_1, m_2, \cdots, m_t \ge 1$ with probability at least $1/m_1 m_2 \cdots m_t$. With this same probability he earns

$$c_t(\vec{m},\vec{x})[(1-c_0(\vec{m},\vec{x}))\cdots(1-c_{t-1}(\vec{m},\vec{x}))][x_1\cdots x_t][m_1\cdots m_t]s_0 .$$

Thus, since his earnings are always nonnegative, his *expected earnings* in period t must be at least $c_t(\vec{m},\vec{x})[(1-c_0(\vec{m},\vec{x}))\cdots(1-c_{t-1}(\vec{m},\vec{x}))][x_1\cdots x_t]s_0$. This concludes the proof of proposition 1.[6]

[6] This construction is somewhat reminiscent of the doubling-up strategy in which one keeps doubling one's stake until a win occurs (Harrison and Kreps, 1979). Our set-up differs in several crucial respects however: the manager only enters into a finite number of gambles and he cannot borrow to finance them. More generally, the mimicking strategy is not a method for beating the odds in the options markets; it is a method for manipulating the distribution of returns in order to earn large fees from investors.

V. Discussion

Mimicking strategies are straightforward to implement using standard derivatives, and they generate returns that look good for extended periods while providing no value-added to investors. (Recall that the investors can earn the same expected returns with possibly much lower variance by investing passively in the benchmark asset.) Similar strategies can be used to mimic *distributions* of returns as well as particular sequences of returns.[7] In fact, however, there is no need to mimic a *distribution* of returns. Managers are paid on the basis of *realized* returns, not distributions. Hence all a mimic needs to do is target some particular sequence of excess returns that *might have arisen* from a distribution (and that generates high fees). Proposition 1 shows that he will earn as high a cut in expectation as a skilled manager *would* earn had he generated the same sequence.

Of course, the fund's investors would not necessarily approve if they could see what the mimic was doing. The point of the analysis, however, is to show what

[7] Indeed, let M_t be a nonnegative random variable with expectation $E[M_t] = \overline{m}_t > 1$. Suppose that a mimic wishes to produce the distribution $M_t X_t$ in period *t*, where X_t is the return from the benchmark. The random variable $M_t' = (1/\overline{m}_t)M_t$ represents a fair bet. The mimic can therefore implement $M_t X_t$ with probability at least $1/\overline{m}_t$ by first placing the fair bet M_t' and then inflating the fund by the factor $\overline{m}_t$ using the strategy described in the Lemma.

can happen when investors cannot observe the managers' underlying strategies -- a situation that is quite common in the hedge fund industry. Performance contracts that are based *purely* on reported returns, and that place no restrictions on managers' strategies, are highly vulnerable to manipulation.

Expression (7) in proposition 1 shows how much fee capture is possible, and why it is very difficult to eliminate this problem by restructuring the compensation contract. One common proposal, for example, is to delay paying a performance bonus for a substantial period of time. To be concrete, let us suppose that a manager can only be paid a performance bonus after five years, at which point he will earn 20% of the total return from the fund in excess of the risk-free rate compounded over five years. For example, with a risk-free rate of 4% he will earn a performance bonus equal to $.20[s_5 - (1.04)^5]_+$, where s_5 is the value of the fund at the end of year 5.

Consider a hypothetical manager who earns multiplicative excess returns equal to 1.10 each year. Under the above contract his bonus in year 5 would be $.20[(1.10)^5(1.04)^5 s_0 - (1.04)^5 s_0] \approx .149 s_0$, that is, about 15% of the amount initially invested. Let us compare this to the expected earnings of someone who generates apparent 10% excess returns using the mimicking strategy. The mimic's strategy runs for five years with probability $(1.10)^{-5} = .621$, hence his expected bonus is about $(.621)(.149)s_0 = .0925s_0$. Thus, with a five-year postponement, the mimic earns an expected bonus equal to more than 9% of the amount initially invested.

Now consider a longer postponement, say ten years. The probability that the mimic's strategy will run this long is $(1.10)^{-10} \approx .386$. However, the bonus will be calculated on a larger base. Namely, if the mimic's fund does keep running for ten years, the bonus will be $.20[(1.10)^{10}(1.04)^{10} - (1.04)^{10}]s_0 \approx .472s_0$. Therefore the *expected* bonus will be approximately $(.386)(.472s_0) = .182s_0$ or about 18% of the amount initially invested. Indeed, it is straightforward to show that under this particular bonus scheme, the expected payment to the mimic *increases* the longer the postponement is.[8]

It is, of course, true that the longer the postponement, the greater the risk that the fund will go bankrupt before the mimic can collect his bonus. Thus postponement may act as a deterrent for mimics who are sufficiently risk averse.

[8] The bonus in the final period T is $.20[(1.10)^T(1.04)^T - (1.04)^T]$ and the probability of earning it is $(1.10)^{-T}$. Hence the expected bonus is $.20[(1.10)^T(1.04)^T - (1.04)^T]/(1.10)^{-T} = .20[1-(1.1)^{-T}][1.04]^T$, which is increasing in T.

However, this does not offer much comfort for several reasons. First, as we have just seen, the postponement must be quite long to have much of an impact. Second, not all mimics need to be risk-neutral; it suffices that *some* of them are. Third, there is a simple way for a risk-averse mimic to diversify away his risk: run several funds in parallel (under different names) using independent mimicking strategies. Suppose, for example, that a mimic runs n independent funds of the type described above, each yielding 10% annual excess returns with probability 1/1.1 = 0.091. The probability that at least one of the funds survives for T years or more is $1-(1-1/1.1^T)^n$. This can be made as close to one as we like by choosing n to be sufficiently large.[9]

VI. Performance measures versus performance payments

The preceding analysis leaves open the possibility that performance contracts with negative payments might solve the problem. Before turning to this case, however, it will be useful to consider the relationship between statistical

[9] A related point is that, in any large population of funds run by mimics, the probability is high that *at least one of them* will look extremely good, perhaps better than many funds run by skilled managers (though not necessarily better than the best of the funds run by skilled managers). Correcting for multiplicity poses quite a challenge when testing for excess returns in financial markets; for a further discussion of this issue see Foster, Stine, and Young (2008).

measures of performance and performance-based compensation contracts. Some standard measures of performance, such as Jensen's alpha or the Sharpe ratio, are easily gamed by manipulating the returns distribution. Other measures avoid some forms of manipulation, but (as we shall see) they do not solve the problem of how to pay for performance. Consider, for example, the following class of measures proposed by Goetzmann et al. (2007) . Let $u(x) = (1-\rho)^{-1} x^{1-\rho}$ be a constant relative risk aversion (CRR) utility function with $\rho > 1$. If a fund delivers the sequence of returns $M_t(1+r_{ft}), 1 \le t \le T$, one can define the performance measure

(8) $$G(\vec{m}) = (1-\rho)^{-1} \ln[(1/T) \sum_{1 \le t \le T} m_t^{1-\rho}] \text{ , } \rho > 1 .$$

A variant of this approach that is used by the rating firm Morningstar (2006) is

(9) $$G^*(\vec{m}) = [(1/T) \sum_{1 \le t \le T} 1/m_t^2]^{-1/2} - 1 .$$

These and related measures rank managers according to their ability to generate excess returns *in expectation*. But to translate these (and other) statistical measures into monetary payments for performance leads to trouble. First, payments must be made on realized returns; one cannot wait forever to see whether the returns are positive in expectation. Second, if the payments are always nonnegative, then the mimic can capture some of them, as proposition 1 shows. Moreover, if the payments are allowed to be negative, then they are constrained by the managers' ability to pay them.[10] In the next section we shall show that this leads to an impossibility theorem: if the penalties are sufficient to screen out the mimics, then they also screen out skilled managers of arbitrarily high ability.

VII. Penalties

Consider a general compensation mechanism ϕ that sometimes imposes penalties, that is, $\phi_t(\vec{m}, \vec{x}) < 0$ for some values of $\vec{m}, \vec{x}$ and t. To simplify the exposition we shall assume throughout this section that the benchmark asset is risk-free, that is, $x_t = 1 + r_{ft}$ for all t. Suppose that a fund starts with an initial amount s_0, which we can assume without loss of generality is $s_0 = 1$. To illustrate the issues that arise when penalties are imposed, let us begin by considering the one-period case. Let $(1 + r_{f1})m \geq 0$ be the fund's total return in

[10] Note that if payments are linear and increasing in the performance measure (8), then arbitrarily large penalties will be imposed when m_t is close to zero.

period 1, and let $\phi(m)$ be the manager's fee as a function of m. The worst-case scenario (for the investors) is that $m = 0$. Assume that in this case the manager suffers a penalty $\phi(0) < 0$. There are two cases to consider: i) the penalty arises because the manager holds an equity stake of size $|\phi(0)|$ in the fund, which he loses when the fund goes bankrupt; or ii) the penalty is held in escrow in a safe asset earning the risk-free rate, and is paid out to the investors if the fund goes bankrupt.

The first case -- the equity stake – would be an effective deterrent provided the mimic were sufficiently risk-averse and were prevented from diversifying his risk across different funds. But an equity stake will not deter a risk-neutral mimic, because the *expected return* from the mimic's strategy is precisely the risk-free rate, so his stake actually earns a positive amount in expectation, namely $(1+r_{f1})|\phi(0)|$, and in addition he earns positive fees from managing the portion of the fund that he does not own.

Now consider the second case, in which future penalties are held in an escrow account earning the risk-free rate of return. For our purposes it suffices to consider the penalty when the fund goes bankrupt. To cover this event the amount placed in escrow must be at least $b = -\phi(0)/(1+r_{f1}) > 0$. Fix some $m^* \geq 1$

and consider a risk-neutral mimic who generates the return $m^*(1+r_{f1})$ with probability $1/m^*$ and goes bankrupt with probability $1-1/m^*$. To deter such a mimic, the fees earned during the period must be nonpositive in expectation, that is,

(10) $$\phi(m^*)/m^* + \phi(0)(1-1/m^*) \le 0 .$$

Since a mimic can target any such m^*, (10) must hold *for all* $m^* \ge 1$.

Now consider a skilled manager who can generate the return m^* with certainty. This manager must also put the amount b in escrow, because ex ante all managers are treated alike and the investors cannot distinguish between them. However, this involves an opportunity cost for the skilled manager, because by investing b in her own private fund she could have generated the return $m^*(1+r_{f1})b$. The resulting *opportunity cost* for the skilled manager is $m^*(1+r_{f1})b-(1+r_{f1})b = -(m^*-1)\phi(0)$. Assuming that utility is linear in money (i.e., the manager is risk-neutral), she will not participate if the opportunity cost exceeds the fee, that is, if

(10′) $$\phi(m^*) + (m^*-1)\phi(0) \le 0 .$$

Dividing (10′) by m^*, we see that it follows immediately from (10), which holds for *all* $m^* \geq 1$. We have therefore shown that, *if a one-period contract deters all risk-neutral mimics, it also deters any risk-neutral manager who generates excess returns.* The following generalizes this result to the case of multiple periods and randomly generated return sequences.

Proposition 2. *There is no compensation mechanism that separates skilled from unskilled managers solely on the basis of their returns histories. In particular, any compensation mechanism that deters unskilled risk-neutral mimics also deters all skilled risk-neutral managers who consistently generate returns in excess of the risk-free rate.*

Proof. Let $x_t = 1 + r_{ft}$ be the risk-free rate of return in period *t*. To simplify the notation we shall drop the x_t's and let $\phi_t(\vec{m})$ denote the payment (possibly negative) in period *t* when the manager delivers the excess return sequence $\vec{m}$. The previous argument shows why holding an equity stake in the fund itself does not act as a deterrent for a risk-neutral mimic. We shall therefore restrict ourselves to the situation where future penalties must be held in escrow.

Consider an arbitrary excess returns sequence $\vec{m} \geq \vec{1}$. Let the mimic's strategy $\vec{M}^0(\vec{m})$ be constructed so that it goes bankrupt in each period *t* with probability exactly $1/(m_1 \cdots m_t)$. Consider some period $t \le T$. The probability that the fund survives to the start of period *t* without going bankrupt is $1/(m_1 \cdots m_{t-1})$. At the end of period *t*, the mimic earns $\phi_t(\vec{m})$ with probability $1/m_t$ and $\phi_t(m_1,...,m_{t-1},0,...,0)$ with probability $(m_t - 1)/m_t$. Hence the *net present value* of the period-*t* payments is

$$(11) \qquad \frac{\phi_t(\vec{m})}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})} + \frac{(m_t - 1)\phi_t(m_1,...,m_{t-1},0,...,0)}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})}.$$

To deter a risk-neutral mimic, the net present value $V^0(\vec{m})$ of all payments must be nonpositive:

$$(12) \qquad V^0(\vec{m}) = \sum_{1 \le t \le T} [\frac{\phi_t(\vec{m})}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})} + \frac{(m_t - 1)\phi_t(m_1,...,m_{t-1},0,...,0)}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})}] \le 0.$$

(Although some of these payments may have to be held in escrow, this does not affect their net present value to the mimic because they earn the risk-free rate until they are paid out.)

Now consider a skilled manager who can deliver the sequence $\vec{m} \geq \vec{1}$ with certainty. (We shall consider distributions over such sequences in a moment.) Let $B(\vec{m})$ be the set of periods t in which a penalty must be paid if the fund goes bankrupt during that period and not before:

(13) $$B(\vec{m}) = \{t : \phi_t(m_1, ..., m_{t-1}, 0, ..., 0) < 0\}.$$

For each $t \in B(\vec{m})$ let

(14) $$b_t(\vec{m}) = -\phi_t(m_1, ..., m_{t-1}, 0, ..., 0)/(1 + r_{f\,t}) > 0.$$

This is amount that must be escrowed during the t^{th} period to ensure that the investors can be paid if the fund goes bankrupt by the end of the period. The skilled manager evaluates the present value of all future fees, penalties, and escrow payments using his *personal discount factor*, which for period-t payments is $\delta_t = \delta_t(\vec{m}) = 1/\prod_{1 \leq s \leq t} m_s(1 + r_{f\,s})$.

Consider any period $t \in B(\vec{m})$. To earn the fee $\phi_t(\vec{m})$ at the end of period *t*, the manager must put $b_t(\vec{m})$ in escrow at the start of the period (if not before).[11] Conditional on delivering the sequence $\vec{m}$, he knows he will get this back with interest at the end of period *t*, that is, he will get back the amount $(1+r_{ft})b_t(\vec{m})$. For the skilled manager, the present value of this period-*t* scenario is

$$(15) \qquad \delta_t\phi_t(\vec{m}) + \delta_t(1+r_{ft})b_t(\vec{m}) - \delta_{t-1}b_t(\vec{m})$$

$$= \delta_t\phi_t(\vec{m}) + \delta_t[(m_t - 1)\phi_t(m_1,...,m_{t-1},0,...,0)]$$

$$= \frac{\phi_t(\vec{m})}{(m_1\cdots m_t)(1+r_{f1})\cdots(1+r_{ft})} + \frac{(m_t-1)\phi_t(m_1,...,m_{t-1},0,...,0)}{(m_1\cdots m_t)(1+r_{f1})\cdots(1+r_{ft})}.$$

Now consider a period $t \notin B(\vec{m})$. This is a period in which the manager earns a nonnegative fee even though the fund goes bankrupt, hence nothing must be held in escrow. The net present value of the fees in any such period is $\phi_t(\vec{m})/[(m_1\cdots m_t)(1+r_{f1})\cdots(1+r_{ft})]$. Thus, summed over all periods, the net present value of the fees for the skilled manager comes to

[11] If penalties must be escrowed more than one period in advance, the opportunity cost to the skilled manager will be even greater and the contract even more unattractive, hence our conclusions still hold.

$$(16) \qquad V(\vec{m}) = \sum_{t \in B(\vec{m})} \left[\frac{\phi_t(\vec{m})}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})} + \frac{(m_t - 1)\phi_t(m_1,\ldots,m_{t-1},0,\ldots,0)}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})}\right]$$

$$+ \sum_{t \notin B(\vec{m})} \frac{\phi_t(\vec{m})}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})}.$$

Since $m_t \geq 1$ and $\phi_t(\vec{m}) \geq 0$ for all $t \notin B(\vec{m})$, we know that

$$(17) \qquad \sum_{t \notin B(\vec{m})} \frac{(m_t - 1)\phi_t(\vec{m})}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})} \geq 0.$$

From (16) and (17) it follows that

$$(18) \qquad V(\vec{m}) \leq \sum_{1 \leq t \leq T} \left[\frac{\phi_t(\vec{m})}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})} + \frac{(m_t - 1)\phi_t(m_1,\ldots,m_{t-1},0,\ldots,0)}{(m_1 \cdots m_t)(1+r_{f1})\cdots(1+r_{ft})}\right].$$

But the right-hand side of this expression must be nonpositive in order to deter the risk-neutral mimics (see expression (12)). It follows that *any contract that is unattractive for the risk-neutral mimics is also unattractive for any risk-neutral skilled manager no matter what excess returns sequence* $\vec{m} \geq \vec{1}$ *he generates*. Since this statement holds for every excess returns sequence, it also holds for any distribution over excess return sequences. This concludes the proof of proposition 2.

VIII. Attracting new money

The preceding analysis shows that any compensation mechanism that rewards highly skilled portfolio managers can be gamed by mimics without delivering any value-added to investors. To achieve this, however, the mimic takes a calculated risk in each period that his fund will suffer a total loss. A manager who is concerned about building a long-term reputation may not want to take such risks; indeed he may make more money in the long run if his returns are lower and he stays in business longer, because this strategy will attract a steady inflow of new money. However, while there is empirical evidence that past performance does affect the inflow of new money to some extent, the precise relationship between performance and flow is a matter of debate.[12] Fortunately we can incorporate flow-performance relationships into our framework without committing ourselves to a specific model of how it works and the previous results remain essentially unchanged.

To see why, consider a benchmark asset generating returns series $\vec{X}$ and a manager who delivers excess returns $\vec{M}$ relative to $\vec{X}$. Let

[12] See for example Gruber, 1996; Massa, Goetzman, and Rouwenhorst, 1999; Chevalier and Ellison, 1997; Sirri and Tufano, 1998; Berk and Green, 2004.

$Z_t = Z_t(m_1,...,m_{t-1}; x_1,...,x_{t-1})$ be a random variable that describes how much net new money flows into the fund at the start of period *t* as a function of the returns in prior periods. In keeping with our general set-up we shall assume that Z_t is a multiplicative random variable, that is, its realization z_t represents the *proportion* by which the fund grows (or shrinks) at the start of period *t* compared to the amount that was in the fund at the end of period $t-1$. Thus, if a fund starts at size 1, its total value at the start of period *t* is

(19) $$Z_t \prod_{1 \le s \le t-1} M_s X_s Z_s .$$

Given any excess returns sequence $\vec{m} \ge \vec{1}$ over *T* years, a mimic can reproduce it with probability $1/\prod_{1 \le t \le T} m_t$ for all realizations of the benchmark returns. Since by hypothesis the flow of new money depends only on $\vec{m}$ and $\vec{x}$, it follows that the probability is at least $1/\prod_{1 \le t \le T} m_t$ that the mimic will attract the same amount of new money into the fund as the skilled manager.

The question of what *patterns* of returns attract the largest inflow of new money is an open problem that we shall not attempt to address here. However, there is some evidence to suggest that investors are attracted to returns that are steady even though they are not spectacular. Consider, for example, a fund that grows at 1% per month year in and year out. (The recent Ponzi scheme of Bernard Madoff grew to some $50 billion by offering returns of about this magnitude.) This can be generated by a mimic who generates a monthly return of 0.66% on top of a risk-free rate of 0.33%. The probability that such a fund will go under in any given year is $1-(1.0066)^{-12}=.076$ or about 7.6%. In expectation, such a fund will stay in business and continue to attract new money for about 13 years.

One could of course argue that portfolio managers might not want to take the risk involved in such schemes if they care sufficiently about their reputations. Some managers might want to stay in business much longer than 13 years, others might be averse to the damage that bankruptcy would do to their personal reputation or self-esteem. We do not deny that these considerations may serve as a deterrent for many people. But our argument only requires the existence of *some* people for whom the prospect of high expected earnings outweighs such concerns. The preceding results show that it is impossible to keep these types of managers out of the market without keeping everyone out.

IX. Conclusion

In this paper we have shown how mimicry can be used to game performance fees by portfolio managers. The framework allows us to estimate how much a mimic can earn under different incentive structures; it also shows that commonly advocated reforms of the incentive structure cannot be relied upon to screen out unskilled risk-neutral managers who do not deliver excess returns to investors. The analysis is somewhat unconventional from a game-theoretic standpoint, because we did not identify the set of players, their utility functions, or their strategy spaces. The reason is that we do not know how to specify any of these components with precision. To write down the players' utility functions, for example, we would need to know their discount factors and degrees of risk aversion, and we would also need to know how their track records generate inflows of new money. While it might be possible to characterize the equilibria of a fully-specified game among investors and managers of different skill levels, this poses quite a challenge that would take us well beyond the framework of the present paper. The advantage of the mimicry argument is that we can draw inferences about the *relationship* between different players' earnings without knowing the details of their payoff functions or how their track records attract new money. The argument is that, if someone is producing returns that earn large fees in expectation, then someone else (with no skill) can mimic the first type and also earn large fees in expectation without knowing anything about how the first type is actually doing it. In this paper we have shown how to apply this idea to financial markets. We conjecture that it may prove useful in other situations where there are many players, the game is complex, and the equilibria are difficult to pin down precisely.

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Appendix

Here we shall show explicitly how to implement the mimicking strategy that was described informally in the text, using puts and calls. We shall consider two situations: i) the benchmark asset is risk-free such as US Treasury bills; ii) the benchmark asset is a market index such as the S&P 500. We shall call the first case the *risk-free model* and the second case the *market index model.*

As is customary in the finance literature, we shall assume that the price of the market index evolves in continuous time τ according to a stochastic process of form

(A1) $$dP_\tau = \mu P_\tau d\tau + \sigma P_\tau dW_\tau ,$$

that is, P_τ is a geometric Brownian motion with mean μ and variance σ^2. The reporting of results is done at discrete time periods, such as the end of a month or a quarter. Let $t = 1, 2, 3...$ denote these periods, and let r_{ft} denote the risk free rate during period *t*. Similarly, let $\tilde{r}_{ft}$ denote the continuous-time risk free rate during period *t*, which we shall assume is constant during the period and satisfies $\tilde{r}_{ft} \leq \mu$. Without loss of generality we may assume that each period is of length one, in which case $e^{\tilde{r}_{ft}} = 1 + r_{ft}$.

Mimicking strategies will be implemented using puts and calls on the market index, whose prices are determined by the Black-Scholes formula (see for example Hull, 2009).

Lemma. *Consider any target sequence of excess returns* $\vec{m} = (m_1, ..., m_T) \geq (1,1,...,1)$ relative to a benchmark asset, which can be either risk-free or a market index. *A mimic has a strategy* $\vec{M}^0(\vec{m})$ *that, for every realized sequence of returns* $\vec{x}$ *of the benchmark asset, generates the returns sequence* $(m_1 x_1, ..., m_T x_T)$ *with probability at least* $1 / \prod_{1 \leq t \leq T} m_t$.

Proof. The options with which one implements the strategy depend on whether the benchmark asset is risk-free or the market index. We shall treat the risk-free case first.

Fix a target sequence of excess returns $\vec{m} = (m_1, ..., m_T) \geq (1,1,...,1)$. We need to show that the mimic has a strategy that in each period t delivers the return $m_t(1+r_{ft})$ with probability at least $1/m_t$.

At the start of period t, the mimic invests everything in the risk-free asset (e.g., US Treasury bills). He then writes (or shorts) a certain quantity q of *cash-or-nothing puts* that expire before the end of the period. Each such option pays one dollar if the market index is below the strike price at the time of expiration. Let Δ be the length of time to expiration and let s be the strike price divided by the index's current price; without loss of generality we may assume that the current price is 1. Let Φ denote the cumulative normal distribution function. Then (see Hull, 2009, section 24.7) the option's present value is $e^{-\tilde{r}_{ft}\Delta}v$, where

(A2) $$v = \Phi[(\ln s - \tilde{r}_{ft}\Delta + \sigma^2\Delta/2)/\sigma\sqrt{\Delta}],$$

and the probability the put will be exercised is

(A3) $$p = \Phi[(\ln s - \mu\Delta + \sigma^2\Delta/2)/\sigma\sqrt{\Delta}].$$

Assume that the value of the fund at the start of the period is w dollars. By selling q options the mimic collects an additional $e^{-\tilde{r}_{ft}\Delta} vq$ dollars. By investing everything (including the proceeds from the options) in the risk-free asset, he can cover up to q options when they are exercised provided that $e^{\tilde{r}_{ft}\Delta} w + vq \geq q$. Thus the maximum number of covered options the mimic can write is $q = we^{\tilde{r}_{ft}\Delta} / (1-v)$.

He chooses the time to expiration Δ and the strike price s so that v satisfies $v = 1 - 1/m_t$. With probability p the options are exercised and the fund is entirely cleaned out (i.e., paid to the option-holders). With probability $1-p$ the options expire without being exercised, in which case the fund has grown by the factor $m_t e^{\tilde{r}_{ft}\Delta}$ over the time interval Δ. The mimic enters into this gamble only once per period, and the funds are invested in the risk-free asset during the remaining time. Hence the total return during the period is $m_t(1+r_{ft})$ with probability $1-p$ and zero with probability p.

We claim that $p \leq v$; indeed this follows immediately from (A2) and (A3) and the assumption that $\tilde{r}_{ft} \leq \mu$. Therefore, if the mimic had $w_{t-1} > 0$ dollars in the fund at the start of period *t*, then by the end of the period he will have $m_t(1+r_{ft})w_{t-1}$ dollars with probability *at least* $1/m_t = 1-v$ and zero dollars with probability *at most* $1-1/m_t$. Therefore after T periods, he will have generated the target sequence of excess returns $(m_1,...,m_T)$ with probability at least $1/\prod_{1\le t\le T} m_t$, as asserted in the lemma.

Next we consider the case where the benchmark asset is the market index. The basic idea is the same as before, except that in this case the mimic invests everything in the market index (rather than in Treasury bills), and he shorts *asset-or-nothing* options rather than cash-or-nothing options. (An asset-or-nothing option pays the holder one share if the market index closes above the strike price in the case of a call, or below it in the case of a put; otherwise the payout is zero.)

As before the mimic shorts the maximum number of options that he can cover, where the strike price and time to expiration are chosen so that the probability they are exercised is at most $1-1/m_t$. With probability *at least* $1/m_t$, this strategy increases the number of shares of the market index held in the fund by the factor m_t. Hence, with probability at least $1/m_t$, it delivers a total return equal to $m_t(P_t / P_{t-1}) = m_t x_t$ for every realization of the market index.

It remains to be shown that the strike price s and time to expiration Δ can be chosen so that the preceding conditions are satisfied. There are two cases to consider: $\mu - \tilde{r}_{ft} \geq \sigma^2$ and $\mu - \tilde{r}_{ft} < \sigma^2$. In the first case the mimic shorts an asset-or-nothing put, whose present value is

(A4) $$v = \Phi[(\ln s - \tilde{r}_{ft}\Delta - \sigma^2\Delta/2)/\sigma\sqrt{\Delta}],$$

and whose probability of being exercised is

(A5) $$p = \Phi[(\ln s - \mu\Delta + \sigma^2\Delta/2)/\sigma\sqrt{\Delta}].$$

(See Hull, 2009, section 24.7.) From our assumption that $\mu - \tilde{r}_{ft} \geq \sigma^2$, it follows that $p \leq v$,which is the desired conclusion. If on the other hand $\mu - \tilde{r}_{ft} < \sigma^2$, then the mimic shorts asset-or-nothing *calls* instead of asset-or-nothing puts. In this case the analog of formulas (A4) and (A5) assure that $p \leq v$ (Hull, 2009, section 24.7).

Given any target sequence $(m_1, m_2, ..., m_T) \geq (1,1,...,1)$, this strategy produces returns $(m_1 x_1, ..., m_T x_T)$ with probability at least $1/\prod_{1 \leq t \leq T} m_t$ for *every* realization $\vec{x}$ of the benchmark asset. This concludes the proof of the lemma.

We remark that the probability bound $1/\prod_{1 \leq t \leq T} m_t$ is conservative. Indeed the proof shows that the probability that the options are exercised may be strictly *less* than is required for the conclusion to hold. Furthermore, in practice, the pricing formulas are not completely accurate for out-of-the-money options, which tend to be overvalued (the so-called 'volatility smile'). This implies that the seller can realize an even larger premium for a given level of risk than is implied by the Black-Scholes formula.

Of course, there will be some transaction costs in executing these strategies, and these will work in the opposite direction. While it is beyond the scope of this paper to try to estimate such costs, the fact that the mimicking strategy requires only one trade per period in a standard market instrument suggests that these costs will be very low. In any event, it is easy to modify the argument to take such costs into account. Suppose that the cost of taking an options position is some fraction ε of the option's payoff. In order to inflate the fund's return in a given period t by the factor $m_t \geq 1$ *after* transaction costs, one would have to inflate the return by the factor $m_t' = m_t / (1 - \varepsilon)$ *before* transaction costs. To illustrate: the transaction cost for an out-of-the-money option on the S&P 500 is typically less than 2% of the option price. Assuming the exercise probability is around 10%, the payout if it is exercised will be about ten times as large, so ε will be about 0.2% of the option's payoff. Thus, in this case, the mimicking strategy would achieve a given target m_t *net of costs* with probability $.998 / m_t$ instead of with probability $1 / m_t$.

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