A Theory of Medical Decision Making

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Abstract

This paper presents an axiomatic model of medical decision making and discusses its potential applications. The medical decision problems envisioned concern the choice of a medical treatment following a diagnosis in situations in which data allow construction of an empirical distribution over the potential outcomes associated with the alternative treatments. In its descriptive interpretation, the model is an hypothesis about the patient's choice behavior. The theory also aims to aid physicians recommend treatments in a coherent manner.

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1 Introduction

For the purpose of this paper, the term *medical decision making* refers to the choice of a course of action (action, for short) following a diagnosis of a patient's condition. An action consists of the medical treatment itself; the facility in which it is to be administered; and, if perceived relevant, the individuals who administer it. Consider, for example, a patient diagnosed with prostate cancer. Given his specific personal characteristics (medical history, age, physical condition, and so forth), the patient must choose among various treatments (surgery, radiation therapy); the medical facilities in which he is to be treated (the local hospital, a medical center in another city); and the physician who performs the surgery or administers the therapy of choice. The consequences consist of the patient's post-treatment state of health, including the side effects of treatment; the associated pain and inconvenience; the direct monetary expenses; and the potential loss of income.

In many situations involving medical decision making, the empirical probability (that is, the relative frequency) of the different outcomes conditional on the treatments, characteristics of the patient, and choice of hospital and physician are known. The question is, how does (or how should) an informed patient choose among the possible courses of action?

In this paper I propose a theory of medical decision making in which the patient's preferences are represented by an outcome-dependent expected utility function. More formally, let a denote an action and denote by c a vector of the patient's characteristics (medical history, age, gender, race, profession, family situation, physical state, and any other personal attributes that may bear on the outcome of the medical treatments under consideration). I examine the structure of a preference relation, $\succcurlyeq$, on the set of actions that is necessary and sufficient for the following representation:

$$(a,c) \mapsto \lambda(a) \sum_{\omega \in \Omega} U(f(\omega; a, c), \omega)\, p(\omega \mid a, c) + v(a),$$

where U is the utility function; ω denotes the post-treatment health state (or outcome); Ω is the set of all outcomes associated with a given diagnosis; $f(\omega; a, c)$ denotes the financial consequence associated with the outcome ω conditional on the patient's characteristics and the action; $p(\cdot \mid a, c)$ is the probability distribution on Ω conditional on the action and the patient's personal characteristics; and λ and v represent the "utility cost," including the pain or discomfort associated with different actions. Note that the patient's risk attitudes, captured by the utility functions of money, $U(\cdot, \omega), \omega \in \Omega$, are outcome dependent but not action dependent.

The application of this model to medical decision making requires the elicitation of the utility functions $U(\cdot, \omega), \omega \in \Omega$; their alignment; and the elicitation of the coefficients $\lambda(a), v(a)$ for all actions, a. Because the model is preference based, the information needed to implement it is, in principle, obtainable from the patient's expressed preferences over conceivable actions and payoff functions f.

The next section presents the theory. Section 3 examines the issues involved in implementing this model. Concluding remarks appear in Section 4. The proof of the main result is given in the appendix.

2 A Model of Medical Decision Making

2.1 The analytical framework

Let Θ denote a finite set whose elements are health diagnoses.[1] For every $\theta \in \Theta$ let $A(\theta)$ denote a finite set of actions, that is, descriptions of the medical aspects of the procedures in all their relevant aspects.[2] For instance, when the diagnosis calls for surgery, an action includes specification of the surgical procedure itself, the facility in which the operation is to take place, the surgeon who is to perform the surgery, the hospitalization and medical follow-up. Let $\Omega_a(\theta)$ denote the finite set of possible outcomes that might result when the diagnosis is θ and the action taken is $a \in A(\theta)$, and let $\Omega(\theta) = \cup_{a \in A(\theta)} \Omega_a(\theta)$.

Denote by $P(\theta)$ the set of all probability distributions on $\Omega(\theta)$ and assume that it is endowed with the $\mathbb{R}^{|\Omega(\theta)|}$ topology. Clearly, $P(\theta)$ contains the set of $\{p_\theta(\cdot \mid a) \mid a \in A(\theta)\}$ of probability distributions on $\Omega(\theta)$ conditional on the available actions. For each $\omega \in \Omega(\theta)$, let I_ω be a closed and bounded interval in $\mathbb{R}$. A *bet,* f, is an element of the product set $F(\theta) := \Pi_{\omega \in \Omega(\theta)} I_\omega$, representing outcome-contingent monetary payoffs. For instance, one may bet on the outcome of a bypass surgery according to which he wins x dollars if he survives the opration and losses y dollars if he does not, to be paid by his estate. Assume

[1] In view of our definition of medical decision problems, the interpretation of θ is the doctor's diagnosis rather than the patient's true state of health.

[2] The specifications of the actions do not include the financial dimentions of the medical procedure, which is handled separately.

that $F(\theta)$ is endowed with the $\mathbb{R}^{|\Omega(\theta)|}$ topology. (Note that a pair (p, f) defines a lottery that, for every $\omega \in \Omega$, assigns the probability $p(\omega)$ to the monetary prize $f(\omega)$).

For every θ, the patient is supposed to be able to conceive of having to choose among elements of $\mathbb{C}(\theta) := A(\theta) \times P(\theta) \times F(\theta)$ consisting of an action in $A(\theta)$, a probability in $P(\theta)$, and a bet in $F(\theta)$. Then $\mathbb{C}(\theta)$ is the *conceivable choice set*. Since a medical decision problem always begins with a diagnosis which is then fixed, to simplify the notation, henceforth I suppress the diagnosis θ.

A preference relation $\succeq$ on $\mathbb{C}$ is a binary relation that has the following interpretation: $(a, p, f) \succeq (a', p', f')$ means that if the patient were in a position that requires him to choose between (a, p, f) and (a', p', f'), he would choose (a, p, f) or be indifferent between the two alternatives. The induced strict preference relation, $\succ$, and indifference relation, $\sim$, are defined as usual and have the usual interpretation.

I assume throughout that $\succeq$ is a weak order, that is,

(A.1) $\succeq$ *is complete and transitive.*

To describe the structure of the preference relation, it is convenient to dissect it and examine each of its components separately.

2.2 Treatment-contingent preferences

For each action, a, define a conditional preference relation $\succeq_a$ on $P \times F$ by $(p, f) \succeq_a (p', f')$ if $(a, p, f) \succeq (a, p', f')$. By definition and (A.1), $\succeq_a$ is a weak order.

For the conditional preferences $\succeq_a$, I adopt the structure of Karni and Safra (2000). Specifically, I assume that the conditional preference relations in the set $\{\succeq_a \mid a \in A\}$ satisfy the following axioms:

(A.2) Continuity - *For all $(p, f) \in P \times F$ the sets $\{(p', f) \mid (p', f) \succeq_a (p, f)\}$ and $\{(p', f) \mid (p, f) \succeq_a (p', f)\}$ are closed in the product topology.*

The second axiom requires that every outcome matters. Formally, let e^ω be the $\omega-$th unit vector in $\mathbb{R}^{|\Omega|}$ (that is, $e^\omega \in P$ is the degenerate probability distribution that assigns the unit probability mass to ω) then,

(A.3) Coordinate Essentiality - *For all $\omega \in \Omega$, there are $f, f' \in F$ such that $(e^\omega, f) \succ_a (e^\omega, f')$.*

The next axiom requires that the evaluation of outcome-contingent payoffs be independent in the sense that preferences among alternatives of the form $(e^\omega, (r, f_{-\omega}))$, where $(r, f_{-\omega}) := (f(\omega_1), ..., f(\omega_{i-1}), r, f(\omega_{i+1}), ..., f(\omega_n))$, depend solely on the payoff of the bet f if the outcome ω obtains. Formally,

(A.4) Certainty Principle - *For all* $f, f', f'', f''' \in F$, $(e^{\omega}, (x, f_{-\omega})) \succeq_a (e^{\omega}, (y, f'_{-\omega}))$ *if and only if* $(e^{\omega}, (x, f''_{-\omega})) \succeq_a (e^{\omega}, (y, f'''_{-\omega}))$.

Define the partial mixture operation on $P \times F$ as follows: for every given $f \in F$, $(p, f), (p', f)$ and $\alpha \in [0, 1]$, $\alpha (p, f) + (1 - \alpha) (p', f) = (\alpha p + (1 - \alpha) p', f)$. This may be interpreted as a two-stage lottery in which, in the first stage, the alternatives (p, f) and (p', f) obtain with probabilities α and $(1 - \alpha)$, respectively. In the second stage, the payoff of f is determined by the lottery, p or p', that was chosen in the first stage. With this interpretation in mind, assume that the decision maker prefers (p, f) over (p', f') and (q, f) over (q', f'). Moreover, assume that if a decision maker faces a choice between the alternatives $L = (\alpha p + (1 - \alpha) q, f)$ and $L' = (\alpha p' + (1 - \alpha) q', f')$ he reasons as follows: if the event whose probability is α is realized, he participates in the lottery (p, f) if he has chosen L and in the lottery (p', f') if he has chosen L'. Conditional on the realization of this event, he is better off with L. By the same logic, he would also prefer L over L' conditional on the realization of the event whose probability is $1 - \alpha$. Consequently, he prefers L over L' unconditionally. Formally,

(A.5) Constrained Independence - *For all* (p, f), (q, f), (p', f'), (q', f') *in* $P \times F$ *and* $\alpha \in [0, 1)$ *if* $(p, f) \sim_a (p', f')$ *then* $(q, f) \succeq_a (q', f')$ *if and only if* $(\alpha p + (1 - \alpha) q, f) \succeq_a (\alpha p' + (1 - \alpha) q', f')$.

A real valued function V_a on $P \times F$ is said to *represent* $\succeq_a$ if, for all (p, f) and (p', f') in $P \times F$, $(p, f) \succeq_a (p', f')$ if and only if $V_a (p, f) \geq V_a (p', f')$. If $V_a (p, f) := \sum_{\omega \in \Omega} p (\omega) U_a (f (\omega), \omega)$

for some functions, $U_a(\cdot,\omega):\mathbb{R}\rightarrow\mathbb{R},\omega\in\Omega$, it is a *linear representation* of $\succeq_a$. Let $\sum_{\omega\in\Omega}p(\omega)U_a(f(\omega),\omega)$ represent $\succeq_a$. The functions $U_a(\cdot,\omega),\omega\in\Omega$ are said to be unique up to *uniform positive linear transformation* if, for any other linear representation of $\succeq_a$, $\sum_{\omega\in\Omega}p(\omega)\hat{U}(f(\omega),\omega)$, $\hat{U}(\cdot,\omega)=\beta U(\cdot,\omega)+\gamma,\beta>0$, for all $\omega\in\Omega$.

The next theorem restates, in the terminology of this paper, Theorem 2 of Karni and Safra (2000).

Theorem 1: *Let $\succeq_a$ be a binary relation on $P\times F$. Then the following conditions are equivalent:*

(a) $\succeq_a$ is a weak order satisfying (A.2) – (A.5).

(b) There exist continuous, non-constant, functions $V_a:P\times F\rightarrow\mathbb{R}$ and $U_a(\cdot,\omega):\mathbb{R}\rightarrow\mathbb{R}$, $\omega\in\Omega$, such that V_a represents $\succeq_a$ and, for all $(p,f)\in P\times F$,

$$V_a(p,f)=\sum_{\omega\in\Omega}p(\omega)U_a(f(\omega),\omega).$$

Moreover, the functions $U_a(\cdot,\omega),\omega\in\Omega(\theta)$, are unique up to uniform positive linear transformation.

The proof is given in Karni and Safra (2000).[3]

[3]The uniqueness part of the theorem in Karni and Safra (2000) states that $U_a(\cdot,\omega)$ are unique up to the following transformations: $\beta U_a(\cdot,\omega)+\gamma(\omega)$, $\beta>0$ and $\sum_{\omega\in\Omega}\gamma(\omega)=\gamma$. This is a mistake. The uniqueness requires that $\gamma(\omega)=\gamma$ for all $\omega\in\Omega$, hence the uniformity.

2.3 Action-independent risk attitudes and the representation of $\succeq$

Medical treatments are costly in terms of time and discomfort. These are temporary, however, and unlikely to alter the patient's risk attitudes. To capture this aspect of the patient's preferences, the next axiom asserts that the risk attitudes are action independent.

(A.6) Action-independent risk attitudes - *For all* $a, a' \in A$, $\succeq_a = \succeq_{a'}$.

A certain richness of the choice space is necessary to link distinct action-contingent preferences. Specifically, there must be some staggered utility overlap among the actions. Formalizing this idea, it is useful to use the following additional terminology: two actions, a and a', are said to be *elementarily linked* at $f \in F$ if there are $\bar{p}, \underline{p}, \bar{p}', \underline{p}' \in P$ satisfying $(\bar{p}, f) \succ_a (\underline{p}, f)$ such that $(a, \bar{p}, f) \sim (a', \bar{p}', f)$ and $(a, \underline{p}, f) \sim (a', \underline{p}', f)$. (Note that, by transitivity, $(\bar{p}', f) \succ_{a'} (\underline{p}', f)$). The treatments a and a' are *linked* if there is a sequence of actions $a_1, ..., a_n$ such that $a = a_1, a' = a_n$ and the actions a_i and a_{i+1} are elementarily linked at $f_i \in F$, $i = 1, ..., n-1$. The set of actions is *linked* if all its elements are linked.

The next theorem is the main result.

Theorem 2: *Let* $\succeq$ *be a preference relation on* $\mathbb{C}$ *and denote by* $\{\succeq_a \mid a \in A\}$ *the induced action-contingent preference relations on* $P \times F$. *If* A *is linked, then the following conditions are equivalent:*

(a) $\succeq$ *is a weak order and the induced preference relations* $\{\succeq_a \mid a \in A\}$ *satisfy (A.2) – (A.6).*

(b) There exist continuous nonconstant functions $V : \mathbb{C} \to \mathbb{R}$, $U(\cdot, \omega) : I_\omega \to \mathbb{R}$, $\omega \in \Omega$, $\lambda : A \to \mathbb{R}_{++}$ and $v : A \to \mathbb{R}$ *such that* V *represents* $\succeq$ *and for all* $(a, p, f) \in \mathbb{C}$,

$$V(a, p, f) = \lambda(a) \sum_{\omega \in \Omega} p(\omega) U(f(\omega), \omega) + v(a).$$

Moreover, the functions $U(\cdot, \omega), \omega \in \Omega$ *are unique up to a uniform positive linear transformation and, given* $U(\cdot, \omega), \omega \in \Omega$, λ *and* v *are unique.*

2.4 Medical decision making

The probabilities and the financial consequences of the different outcomes contingent on patient characteristics and available actions are determined by the "state of the art," or technology. Formally, a *technology* is a function $t : C \times A(\theta) \to P(\theta) \times F(\theta)$ that associates with each vector of personal characteristics and action a probability distribution on $\Omega(\theta)$ and a bet in $F(\theta)$ depicting the financial consequences associated with the different outcomes. These consequences depend on the patient's health, disability, and life insurance coverage and occupation which, in turn, determine the potential loss of income. A medical decision entails a choice among alternatives in $A(\theta)$. Given the patient's characteristics, c, and the technology, t, define a preference relation on $A(\theta)$ by $a \succcurlyeq_c a'$ if and only if $(a, t(a; c)) := (a, p(a; c), f(a; c)) \succeq (a', p(a', c), f(a, c)) := (a', t(a'; c))$. Thus, given the patient characteristics and technology, the application of Theorem 2 implies that, for all $a, a' \in A(\theta)$, $a \succcurlyeq_c a'$ if and only if

$$\lambda(a) \sum_{\omega \in \Omega(\theta)} p(\omega; a, c)\, U(f(\omega; a, c), \omega) + v(a) \geq \lambda(a') \sum_{\omega \in \Omega(\theta)} p(\omega; a', c)\, U(f(\omega; a', c), \omega) + v(a'). \tag{1}$$

Note that the choice of a affects the patient's well-being in two distinct ways. First, as already mentioned, the alternatives actions may involve different degrees of pain, suffering, and inconvenience. This aspect of the choice of action is captured by the functions λ and v. Second, the patient's insurance may cover some alternatives fully and some others only partially or not at all and, in addition, depending on his occupation, the various outcomes may have distinct financial implications. These financial aspects of the decision is captured by the dependence of $f(\cdot; a, c)$ on a. For instance, if the patient's insurance fully covers the medical costs of the action a then $f(\cdot; a, c) = f(\cdot; c)$, where $f(\cdot; c)$ depicts the contingent loss of income (uncovered by insurance). If the medical costs of some actions are coinsured (for instance, under coinsurance, only x percent of the cost of a is covered) then $f(\cdot; a, c) = f(\cdot; c) - (1 - x)g(a)$, where $g(a)$ denotes the full financial cost of a.

3 Application

3.1 Outline of the procedures

The most important and immediate application of this model is helping physicians and patients decide which course of action is most appropriate in a given situation. Such decisions are based on information from two sources: (a) medical information, provided by the physician, specifying the set of outcomes Ω and the probabilities $\{p(\cdot, a, c) \mid a \in A\}$ conditional on the action and patient's characteristics, and (b) personal information, provided by the patient, concerning his characteristics and preferences, on the basis of which the relevant utility functions U, λ and v are to be chosen.

The elicitation of the subjective "parameters," (that is, the outcome-dependent utility functions and action-dependent cost coefficients) involves three distinct procedures. First, for every given outcome, it is necessary to elicit the outcome-dependent utility function (that is, for all $\omega \in \Omega$, the functions $U(\cdot, \omega)$ must be determined). Second, the outcome-dependent utility functions need to be aligned, so that they agree on the evaluation of the monetary payoff across outcomes. Third, the expected utilities of the distinct actions must be calibrated to allow comparisons among them.

3.2 Elicitation of the patient's risk attitudes

The elicitation of von Neumann-Morgenstern utility functions can be done using distinct methods. A well-known method is based on the elicitation the certainty equivalents of lotteries, using direct comparisons (see Abdellaoui et al. [2007]) or a technique introduced by Becker, DeGroot, and Marchak (1964), according to which, under expected utility preferences, true revelation of certainty equivalent is incentive compatible. Repeated elicitations of certainty equivalents of lotteries allow the construction of a utility function.

The economic and financial literature pays special attention to certain parametric families of utility functions, including the power family; the exponential family; the expo-power family (due to Saha [1993]); and the hyperbolic absolute risk aversion family (HARA), introduced by Merton (1971).[4] From an empirical point of view, the use of parametric utility functions offers a reasonable trade-off between generality and economy of observations. This is especially true when one is interested in local approximations.

Arrow (1965) advocates the use of utility functions displaying decreasing absolute risk aversion and increasing relative risk aversion. More recently, experimental studies of risk attitudes by Holt and Laury (2002) and Abdellaoui et al. (2007) argue in favor of using variations of the expo-power family. Holt and Laury (2002) lend support to a parametric

[4] Abdellaoui et al. (2007) introduce and used a special, one-parameter, variation of the expo-power family. Holt and Laury (2002) used the expo-power family of Saha (1993) to study the nature of risk aversion and its dependence on the stakes.

family of utility functions that take the form

$$u(w) = \frac{1 - \exp\left(-\alpha w^{1-r}\right)}{\alpha}, \tag{2}$$

where w denotes the decision maker's wealth; $\alpha \geq 0$ and $1 \geq r \geq 0$. In the limit, as α tends to zero, this utility function becomes linear in w. The de Finetti (1952), Arrow (1965) and Pratt (1964) measure of relative risk aversion for this family of utility functions is

$$-\frac{u''(w)\, w}{u'(w)} = r + \alpha(1-r)\, w^{1-r}. \tag{3}$$

Hence, the utility function displays constant relative risk aversion, r, when $\alpha = 0$ and constant absolute risk aversion when $r = 0$. When both $\alpha > 0$ and $r > 0$, the utility functions display decreasing absolute risk aversion and increasing relative risk aversion. Holt and Laury (2002) estimate $r = 0.269$ and $\alpha = 0.029$.

Abdellaoui et al. (2007) one-parameter version of the expo-power family is

$$u(w) = -\exp\left(-w^r/r\right), \text{ for } r \neq 0 \text{ and } u(w) = -1/w \text{ for } r = 0. \tag{4}$$

For $r \in [0,1]$, this function displays decreasing absolute and increasing relative risk aversion. Using the trade-off elicitation procedure in an experimental setting, their estimate of r based on group average is $r = 1.242$. This implies a utility function that is slightly convex at low levels of wealth and slightly concave at high levels of wealth.

Adopting the expo-power parametric family of utility functions to the present context, the outcome dependence of the preference relation requires that the parameter values depend on the outcomes. In the two-parameter case, for instance, this amounts to specifying utility functions as follows,

$$U(w,\omega)=\frac{1-\exp\left(-\alpha(\omega)\,w^{1-r(\omega)}\right)}{\alpha(\omega)},\quad \omega\in\Omega. \tag{5}$$

The corresponding outcome-dependent degrees of relative risk aversion are given by

$$-\frac{U''(w,\omega)\,w}{U'(w,\omega)}=r(\omega)+\alpha(\omega)\,(1-r(\omega))\,w^{1-r(\omega)},\ \ \omega\in\Omega. \tag{6}$$

To apply this model, it is necessary to estimate the parameter values $\{\alpha(\omega), r(\omega) \mid \omega\in\Omega\}$, which raises a methodological issue. The elicitation of an outcome-independent von Neumann-Morgenstern utility function requires that the decision maker choose among lotteries and evaluates their payoffs from "where he stands." The elicitation of outcome-dependent von Neumann-Morgenstern utility functions requires that the decision maker evaluate the lottery payoffs contingent on outcomes not yet experienced by him. For example, a patient who needs to undergo prostate cancer surgery that may result in incontinence must evaluate lottery payoffs conditional on physical conditions which are not part of his experience, and in some sense, are "life-changin". It is possible that the ex ante perceived and ex post actual evaluations differ.[5] If the relevant valuation is the ex post one and if it depends on per-

[5] See for instance Brickman, Coates and Janoff-Bulman (1978) for a study showing that the ex-post level of happiness reported by lottery winners and accident victims were not markedly different from that of a individuals belonging to randomly selected control group. According to these authors their findings suggest that "we tend to overestimate the magnitude, generality, and the duration of people feelings." (Brickman et. al. (1978) p. 926). This tendency is related to pheomenon, reported in Deutch (1960) and Andrew and Withey (1976), of observers who see actors as more distress by their misfortune than the actors see themselves.

sonal characteristics such as age, gender, marital status, number of children, education, and profession, it may be possible to elicit the utility functions of individuals with the relevant health condition and ascribe the resulting utility to individuals with similar characteristics.

3.3 Alignment of the utility functions

Suppose that the estimated parameter values of the utility functions in (5) are obtained. For every given outcome, ω, the elicited utility function is unique up to a positive linear transformation. The next step requires aligning the utility functions across outcomes. This involves a simple procedure. Fix ω_0 and $w' > w$, and set $U(w', \omega_0) = 1$, $U(w, \omega_0) = 0$. For each $\omega \in \Omega - \{\omega_0\}$, let the decision maker indicate the wealth levels $w(\omega)$ and $w'(\omega)$ that would leave him indifferent between the payoff-outcome pairs $(w(\omega), \omega)$ and (w, ω_0) and between the payoff-outcome pairs $(w'(\omega), \omega)$ and (w', ω_0). Formally, let $w(\omega)$ and $w'(\omega)$ be defined by $\delta_{(w(\omega),\omega)} \sim \delta_{(w,\omega_0)}$ and $\delta_{(w'(\omega),\omega)} \sim \delta_{(w',\omega_0)}$. By (A.6), this indifference relation is independent of the action. For each $\omega \in \Omega$, let $b(\omega)$ and $a(\omega)$ be the solution to the equations

$$\frac{b(\omega)\left[1 - \exp\left(-\alpha(\omega)(w')^{1-r(\omega)}\right)\right]}{\alpha(\omega)} + a(\omega) = 1 \tag{7}$$

and

$$\frac{b(\omega)\left[1 - \exp\left(-\alpha(\omega) w^{1-r(\omega)}\right)\right]}{\alpha(\omega)} + a(\omega) = 0. \tag{8}$$

For every $w \in I_\omega$ and $\omega \in \Omega$, let

$$U(w, \omega) := \frac{b(\omega)\left[1 - \exp\left(-\alpha(\omega) w^{1-r(\omega)}\right)\right]}{\alpha(\omega)} + a(\omega). \tag{9}$$

3.4 Calibration of utility across actions

With the utility functions given in equations (9) and invoking the linkage of the set of actions, arrange A in a sequence, $a_1, ..., a_n$, such that a_i and a_{i+1} are elementarily linked. For every $i = 1, ..., n-1$, choose $f_i \in F$ and $\bar{p}_i, \underline{p}_i, \bar{p}_{i+1}, \underline{p}_{i+1} \in P$ such that $(\bar{p}_i, f_i) \succ_{a_i} \left(\underline{p}_i, f_i\right)$, $(a_i, \bar{p}_i, f_i) \sim (a_{i+1}, \bar{p}_{i+1}, f_i)$ and $\left(a_i, \underline{p}_i, f_i\right) \sim \left(a_{i+1}, \underline{p}_{i+1}, f_i\right)$. To simplify the notation let $\bar{\zeta}(a_i) := \sum_{\omega \in \Omega(\theta)} \bar{p}_i(\omega; a_i, c) U(f_i(\omega; a_i, c), \omega)$ and $\underline{\zeta}(a_i) := \sum_{\omega \in \Omega(\theta)} \underline{p}_i(\omega; a_i, c) U(f_i(\omega; a_i, c), \omega)$.

Setting $\lambda(a_1) = 1$ and $v(a_1) = 0$ and invoking Theorem 2 and equation (1), solve sequentially for $\lambda(a_i)$ and $v(a_i)$, $i = 2, ..., n$, using, at each stage, the pairs of equations

$$\lambda(a_i) \bar{\zeta}(a_i) + v(a_i) = \lambda(a_i) \bar{\zeta}(a_{i+1}) + v(a_{i+1}) \tag{10}$$

and

$$\lambda(a_i) \underline{\zeta}(a_i) + v(a_i) = \lambda(a_i) \underline{\zeta}(a_{i+1}) + v(a_{i+1}). \tag{11}$$

The estimation of the utility of action coefficients, λ and v, relies on the patient's assessment of the pain and discomfort associated with procedures that he may have never experienced before. However, unlike with the estimation of the outcome-dependent utility, the discomfort is not a "life-changing" event, which may affect his long-term well-being and attitudes. In this respect, the estimation of these coefficients is more like the estimation of a consumer's utility function over regions of the commodity space that, because of budget constraints, he never experienced.

4 Concluding Remarks

The model presented here can be interpreted as an hypothesis about decision makers' choice behavior in situations requiring medical decision making. The decision makers are patients (or their guardians where patients are unable to make decisions themselves). The decision makers are supposed to be informed about the diagnosis, the available courses of actions, their consequences, and the probabilities of the associated outcomes.

It often happens that, upon receiving a diagnosis, the informed decision maker asks the physician to recommend a course of action. Such a recommendation entails a normative judgment, involving an assessment of the implications of the alternative actions on the patient's well-being, presumably incorporating his or her personal characteristics and values. In such cases, the use of the expected utility model, whose axiomatic foundations are normatively compelling, seems appropriate. The model thus help physicians identify, organize, and integrate the relevant data to attain consistency and coherence in their recommendations.

The model presented here applies to medical decision problems for which the data may be summarized in the form of empirical distributions over outcomes conditional on actions and patients' characteristics. Medical decision problems in which such data are not available require different treatment. In particular, they require the parallel assessments of the subjective probabilities of the physician making the recommendation and the patient's valuation of outcomes, which are then integrated to construct a decision criterion. The modeling of the physician's subjective beliefs regarding the likely realization of the alternative outcomes following each treatment can be developed along the lines explore in Karni (2006, 2007). Treatment of this important subject is beyond the scope of this paper.

BIBLIOGRAPHY

Abdellaoui, M., C. Barrios and P. P. Wakker (2007) "Reconciling Introspective Utility with Revealed Preference: Experimental Arguments Based on Prospect Theory." *Journal of Econometrics*, 138, 356-378.

Andrews, F. M. and S. B. Withey (1976) *Social Indicators of Well-Being.* New York: Plenum Press.

Arrow, K. J. (1965) "Aspects of the Theory of Risk-Bearing," Helsinki: Yrjö Hahnsson Foundation.

Becker, G. M., M. H. DeGroot, and J. Marschak (1964) "Measuring Utility by a Single Response Sequential Method," *Behavioral Science* 9 226-232.

Brickman, P. D., Coates, and R. Janoff-Bulman (1978) "Lottery Winners and Accident Vctims: Is Happiness Relative?" *Journal of Personality and Social Psychology,* 36, 917-927.

de Finetti, B. (1952) "Sulla preferibilita." *Giornale degli Economisti e Annali di Economia,* 11, 685–709.

Deutch, M. (1960) "The Pathetic Fallacy: An Observer Error in Social Perception," *Journal of Personality*, 28, 317-332.

Holt, C. A. and S. K. Laury (2002) "Risk Aversion and Incentive Effects," *American Economic Review,* 92, 1644–1655.

Karni, E. (2006) “Subjective Expected Utility Theory without States of the World.” *Journal of Mathematical Economics*, 42, 325–342.

Karni, E. (2007) “Bayesian Decision Making and the Representation of Beliefs.” Unpublished manuscript.

Karni, E. and Z. Safra (2000) “An Extension of a Theorem of von Neumann and Morgenstern with Application to Social Choice Theory,” *Journal of Mathematical Economics*, 34, 315–327.

Merton, R. C. (1971) “Optimum Consumption and Portfolio Rules in Continuous-Time Model.” *Journal of Economic Theroy*, 3, 373–413.

Pratt, J. W. (1964) “Risk Aversion in the Small and in the Large.” *Econometrica* 32, 122–136.

Saha, A. (1993) “Expo-Power Utility: A Flexible Form for Absolute and Relative Risk Aversion.” *American Journal of Agricultural Economics,* 75, 905–913.

APPENDIX

Proof of Theorem 2: $(a) \Rightarrow (b)$. By Theorem 1, $\succeq_a$ is represented by

$$V_a(p, f) = \sum_{\omega \in \Omega} p(\omega) U_a(f(\omega), \omega). \tag{12}$$

Action-independent risk attitudes, (A.6), and the uniqueness part of Theorem 1 imply that for all a, $a' \in A$, $U_a(\cdot, \omega)$ and $U_{a'}(\cdot, \omega)$ are linear transformations of one another. Fix a^0 and let $U(\cdot, \omega) := U_{a^0}(\cdot, \omega)$ for all $\omega \in \Omega$. Then

$$U_a(f(\omega), \omega) = \lambda(a) U(f(\omega), \omega) + v(a), \text{ for all } a \in A, f \in F \text{ and } \omega \in \Omega. \tag{13}$$

Let $a_0, a \in A$ be elementarily linked, and define $\lambda(a)$ and $v(a)$ by the unique solution to the following equations:

$$\lambda(a) \sum_{\omega \in \Omega} \bar{p}(\omega) U(f(\omega), \omega) + v(a) = \sum_{\omega \in \Omega} \bar{p}'(\omega) U(f(\omega), \omega) \tag{14}$$

and

$$\lambda(a) \sum_{\omega \in \Omega} \underline{p}(\omega) U(f(\omega), \omega) + v(a) = \sum_{\omega \in \Omega} \underline{p}'(\omega) U(f(\omega), \omega). \tag{15}$$

Let a and a' be elementarily linked, and define $\lambda(a')$ and $v(a')$ by the unique solution to the equations

$$\lambda(a') \sum_{\omega \in \Omega} \bar{p}'(\omega) U(f'(\omega), \omega) + v(a') = \lambda(a) \sum_{\omega \in \Omega} \bar{p}(\omega) U(f'(\omega), \omega) + v(a) \tag{16}$$

and

$$\lambda(a') \sum_{\omega \in \Omega} \underline{p}'(\omega) U(f'(\omega), \omega) + v(a') = \lambda(a) \sum_{\omega \in \Omega} \underline{p}(\omega) U(f'(\omega), \omega) + v(a). \tag{17}$$

Because A is finite and linked, repeating this process, it is possible to solve $(\lambda(a), v(a))$ for all $a \in A$.

For every $a \in A$, define:

$$B_a = \{(p, f) \in P \times F \mid (p, f) \succeq_a (p', f') \ \forall (p', f') \in P \times F\}$$

and

$$W_a = \{(p, f) \in P \times F \mid (p', f') \succeq_a (p, f) \ \forall (p', f') \in P \times F\}.$$

By the compactness of $P \times F$ and continuity of $\succeq_a$, the sets B_a and W_a are closed and nonempty. Moreover, since $(p, f) = (\Sigma_{\omega \in \Omega} p(\omega) e^{\omega}, f)$, constraint independence and transitivity imply that there are $\omega, \omega' \in \Omega$ such that $(e^{\omega}, f) \in B_a$ and $\left(e^{\omega'}, f'\right) \in W_a$. Define

$$B_a^0 = \{(p, f) \in B_a \mid p = e^{\omega} \text{ for some } \omega \in \Omega\}$$

and

$$W_a^0 = \{(p, f) \in W_a \mid p = e^{\omega} \text{ for some } \omega \in \Omega\}.$$

By coordinate essentiality, $B_a^0 \cap W_a^0 \neq \varnothing$ for all $a \in A$.

By Theorem 1 $V_a(e^{\omega}, f) = U_a(f(\omega), \omega)$, for all $a \in A$, and, by equation (13), $U_a(f(\omega), \omega) = \lambda(a)U(f(\omega), \omega) + v(a)$, where $\lambda(a) > 0$. Similarly, $V_a\left(e^{\omega'}, f'\right) = U_a(f'(\omega'), \omega') = \lambda(a)U(f'(\omega'), \omega') + v(a)$. Consequently, $(e^{\omega}, f) \in B_a^0$ and $\left(e^{\omega'}, f'\right) \in W_a^0$ if and only if $(e^{\omega}, f) \in B_{a'}^0$ and $\left(e^{\omega'}, f'\right) \in W_{a'}^0$, for all $a, a' \in A$. Given $(e^{\omega}, f) \in B_a^0$, let $j > i$ if $(a_j, e^{\omega}, f) \succ (a_i, e^{\omega}, f)$. (If $(a_j, e^{\omega}, f) \sim (a_i, e^{\omega}, f)$, then the order is arbitrary.) Hence A can be written as an $n-$tuple $(a_1, ..., a_n)$, and a_i and a_{i+1} are elementarily linked, $i = 1, ..., n-1$.

Let $f, f' \in F$ and ω,$\omega' \in \Omega$ be such that $(e^{\omega}, f) \in B_a^0$ and $\left(e^{\omega'}, f'\right) \in W_a^0$. Define $\hat{f} = \left(f(\omega), \left(f'(\omega'), f_{-\omega'}\right)_{-\omega}\right)$. Because $f(\omega)$ is the $\omega-$th coordinate of both f and $\hat{f}$, and $f'(\omega')$ is the $\omega'-$th coordinate of both f' and $\hat{f}$, applying the certainty principle with $x = y$, $f = f''$ and $f' = f'''$, $\left(e^{\omega}, \hat{f}\right) \in B_a^0$ and $\left(e^{\omega'}, \hat{f}\right) \in W_a^0$.

Let a and a' be elementarily linked at $f^* \in F$ with $\bar{p}, \underline{p}, \bar{p}', \underline{p}' \in P$ satisfying $(\bar{p}, f^*) \succ_a \left(\underline{p}, f^*\right)$ such that $(a, \bar{p}, f^*) \sim (a', \bar{p}', f^*)$ and $\left(a, \underline{p}, f^*\right) \sim \left(a', \underline{p}', f^*\right)$. There are then $\bar{\alpha}_a, \underline{\alpha}_a, \bar{\alpha}_{a'}, \underline{\alpha}_{a'} \in [0,1]$ such that $(\bar{p}, f^*) \sim_a \left(\left(\bar{\alpha}_a e^{\omega} + (1-\bar{\alpha}_a) e^{\omega'}\right), \hat{f}\right)$, $\left(\underline{p}, f^*\right) \sim_a \left(\left(\underline{\alpha}_a e^{\omega} + (1-\underline{\alpha}_a) e^{\omega'}\right), \hat{f}\right)$, $(\bar{p}', f^*) \sim_{a'} \left(\left(\bar{\alpha}_{a'} e^{\omega} + (1-\bar{\alpha}_{a'}) e^{\omega'}\right), \hat{f}\right)$, and $\left(\underline{p}', f^*\right) \sim_{a'} \left(\left(\underline{\alpha}_{a'} e^{\omega} + (1-\underline{\alpha}_{a'}) e^{\omega'}\right), \hat{f}\right)$. By transitivity

$$(a, \left(\bar{\alpha}_a e^{\omega} + (1-\bar{\alpha}_a) e^{\omega'}\right), \hat{f}) \sim (a', \left(\bar{\alpha}_{a'} e^{\omega} + (1-\bar{\alpha}_{a'}) e^{\omega'}\right), \hat{f}) \tag{18}$$

and

$$(a', \left(\underline{\alpha}_a e^{\omega} + (1-\underline{\alpha}_a) e^{\omega'}\right), \hat{f}) \sim (a', \left(\underline{\alpha}_{a'} e^{\omega} + (1-\underline{\alpha}_{a'}) e^{\omega'}\right), \hat{f}). \tag{19}$$

To simplify the notation, let $\left(\alpha e^{\omega} + (1-\alpha) e^{\omega'}\right) \equiv \hat{q}(\alpha)$. Then, by (18) and (19), a and a' are elementarily linked with $\hat{f} \in F$ and $\hat{q}(\bar{\alpha}_a), \hat{q}(\underline{\alpha}_a), \hat{q}(\bar{\alpha}_{a'}), \hat{q}(\underline{\alpha}_{a'}) \in P$.

Consider next (a, p, f). By definition $\left(a, e^{\omega}, \hat{f}\right) \succeq (a, p, f) \succeq \left(a, e^{\omega'}, \hat{f}\right)$. Thus, by (A.2) and (A.5), there is a unique α_p such that $(a, p, f) \sim \left(a, q(\alpha_p), \hat{f}\right)$.

Consider the alternatives (a, p, f) and (a', p', f') and, without loss of generality, suppose that $(a', p', f') \succeq (a, p, f)$. Three cases need to be considered:

Case 1: $(a', p', f') \succeq \left(a, e^{\omega}, \hat{f}\right)$. Then $V_{a'}(p', f') \geq U_a\left(\hat{f}(\omega), \omega\right) \geq V_a(p, f)$.

Case 2: $\left(a', e^{\omega'}, \hat{f}\right) \succeq (a, p, f)$. Then $V_{a'}(p', f') \geq U_{a'}\left(\hat{f}(\omega'), \omega'\right) \geq V_a(p, f)$.

Case 3: $\left(a', e^{\omega}, \hat{f}\right) \succ (a', p', f') \succeq (a, p, f) \succ \left(a', e^{\omega'}, \hat{f}\right)$. Then, by (A.2) and (A.5), there are unique $\hat{q}(\alpha_{p'})$ and $\hat{q}(\alpha_p)$ such that $(a', p', f') \sim \left(a', \hat{q}(\alpha_{p'}), \hat{f}\right)$ and $(a, p, f) \sim \left(a', \hat{q}(\alpha'_p), \hat{f}\right)$, respectively. Moreover, by the same argument, there is a unique $\hat{q}(\alpha_a)$ satisfying $(a, p, f) \sim \left(a, \hat{q}(\alpha_p), \hat{f}\right)$. By transitivity, $\left(a', \hat{q}(\alpha'_p), \hat{f}\right) \sim \left(a, \hat{q}(\alpha_p), \hat{f}\right)$. Hence, by transitivity,

$$(a', p', f') \sim \left(a', \hat{q}(\alpha_{p'}), \hat{f}\right) \succeq \left(a, \hat{q}(\alpha'_p), \hat{f}\right) \sim \left(a, \hat{q}(\alpha_p), \hat{f}\right) \sim (a, p, f). \tag{20}$$

By equation (12),

$$\begin{gathered} V_{a'}(p', f') = V_{a'}\left(\hat{q}(\alpha_{p'}), \hat{f}\right) = \\ \lambda(a')\left[\alpha_{p'} U(f(\omega), \omega) + (1 - \alpha_{p'}) U(f(\omega'), \omega')\right] + v(a') \geq \\ \lambda(a)\left[\alpha'_p U(f(\omega), \omega) + \left(1 - \alpha'_p\right) U(f(\omega'), \omega')\right] + v(a) = \\ \lambda(a)\left[\alpha_p U(f(\omega), \omega) + (1 - \alpha_p) U(f(\omega'), \omega')\right] + v(a) = \\ V_a\left(\hat{q}(\alpha_p), \hat{f}\right) = V_a(p, f) \end{gathered} \tag{21}$$

Thus, by Theorem 1 and equations (16) and (17),

$$\lambda(a') \sum_{\omega \in \Omega} p'(\omega) U(f'(\omega), \omega) + v(a') \geq \lambda(a) \sum_{\omega \in \Omega} p(\omega) U(f(\omega), \omega) + v(a). \tag{(5)}$$

If a and a' are not elementarily linked, then, a and a' are linked since A is linked. Define $\alpha_1, ..., \alpha_{n-1}$ by $\left(a_i, \alpha_i \bar{p}_i + (1 - \alpha_i) \underline{p}_i, f^*\right) \sim \left(a_{i+1}, \alpha_i \bar{p}'_i + (1 - \alpha_i) \underline{p}'_i, f^*\right)$, where $a = a_1$ and $a' = a_{n-1}$. The conclusion follows by repeated application of the representation.

$(b) \Rightarrow (a)$. That (b) implies (A.1) – (A.5) is an implication of Theorem 1. That it implies (A.6) is immediate.

To prove the uniqueness part, note that, by Theorem 1, the functions $U(\cdot, \omega), \omega \in \Omega$, are unique up to a uniform positive linear transformation. Given $U(\cdot, \omega), \omega \in \Omega$, the uniqueness of $\lambda(\cdot)$ and $v(\cdot)$ follow from equations (14) – (17).■