Reverse Bayesianism: A Generalization

Edi Karni*
Johns Hopkins University and
Warwick Business School

Quitzé Valenzuela-Stookey†
Northwestern University

Marie-Louise Vierø‡
Queen's University

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Abstract

In this note we generalize the main result in Karni and Vierø (2013) by allowing the discovery of new consequences to nulllify some states that were non-null before the discovery.

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*karni@jhu.edu
†quitze.vs@u.northwestern.edu
‡viero@econ.queensu.ca. Vierø acknowledges financial support from SSHRC grant number 435-2014-0642.

1 Introduction

Karni and Vierø (2013) provides an approach to modelling increasing awareness of a decision maker. The approach allows for the decision maker's state space to expand as she becomes aware of new possible actions and consequences. They named their approach to modelling increasing awareness "reverse Bayesianism," motivated by a consistency property of the decision maker's beliefs over the state spaces before and after the expansion of her awareness. Under reverse Bayesianism the likelihood ratios between originally non-null states remain unchanged upon the discovery of new consequences and the subsequent expansion of the state space.

Karni and Vierø (2013) implicitly assumes that any state that was non-null before the increase in awareness resulting from the discovery of a new consequence had to remain non-null after the discovery. Under monotonicity this is equivalent to assuming that the prior set of non-null states – what Karni and Vierø (2013) calls the feasible state space – is included in the posterior feasible state space. However, there are important situations in which such inclusion is violated. For example, situations of scientific discoveries that falsify prior beliefs. To illustrate, consider the famous Michelson–Morley experiment. The experiment compared the velocity of light traveling in perpendicular directions in an attempt to detect difference in the return time that would indicate motion of matter through the substance aether, which was hypothesized to fill empty space. The failure to detect such difference provided strong evidence against the aether theory, contradicted the predictions of Newtonian mechanics, and prompted research that eventually led to Einstein's special relativity theory.

In terms of reverse Bayesianism, the a-priori (i.e. before the Michelson–Morley experiment) feasible state-space includes states in which the velocity of light obeys the of rules of Newtonian mechanics. The Michelson-Morley experiment resulted in a consequence that required a revision of the Newtonian outlook. This revision nullified some of the a-priori feasible states simultaneous to an expansion of the conceivable state space. The present note modifies the axiomatization of Karni and Vierø (2013) so as to allow some states that were non-null before the discovery of new consequences to become null after it.

2 The Main Result

2.1 Preliminaries

We briefly restate the framework of Karni and Vierø (2013). Let F be a finite, nonempty set of *feasible acts*, and C be a finite, nonempty set of *feasible consequences.* Together these sets determine a *conceivable state space*, C^F, whose elements depict the resolutions of uncertainty.

On this conceivable state space, we define what we refer to as *conceivable acts*. Formally,

$$\hat{F} := \{f : C^F \to \Delta(C)\}, \tag{1}$$

where $\Delta(C)$ is the set of all lotteries over C. As is usually done, we abuse notation and use p to also denote the constant act that returns the lottery p in each state. We use both c and δ_c to denote the lottery that returns consequence c with probability 1, depending on the context.

Discovery of new consequences expands the conceivable state space. Let C denote the initial set of consequences and suppose that a new consequence, $\bar{c}$, is discovered. The set of consequences of which the decision maker is aware then expands to $C' = C \cup \{\bar{c}\}$. We denote by F^* the set of feasible acts with range C'. Using these notations, the expanded conceivable state space is $(C')^{F^*}$. The corresponding expanded set of conceivable acts is given by

$$\hat{F}^* := \{f : (C')^{F^*} \to \Delta(C')\}. \tag{2}$$

We consider a decision maker whose choice behavior is characterized by a preference relation $\succcurlyeq_{\hat{F}}$ on the set of conceivable acts $\hat{F}$. We denote by $\succ_{\hat{F}}$ and $\sim_{\hat{F}}$ the asymmetric and symmetric parts of $\succcurlyeq_{\hat{F}}$, with the interpretations of strict preference and indifference, respectively. For any $f \in \hat{F}$, $p \in \Delta(C)$, and $s \in C^F$, let $p_s f$ be the act in $\hat{F}$ obtained from f by replacing its $s-th$ coordinate with p. A state $s \in C^F$ is said to be *null* if $p_s f \sim_{\hat{F}} q_s f$ for all $p, q \in \Delta(C)$. A state is said to be *nonnull* if it is not null. Denote by E^N the set of null states and let $S(F, C) = C^F - E^N$ be the set of all nonnull states. Henceforth we refer to $S(F, C)$ as the *feasible state space*.

When the state space expands, so does the set of conceivable acts, which means that the preference relations must be redefined on the extended domain. Specifically, if $\hat{F}^*$ is the expanded set of conceivable acts in the wake of discoveries of new feasible consequences, then the corresponding preference relation is denoted by $\succcurlyeq_{\hat{F}^*}$. Let $\mathcal{F}$ be a family of sets of conceivable acts corresponding to increasing awareness of consequences.[1]

In Karni and Vierø (2013) it was implicitly assumed that upon the expansion of the state space following the discovery of a new consequence, non-null states remain non-null. Formally, for all $f \in \hat{F}$ and $f' \in \hat{F}^*$, if $p_s f \succ_{\hat{F}} q_s f$ then $p_s f' \succ_{\hat{F}^*} q_s f'$, for all $s \in S(F, C)$. Under monotonicity this is equivalent to assuming that $S(F, C) \subseteq S(F^*, C')$. However, as we discussed in the introduction, there are important situations in which such inclusion is violated.

For each $\hat{F} \in \mathcal{F}$, $f, g \in \hat{F}$, and $\alpha \in [0, 1]$ define the convex combination $\alpha f + (1-\alpha) g \in \hat{F}$ by: $(\alpha f + (1-\alpha) g)(s) = \alpha f(s) + (1-\alpha) g(s)$, for all $s \in C^F$. Then, $\hat{F}$ is a convex subset

[1]For the preference relation $\succcurlyeq_{\hat{F}^*}$ as well as for preference relations associated with any other awareness levels, we make the corresponding definitions to those stated in the previous paragraph.

in a linear space.[2] We assume that, for each $\hat{F} \in \mathcal{F}$, $\succcurlyeq_{\hat{F}}$ abides by the axioms of Anscombe and Aumann (1963). Formally,

(A.1) (**Weak order**) For all $\hat{F} \in \mathcal{F}$, the preference relation $\succcurlyeq_{\hat{F}}$ is transitive and complete.

(A.2) (**Archimedean**) For all $\hat{F} \in \mathcal{F}$ and $f, g, h \in \hat{F}$, if $f \succ_{\hat{F}} g$ and $g \succ_{\hat{F}} h$ then $\alpha f + (1 - \alpha) h \succ_{\hat{F}} g$ and $g \succ_{\hat{F}} \beta f + (1 - \beta) h$, for some $\alpha, \beta \in (0, 1)$.

(A.3) (**Independence**) For all $\hat{F} \in \mathcal{F}$, $f, g, h \in \hat{F}$, and $\alpha \in (0, 1]$, $f \succcurlyeq_{\hat{F}} g$ if and only if $\alpha f + (1 - \alpha) h \succcurlyeq_{\hat{F}} \alpha g + (1 - \alpha) h$.

(A.4) (**Monotonicity**) For all $\hat{F} \in \mathcal{F}$, $f \in \hat{F}$, $p, q \in \Delta (C)$ and nonnull event $E \subseteq C^F$, $f_{-E}p \succcurlyeq_{\hat{F}} f_{-E}q$ if and only if $p \succcurlyeq_{\hat{F}} q$.

(A.5) (**Nontriviality**) For all $\hat{F} \in \mathcal{F}$, $\succ_{\hat{F}} \neq \varnothing$.

Karni and Vierø (2013) postulated the following awareness consistency axiom to characterize the decision maker's reaction to expansion in his awareness of consequences:

(A.7) (**Awareness consistency**) For every given F, for all C, C' with $C \subset C'$, $S(F, C) \subseteq S(F^*, C')$, $f, g \in \hat{F}$, and $f', g' \in \hat{F}^*$, such that $f' = f$ and $g' = g$ on $S(F, C)$ and $f' = g'$ on $S(F^*, C') - S(F, C)$ it holds that $f \succcurlyeq_{\hat{F}} g$ if and only if $f' \succcurlyeq_{\hat{F}^*} g'$.

To allow for the possibility that some non-null states do become null upon the discovery of new consequences, the awareness consistency axiom (A.7) above should be modified as follows:

(A.7r) (**Revised awareness consistency**) For every given F, for all C, C' with $C \subset C'$, and for $f, g \in \hat{F}$, and $f', g' \in \hat{F}^*$, such that $f' = f$ and $g' = g$ on $S(F, C) \cap S(F^*, C')$, $f = g$ on $S(F, C) - [S(F, C) \cap S(F^*, C')]$ and $f' = g'$ on $S(F^*, C') - [S(F, C) \cap S(F^*, C')]$ it holds that $f \succcurlyeq_{\hat{F}} g$ if and only if $f' \succcurlyeq_{\hat{F}^*} g'$.

To grasp the nature of the extension of the "Reverse Bayesianism" result, consider the awareness consistency axiom (A.7). Like axiom (A.7r) it implies that the evaluation of acts is separable across nonnull events. However, it is more restrictive in the sense that it requires that equalities $f' = f$ and $g' = g$ hold on the set $S(F, C)$ while (A.7r) requires that these equalities only hold on the event $S(F^*, C') \cap S(F, C)$. This difference is significant because if, in (A.7r), we do not insist that $f = g$ on $S(F, C) - [S(F, C) \cap S(F^*, C')]$, there could arise a preference reversal $g \succ_{\hat{F}} f$ and $f' \succcurlyeq_{\hat{F}^*} g'$. This reversal reflects the fact that the

[2]Throughout this paper we use Fishburn's (1970) formulation of Anscombe and Aumann (1963). According to this formulation, mixed acts, (that is, $\alpha f + (1 - \alpha) g$) are, by definition, conceivable acts.

decision maker initially believes that all the states in $S(F,C)$ are nonnull, and following the revision of her beliefs in the wake of the discovery of new consequences, some states in $S(F,C)$ become null. Hence, it is possible that initially $g \succ_{\hat{F}} f$ because the payoffs of g in the states that would later become null make it more attractive than f but once these states are nullified, f dominates g. The awareness consistency axiom (A.7) is compelling if none of the states in $S(F,C)$ becomes null after the new consequences are discovered.

2.2 Representation theorem

Dominiak and Tserenjigmid (2018) have shown that the invariant risk preferences axiom (A.6) in Karni and Vierø (2013) is redundant. Therefore, in addition to invoking the revised awareness consistency axiom, we state and prove the theorem below without the invariant risk preferences axiom. Since the proof in Dominiak and Tserenjigmid (2018) was based on the awareness consistency axiom, the revised awareness consistency axiom requires a proof of the below theorem that differs from the proofs in both Karni and Vierø (2013) and Dominiak and Tserenjigmid (2018).

Theorem. *For each $\hat{F} \in \mathcal{F}$, let $\succcurlyeq_{\hat{F}}$ be a binary relation on $\hat{F}$ then, for all $\hat{F}, \hat{F}^* \in \mathcal{F}$, the following two conditions are equivalent:*

(i) Each $\succcurlyeq_{\hat{F}}$ satisfies (A.1) - (A.5) in Karni and Vierø (2013) and, jointly, $\succcurlyeq_{\hat{F}}$ and $\succcurlyeq_{\hat{F}^*}$ satisfy (A.7r).

(ii) There exist real-valued, non-constant, affine functions, U on $\Delta(C)$ and U^* on $\Delta(C')$, and for any two $\hat{F}, \hat{F}^* \in \mathcal{F}$, there are probability measures, $\pi_{\hat{F}}$ on C^F and $\pi_{\hat{F}^*}$ on $(C')^{F^*}$, such that for all $f, g \in \hat{F}$,

$$f \succcurlyeq_{\hat{F}} g \Leftrightarrow \sum_{s \in C^F} U\left(f(s)\right) \pi_{\hat{F}}(s) \geq \sum_{s \in C^F} U\left(g(s)\right) \pi_{\hat{F}}(s). \tag{3}$$

and, for all $f', g' \in \hat{F}^*$,

$$f' \succcurlyeq_{\hat{F}^*} g' \Leftrightarrow \sum_{s \in (C')^{F^*}} U^*\left(f'(s)\right) \pi_{\hat{F}^*}(s) \geq \sum_{s \in (C')^{F^*}} U^*\left(g'(s)\right) \pi_{\hat{F}^*}(s). \tag{4}$$

Moreover, U and U^* are unique up to positive linear transformations, and there exists such transformations for which $U(p) = U^*(p)$ for all $p \in \Delta(C)$. The probability distributions $\pi_{\hat{F}}$ and $\pi_{\hat{F}^*}$ are unique, $\pi_{\hat{F}}\left(S(F,C)\right) = \pi_{\hat{F}^*}\left(S(F^*,C')\right) = 1$, and

$$\frac{\pi_{\hat{F}}(s)}{\pi_{\hat{F}}(s')} = \frac{\pi_{\hat{F}^*}(s)}{\pi_{\hat{F}^*}(s')}, \tag{5}$$

for all $s, s' \in S(F,C) \cap S(F^*,C')$.

Note that if $S(F,C) \subseteq S(F^*,C')$, then $S(F,C) \cap S(F^*,C') = S(F,C)$, and we have the result in Theorem 1 in Karni and Vierø (2013).

The generalization in the theorem allows for a wider range of applications of reverse Bayesianism than Karni and Vierø (2013). The generalization permits a particular type of belief revision on the prior feasible state space, namely extreme belief revision in which a state is nullified. For prior feasible states that are still considered feasible after the expansion in awareness, beliefs are updated according to reverse Bayesianism. This can be justified from a philosophical viewpoint. It is possible to falsify a hypothesis, but one can only gain statistical evidence that supports that something is true. Therefore, it is reasonable that one would nullify a state when presented with evidence that falsifies what is called a link in Karni and Vierø (2013), but that with other types of evidence one maintains the relative beliefs.

It is worth mentioning that with a strengthening of axiom (A.7r), (see (A.7r') below) in conjunction with monotonicity, nullification of prior feasible states will not occur.

(A.7r') For every given F, for all C, C' with $C \subset C'$, $f, g \in \hat{F}$ and $f', g' \in \hat{F}^*$, such that $f' = f$ and $g' = g$ on $S(F^*,C') \cap S(F,C)$ and $f' = g'$ on $S(F^*,C') - [S(F,C) \cap S(F^*,C')]$ it holds that $f \succcurlyeq_{\hat{F}} g$ if and only if $f' \succcurlyeq_{\hat{F}^*} g'$.

Proposition 1 *If $\succcurlyeq_{\hat{F}}$ and $\succcurlyeq_{\hat{F}^*}$ satisfy axioms (A.7r') and (A.4) then $S(F,C) \subseteq S(F^*,C')$.*

The difference between axioms (A.7r) and (A.7r') is that axiom (A.7r) allows for preference reversals for acts that differ on $S(F,C) - [S(F^*,C') \cap S(F,C)]$, while axiom (A.7r') does not. By forcing the prior and posterior preference relations to agree on the ranking of acts, even if the acts differ in that event, axiom (A.7r') together with monotonicity, has the implication that $S(F,C) - S(F^*,C')$ must be null under the posterior preference relation.

Karni and Vierø (2013) allows for the possibility of states that were previously conceivable but null to become feasible upon the discovery of new consequences. The following revision of (A7r') rules out such belief revisions. That is, it implies that prior conceivable but null states remain null. In this case the likelihood ratios of all original conceivable states remain unchanged when beliefs are updated according to reverse Bayesianism.

(A.7r") For every given F, for all C, C' with $C \subset C'$, $f, g \in \hat{F}$ and $f', g' \in \hat{F}^*$, such that $f' = f$ and $g' = g$ on $S(F^*,C') \cap C^F$ and $f' = g'$ on $S(F^*,C') - [S(F^*,C') \cap S(F,C)]$ it holds that $f \succsim_{\hat{F}} g$ if and only if $f' \succsim_{\hat{F}^*} g'$.

Proposition 2 *Let $C \subset C'$, then (A.7r") and (A.4) imply that $S(F^*,C') \cap C^F = S(F,C)$.*

3 Proofs

3.1 Proof of Theorem:

(Sufficiency) Fix F and C. By (A.1) - (A.5), the theorem of Anscombe and Aumann (1963) and the von Neumann-Morgenstern expected utility theorem, there exists a real-valued, non-constant, function $u_{\hat{F}}$ on C such that for all $p, q \in \Delta(C)$

$$p \succsim_{\hat{F}} q \Leftrightarrow \sum_{c \in Supp(p)} u_{\hat{F}}(c)p(c) \geq \sum_{c \in Supp(q)} u_{\hat{F}}(c)q(c). \tag{6}$$

Let $C' \supset C$ and $\hat{F}^* \in \mathcal{F}$. Then, by the same argument as above, there exists a real-valued function $u_{\hat{F}^*}$ on C' such that for all $p', q' \in \Delta(C')$

$$p' \succsim_{\hat{F}^*} q' \Leftrightarrow \sum_{c \in Supp(p')} u_{\hat{F}^*}(c)p'(c) \geq \sum_{c \in Supp(q')} u_{\hat{F}^*}(c)q'(c). \tag{7}$$

The functions $u_{\hat{F}}$ and $u_{\hat{F}^*}$ are unique up to positive linear transformations. Define $U(f(s)) := \sum_{c \in Supp(f(s))} u_{\hat{F}}(c)f(s)(c)$, for all $f \in \hat{F}$ and $s \in S(\hat{F}, C)$ and define $U^*(f(s)) := \sum_{c \in Supp(f(s))} u_{\hat{F}^*}(c)f(s)(c)$, for all $f \in \hat{F}^*$ and $s \in S(\hat{F}^*, C')$.

Let b and w be a best and worst consequence in C, respectively. Without loss of generality, normalize $u_{\hat{F}}(b) = u_{\hat{F}^*}(b) = 1$ and $u_{\hat{F}}(w) = u_{\hat{F}^*}(w) = 0$.

Take any lottery $q \in \Delta(C)$ and acts $f, g \in \hat{F}$ and $f', g' \in \hat{F}^*$ such that $f' = f$ and $g' = g = q$ on $S(F, C) \cap S(F^*, C')$, $f = g = \delta_w$ on $S(F, C) - [S(F, C) \cap S(F^*, C')]$ and $f' = g' = \delta_w$ on $S(F^*, C') - [S(F, C) \cap S(F^*, C')]$. Then, by axiom (A.7r), we have that $f \succsim_{\hat{F}} g$ if and only if $f' \succsim_{\hat{F}^*} g'$, which implies that

$$\begin{aligned} &\sum_{s \in S(F,C) \cap S(F^*,C')} U(f(s))\pi_{\hat{F}}(s) = U(q)\pi_{\hat{F}}(S(F, C) \cap S(F^*, C')) \\ \Leftrightarrow &\sum_{s \in S(F,C) \cap S(F^*,C')} U^*(f(s))\pi_{\hat{F}^*}(s) = U^*(q)\pi_{\hat{F}^*}(S(F, C) \cap S(F^*, C')) \end{aligned} \tag{8}$$

The next step will show that beliefs are updated according to reverse Bayesianism for all states in $S(F, C) \cap S(F^*, C')$. For any $s \in S(F, C) \cap S(F^*, C')$, let $f(s) = \delta_c$ for some $c \in C$ and $f(\tilde{s}) = \delta_w$ for all $\tilde{s} \neq s$. Let $q = \frac{\pi_{\hat{F}}(s)}{\pi_{\hat{F}}(S(F,C) \cap S(F^*,C'))}\delta_c + (1 - \frac{\pi_{\hat{F}}(s)}{\pi_{\hat{F}}(S(F,C) \cap S(F^*,C'))})\delta_w$. Then the utility of f is given by

$$\sum_{s \in S(F,C)} U(f(s))\pi_{\hat{F}}(s) = \pi_{\hat{F}}(s)U(\delta_c) \tag{9}$$

while the utility of g is given by

$$\sum_{s \in S(F,C)} U(g(s))\pi_{\hat{F}}(s) = \frac{\pi_{\hat{F}}(s)}{\pi_{\hat{F}}(S(F, C) \cap S(F^*, C'))}U(\delta_c)\pi_{\hat{F}}(S(F, C) \cap S(F^*, C')). \tag{10}$$

Equations (9) and (10) imply that $f \sim_{\hat{F}} g$.

The utility of f' is given by

$$\sum_{s \in S(F^*,C')} U^*(f'(s))\pi_{\hat{F}^*}(s) = \pi_{\hat{F}}(s)U^*(c) \tag{11}$$

while the utility of g' is given by

$$\sum_{s \in S(F^*,C')} U^*(g'(s))\pi_{\hat{F}}(s) = \frac{\pi_{\hat{F}}(s)}{\pi_{\hat{F}}(S(F,C) \cap S(F^*,C'))} U^*(\delta_c)\pi_{\hat{F}^*}(S(F,C) \cap S(F^*,C')). \tag{12}$$

By axiom (A.7r), $f' \sim_{\hat{F}^*} g'$, since $f \sim_{\hat{F}} g$. Hence, (8) implies that

$$\pi_{\hat{F}^*}(s)U^*(\delta_c) = \frac{\pi_{\hat{F}}(s)}{\pi_{\hat{F}}(S(F,C) \cap S(F^*,C'))} U^*(\delta_c)\pi_{\hat{F}^*}(S(F,C) \cap S(F^*,C')). \tag{13}$$

Equivalently,

$$\frac{\pi_{\hat{F}^*}(s)}{\pi_{\hat{F}^*}(S(F,C) \cap S(F^*,C'))} = \frac{\pi_{\hat{F}}(s)}{\pi_{\hat{F}}(S(F,C) \cap S(F^*,C'))} \tag{14}$$

Next we show that under the normalization of the utility functions $u_{\hat{F}}$ and $u_{\hat{F}^*}$ it holds that $U^*(\delta_c) = U(\delta_c)$ for all $c \in C$. For any $c \in C$ and $s \in S(F,C) \cap S(F^*,C')$), let $f(s) = \alpha\delta_c + (1-\alpha)\delta_w$ and $f(\tilde{s}) = \delta_w$ for all $\tilde{s} \neq s$. Let $q = \beta\delta_b + (1-\beta)\delta_w$. By the normalization, $U(\delta_b) = U^*(\delta_b) = 1$. Hence, (8) implies that

$$\alpha U(\delta_c)\pi_{\hat{F}}(s) = \beta\pi_{\hat{F}}(S(F,C) \cap S(F^*,C'))) \tag{15}$$

$$\Leftrightarrow \pi_{\hat{F}^*}(s)\alpha U^*(\delta_c) = \pi_{\hat{F}^*}(S(F,C) \cap S(F^*,C')))\beta \tag{16}$$

Equations (14), (15), and (16) now imply that $U^*(\delta_c) = U(\delta_c)$ for all $c \in \Delta(C)$.

(Necessity) The necessity of (A.1)-(A.5) is an implication of the Anscombe and Aumann (1963) theorem. The necessity of (A.7r) is immediate.

The uniqueness part is an implication of the uniqueness of the utility and probability in Anscombe and Aumann (1963). ■

3.2 Proof of Proposition 1:

Suppose there exists s such that $s \in S(F,C), s \notin S(F^*,C')$. Let $p,q \in \Delta(C)$ be such that $p \succ_{\hat{F}} q$ (Note that if no such p,q exist then by Monotonicity $S(F,C) = \varnothing$, so the result is trivial). Define the conceivable acts f,g, and f',g' as follows: $f = g$ on $S(F,C) \setminus s$, $f(s) = p$, $g(s) = q$. Furthermore $f' = f$, $g' = g$ on $S(F,C)$, and $f' = g'$ on $S(F^*,C') - [S(F^*,C') \cap S(F,C)]$. Note that f' and g' coincide exactly on $S(F^*,C')$, so $f' \sim_{\hat{F}^*} g$. Moreover, by Monotonicity $f \succ_{\hat{F}} g'$. Since f,g,f',g' satisfy the conditions of (A.7r') we have a contradiction. ■

3.3 Proof of proposition 2

First show $S(F^*, C') \cap C^F \subseteq S(F, C)$. Suppose there exists s such that $s \in S(F^*, C') \cap C^F$, $s \notin S(F, C)$. Let $p, q \in \Delta(C)$ be such that $p \succ_{\hat{F}^*} q$ (if no such p, q exist then, by (A.4), $S(F^*, C') = \varnothing$, so the result is trivial). Define the conceivable acts f, g, and f', g' as follows: $f = g$ on $C^F \setminus s$, $f(s) = p$, $g(s) = q$. Furthermore $f' = f$, $g' = g$ on $S(F^*, C') \cap C^F$, and $f' = g'$ on $S(F^*, C') - [S(F^*, C') \cap C^F]$. But f and g coincide on $S(F, C)$,hence, $f \sim_{\hat{F}} g$. By (A.4) $f' \succ_{\hat{F}^*} g'$. Since f, g, f', g' satisfy the conditions of (A.7r") we have a contradiction.

For the reverse direction, suppose there exists s such that $s \in S(F, C), s \notin S(F^*, C')$. Let $p, q \in \Delta(C)$ be such that $p \succ_{\hat{F}} q$ (if no such p, q exist then by (A.4) $S(F, C) = \varnothing$, so the result is trivial). Define the conceivable acts f, g, and f', g' as follows: $f = g$ on $S(F, C) \setminus s$, $f(s) = p$, $g(s) = q$. Furthermore $f' = f$, $g' = g$ on $S(F, C)$, and $f' = g'$ on $S(F^*, C') - [S(F^*, C') \cap S(F, C)]$. But f' and g' coincide on $S(F^*, C')$, hence $f' \sim_{\hat{F}^*} g$. By (A.4) $f \succ_{\hat{F}} g'$. Since f, g, f', g' satisfy the conditions of (A.7r") we have a contradiction. ■

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