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Contagion in Derivatives Markets

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Abstract. A major credit shock can induce large intraday variation margin payments between counterparties in derivatives markets, which may force some participants to default on their payments. These payment shortfalls become amplified as they cascade through the network of exposures. Using detailed Depository Trust & Clearing Corporation data, we model the full network of exposures, shock-induced payments, initial margin collected, and liquidity buffers for about 900 firms operating in the U.S. credit default swaps market. We estimate the total amount of contagion, the marginal contribution of each firm to contagion, and the number of defaulting firms for a systemic shock to credit spreads. A novel feature of the model is that it allows for a range of behavioral responses to balance sheet stress, including delayed or partial payments. The model provides a framework for analyzing the relative effectiveness of different policy options, such as increasing margin requirements or mandating greater liquidity reserves.

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1. Introduction

The recent financial crisis highlighted the potential risks posed by derivatives markets. As the crisis unfolded, large sellers of protection, such as American International Group, Inc. (AIG), became liable for payments on credit default swap (CDS) contracts that they had previously sold to banks and dealers as protection against credit defaults. Until the crisis, these protection sellers received a steady stream of payments from protection buyers and rarely had to pay out claims. When the crisis hit, the sudden calls for payments put great pressure on the sellers, some of which had a thin capital base. In particular, AIG had to be rescued by the U.S. Federal Reserve to meet its margin obligations on CDS contracts to dealers that, in turn, were threatened if the payments were not made.

This incident casts a spotlight on the potential risks posed by the CDS market to the stability of the financial system. Before the crisis, this market grew rapidly. From its inception in the 1990s to 2007, the total notional value of CDS contracts rose to about $60 trillion, although it has since declined to about $9.4 trillion (Bank for International Settlements 2018). Under a shock to credit markets, the sellers of CDS protection become liable for payments to the buyers. AIG got into trouble, because it had sold protection on large pools of subprime mortgages to a variety of banks and broker dealers. During the crisis, the value of these mortgages deteriorated sharply, which triggered margin calls that AIG was unable to meet (Financial Crisis Inquiry Commission 2011).

Since the crisis, various reforms have been introduced to mitigate destabilizing risks in the financial system. First, requirements for posting initial margin (IM) as security against nonpayment have been strengthened. This has occurred both in the scope of firms that are required to post initial margin and in the segregation of initial margin accounts so that one counterparty's losses cannot be covered by another counterparty's funds. These developments have facilitated a movement toward exchanging variation margin (VM) on a daily basis. Variation margin payments or bilateral exchanges in contractual value were more infrequent before the financial crisis and were not the subject of regulation.

Second, another reform prompted by the crisis was to incentivize firms to trade contracts through central counterparties (CCPs). Central clearing has been encouraged through higher capital and margin requirements for noncentrally cleared contracts.[1] An advantage of central clearing is that it fosters contractual standardization and shortens intermediation chains, which can help to reduce network contagion (Cont and Minca 2016). A disadvantage is that it concentrates risk at a single point of failure and imposes on the CCPs much shorter variation margin compliance windows than existed before the crisis.

In this paper, we model the propagation of losses in derivatives markets through network spillover effects, taking account of the short timeframe within which payments must be made (typically a few hours) and also, the key role played by the CCP. The focus of our analysis is the CDS market. Given a shock to credit spreads, we estimate the total amount of contagion and the contribution of individual firms to contagion under a range of assumptions about their response to balance sheet stress. The model builds on the framework of Eisenberg and Noe (2001), but it has several novel elements that are specific to over-the-counter derivatives markets. In particular, the model incorporates two safety valves that mitigate network spillover effects. The first is the posting of initial margin as a security deposit against potential default. The second is the maintenance of in-house cash reserves and dedicated lines of credit to manage daily fluctuations in the demand for variation margin payments. Initial margins and cash reserves vary substantially among firms depending on the risk characteristics of their portfolios, which we observe in the data. The model, therefore, incorporates heterogeneity in firms' balance sheets as well as their positions in the network.

Another contribution of this paper is to allow for a range of possible strategies to cope with short-term liquidity stress, including delayed or partial payment and payment with illiquid collateral. We show how to accommodate a range of such responses in the model and then, explore their implications for contagion using detailed exposure data provided to the Office of Financial Research by the Depository Trust & Clearing Corporation (DTCC). The data provide a detailed picture of the network of counterparty exposures in the U.S. CDS market at particular dates, including exposures between banks, dealers, hedge funds, asset managers, and insurance companies. We also use the data to estimate the initial margin posted by each counterparty and the liquidity buffers that they maintain to manage daily fluctuations in margin calls.

We then apply the framework to estimate the total payment shortfalls that would result from a severe but not implausible market shock, namely the Federal Reserve's 2015 Comprehensive Capital Analysis and Review (CCAR) trading book shock. This shock was designed to test the robustness of the financial system under severe conditions while embedding comovements in the value of credit instruments that we are not in a position to estimate ourselves. The shock implies a sudden decrease in the value of corporate and sovereign debt on which CDS contracts are written, and thus, it results in large demands for variation margin payments between counterparties to these contracts.

The plan of the paper is as follows. In the next section, we provide an overview of the literature on network contagion, including recent papers on CDS markets. In Section 3, we introduce the network model and demonstrate the existence of a payments equilibrium under very general conditions on the firms' response functions. In Section 4, we discuss specific examples of such response functions. Sections 5 and 6 describe the DTCC data in detail and show how it can be used to estimate the initial margins posted by counterparties, the amount of cash reserves that firms need to manage their own accounts, and the variation margin payments induced by a shock.

In Section 7, we bring these elements of the model together and estimate the total amount of contagion that would be produced by the CCAR shock. We also conduct a sensitivity analysis to assess how the amount of contagion is affected by the size of the liquidity buffers, the amount that firms post in initial margin, and the strategies that they use to manage balance sheet stress. We find that, even when liquidity buffers are large enough to handle daily fluctuations in variation margin payments with 99.7% probability and initial margins are large enough to cover payment delinquencies with 99.5% probability (based on historical DTCC data), the amount of default contagion under the CCAR shock could be very substantial. In particular, the shortfall in payments is on the order of 5%–12% of total payment obligations and 15%–17% of all market participants default. Interestingly, the CCP does not default, although it must reach into its guarantee fund to cover its obligations to member firms.

The model also permits a forensic analysis of the sources of contagion. In particular, we can estimate each firm's marginal contribution to contagion by calculating the amount by which payment shortfalls would be reduced throughout the system if that firm's obligations could be fully guaranteed by government intervention. We first find that, by this measure, exactly one (nonmember) firm is responsible for a very significant proportion of total systemic losses owing to its large size and the imbalance between its buy and sell positions. A second key finding is that, although the member firms contribute less to contagion at the margin because of their balanced portfolios, they tend to amplify contagion (through spillover effects) because of their central position in the network. A third finding is that, although the CCP is the most centrally located of the nodes, it contributes relatively little to contagion, because it has a perfectly matched buy and sell portfolio, stringent initial margin requirements, and a large guarantee fund that acts as liquidity buffer.

2. Related Literature

The financial crisis of 2007–2009 has sparked a rapidly growing literature, both theoretical and empirical, on financial networks and systemic risk. A central theme of this literature is the relationship between network structure and the risk of contagion. Network connections can have a positive effect by diversifying the risk exposures of individual market participants, but they can also have a negative effect by creating channels through which shocks can spread. The tension between these two forces has been explored in a variety of papers; see, among others, Allen and Gale (2000), Freixas et al. (2000), Gai and Kapadia (2010), Blume et al. (2011), Gai et al. (2011), Haldane and May (2011), Cont et al. (2012), Elliott et al. (2014), and Acemoglu et al. (2015).

A key methodology for analyzing how networks propagate contagion is from Eisenberg and Noe (2001), who show how payment shortfalls that originate at some nodes can cascade through the network, causing an ever-widening series of defaults. There is now a substantial literature that builds on their approach (as we do here); see, in particular, Upper and Worms (2004), Elsinger (2009), Rogers and Veraart (2013), Elliott et al. (2014), Glasserman and Young (2015), and Acemoglu et al. (2015). For general surveys of the literature, see Bisias et al. (2012) and Glasserman and Young (2016).

There is also a literature that focuses specifically on the network structure of CDS markets. The potential for contagion in these markets was first highlighted by Cont (2010), who emphasized the importance of adequate liquid reserves to cope with large and sudden demand for variation margin. Cont (2010) also analyzed the extent to which a CCP can mitigate contagion, a topic that was subsequently treated by Duffie and Zhu (2011), Cont and Kokholm (2014), and Cont and Minca (2016). Of crucial importance is how the CCP sets margin requirements. Capponi and Cheng (2018) examine the tension between setting member fees and collateral levels and how, if made effectively, these choices limit contagion from portfolio shocks. Various empirical studies examine how CCPs set margin levels in practice (Duffie et al. 2015, Capponi et al. 2017). These studies find that value-at-risk (VaR) approaches tend to underestimate CCP collateral levels. For this reason, we shall use the CCP's reported collateral instead of attempting to estimate it.

Various authors have studied the structure of CDS exposures and the potential for contagion among European banks; see, in particular, Brunnermeier et al. (2013), Clerc et al. (2014), Peltonen et al. (2014), and Vuillemey and Peltonen (2015). Cont and Minca (2016) analyze the combined network of exposures in the CDS and interest rate swap markets together and argue that central clearing in both markets can significantly reduce the probability and magnitude of illiquidity spirals. Their work differs from this paper in the methodologies used to study contagion and in the focus on the European rather than the U.S. market.[2] More recently, Ali et al. (2016) examined the network structure of the CDS market in the United Kingdom. These authors argue, as do Glasserman and Young (2016), that the systemic importance of market participants is not fully captured by conventional measures of centrality. The size and structure of financial firms' balance sheets in addition to their position in the network are crucial to understanding how much risk they pose to the system as a whole.

3. Network Contagion Model

We take as given the network of CDS contracts that exist between counterparties at a given point in time and study the impact of a sudden shock to credit markets. Such a shock triggers large VM payments from the sellers to the buyers of CDS protection. Suppose, for example, that a firm sold \$100 million in protection against the default of corporation C over the next five years. The shock decreases the value of C's debt and correspondingly, increases the implied probability that C will default within the contract period. This increases the value of the CDS contract to the buyer and increases the liability of the seller to the buyer. This change in value must be paid by the seller to the buyer as variation margin; moreover, the payment is typically due within one day. The upshot is that shocks to credit markets impose expectations of rapid cash payments between participants in the CDS market.

Let $\bar{p}_{ij}$ denote the VM payment obligation from firm i to firm j that results from a given shock. Let $\bar{p}_i = \sum_{j \neq i} \bar{p}_{ij}$ denote the total payment obligation of i to all other firms. In what follows, we shall restrict attention to those firms i such that $\bar{p}_i > 0$. The others do not transmit payment shortfalls; instead, they act as shock absorbers. In this context, these firms are solely buyers of protection (not sellers), and under a shock, they will have no VM obligations.

The *relative liability* of firm i to firm j is

$$a_{ij} = \bar{p}_{ij} / \bar{p}_i. \tag{1}$$

Note that, for each i, $\sum_{j \neq i} a_{ij} \leq 1$; moreover, $\sum_{j \neq i} a_{ij} < 1$ if firm i owes payments to one or more firms with no obligations (which are not indexed). It follows that the matrix $A = (a_{ij})$ is row substochastic.

Consider a node i, and let c_{ki} denote the amount of IM that it collects from counterparty k. The purpose of the IM is to cover the shortfall in VM payments. In particular, if counterparty k fails to pay VM to i in a timely manner, the position will be closed out or novated to another firm, and the IM will be applied to any losses that are incurred between the time that the payment was due and the time that it takes to novate or close out the position.

In addition, each firm i maintains cash reserves and short-term lines of credit that allow it to manage daily fluctuations in VM payments and receipts. These constitute i's *liquidity buffer* b_i. In Section 5.3, we shall show how to estimate these buffers from the DTCC data.

Given a shock, let $p_{ki} \leq \bar{p}_{ki}$ denote the actual payment made by k to i. If $p_{ki} < \bar{p}_{ki}$, the difference will be made up out of the IM sitting in k's account at firm i provided that $\bar{p}_{ki} - p_{ki} \le c_{ki}$. If $\bar{p}_{ki} - p_{ki} > c_{ki}$, however, then the difference $\bar{p}_{ki} - (p_{ki} + c_{ki})$ must be borne by i. We define the *initial stress* at i, s_i^0, to be the difference between i's payment obligations to others and its liquidity buffer plus payment obligations from others:

$$s_i^0 = \bar{p}_i - \sum_{k \neq i} \bar{p}_{ki} - b_i. \tag{2}$$

The *stress* at i, s_i, is the amount in equilibrium by which i's payment obligations exceed its incoming payments (supplemented by the counterparties' initial margins) plus i's liquidity buffer: that is,

$$s_i = s_i(\boldsymbol{p}) = \bar{p}_i - \sum_{k \neq i} ((p_{ki} + c_{ki}) \wedge \bar{p}_{ki}) - b_i. \tag{3}$$

Note that, even when all of i's counterparties pay in full (that is, $p_{ki} = \bar{p}_{ki}$ for all k), stress will still be positive if i's payments obligations exceed its incoming payments by more than its liquidity buffer b_i.

The final element of the model is to specify how balance sheet stress translates into actual payments. We argue that, in practice, firms will respond to stress in diverse ways depending on their access to credit, their relationship with counterparties, and their risk management practices. We do not have enough information about individual firms to model these responses explicitly. Instead, we shall consider a range of possible responses, and then, we show how they can be used to bound the total amount of network contagion holding other parameters fixed.

Given a level of stress s_i at firm i, let $s_{ij}(\boldsymbol{p}) = a_{ij}s_i(\boldsymbol{p})$, and let $p_{ij} = f_{ij}(s_{ij}, \bar{p}_{ij}) \in [0, \bar{p}_{ij}]$ be the *expected payment* that i makes to counterparty j. We call f_{ij} a *stress response function.*[3] We shall impose two regularity conditions. (i) f_{ij} is monotone nonincreasing in s_{ij}: that is, higher levels of stress lead to lower (or at least not higher) payments. (ii) f_{ij} is *upper semicontinuous* in s_{ij}: that is, for every convergent sequence $\{s_{ij}^k\} \to s_{ij}^*$ and for every $\bar{p}_{ij}$,

$$\limsup_{s_{ij}^k \to s_{ij}^*} f_{ij}(s_{ij}^k, \bar{p}_{ij}) \le f_{ij}(s_{ij}^*, \bar{p}_{ij}). \tag{4}$$

In the next section, we shall describe specific examples of such stress response functions. Given the functions f_{ij}, we are now in a position to define the notion of payments equilibrium. For every payment vector $\boldsymbol{p} = (p_{ij})_{1 \le i,j \le n}$ such that $0 \le p_{ij} \le \bar{p}_{ij}$, define the function

$$\forall i \neq j, \quad \Phi(\boldsymbol{p})_{ij} = f_{ij}(s_{ij}(\boldsymbol{p}), \bar{p}_{ij}). \tag{5}$$

It is straightforward to show that $s_{ij}(\boldsymbol{p})$ is continuous in $\boldsymbol{p}$; hence, $\Phi(\boldsymbol{p})$ is upper semicontinuous in $\boldsymbol{p}$. (The composition of a continuous function and an upper-semicontinuous function is upper semicontinuous.) The function $\Phi(\boldsymbol{p})$ is monotone and order preserving on the bounded lattice $L = \Pi_{1 \le i,j \le n}[0, \bar{p}_{ij}] \in \mathbb{R}^{n^2}$, where the (partial) order on L is defined by $\boldsymbol{p} \le \boldsymbol{p}'$ if $p_{ij} \le p'_{ij}$ for all i, j.

Consider the recursively defined sequence

$$\boldsymbol{p}^1 = \Phi(\bar{\boldsymbol{p}}), \boldsymbol{p}^2 = \Phi(\boldsymbol{p}^1), \boldsymbol{p}^3 = \Phi(\boldsymbol{p}^2), \ldots \tag{6}$$

Proposition 1. *The sequence in (6) converges to the greatest fixed point of Φ.*

Proof. Because $\bar{\boldsymbol{p}}$ is the maximal element in L, $\boldsymbol{p}^1 = \Phi(\bar{\boldsymbol{p}}) \le \bar{\boldsymbol{p}}$. Because Φ is order preserving, $\boldsymbol{p}^2 = \Phi(\boldsymbol{p}^1) \le \Phi(\bar{\boldsymbol{p}}) = \boldsymbol{p}^1$. Proceeding inductively, we see that the sequence $\{\boldsymbol{p}^k\}$ in (6) is monotone nonincreasing. It is also bounded below by the zero vector. By the monotone convergence theorem, $\{\boldsymbol{p}^k\}$ converges to its greatest lower bound, $\boldsymbol{p}^*$.

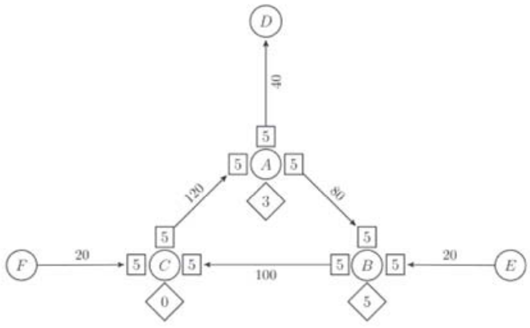
We claim that $\boldsymbol{p}^*$ is a fixed point of Φ. Because Φ is order preserving and $\boldsymbol{p}^* \le \boldsymbol{p}^k$ for all k, $\Phi(\boldsymbol{p}^*) \le \lim_{k \to \infty} \Phi(\boldsymbol{p}^k)$. Because Φ is upper semicontinuous, $\Phi(\boldsymbol{p}^*) = \Phi(\lim_{k \to \infty} \boldsymbol{p}^k) \ge \lim_{k \to \infty} \Phi(\boldsymbol{p}^k)$. It follows that $\Phi(\boldsymbol{p}^*) = \lim_{k \to \infty} \Phi(\boldsymbol{p}^k)$. By construction, $\boldsymbol{p}^{k+1} = \Phi(\boldsymbol{p}^k)$ for all k; hence, $\Phi(\boldsymbol{p}^*) = \lim_{k \to \infty} \Phi(\boldsymbol{p}^k) = \lim_{k \to \infty} \boldsymbol{p}^{k+1} = \boldsymbol{p}^*$, and therefore, $\boldsymbol{p}^*$ is fixed point of Φ.

By Tarski's Theorem (Tarski 1955), Φ has the greatest fixed point $\boldsymbol{p}^+ \in L$. Clearly, $\boldsymbol{p}^+ \le \bar{\boldsymbol{p}}$; hence, $\boldsymbol{p}^+ = \Phi(\boldsymbol{p}^+) \le \Phi(\bar{\boldsymbol{p}}) = \boldsymbol{p}^1$. Applying Φ repeatedly, we conclude that $\boldsymbol{p}^+ \le \boldsymbol{p}^k$ for all k; hence, $\boldsymbol{p}^+ \le \boldsymbol{p}^* = \lim_{k \to \infty} \boldsymbol{p}^k$. By assumption, $\boldsymbol{p}^+$ is the maximal fixed point, and we have just shown that $\boldsymbol{p}^*$ is also a fixed point; hence, $\boldsymbol{p}^+ = \boldsymbol{p}^*$. □

A number of fixed point results in the literature are particular instances of this proposition; see, among others, Rogers and Veraart (2013) and Elliott et al. (2014).

To illustrate the recursive computation of payments equilibrium, consider the three-firm example shown in Figure 1. Each firm collects IM from each

Figure 1. Example Payment Network



Note. Arc labels, diamonds, and squares denote variation margin payment obligations, liquidity buffers, and initial margin stocks, respectively.

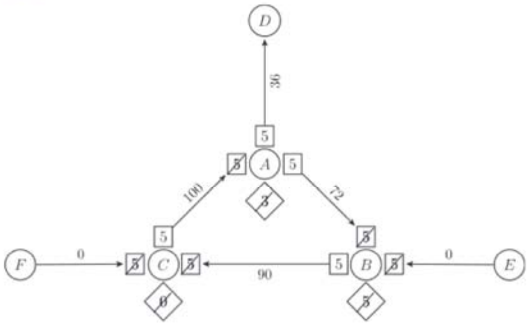
of its counterparties, and these amounts are held in separate boxes. In addition, each firm has a liquidity buffer that is held in a diamond. The amounts owed are shown next to the corresponding arrows. For example, firm A is owed 120 by C and owes 80 to B and 40 to D. In what follows , we shall assume that, when a firm defaults, it pays its counterparties the amount that it owes them minus the pro rata stress: that is, $f_{ij}(s_{ij}, \bar{p}_{ij}) = \bar{p}_{ij} - s_{ij}$ for all i and j.

Now suppose that firms E and F default completely. Then, B can seize the five units of IM posted by E but not the IM posted by its other two counterparties (A and C). In the first iteration of the algorithm, assume that B pays whatever it can to C: 5 in IM that it collected from E plus 5 in B's liquidity buffer plus the anticipated 80 in payments from A. Thus, in the first round, we find that $p^1_{BC} = 90$. This is less than B owes C; hence, C can seize the 5 in IM that it collected from B plus the 5 in IM that it collected from F (which is in default) plus the 90 in anticipated payments from B, and it can pay 100 to A: $p^1_{CA} = 100$. This is less than what C owes A; hence, A seizes the 5 in IM that it collected from C, adds 3 from its liquidity diamond, and pays 108 to D and B in the proportions of one to two. Thus, at the end of round 1 of the algorithm, we have the (hypothetical) payments shown in Figure 2. Note, however, that the ingoing and outgoing payments, supplemented by the IM and liquidity buffers, are not in balance. Therefore, we need to apply the function Φ again to determine the next round of payments. This process converges to the payments equilibrium shown in Figure 3.

4. Stress Response Functions

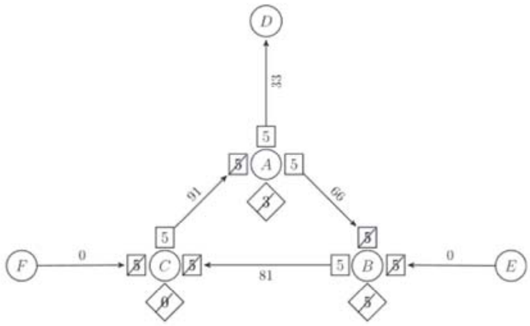
We now consider the form of the stress response functions in more detail. Recall that, when firm i is

Figure 2. Example Payment Network at the End of Round 1



Notes. Firms E and F default, which results in reduced payments (relative to obligations) from B to C, from C to A, and from A to B. B and C both seize their failing counterparties' IM stocks and resort to their own liquidity buffers. The resulting payment shortfall generates a corresponding response from A. Exhausted stocks and buffers are indicated with a stricken line.

Figure 3. Example Network Equilibrium



Note. In equilibrium, every firm's incoming payments plus IM seizures and liquidity buffer equal its outgoing payments.

under stress ($s_i > 0$), it is unable to meet its payment obligations in full, even after seizing the IM posted by the counterparties that failed to pay and after exhausting its own liquidity buffer. The question is how much a firm will pay its counterparties (on the day when VM payments are due) and how much it will hold back. In the Eisenberg–Noe model, it is assumed that each firm's assets are fully distributed to its counterparties, such as they would be in resolution. In this case, the stress response function takes the form

$$f_{ij}(s_{ij}, \bar{p}_{ij}) = \bar{p}_{ij} - s_{ij}, \tag{7}$$

where $s_{ij} = a_{ij}s_i$ is the pro rata shortfall from i to j. We shall call this the *soft default* option (see Figure 4(a)).

In this context, however, this scenario seems overly optimistic: if i is unable to pay in full, it will suffer default—and the consequent reputational loss—even if it pays as much as it can. Thus, there would seem to be little benefit to making a partial payment if default is going to ensue in any case. In the extreme, the response would be to pay nothing whenever $s_i > 0$: that is,

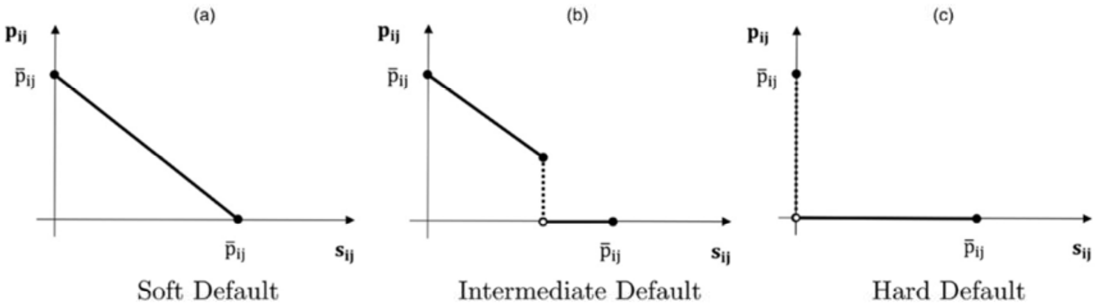
$$f_{ij}(s_{ij}, \bar{p}_{ij}) = \begin{cases} 0 & \text{whenever } s_{ij} > 0, \\ \bar{p}_{ij} & \text{if } s_{ij} = 0. \end{cases} \tag{8}$$

We call this the *hard default* option (see Figure 4(c)).

It seems likely that firms' actual stress response functions are somewhere in between these two extremes. If the anticipated shortfall s_{ij} is sufficiently large (relative to $\bar{p}_{ij}$), i may declare default and pay nothing; if s_{ij} is small, i may make its best effort to close the gap and perhaps receive temporary forbearance from counterparty j, which might prefer to receive some payment immediately rather than foreclose on i.[4] In this case, the stress response function might take the intermediate form shown in Figure 4(b).

In the empirical applications to follow, we shall not attempt to estimate the response functions for

Figure 4. Stress Response Functions



Source. Office of Financial Research analysis.
Notes. Three cases are illustrated. Panel (a) describes the situation where i pays the maximum amount available (as it would in resolution). Panel (c) shows the situation where i defaults and pays nothing. Panel (b) shows an intermediate situation in which i makes a reduced payment if s_{ij} is not too large and pays nothing when s_{ij} exceeds a certain threshold.

particular firms. Rather, we shall treat hard and soft default as bounds on the range of plausible responses and estimate the amount of contagion produced by each.

5. Estimating the Model Parameters

In this section, we show how to estimate the key elements of the network contagion model—variation margin, initial margin, and liquidity buffers—from the DTCC data. The data report the positions on all standardized and confirmed CDS involving U.S. entities since 2010. Positions represent extant swap transactions with comparable risk characteristics between counterparties. They include detailed information about underlying reference entities, notional amounts bought and sold, inception and termination dates, and other terms of contracts. We also use data from Markit to estimate single-name credit spreads for all reference entities in the positions that we observe.

5.1. Variation Margin

VM payments are cash transfers made by a firm to its counterparties to account for changes in the value of the CDS contracts. These variation margin payments are made on a daily basis. From the protection seller's perspective, a CDS derives positive value from premia received until the contract terminates or the underlying reference entity defaults (whichever comes first); in the latter case, the seller's contract value is reduced by the expected protection payment. The sources of value are switched from the standpoint of the protection purchaser: at contract inception, the present value of premia paid is balanced by the expected value of default payments. The value of the contract varies with market credit spreads through their concurrent impact on the present value of premia receipts and the expected value of default payments.

Consider a contract k that is established between counterparties i and j at time t on a set of reference entity characteristics r_k and a notional amount of protection N_k. Through the use of a bootstrapping procedure to value CDS contracts using the term structure of credit spreads at t, we are able to estimate the net present value (NPV) of the contract (Luo 2005). The change in value of contract k between successive periods t and $t+1$ determines the variation margin $\mathrm{VM}_{ij}(N_k, r_k, t, t+1)$ payable on the kth contract. The sum of changes across all contracts between i and j is the bilateral variation margin

$$\mathrm{VM}_{ij}(t, t+1) = \sum_k \mathrm{VM}_{ij}(N_k, r_k, t, t+1). \tag{9}$$

We estimate the weekly change in contract values, and the induced VM payments for each pair of firms in our data set over the period January 1, 2010 to October 21, 2016. The term of the firms' CDS contracts come from DTCC, whereas data on credit spreads and discount rates come from Markit and Bloomberg, respectively. Contracts on indices or portfolios of reference entities are handled by disaggregating them into their single-name equivalents. The details of these calculations are described in the appendix.

5.2. Initial Margin

The role of IM is to cover potential shortfall in VM payments by a firm's counterparties, including the cost of closing out or transforming the position in case of default. Initial margins collected from counterparties are held in segregated accounts and can only be used to cover losses induced by a given counterparty's failure to pay. A portion of the IM is typically held in cash or cash equivalents, and the remainder is held in assets that can be liquidated on short notice but not necessarily at full value. Not all counterparties are required to post IM. For example broker dealers only need to post IM on contracts with other broker dealers and the CCP. Other market participants, such as hedge funds and asset managers, need to post IM with broker dealers, commercial banks, and the CCP but not with each other.

To determine the amount of IM posted (where IM is required), we adopt a conventional portfolio-at-risk measure, namely a 99.5% VaR with a 10-day margin period of risk (BCBS and IOSCO 2015). For each pair of firms i and j, the DTCC data report the portfolio of CDS contracts for which i and j were the counterparties on the date of the shock. Using Markit data, we can infer the price changes and hence, the VM that would have been exchanged between i and j if they had held this same portfolio over the prior 1,000 days. We then find the amount c_{ij} such that, on all but 5 of 1,000 days, the net amount of VM that i owed j was less than c_{ij}.

In the case of the CCP, VaR approaches tend to underestimate CCP collateral levels (Duffie et al. 2015, Capponi et al. 2017). Hence, in this case, we estimate the IM that would be required to meet a 10-day 99.5% VaR bilaterally for each of its counterparties and then, scale up the estimates by a common factor so that the total IM collected corresponds to the CCP's total reported IM at the end of 2014 (ICE 2016).

5.3. Liquidity Buffers

The IM collected by firm i from its counterparties is dedicated to covering shortfalls in payments to i from its counterparties; it cannot be accessed to meet i's obligations to others. To cover its own obligations, the firm maintains a liquidity buffer b_i, which includes cash or cash equivalents and short-term lines of credit. These buffers are not part of the DTCC data, and they are not available from public data sources. Instead, we estimate their magnitude by considering how much cash a prudently managed firm would need to manage its *net* VM obligations. These numbers can be estimated from the weekly inflows and outflows of VM at the firm level, which are derived from DTCC data as described above.[5]

Fix a firm i, and let $X_i(t)$ denote the total net VM payment that i owes to all of its counterparties on a given day t. If $X_i(t) > 0$, then i owes more than it is owed; the reverse holds if $X_i(t) < 0$. Let $N_i(t)$ be the gross notional value of i's CDS contracts at time t, and let $\tilde{X}_i(t) = X_i(t)/N_i(t)$. There is considerable heterogeneity in the volatility of $\tilde{X}_i(t)$ between different types of firms, reflecting differences in their overall risk management policy and degree of hedging (see Table 1). In particular, the members of the CCP have an average portfolio volatility that is an order of magnitude smaller than the portfolio volatility of nonmember commercial banks. Moreover, the latter is less than one-half as volatile as the holdings of hedge funds, asset managers, and insurance companies.

To estimate the size of i's liquidity buffer, we use a portfolio VaR measure. Let $F_i(\tilde{X})$ be the cumulative density function of $\tilde{X}_i$ over the period of observation, and let θ be a VaR level, such as $\theta = 0.99$ or $\theta = 0.997$. In particular, $\theta = 0.99$ corresponds to the third largest net negative change in VM, and $\theta = 0.997$ corresponds to the largest net negative change in VM over the period of observation. Let $\tilde{x}_{i,\theta}$ be the value such that $F_i(\tilde{x}_{i,\theta}) = \theta$. At a given date $t^* \in [0, T]$, we estimate i's liquidity buffer to be $b_{i,\theta} = \tilde{x}_{i,\theta} N_i(t^*)$.

In the case of the CCP, we use the actual size of the guarantee fund reported for the quarter instead of estimating its liquidity buffer. At the date of this study, ICE Clear Credit's guarantee fund had assets worth about \$2.4 billion (ICE 2016).

Although the portfolio volatility of CCP members is on average much smaller than that of nonmembers, the size of their portfolios is so much larger that many of them require higher absolute amounts to manage their VM payments at a given level of risk (see Table 2).

In what follows, we shall adopt the most conservative of these scenarios ($\theta = 0.997$) as our estimate of firms' liquidity buffers.

Table 1. Average Portfolio Volatility by Type of Firm

Firm type	Standard deviation
Members	0.0009
Nonmembers	
Commercial banks	0.0106
Hedge fund/asset managers	0.0235
Insurance and pensions	0.0224

Source. The authors' calculations used data provided by Depository Trust & Clearing Corporation and Markit.
Note. For each firm, we compute the standard deviation of $\tilde{X}_i(t)$ over the period from January 1, 2010 to October 21, 2016 using weekly data from DTCC and take the average of those standard derivations over all firms of a given type.

6. VM Payments Induced by the CCAR Shock

The Federal Reserve's 2015 CCAR severely adverse global market trading shock prescribes a sudden widening of credit spreads for corporate, state, municipal, and sovereign debt according to their rating class (Federal Reserve Board 2016). The shock is applied to all outstanding positions as of October 6, 2014. The

Table 2. Average Liquidity Buffer by Type of Firm (in Millions of Dollars)

Firm type	$\theta = 0.95$	$\theta = 0.99$	$\theta = 0.997$
Members	35.4	91.4	334
Nonmembers			
Commercial banks	1.1	3.2	9.1
Hedge fund/asset managers	4.0	11.0	41.0
Insurance and pensions	5.8	13.7	34.7
CCP	2,400	2,400	2,400

Source. The authors' calculations used data provided by Depository Trust & Clearing Corporation and Markit.

change in credit spreads alters the value of the premium and payment legs of CDS contracts that reference various classes of debt. These changes in CDS contract values induce VM payment obligations between counterparties. The methodology for estimating these VM payments is described in detail in the appendix.

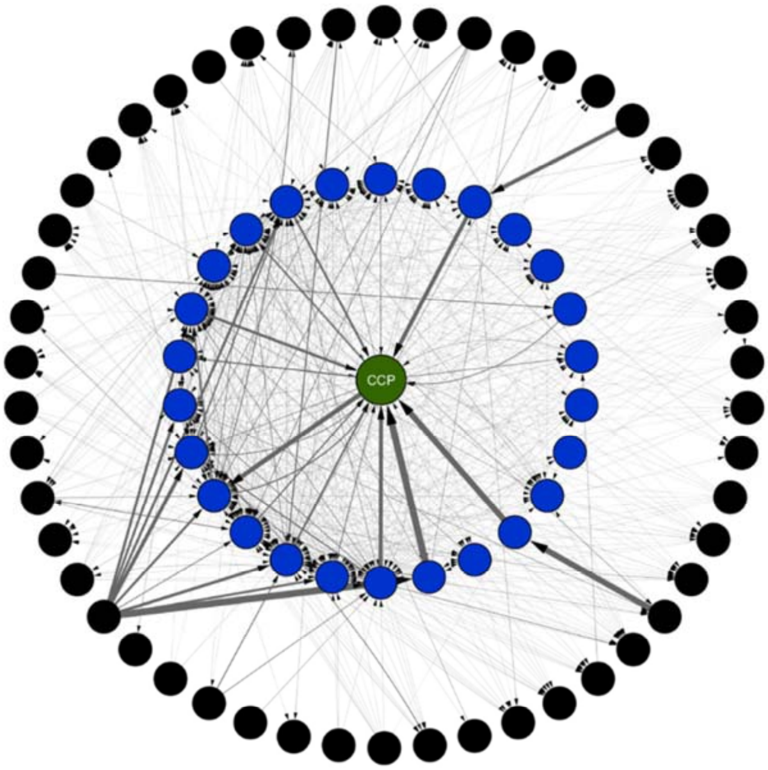
Figure 5 shows the net VM payment obligations between the CCP, members of the CCP, and other nonmember firms on the CCAR shock date.[6] In addition, there are many nonmember firms that have positions directly with members as well as positions with the CCP that are guaranteed by members. There are over 900 such firms, including a wide variety of hedge funds, asset managers, and insurance companies.

Two key features of the network are that (i) nonmembers tend to owe members rather than each other and that (ii) the largest nonmembers contribute substantially more stress than the largest members, because they are large net sellers of CDS protection. These points are highlighted in Table 3, which shows the average VM owed by the top five members versus the average VM owed by the top five commercial banks, the top five hedge funds and asset managers, and the top five insurance companies ordered by the amount of VM owed. The market structure described here affects the results described in Section 7 in several critical ways. First, liquidity buffers (and additionally for the CCP, its default fund) will play critical roles in resolving stress in the network. The exhaustion of such resources for members and the CCP leads to payments contagion. Second, the initial channels for stress are payments due from nonmembers to members, which in some cases, are several times larger than their liquidity buffers.

7. Empirical Analysis

We now apply this framework to estimate the total amount of contagion that would be produced by a large and sudden credit shock as well as the amount that each individual firm contributes to contagion at

Figure 5. (Color online) Variation Margin Payment Network for CCP Members and a Subsample of Nonmembers Based on the 2015 CCAR Shock



Source. The authors' calculations used data provided by the Depository Trust & Clearing Corporation and Markit.
Notes. The network diagram plots the CPP clearing house (green), CCP members (blue), and a sample of CCP nonmembers (black). The width and direction of each arrow indicate the relative size of the net VM payment owed bilaterally between counterparties.

Table 3. Aggregate Variation Margin Payments Under the CCAR Shock

Firms	Gross notional exposure ($ billions)	Variation margin owed by ($ millions)	Variation margin owed to ($ millions)	Net variation margin owed ($ millions)	Liquidity buffer ($ millions)
Top five members	7,151	8,396	5,510	2,886	2,276
Nonmembers					
Top five commercial banks	31	268	8	259	80
Top five hedge fund/asset managers	296	9,730	263	9,466	1,742
Top five insurance and pensions	30	557	48	509	184
CCP	3,344	8,747	8,747	—	2,400

Source. The authors' calculations used data provided by Depository Trust & Clearing Corporation and Markit.

the margin. Define the *impact* of a credit shock to be the total shortfall in VM payments (net of initial margin seized) over all pairs of counterparties in the network. Specifically, if p_{ij} is the equilibrium payment from i to j, then the *payment shortfall* from i to j is $d_{ij} = (\bar{p}_{ij} - p_{ij} - c_{ij})_+$, and the total shortfall is

$$D = \sum_{1 \le i,j \le n} d_{ij}. \tag{10}$$

An alternative measure of impact is the *proportion* of firms that default on their payments. We provide estimates of both measures of impact in the subsequent analysis, although our preferred measure is (10), because it accounts for the magnitude and not just the number of defaults.

Table 4 shows the percentage of firms in default in each of five categories: (i) CCP members (broker dealers), (ii) commercial banks, (iii) hedge funds and asset managers, (iv) insurance companies and pension funds, and (v) the CCP itself. Over the entire population of firms, 15% are in default under the soft default scenario, and 17% are in default under the hard default scenario. Thus, there is surprisingly little difference between the two scenarios in the overall number of firms that default, although there is a marked increase in the number of *members* that default.

Table 5 shows the aggregate payment shortfall for each type of firm. Observe first that the CCP does not default under either scenario, although it must dip into its guarantee fund to meet its payment obligations. A notable difference between the two scenarios is that, under hard default, the members' payment shortfalls are more than seven times larger than under soft default; moreover, in the hard default case, these shortfalls represent over one-half the total shortfall of all firms.

Table 5 also clarifies the extent to which network spillover effects amplify the initial shock. Recall from expression (2) that the initial stress to a firm's balance sheet is the amount of outgoing payments due less the incoming payments due plus the firm's liquidity buffer. These figures are given in the second column of Table 5. In equilibrium, however, the incoming payments are often less than the payments due, which increases balance sheet stress. The ratio of equilibrium stress to initial stress is the *amplification factor* owing to network effects. Observe that the amplification factor is especially large for the CCP members. There are two reasons for this effect. (i) Members have fairly balanced initial positions, and therefore, their initial stress is low relative to the extent of their exposure. (ii) Members are central to the network, and hence, they are particularly vulnerable to waves of defaults that cascade through the network and inundate them with payment shortfalls by their counterparties.

The model can also be used to assess the amount that individual firms contribute to contagion at the margin. Given a default scenario (hard or soft), let $D = \sum_{1 \le i,j \le n} d_{ij}$ be the total shortfall in payments summed over all firms. Now choose a particular firm i, and suppose (hypothetically) that its payment obligations to all counterparties could be guaranteed, say by giving i access to an emergency central bank loan. Let D_i be the total payment shortfalls over all firms in the network under this scenario. We define i's *marginal contribution to contagion* to be the relative difference $(D - D_i)/D$: that is, the percentage by which total payment shortfalls are reduced when i's payments are guaranteed.

Figure 6 shows the marginal contribution of the largest contributors to contagion under hard and soft

Table 4. Percentage of Firms with Payment Shortfall Under the 2015 CCAR Shock

	Soft default	Hard default
Total	15	17
Members	35	58
Nonmembers		
Commercial banks	24	29
Hedge fund/asset managers	12	13
Insurance and pensions	19	26
CCP	0.0	0.0

Source. The authors' calculations used data provided by Depository Trust & Clearing Corporation and Markit.

Table 5. Initial Stress and Equilibrium Payment Shortfall Under the 2015 CCAR Shock

	Initial stress	Soft default	Soft default amplification	Hard default	Hard default amplification
Total	9,985	11,215	1.12	27,112	2.72
Members	812	2,004	2.46	15,285	18.82
Nonmembers					
Commercial banks	244	246	1.01	432	1.77
Hedge fund/asset managers	8,577	8,602	1.00	11,292	1.32
Insurance and pensions	334	345	1.04	968	2.90
CCP	—	—	—	—	—

Source. The authors' calculations used data provided by Depository Trust & Clearing Corporation and Markit.

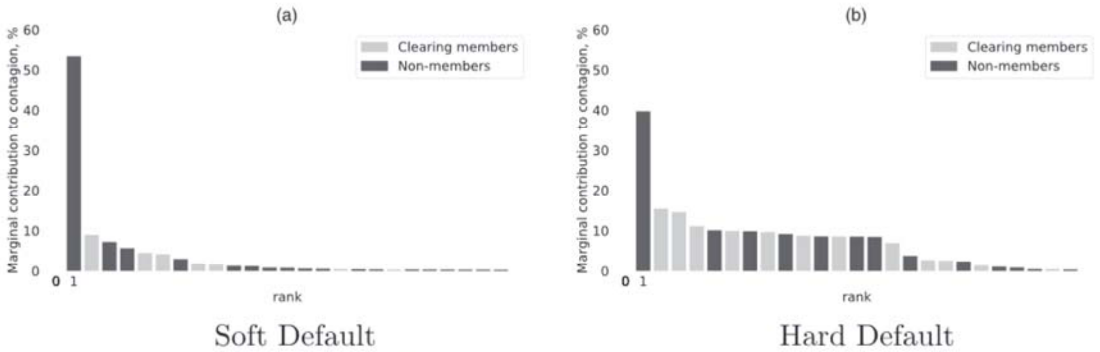
default. In both cases, the largest single contributor to contagion is an asset manager that is a large net seller of protection. Under the CCAR shock, this firm owes much more than it is owed, and the resulting shortfall in payments to its counterparties triggers contagion throughout the system. Figure 6 demonstrates that, if this firm's obligations could be guaranteed, total payment shortfalls would be greatly reduced under either soft or hard default.[7] Figure 6 also shows that several clearing members each contribute over 10% to systemic risk in the hard default scenario. These firms have fairly balanced portfolios, but unlike the large asset manager, they are quite central to the network and therefore, amplify payment defaults by their counterparties.

Our framework can also be used to evaluate the effectiveness of tightening initial margin requirements or increasing firms' liquidity buffers. Table 6 shows the total amount of initial margin posted by all firms under the standards prevailing in 2015. We conduct a sensitivity analysis by computing the equilibrium payments shortfall that would result if all IM requirements were boosted by 50% or 100%.[8] In the soft default scenario, the impact of doubling IM requirements is quite small regardless of whether it is measured by the reduction in payment shortfall ($0.8 billion) or the number of firms in default (0.1%). In the hard default scenario, the impact is still relatively moderate. When IM is doubled, the shortfall is reduced by $2.8 billion, but this comes at a system-wide cost of posting an additional $20.2 billion in collateral; moreover, the percentage of firms in default is only reduced by 0.2%.

We now conduct a similar analysis for increases in liquidity buffers. In theory, each dollar added to a firm's liquidity buffer should go further than a dollar of additional IM posted to one of its counterparties, because the former can be applied to all of its payment obligations, whereas the latter can only be applied to payments to that particular counterparty. Table 7 shows the impact of increasing liquidity buffers at all firms (and the CCP's guarantee fund) by 50% and also, 100%. Note that the absolute dollar amounts are roughly twice that of the corresponding increases in IM in Table 6.

In the soft default scenario, the shortfall is reduced by about $4.1 billion when liquidity buffers are doubled. In the hard default scenario, the shortfall is reduced by about $5.8 billion. Thus, on a dollar for dollar basis, there is not much difference in the effectiveness of liquidity buffers versus initial margin in reducing the shortfall.[9] Moreover, in both cases, the benefit (in reduced shortfall) is quite modest given the large amount of additional liquid collateral that would be required.

Figure 6. Distribution of Firms' Marginal Contributions to Contagion



Source. The authors' calculations used data provided by the Depository Trust & Clearing Corporation and Markit.

Table 6. Payment Shortfall and Firms in Default Under Increased Initial Margins

		Soft default		Hard default	
	Total initial margin ($ billions)	Shortfall ($ billions)	Firms in default (%)	Shortfall ($ billions)	Firms in default (%)
Current	20.3	11.1	14.7	27.1	16.8
Plus 50%	30.4	10.7	14.6	25.6	16.7
Plus 100%	40.5	10.3	14.6	24.3	16.6

Source. The authors' calculations used data provided by Depository Trust & Clearing Corporation and Markit.

Table 7. Payment Shortfall and Firms in Default Under Increased Liquidity Buffers

		Soft default		Hard default	
	Total liquidity buffer ($ billions)	Shortfall ($ billions)	Firms in default (%)	Shortfall ($ billions)	Firms in default (%)
Current	42.1	11.1	14.7	27.1	16.8
Plus 50%	63.7	8.6	11.4	24.1	12.4
Plus 100%	84.2	7.0	9.0	21.3	9.3

Source. The authors' calculations used data provided by Depository Trust & Clearing Corporation and Markit.

What is the explanation for this somewhat surprising result? It turns out that, even when liquidity buffers are doubled, there is still one nonmember firm that is under severe stress and triggers a great deal of contagion; moreover, there are still several members that amplify contagion because of their central position in the network. The basic problem is that, even when liquidity buffers are set at a very high level, they may be insufficient to handle extreme tail events (such as CCAR) and their network spillover effects. Whether this warrants an increase in liquidity buffers (or initial margins) is for policy makers to decide, but our results suggest that the adequacy of these safety valves can only be fully evaluated by constructing stress tests within the framework of a network model.

8. Conclusion

In this paper, we have analyzed the network of counterparty exposures in the CDS market. In contrast to much of the prior work on stress testing, contagion, and fire sales, we track the potential effects of a shock using actual financial exposures and the Federal Reserve's supervisory stress scenario as of a particular date. We estimate the impact of the shock on the value of market participants' portfolios and by implication, the VM owed between the contracting parties. A significant feature of this market is that demands for VM must be met over very short time horizons. A failure to meet these demands leads to payment shortfalls that become amplified as they cascade through the network.

We have examined the potential contribution to network contagion of the 30 members of the major CCP in this market (ICE Clear Credit) and also, the potential contribution of the major nonmembers. We find that network exposures significantly increase the amount of contagion. Furthermore, there are many members (and some nonmembers) that contribute substantially more to contagion than the CCP. Our analysis suggests that more attention should be paid to firms that are very large, have highly unbalanced CDS positions, and in which failure can trigger large systemic losses as happened with AIG in 2008. It also highlights the key role of initial margin and liquidity buffers in coping with large and sudden demands for variation margin that result from a credit shock.

Our study is limited to the analysis of a specific part of the derivatives market and does not encompass the full range of shocks to which firms may be exposed. In particular, we have not included exposures to interest rate swaps, which form a substantially larger market (in notional terms) than the CDS market but that are not part of our data set. Under a severe credit shock, firms may be subjected to simultaneous payment demands over multiple lines of business, increasing the stress on their resources and possibly leading to even higher losses than we have estimated here.

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Appendix A. Evaluating CDS Portfolios

Here, we describe the methodology for estimating the mark-to-market value of each counterparty's exposures at a given date for both single-name and index positions. We describe a bootstrap procedure to generate a schedule of hazard rates consistent with the market for all traded credit curves. We then describe the process of index disaggregation to single-name equivalents. Finally, we describe how we arrive at expressions for variation margin payments under stress.

A.1. Bootstrapping Credit Curves

We calibrate hazard rate schedules associated with each of the reference entities in the contracts that we observe. The CDS market quotes credit spreads through a range of standard terms: 1-, 3-, 5-, 7-, and 10-year maturities and sometimes, longer maturities. Each additional term generates a hazard rate over a corresponding increment: the 3-year term generates a 1- to 3-year increment, the 5-year term generates a 3- to 5-year increment, and so on through 10 years. The bootstrapping technique that we use here generates a piecewise constant schedule of hazard rates. A CDS contract struck at inception at the market spread for a standard maturity and valued using the schedule of hazard rates through that maturity will have equal default and premium legs. Bootstrapping permits us to value any position with remaining maturity at the time of stress that may not correspond to market-quoted maturities.

The CDS payment and premium legs are implicit functions of a hazard rate, λ, which enters the expression for the survival probability $S(0,t,\lambda)$ through its definition as $\exp\left(-\int_t^u \lambda ds\right)$. Let $Z(0,t_i)$ denote the risk-free discount factor through period i, which we compute from London Interbank Offered Rates from 1 to 6 months and swap rates from 9 months to 30 years. We assume that CDS premia are paid on International Money Market (IMM) payment dates, consistent with market convention. Finally, to allow for the possibility that a default occurs intraperiod, CDS premia are prorated to the time of default. For simplicity, assume that $\alpha = 0.5$ (i.e., that any default occurs at the interperiod halfway point). In subsequent notation, Δ_t represents the day count fraction for the time interval $(t-1,t)$ such that $\sum_{t=1}^{T}\Delta_t = T$. We use the Actual/365 day count convention standard in the CDS market.

We express the CDS premium leg through maturity T as follows:

$$V_{prem}^{0\to T}(\lambda,\ s_T) = s_T\sum_{t=1}^{T} Z(0,t)\Delta_t((1-\alpha)S(0,t) + \alpha S(0,t-1)). \tag{A1}$$

In any period, the payment leg derives its value from the incremental default probability over that time. Given the relationship between the default probability $P(t,u,\lambda)$ and the survival probability $S(t,u,\lambda)$, $S(t,u,\lambda) = 1 - P(t,u,\lambda)$, we express the payment leg as follows:

$$V_{pay}^{0\to T}(\lambda) = (1-R)\sum_{t=1}^{T} Z(0,t)(P(0,t) - P(0,t-1)). \tag{A2}$$

Let λ^* be the solution that sets the CDS payment and premia legs to fair value (equality) at inception: that is, $V_{contract}^{0\to T}(\lambda^*,\ s_T) = V_{pay}^{0\to T}(\lambda) - V_{prem}^{0\to T}(\lambda,\ s_T) = 0$.

Credit spreads are quoted for a sequence of maturities $T_1, T_2, \ldots T_k$. Quotes consistent across the term structure require that, for each $T_i \geq T$, a vector of hazard rates $\boldsymbol{\lambda}^* = \{\lambda^*_{0,T_i}, \lambda^*_{T_i,T_{i+k}}\}$ exists such that $V_{contract}^{0\to T_{i+k}}(\boldsymbol{\lambda}^*,\ s_{T_{i+k}}) = V_{contract}^{0\to T_i}(\lambda^*_{0,T_i}, s_{T_{i+k}}) + V_{contract}^{T_i\to T_{i+k}}(\lambda^*_{T_i,T_{i+k}},\ s_{T_{i+k}})$ for all k. We adopt a bootstrap procedure from Luo (2005) that ensures this by construction. The procedure generates hazard rates $\left\{\lambda^*_{0,T_1}, \lambda^*_{T_1,T_2}, \ldots \lambda^*_{T_{k-1},T_k}\right\}$ that correspond to quoted maturities of $\{T_1, T_2, \ldots T_k\}$. The default probability at any time t such that $T_k \leq t \leq T_m$ can be expressed as a function of bootstrapped hazard rates as follows:

$$P(0,t,\boldsymbol{\lambda}^*) = P(0,T_k,\lambda^*_{0,T_k})P(T_k,t,\lambda^*_{T_k,T_m}). \tag{A3}$$

For notational convenience, we will refer to $P(0,t,\boldsymbol{\lambda}^*)$ from here on as $P(t)$. We make the simplifying assumption that maturity dates fall on IMM payment dates. We start from the premise that $\boldsymbol{\lambda}^*_{0,T_k}$ is known and is either (i) the solution that equates (A.1) and (A.2) or (ii) the bootstrapped solution vector described by the preceding recursive procedure. The parameters s_{T_m} are derived from market quotes. The conditional premia and payment legs are given as follows:

$$\begin{aligned} V_{prem}^{0\to T_m}&(\lambda_{T_k,T_m};\ s_{T_m},\ \boldsymbol{\lambda}^*_{0,T_k}) \\ &= s_{T_m}\Bigg\{C(\boldsymbol{\lambda}^*_{0,T_k}) + D(\lambda_{T_k,T_m}) - \sum_{t=T_k+1}^{T_m} Z(0,t)\Delta_t \\ &\quad\cdot\left(P(t) - P(T_k) - \alpha\frac{P(t)-P(t-1)}{2}\right)\Bigg\}, \end{aligned} \tag{A.4}$$

where

$$C(\boldsymbol{\lambda}^*_{0,T_k}) = \sum_{\substack{(T_i,T_j)\in\\ \{(0,T_1)\ldots\\ (T_{k-1},T_k)\}}}\sum_{t=T_i}^{T_j} Z(0,t)\Delta_t\left[(1-P(t)) + \alpha\frac{(P(t)-P(t-1))}{2}\right],$$

$$D(\lambda_{T_k,T_m}) = \sum_{t=T_k}^{T_m} Z(0,t)\Delta_t(1-P(T_k)).$$

Similarly,

$$\begin{aligned} V_{pay}^{0\to T_m}&(\lambda_{T_k,T_m};\ \boldsymbol{\lambda}^*_{0,T_k}) \\ &= A(\boldsymbol{\lambda}^*_{0,T_k}) + \sum_{t=T_k+1}^{T_m}(1-R)Z(0,t)(P(t) - P(t-1)), \end{aligned} \tag{A5}$$

where

$$A(\boldsymbol{\lambda}^*_{0,T_k}) = \sum_{\substack{(T_i,T_j)\in\\ \{(0,T_1)\ldots\\ (T_{k-1},T_k)\}}}\sum_{t=T_i}^{T_j}(1-R)Z(0,t)(P(t)-P(t-1)).$$

Here, $\lambda^*_{T_k,T_m}$ is the value that sets Equations (A.4) and (A.5) to parity. After this value is determined, the hazard rate vector $\boldsymbol{\lambda}^*_{0,T_m}$ can be derived for subsequent stages of the bootstrap recursion. The resulting bootstrapped hazard

Table A.1. The Impact of the 2015 CCAR Severely Adverse Global Market Trading Shock

	AAA	AA	A	BBB	BB	B	<B or not rated
Corporate credit							
Advanced economies							
Spread widening (%)	130	133	110	202	269	265	265
Emerging markets							
Spread widening (%)	192	217	243	278	402	436	466
State and municipal credit							
Spread widening (bps)	12	17	37	158	236	315	393
Sovereign and supra credit							
See Federal Reserve Board worksheet: CCAR-2015-Severely-Adverse-Market-Shocks-data.xlsx							

Source. Federal Reserve Board (2016).

rate schedule allows us to value a contract of any term by applying its contracted spread and the hazard rate schedule associated with its term.

A.2. Portfolios of Single-Name Equivalents

We disaggregate Markit credit indices to single-name constituents. For each position referencing a Markit credit index, we decompose the index using Markit RED data. This source provides the composition of the index at any point in time, taking into account index revisions and defaults. We use the disaggregation technique described in section 2 of Siriwardane (2019).

Each Markit credit index is described by its series and version. A series may have one or more versions. An index series factor f_i is defined for every version i as $f_i = 1 - D_i/N$, where D_i is the number of defaults for an index series version $i \in \{1, 2, 3 \ldots\}$. Version 1 is characterized by $D_1 = 0$, and therefore, $f_1 = 1$. The weight of a constituent within a version must be computed on a given valuation date and is a function of the index composition on the date that the position was established (trade date). In general, the index composition at the trade date may not be its composition at inception. The constituent u's weight in index version i can be expressed by $w_i(u)$, and its weight at inception is given as $w_1(u) = 1/N$. Subsequent versions' weights are given by

$$w_i(u) = \frac{w_1(u)}{f_i} \quad \forall\, i \geq 1. \tag{A.6}$$

As an example, an index with 43 original constituents at inception would have a per constituent weight of $w_1(u) = 0.0233$. Following the default of one constituent, version 2 of the index would have a per constituent weight of $w_2(u) = 0.0238$. The per constituent weight is scaled by the notional value of the index position to arrive at the effective single-name notional equivalent. We perform all calculations in this paper on firms' single-name equivalent CDS positions.

A.3. Estimating Variation Margin

At the CCAR valuation date, we generate stressed portfolio values using the following approach. The change in the value of exposures under stress follows from their valuation at baseline and revaluation after the market shock, which specifies an increase in credit spreads. These increases are shown in Table A.1 for various classes of debt.

We incorporate counterparty flows in the description of the NPV as follows. Suppose that firm x writes the payment leg, whereas firm y writes the premium leg. Incorporating such flows and suppressing some earlier notation, we express $V_{prem}^{0\to T}(\lambda_m{}^*; s, \boldsymbol{\lambda}_{m-1}{}^*)$ as $V_x^{prem}(\lambda; s, T)$, and similarly, we express the payment leg as $V_y^{pay}(\lambda, T_m)$. The hazard rate environment that exists at valuation date t is given by $\boldsymbol{\lambda}^t$. Analogously, the environment at t under stress is $\boldsymbol{\lambda}^{shock}$.

The NPV of a swap of notional on the as of date T is defined as follows:

$$\begin{aligned} &NPV_{xy}(N, \boldsymbol{\lambda}^T, s, T) \\ &\quad = N\left[V_x^{prem}(\boldsymbol{\lambda}^T; s, T) - V_y^{pay}(\boldsymbol{\lambda}^T, T)\right]. \end{aligned} \tag{A7}$$

Similarly, the NPV of the swap under stress is

$$\begin{aligned} &NPV_{xy}(N, \boldsymbol{\lambda}^{shock}, s, T) \\ &\quad = N\left[V_x^{prem}(\boldsymbol{\lambda}^{shock}, s, T) - V_y^{pay}(\boldsymbol{\lambda}^{shock}, T)\right]. \end{aligned} \tag{A8}$$

The difference between these values determines the shock-induced variation margin payment due from x to y for each contract, from which we deduce the net payment due by summing over all contracts between the two parties.

Endnotes

[1] However, these incentives may not be sufficient to induce firms to clear in all cases; for an analysis of this issue, see Ghamami and Glasserman (2017).

[2] There has been less research on the CDS market in the United States, mainly because of limited data access. To get around this difficulty Markose et al. (2012) use imputation techniques to estimate CDS exposures from publicly reported balance sheet data (see also Markose 2012, Bank for International Settlements 2013).

[3] Note that $f_{ij}(s_{ij}, \bar{p}_{ij}) \geq \bar{p}_{ij} - s_{ij}$, because the latter is the maximum that i can pay j.

[4] Counterparty j's willingness to extend forbearance to i will depend on how much collateral it collected from i, how likely it is that i will be able to close the gap if given extra time, and how much pressure j is under from its counterparties. These factors will vary significantly among firms.

[5] Similarly, bank holding companies compute their liquidity coverage ratios by estimating the largest net outflow of funds that they would have incurred over a look-back period of several years.

[6] The CCP actually has 30 members, but the exposures of several of them are aggregated in the data at the bank holding company level, which leads to 26 observable members.

[7] It is an interesting fact that, although this firm triggers a great deal of contagion in the system, its initial stress is very nearly the same as its final equilibrium stress: that is, the stress at this particular firm is not greatly amplified. The reason is that it is relatively peripheral to the network and does not suffer much in the way of payment shortfalls by its counterparties.

[8] Because the CCP does not post IM to anyone, its posted margin is not affected. However, members' posted margins to the CCP are increased by 50% and 100%, and therefore, the CCP *collects* more IM.

[9] Doubling of IM "costs" an additional \$20.2 billion in collateral and "saves" \$2.8 billion in reduced contagion, whereas a doubling of liquidity buffers costs an additional \$42.1 billion and saves \$5.8 billion. These results are nearly identical on a dollar for dollar basis.

References

Acemoglu D, Ozdaglar A, Tahbaz-Salehi A (2015) Systemic risk and stability in financial networks. *Amer. Econom. Rev.* 105(2): 564–608.

Ali R, Vause N, Zikes F (2016) Systemic risk in derivatives markets: A pilot study using CDS data. Bank of England Financial Stability Paper 38, Bank of England, London.

Allen F, Gale D (2000) Financial contagion. *J. Political Econom.* 108(1): 1–33.

Bank for International Settlements (2013) Macroeconomic impact assessment of OTC derivatives regulatory reforms. OTC Derivatives Assessment Team, Bank for International Settlements, Basel, Switzerland.

Bank for International Settlements (2018) BIS statistics: OTC derivatives outstanding. Accessed August 24, 2018, https://www.bis.org/statistics/derstats.htm.

BCBS and IOSCO (2015) Margin requirements for non-centrally-cleared derivatives. Technical report, BIS and OICU-IOSCO, Basil, Switzerland.

Bisias D, Flood M, Lo AW, Valavanis S (2012) A survey of systemic risk analytics. *Annual Rev. Financial Econom.* 4(1):255–296.

Blume L, Easley D, Kleinberg J, Kleinberg R, Tardos E (2011) Network formation in the presence of contagious risk. *Proc. 12th ACM Conf. Electronic Commerce* (ACM, New York), 1–10.

Brunnermeier M, Clerc L, Scheicher M (2013) Assessing contagion risks in the CDS market. *Financial Stability Rev. (Banque de France)* 17:123–134.

Capponi A, Cheng WA (2018) Clearinghouse margin requirements. *Oper. Res.* 66(6):1542–1558.

Capponi A, Cheng WA, Giglio S, Haynes R (2017) The collateral rule: An empirical analysis of the CDS market. Working paper, Columbia University, New York.

Clerc L, Gabrieli S, Kern S, El Omari Y (2014) Monitoring the European CDS market through networks: Implications for contagion risks. Banque de France Working Paper No. 477, Banque de France, Paris.

Cont R (2010) Credit default swaps and financial stability. *Financial Stability Rev. (Banque de France)* 14:35–43.

Cont R, Kokholm T (2014) Central clearing of OTC derivatives: Bilateral vs multilateral netting. *Statist. Risk Model.* 31(1):3–22.

Cont R, Minca A (2016) Credit default swaps and systemic risk. *Ann. Oper. Res.* 247(2):523–547.

Cont R, Moussa A, Santos EB (2012) Network structure and systemic risk in banking systems. Fouque J.-P., Langsam J, eds. *Handbook of Systemic Risk* (Cambridge University Press, Cambridge, UK), 327–368.

Duffie D, Zhu H (2011) Does a central clearing counterparty reduce counterparty risk? *Rev. Asset Pricing Stud.* 1(1):74–95.

Duffie D, Scheicher M, Vuillemey G (2015) Central clearing and collateral demand. *J. Financial Econom.* 116(2):237–256.

Eisenberg L, Noe T (2001) Systemic risk in financial systems. *Management Sci.* 47(2):236–249.

Elliott M, Golub B, Jackson M (2014) Financial networks and contagion. *Amer. Econom. Rev.* 104(10):3115–3153.

Elsinger H (2009) Financial networks, cross joldings, and limited liability. Oesterreichische Nationalbank Working Paper 15, Oesterreichische Nationalbank, Vienna.

Federal Reserve Board (2016) Stress tests and capital planning: Comprehensive capital analysis and review. Technical report, Board of Governors of the Federal Reserve System, Washington, DC.

Financial Crisis Inquiry Commission (2011) The financial crisis inquiry report: Final report of the national commission on the causes of the financial and economic crisis in the United States. Report, U.S. Government Printing Office, Washington, DC.

Freixas X, Parigi BM, Rochet J-C (2000) Systemic risk, interbank relations, and liquidity provision by the central bank. *J. Money Credit Banking* 32(3):611–638.

Gai P, Kapadia S (2010) Contagion in financial networks. *Proc. Roy. Soc. A: Math. Physical Engrg. Sci.* 466(2120):2401–2423.

Gai P, Haldane A, Kapadia S (2011) Complexity, concentration and contagion. *J. Monetary Econom.* 58(5):453–470.

Ghamami S, Glasserman P (2017) Does OTC derivatives reform incentivize central clearing? *J. Financial Intermediation* 32:76–87.

Glasserman P, Young HP (2015) How likely is contagion in financial networks? *J. Banking Finance* 50:383–399.

Glasserman P, Young HP (2016) Contagion in financial networks. *J. Econom. Literature* 54(3):779–831.

Haldane A, May R (2011) Systemic risk in banking ecosystems. *Nature* 469(7330):351–355.

ICE (2016) ICE Clear Credit LLC: Financial statements: Years ended December 31, 2015 and 2014. Technical report, Intercontinental Exchange, New York.

Luo LS (2005) Bootstrapping default probability curves. *J. Credit Risk* 1(4):169–179.

Markose S (2012) Systemic risk from global financial derivatives: A network analysis of contagion and its mitigation with super-spreader tax. IMF Working Paper WP/12/282, International Monetary Fund, Washington, DC.

Markose S, Giansante S, Shaghaghi AR (2012) Too interconnected to fail: Financial network of US CDS market: Topological fragility and systemic risk. *J. Econom. Behav. Organ.* 83(3): 627–646.

Peltonen TA, Scheicher M, Vuillemey G (2014) The network structure of the CDS market and its determinants. *J. Financial Stability* 13: 118–133.

Rogers LC, Veraart LA (2013) Failure and rescue in an interbank network. *Management Sci.* 59(4):882–898.

Siriwardane E (2019) Limited investment capital and credit spreads. *J. Finance.* Forthcoming.

Tarski A (1955) A lattice-theoretical fixpoint theorem and its applications. *Pacific J. Math.* 5(2):285–309.

Upper C, Worms A (2004) Estimating bilateral exposures in the German interbank market: Is there a danger of contagion? *Eur. Econom. Rev.* 48(4):827–849.

Vuillemey G, Peltonen TA (2015) Disentangling the bond–CDS nexus: A stress test model of the CDS market. *Econom. Model.* 49:32–45.