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RESEARCH ARTICLE

Preventive-service fraud in credence good markets

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Abstract
Preventive dental care and automotive service is intended to preempt problems that, if they materialized, would require costly treatment or repair. In these markets fraud is both persistent and pervasive. This paper analyzes these markets invoking the notion of perfect Bayesian equilibrium of a stochastic dynamic games of incomplete information in which the players are customers and service providers. The services provided are credence because diagnosis and service are bundled and the customers lack the expertise necessary to assess the need for the prescribed service both ex ante and ex post. The analysis shows that fraud is a prevalent equilibrium phenomenon that is somewhat mitigated by customers' loyalty.

Keywords Credence goods · Preventive-services fraud · Stochastic game of incomplete information

JEL Classification D40 · D41 · D69 · D82

1 Introduction

In May 2019 the *Atlantic* ran an article entitled "The Trouble With Dentistry," which exposed the practice of excessive diagnosis and treatment endemic to the dental industry.

> ... dentistry's struggle to embrace scientific inquiry has left dentists with considerable latitude to advise unnecessary procedures—whether intentionally or not. The standard euphemism for this proclivity is overtreatment. Favored procedures, many of which are elaborate and steeply priced, include root canals,

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> the application of crowns and veneers, teeth whitening and filing, deep cleaning, gum grafts, fillings for “microcavities”—incipient lesions that do not require immediate treatment—and superfluous restorations and replacements, such as swapping old metal fillings for modern resin ones.[1]

Gottschalk et al. (2020) reported a recent field experiment that provides some clues about its pervasiveness.

> A test patient who does not need treatment is sent to 180 dentists to receive treatment recommendations. In the experiment, we vary the socio-economic status (SES) of the patient and whether a second opinion signal is sent. We observe an overtreatment recommendation rate of 28% and a striking heterogeneity in treatment recommendations. Furthermore, we find significantly less overtreatment recommendations for patients with higher SES compared to lower SES for standard visits, suggesting a complex role of patients’ SES. Dentists with shorter waiting times are more likely to propose unnecessary treatment.

The practices of excessive diagnosis and overtreatment are also found in other industries, such as car service and maintenance.[2] Both dental patients and car owners, having routine checkup or undergoing emergency treatment or service, may receive diagnoses that include recommendations for treatments intended to preempt problems looming on the horizon. Customers usually lack the expertise to assess the necessity of the recommended treatment, both ex ante and ex post. Their only recourse is to accept the recommended treatment or service or seek a second opinion. The service providers’ incentive to prescribe overtreatment is the business it generates.

Darby and Karni (1973) were the first to identify the crucial ingredients of the problem of fraudulent prescription of unnecessary services, in what they dubbed, “markets for credence-quality good and services,”—information asymmetry and the bundling of diagnosis and service. The information asymmetry is the result of the fact that the providers have the expertise required to assess the need for service while the customers’ are unable to assess the necessity of the prescribed service before and after it is delivered. Numerous studies confirm the prevalence of fraudulent behavior in such markets.[3]

The nature and extent of prescription of unnecessary services depend on the characteristics of the credence services markets. The modeling and analysis of these markets must therefore be based on the specific features of the market under consideration. In this paper, I model and analyze the dentistry and auto-maintenance industries based on their distinct characteristics.[4] The features of this market that

[1] Included in the article was the following testimony by Trish Walraven, who worked as a dental hygienist for 25 years. “We would see patients seeking a second opinion, and they had treatment plans telling them they need eight fillings in virgin teeth. We would look at X-rays and say, ‘You’ve got to be kidding me.’ It was blatantly overtreatment—drilling into teeth that did not need it whatsoever.”

[2] See Beck et al. (2014).

[3] See Dulleck and Kerschbamer (2006) and Balafoutas and Kerschbamer (2020) for extensive surveys.

[4] See, for example, the analysis of Chiu and Karni (2021) of the market for emergency mechanical or medical services.

are highlighted in this study include what Darby and Karni (1973) referred to as "client relationship." To capture this aspect of the market, I assume that service providers have loyal clienteles. The customers' loyalty is manifested by considering their regular providers the default option for routine maintenance or emergency service and they return to this provider for service if they obtain a second opinion that agrees with the first provider's prescription. Ex ante the providers are assumed to be identical in every respect. Ex post, however, the providers may, endogenously, cultivate different-size clienteles, which affect their behavior, and at any point in time they may have different queues of customers waiting to be served. Both the size of the clientele and the length of the queues are the providers' private information. Customers have different incomes and, consequently, distinct risk attitudes. In addition, the customer's cost of seeking a second opinion is a realization of a random variable. Knowledge of her income and search cost is the customer's private information.

Some aspects of the market for experts' advice and services have received growing attention in recent years. In particular, a strand of the literature emphasize the need to incentivize the expert to exert the necessary effort to diagnose the client's problem. Pesendorfer and Wolinsky (2003) analyzed the equilibrium effort of experts operating in competitive markets in which they offer contracts to customers who may seek multiple opinions. Their main concern is the inefficiency that result from price competition. More recently, Chen et al. (2022) analyzed the role of liability in the provision of expert services in a model in which the quality of the services provided may be detected with some probability after the fact. They consider the role of liability in incentivizing the expert to exert diagnostic effort as well as to prescribe services that constitute undertreatment and overtreatment of the customer's problem. The case of credence services is an special case of their more general model. While these are important issues in some markets they do not seem essential in the market I am dealing with in this paper. In particular, it seems reasonable to suppose that the diagnosis of preventive treatment is done while providing maintenance services, so even if there is cost involved it is not a critical aspect of the markets under consideration. Typically in these markets it is the provider who takes the initiative of advising the customers of the need for preventive services that they may be unaware off. Because insufficient treatment may result in detectable problems down the road, liability is a feature of this market designed to deter undertreatment. Consequently, in this paper, I invoke liability as deterrence against undertreatment of perceived problems.

The interaction among service providers and customers in this paper is modeled as a stochastic dynamic games of incomplete information. The analysis is based on the notion of perfect Bayesian equilibrium.

The next section describes the preventive-service market and the game that depicts the interactions among the market participants. Section 3 includes the equilibrium analysis and discusses its economic implications. Section 4 discusses the robustness of the model and possible extensions. Section 5 provides the proofs.

2 The credence preventive-service market

2.1 An overview

Consider a market for credence preventive service populated by finite number, n, of customers and, m, of service providers, where $n \gg m$. An important feature of this market is the "customer-provider relationship." These relationships, built and maintained through repeated interactions, are manifested by the customers' inclination to seek periodic maintenance service (e.g., teeth cleaning, oil change) and, when necessary, emergency service from their regular providers. To capture this feature of the market, I assume that each provider has loyal customers who schedule the routine maintenance service and visit the provider first in case of an emergency. Formally, let $(C_1, ..., C_m)$ be a partition of the set C of customers and $i \in C_j$ indicates that customer i belongs to the clientele of provider j. The value of a customer's loyalty to the provider is the expected present value of the future services the customer purchases. This value depends on the anticipated longevity of the relation which, in turn, depends on the strategies of the providers and the customers. Denote this value by $v_i \in [0, \bar{v}]$ and assume that, in equilibrium, it is common knowledge of the customer and the provider with whom she interacts.

Customers seeking a second opinion schedule an appointment with another provider. If they detect fraudulent behavior on the part of the provider to whose clientele they originally belonged, they switch their loyalty and join the clientele of the provider from whom they sought the second opinion.

The information asymmetry in this market is two-sided. The customers' private information is their risk preferences and the realization of random cost of seeking a second opinion. The service providers' private information is the size of their clienteles and length of their queues. In addition, after the inspection, there is additional information asymmetry. The provider is informed about the potential problems whereas the customer is not. The supposition that the customer is not aware of the length of the supplier's queue seems an appropriate description of the market under consideration. Typically, preventive care can be scheduled for some future, convenient, date. Consequently, unlike in the situations, (e.g., mechanical breakdown or health issues that require urgent care and whose delay has severe consequences), in which waiting time is critical and the customers inquire about the waiting time before choosing the provider, inquiring about the length of the supplier queue is not essential.[5]

To simplify the exposition, I assume that there are two states, ω_0 and ω_1.[6] In state ω_0, no imminent problem is detected and no preventive treatment or service is required. In state ω_1, a looming problem is detected that requires preventive intervention to avoid a more costly treatment in the future. Let $\Omega = \{\omega_0, \omega_1\}$ denote the state space.

When a customer arrives at the service facility for a scheduled appointment or emergency service, the provider delivers the required service and observes (i.e., diagnoses) the state. Based on the diagnosis, the provider may prescribe, ω_0, no preventive

[5] Chiu and Karni (2021) model credence goods market in which the length of the provider's queue is essential ingredient of the model.

[6] For a discussion of the implications of relaxing this assumption, see Sect. 4.

treatment necessary or, ω_1, preventive treatment is necessary. Lacking the required expertise, the customer have no way of knowing ex ante whether the proposed treatment is necessary and if she accepts the recommended service, she cannot ascertain, ex post, whether the service was indeed necessary.

Upon receiving a prescription, the customer must decide whether to accept it or seek a second opinion. If the customer chooses to seek a second opinion, she receives a second prescription and must decide whether to accept it, return to the first provider, or seek a third opinion.[7] Assume that if the second opinion agrees with the first, loyalty makes the customer return to her regular provider for service.

Providers maintain facilities with installed service capacities capable of handling the traffic generated by clientele up to a finite size. Once a provider's clientele attains its capacity limits, the provider cannot accept additional customers (i.e., the provider cannot commit to providing routine services). The service market is competitive and the providers are price takers charging the same hourly service price. Using the service hour as the numeraire, the marginal cost of maintaining a service station is c per hour, regardless of whether it is in use or not. The profit generated by a service hour is $1 - c$ if the station is occupied and $-c$ if it is not. At each point in time, t, the queue (i.e., the number of scheduled service hours) of provider j is $Q_j(t) \in [0, \bar{Q}_j]$, where $\bar{Q}_j =| C_j | \omega_1$. Let G_j denote the cumulative distribution function of Q which is determined by the probabilities of the states, the provider's prescription strategies, the customers' arrival rate and their decisions. Let $\mathcal{G}_j$ be the set of cumulative distribution functions on $[0, \bar{Q}_j]$ endowed with the topology of weak convergence.

Customers are assumed to have identical preferences and different incomes, denoted by $y_i \in [0, \bar{y}]$. The income distribution is depicted by a cumulative distribution function, F on $[0, \bar{y}]$, that is differentiable and has full support. If a customer seeks a second opinion she must pay a diagnostic fee, D, and incur random search cost. To simplify the exposition, I assume that, at each point in time, depending how busy the customer happens to be, the search cost may be negligible, $\theta_0 = 0$, or positive, θ_1, with probabilities $\pi(\theta_k)$, $k = 0, 1$. In view of its dependence on her circumstances at the time, the search cost is the customer's private information. Let $\Theta := \{\theta_0, \theta_1\}$.

2.2 The stage game

The credence service market is modeled as a stochastic dynamic game of incomplete information and analyzed using the concept of perfect Bayesian equilibrium. The players are the service providers and the customers. A stage game is initiated when a customer arrives at the service facility of her regular provider. I describe next the players strategies, beliefs and payoffs.

2.2.1 The extensive form stage game

The stage game begins with nature assigning the customer a state, $\omega \in \Omega$. The probabilities $\mu(\omega_k)$, $k \in \{0, 1\}$, of being assigned the states ω_k, are assumed to be common

[7] As will become clear, when there are only two states, the costomer finds seeking a third opinion unnecessary.

knowledge. At time t, a customer, $i \in C_j$, shows up for a routine maintenance or emergency service, initiating the stage game $\Gamma\left(\omega, Q_j, v_i\right)$. The players in this stage game are the provider, j; the customer, $i \in C_j$; and a provider, h, whom the customer selects, at random, if she decides to seek a second opinion.[8] Henceforth, I refer to the provider to whose clientele the customer belongs, as the first provider and to the second opinion provider as the second provider and denote by p_1 and p_2 their prescriptions, respectively.

The first provider observes the state (i.e., his information sets are $H_j^0 = \{(\omega_0, y, \theta) \mid (y, \theta) \in [0, \bar{y}] \times \Theta\}$ and $H_j^1 = \{(\omega_1, y, \theta) \mid (y, \theta) \in [0, \bar{y}] \times \Theta\}$) and chooses a prescription $p_1 \in \Omega$, where $p_1 = \omega_0$ means that no preventive service is recommended and $p_1 = \omega_1$ means that preventive ω_1 hours of service are recommended. I assume that if the state is ω_1, then the provider never finds it in his interest to prescribe ω_0. This assumption, referred to in the literature as *liability*, is justified if the customer can sue for malpractice if the state is revealed to be ω_1 and the provider prescribed ω_0. In such case the provider pays a penalty x large enough to deter him from prescribing undertreatment (e.g., may lose his licence). If the customer is compensated for the damage done by the wrong prescription, the compensation does not cover the cost of the damage so the customer suffers a net loss $z < 0$.

Unable to observe the state, and not knowing the provider's total number of scheduled maintenance service hours, Q_j, the customer's initial information set is $H_c = \{\left(\omega, Q_j\right) \in \Omega \times [0, \bar{Q}_j]\}$. Her prior beliefs on Ω and $[0, \bar{Q}_j]$ are μ and G_j. After having received the first provider's prescription, the customer updates her beliefs invoking Bayes' rule.

Having been prescribed p_1, if the customer seeks a second opinion she must pay a diagnosis fee, $D < \mu(\omega_0)\,\omega_1$, and incur a search cost $\theta \in \Theta$.[9] Assume that the search for prescription is with recall—in other words, having received a second prescription, the customer can either accept it or return to the first provider and accept his prescription or continue the search seeking a third opinion. As the service price is the same, a customer seeking a second opinion selects a provider at random, with equal probabilities. The assumed client's loyalty implies that if the second prescription agrees with the first, then the customer buys the service from the first provider.

Providers know their clients; when an unfamiliar customer shows up, they infer that she is seeking a second opinion. The second provider, h, implements a prescription policy $p_2 \in \Omega$. Not knowing the prescription that the customer receives on her first visit, the second provider's information sets are $H_h^0 = \{(\omega_0, p_1 = \omega_0), (\omega_0, p_1 = \omega_1)\}$ and $H_h^1 = \{(\omega_1, p_1 = \omega_0), (\omega_1, p_1 = \omega_1)\}$.[10] However, the second provider believes that if the first provider prescribed $p_1 = \omega_0$ then the customer would never seek a second opinion. Thus, the node $(\omega_0, p_1 = \omega_0)$ in H_h^0 is assigned zero probability.

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[8] As will become clear later, the equilibrium strategy of the second-opinion provider is to prescribe truthfully. Consequently, the customer never seeks a third opinion in equilibrium.

[9] As will become clear later, this assumption guarantees that some risk-averse customers will seek second opinion. If $D \geq \mu(\omega_0)\,\omega_1$ no risk averse customer will seek a second opinion and, assuming that the customers are risk averse, the providers will always prescribe ω_1.

[10] Because the provider does not know the customer's income, strictly speaking the information sets should include the incomes. However, this information is irrelevant for the second provider's decisions. To simplify the notations the income is not included in the depiction of the information sets.

Moreover, by liability, the node $(\omega_1, p_1 = \omega_0)$ in H_h^1 is also assigned probability zero. The customer will accept the second provider's prescription if and only if the state is ω_0, the first provider prescribed $p_1 = \omega_1$, and he prescribes $p_2 = \omega_0$. In every other instance, the customer returns to the first provider.

In the final stage, the customer, having obtained two prescriptions, p_1 and p_2, must decide whose prescription to accept and whether to remain loyal to the provider to whose clientele she initially belonged or switch her loyalty to the second provider. *Fraud is said to be committed if the state is* ω_0, *and the first provider prescribes* $p_1 = \omega_1$.

2.2.2 The customers

As we shall see later, prescribing truthfully (i.e., $p_2 = \omega_k$, when ω_k, $k \in \{0, 1\}$, is the true state) is the second provider's dominant strategy. Consequently, customers have no incentive to seek more than two opinions.

The customers' strategies The customers' strategies are pairs of mappings $\sigma_1 : [0, \bar{y}] \times \Theta \times \Omega \to \{0, 1\}$ and $\sigma_2 : [0, \bar{y}] \times \Theta \times \Omega^2 \to \{0, 1\}$ that have the following interpretations, $\sigma_1(y, \theta, p_1) = 1$, means that the customer accepts the first provider's prescription, p_1, and terminates the search; $\sigma_1(y, \theta, p_1) = 0$ means that she seeks a second prescription. Similarly, $\sigma_2(y, \theta, p_1, p_2) = 1$ means that the customer, having sought a second opinion, accepts the second provider's prescription, p_2 and $\sigma_2(y, \theta, p_1, p_2) = 0$ means that she rejects the second provider's prescription and accepts the prescription p_1 of the first provider. Let Σ denote the set of customers' strategies.

The customers' beliefs Given the customer's information set $H_c = \Omega \times [0, \bar{Q}_j]$, her prior beliefs on Ω and $[0, \bar{Q}_j]$ are μ and G_j. Upon obtaining the first prescription, p_1, the customer updates her beliefs about the true state by applying Bayes' rule. Given the customer's value, v_i, her posterior beliefs conditional on the first provider's prescription, p_1, are represented by $\mu^i(\cdot \mid p_1, v_i)$ on Ω.

If the first provider prescribed $p_1 = \omega_1$ and if the customer decides to seek a second opinion, she selects a provider, h, at random. The second provider observes the state. If the state is ω_1 then, by liability, the second provider prescribes $p_2 = \omega_1$ with probability one and the customer, finding no evidence that the first provider overprescribed, returns to him for service. If the state is ω_0, and the second provider prescribes $p_2 = \omega_0$ then, in addition to collecting the diagnosis fee, the second provider adds the customer to his clientele. If the second provider prescribes $p_2 = \omega_1$ then the customer will return to the first provider, in which case the second provider payoff is just the diagnosis fee. Combining these arguments, we conclude that *the second provider's dominant strategy is to prescribes truthfully; consequently, the second prescription reveals the true state. The customer has no incentive to seek a third opinion.*

The customers' payoffs Assume that the customer's utility function displays decreasing absolute risk aversion, and let $\mathbb{R}_{++}$ be the range of the Arrow-Pratt measure of absolute risk aversion.[11] Thus, accepting the first provider's prescrip-

[11] This assumption, which involves some loss of generality, is intended to simplify the exposition using the customer's income to parametrize her risk attitudes. Examples of such utility functions are $u(y) = y^{\alpha}$, where $\alpha \in (0, 1]$, and $u(y) = \log y$.

tion the utility of a customer whose income is y is $u(y - p_1)$, $p_1 \in \Omega$. Seeking a second opinion and accepting the least costly prescription, the customer's utility is $u(y - D - \min\{p_1, p_2\}) - \theta$, $\theta \in \Theta$. Clearly, conditional on $p_1 = \omega_0$, the customer always accepts the prescription and stops the search. Conditional on $p_1 = \omega_1$ the customer's expected utility from seeking a second prescription is:

$$\begin{aligned}\bar{u}\,(y, v_i, \theta) &:= \mu\,(\omega_1 \mid p_1 = \omega_1, v_i)\, u(y - D - \omega_1) \\ &\quad + \mu\,(\omega_0 \mid p_1 = \omega_1, v_i))u(y - D) - \theta, \end{aligned} \tag{1}$$

and the customer's utility of accepting the first provider's prescription is $u\,(y - \omega_1)$.

For each θ and v_i, let $y^*\,(v_i, \theta)$ be the solution to the equation $\bar{u}\,(y, v_i, \theta) = u\,(y - \omega_1)$ and assume that it exists.[12] Then, $\bar{u}\,(y, v_i, \theta) \geq u\,(y - \omega_1)$ if $y \geq y^*\,(v_i, \theta)$, and $\bar{u}\,(y, v_i, \theta) < u\,(y - \omega_1)$ if $y < y^*\,(v_i, \theta)$. In the former case, the customer exhibits a low degree of risk aversion and is willing to take the risk and bear the cost of seeking a second opinion. In the latter case, she is sufficiently risk averse that she prefers to avoid taking the risk and bear the cost involved in seeking a second opinion and accepts the prescription.

The effect of θ on $y^*\,(v_i, \theta)$ is ambiguous. Specifically,

$$\frac{\partial y^*\,(v_i, \theta)}{\partial \theta} = -\frac{\partial \bar{u}\,(y, v_i, \theta)\,/\partial \theta}{\partial \bar{u}\,(y, v_i, \theta)\,/\partial y - \partial u\,(y - \omega_1)\,/\partial y}.$$

Clearly, $\partial \bar{u}\,(y, v_i, \theta)\,/\partial \theta < 0$ but the sign of the denominator may be positive or negative. To grasp this it suffices to observe that

$$\begin{aligned}\frac{\partial \bar{u}\,(y, v_i, \theta)}{\partial y} &= \mu\,(\omega_1 \mid p_1 = \omega_1, v_i)\,\frac{\partial u(y - D - \omega_1)}{\partial y} \\ &\quad + \mu\,(\omega_0 \mid p_1 = \omega_1, v_i))\frac{\partial u(y - D)}{\partial y}.\end{aligned}$$

By the concavity of u we have that $\partial u(y - D - \omega_1)/\partial y > \partial u\,(y - \omega_1)\,/\partial y$ and $\partial u(y - D)/\partial y < \partial u\,(y - \omega_1)\,/\partial y$. Hence, the sign of the denominator depends on the probability $\mu\,(\omega_1 \mid p_1 = \omega_1, v_i)$. For the same reason the effect of v_i on $y^*\,(v_i, \theta)$ is similarly ambiguous.[13]

[12] It is easy to verify that for some $\varepsilon \in (0, 1)$ $D \in (0, \varepsilon\omega_1)$ and $\theta > 0$ sufficiently small the solutions exist and, by decreasing risk aversion is unique.

[13] Note that

$$\frac{\partial y^*\,(v_i, \theta)}{\partial v_i} = -\frac{\partial \bar{u}\,(y, v_i, \theta)\,/\partial v_i}{\partial \bar{u}\,(y, v_i, \theta)\,/\partial y - \partial u\,(y - \omega_1)\,/\partial y}.$$

But

$$\frac{\partial \bar{u}\,(y, v_i, \theta)}{\partial v_i} = \left[u(y - D - \omega_1) - \mu\,(\omega_0 \mid p_1 = \omega_1, v_i))u(y - D)\right]\frac{\partial \mu\,(\omega_1 \mid p_1 = \omega_1, v_i)}{\partial v_i}$$

and $\partial \mu\,(\omega_1 \mid p_1 = \omega_1, v_i)\,/\partial v_i < 0$. Hence, the numerator is positive. However, the sign of the denominator is ambiguous.

2.2.3 The providers

Each provider $j = 1, ..., m$, has a set, C_j, of loyal customers. Denote by Q_j the provider's queue length, expressed in terms of hours committed to serving his customers, when a new customer shows up. When a customer shows up at the service station, the provider responds differently depending on whether or not the customer belongs to his clientele.

The providers' strategies Provider j's *pure prescription strategy* is a mapping $p : \Omega \times \left[0, \bar{Q}_j\right] \times [0, \bar{v}] \times \{C_j, C \backslash C_j\} \rightarrow \Omega$, where $p\left(\omega, Q_j, v_i, i \in C_j\right) = p_1\left(\omega, Q_j, v_i\right)$ and $p\left(\omega, Q_j, v_i, i \in C/C_j\right) = p_2\left(\omega, Q_j, v_i\right)$ denote the provider's prescriptions as a function of the state ω; his queue, Q_j; the customer's value, v_i; and whether or not the customer belongs to his clientele.[14] In particular, $i \in C_j$ implies that j is the first provider and $i \in C/C_j$ implies that the provider was selected at random for a second opinion. The asymmetries between providers is a consequence of their relation to the customer.

The providers' beliefs The first provider knows the customer's value, v_i, and believes that if he prescribes $p_1 = \omega_0$, the probability that the customer seeks a second opinion is zero and that if he prescribes $p_1 = \omega_1$, the probability that the customer seeks a second opinion is $(1 - F\left(y^*\left(v_i, \theta\right)\right))$ (i.e., the probability that the customer's income is sufficiently high and, consequently, her aversion to risk sufficiently low that she is willing to bear the risk and cost of seeking a second opinion). Moreover, the first provider anticipates that, should the customer seek a second opinion, the second provider will prescribe truthfully. This anticipation is sustained in equilibrium.

The second provider believes that the customer shows up for a second opinion only if she was prescribed $p_1 = \omega_1$ on her first visit.

The providers' payoffs It is convenient to study the providers' payoffs starting with the second provider. If the customer seeks a second opinion, the provider she selects infers, correctly, that the customer must have received the prescription $p_1 = \omega_1$ on her first visit. Thus, regardless of the state, if $p_2 = \omega_1$, then the second provider's payoff is the diagnosis fee, D.[15] If the state is ω_0, then prescribing $p_2 = \omega_0$, the provider collects the diagnosis fee and, in addition, the customer will join his clientele, which is worth $\bar{v}_i > 0$ in expected present value. Hence, *prescribing truthfully is the second provider's dominant strategy and his expected payoff is* $D + \Pr\{p_1\left(\omega_0, Q_j, v_i\right) = \omega_1\}\bar{v}_i$.

Consider next the first provider's payoff. At the start of a stage game, $\Gamma\left(\omega, Q_j, v_i\right)$, if the state is ω_1 provider j must prescribe $p_1 = \omega_1$, anticipating that this prescription will be accepted, either immediately or after the customer obtains a second opinion, with probability one. In this case, the length of j's queue at the end of the stage game is $Q_j + \omega_1$. If the state is ω_0 and the provider prescribes $p_1 = \omega_0$, then the prescription will be accepted and the length of his queue at the end of the game will be Q_j. In either case, since the provider prescribed truthfully, the customer's loyalty is retained.

If the state is ω_0 and the provider prescribes $p_1 = \omega_1$ then, with probability

$$F\left(y^*\left(v_i\right)\right) = F\left(y^*\left(v_i, \theta_0\right)\right)\pi\left(\theta_0\right) + F\left(y^*\left(v_i, \theta_1\right)\right)\pi\left(\theta_1\right), \tag{2}$$

[14] As will become clear below, it is sufficient that the second provider assume that $v_i > 0$.

[15] If the state is ω_1 then, by liability, $p_2 = \omega_1$.

the prescription is accepted, the length of the queue at the end of the game is $Q_j + \omega_1$, and the customer loyalty is retained.[16] With probability $1 - F(y^*(v_i))$ the customer seeks a second opinion, following which the first provider's prescription is rejected, the queue remains Q_j, and the customer leaves the provider's clientele.

Consider first a customer $i \in C_j$, arriving at a time when the supplier's queue is Q_j. Just before the start of the stage game, $\Gamma\left(\omega, Q_j, v_i\right)$, provider j expects to earn cash flow from servicing the customers in his queue, yielding a discounted value

$$w\left(Q_j\right) := \int_0^{Q_j} (1-c)\, e^{-r\tau} d\tau, \tag{3}$$

where r denotes the discount rate.

If the new customer exhibits the state ω_1, then $p_1 = \omega_1$ and, with probability one, the prescription is accepted and the provider's payoff is

$$w\left(Q_j + \omega_1\right) = \int_0^{Q_j+\omega_1} (1-c)\, e^{-r\tau} d\tau. \tag{4}$$

If the new customer exhibits the state ω_0 and the provider prescribes $p_1 = \omega_0$, then the customer accepts the prescription with probability one and the supplier's payoff is $w\left(Q_j\right)$.

If the state is ω_0 and the provider prescribes $p_1 = \omega_1$, then the customer accepts the prescription with probability $F(y^*(v_i))$. With probability $1-F(y^*(v_i))$ the customer seeks a second opinion. Because the second provider's prescription reveals the state, the customer accepts the second prescription and switches her loyalty to the second provider. To formalize the the provider's expected profit of prescribing $p_1 = \omega_1$ when the state is ω_0 we need to introduce additional notations.

It is assumed that the arrival of customers needing maintenance service follows an exogenous stochastic process. Denote by $\Phi_j(\tau) = e^{-\beta_j \tau}$, where $\beta_j := C_j/C$, the probability that the elapsed time since the end of the preceding stage game during which no other customer arrives is τ. This probability depends on the exogenous stochastic process of customers' arrival, depicted by $e^{-\tau}$, and the likelihood that the next arriving customer belongs to the clientele of provider j. Thus, $\Phi_j(\tau)$ is a strictly decreasing convex function, has full support on $[0, \infty)$, and is absolutely continuous with respect to the Lebesgue measure and $\Phi_j(0) = 1$. Let J be the CDF on $[0, \bar{v}]$, the range of possible customer's loyalty values and assume that J has full support. Then, the provider's expected profit of prescribing $p_1 = \omega_1$ when the state is ω_0 is

$$\begin{aligned} V\left(Q_j, v_i\right) = {} & F\left(y^*(v_i)\right)\left(w\left(Q_j+\omega_1\right) + A\left(Q_j+\omega_1\right)\right) \\ & + \left(1 - F\left(y^*(v_i)\right)\right)\left(w\left(Q_j\right) - v_i + A\left(Q_j\right)\right), \end{aligned} \tag{5}$$

where, for all Q_j,

[16] Because θ is the customer private information, form the provider's viewpoint the customer reservation income $y^*\left(v_i, \widetilde{\theta}\right)$ is a random variable.

$$A\left(Q_j\right) := \int_0^{\bar{v}} \left[\int_0^{Q_j} e^{-r\tau} V\left(Q_j - \tau, v\right) d\Phi_j(\tau) + V(0, v) \int_{Q_j}^{\infty} e^{-r\tau} d\Phi_j(\tau) \right] dJ(v)$$

denotes the value of the expected future profits following the stage game contingent on the length of the queue.

Let $F(y^*) = \int_0^{\bar{v}} F(y^*(v))\, dJ(v)$ then[17]

$$\begin{aligned}
&A'\left(Q_j+\omega_1\right) - A'\left(Q_j\right) \\
&= (1-c)\left(e^{-r\omega_1}-1\right)\left[F\left(y^*\right)e^{-r(Q_j+\omega_1)} + \left(1-F\left(y^*\right)\right)e^{-rQ_j}\right] \\
&\quad + F\left(y^*\right)\int_0^{Q_j+2\omega_1} e^{-r\tau}\left(A'\left(Q_j+2\omega_1-\tau\right)d\Phi_j(\tau) - \int_0^{Q_j+\omega_1} A'\left(Q_j+\omega_1-\tau\right)\right) \\
&\quad d\Phi_j(\tau) \\
&\quad + \left(1-F\left(y^*\right)\right)\int_0^{Q_j+2\omega_1} e^{-r\tau}\left(A'\left(Q_j+\omega_1-\tau\right)d\Phi_j(\tau) - \int_0^{Q_j+\omega_1} A'\left(Q_j-\tau\right)\right) \\
&\quad d\Phi_j(\tau) \\
&= (1-c)\left(e^{-r\omega_1}-1\right)\left[F\left(y^*\right)e^{-r(Q_j+\omega_1)} + \left(1-F\left(y^*\right)\right)e^{-rQ_j}\right] \\
&\quad + \int_0^{Q_j+\omega_1} e^{-r\tau}[F\left(y^*\right)\left(A'\left(Q_j+2\omega_1-\tau\right) - A'\left(Q_j+\omega_1-\tau\right)\right) \\
&\quad + \left(1-F\left(y^*\right)\right)\left(A'\left(Q_j+\omega_1-\tau\right) - A'\left(Q_j-\tau\right)\right)]d\Phi_j(\tau) \\
&\quad + \int_{Q_j+\omega_1}^{Q_j+2\omega_1} e^{-r\tau}\left[F\left(y^*\right)A'\left(Q_j+2\omega_1-\tau\right) + \left(1-F\left(y^*\right)\right)A'\left(Q_j+\omega_1-\tau\right)\right] \\
&\quad d\Phi_j(\tau).
\end{aligned}$$

Since $e^{-r\omega_1} - 1 < 0$, the first term on the right-hand is negative. By the same argument, for all $\tau \in \left[0, Q_j\right]$, $A'\left(Q_j+\omega_1-\tau\right) - A'\left(Q_j-\tau\right)$ and $A'\left(Q_j+\omega_1-\tau\right) - A'\left(Q_j-\tau\right)$ has a negative first term. Thus, repeated application implies that the difference between the marginal values of the queues at $Q_j+\omega_1$ and Q_j, (i.e.,

[17] Differentiating $A(\cdot)$ we get

$$\begin{aligned}
A'\left(Q_j\right) &= \int_0^{\bar{v}} \{e^{-rQ_j}\left[V\left(Q_j-Q_j, v\right) - V(0, v)\right]\Phi_j'\left(Q_j\right) + \int_0^{Q_j+\omega_1} e^{-r\tau} V'\left(Q_j+\omega_1-\tau, v\right) \\
&\quad d\Phi_j(\tau)\}dJ(v) \\
&= (1-c)\int_0^{Q_j+\omega_1} e^{-r\tau}\{F\left(y^*\right)\left[e^{-r(Q_j+\omega_1-\tau)} + A'\left(Q_j+\omega_1-\tau\right)\right] \\
&\quad + \left(1-F\left(y^*\right)\right)\left[e^{-r(Q_j-\tau)} + A'\left(Q_j-\tau\right)\right]\}d\Phi_j(\tau) \\
&= (1-c)\{F\left(y^*\right)e^{-r(Q_j+\omega_1)} + \left(1-F\left(y^*\right)\right)e^{-rQ_j}\right) \\
&\quad + \int_0^{Q_j+\omega_1} e^{-r\tau}\left[F\left(y^*\right)e^{-r\tau}A'\left(Q_j+\omega_1-\tau\right) + \left(1-F\left(y^*(v)\right)\right)A'\left(Q_j-\tau\right)\right]d\Phi_j(\tau)\},
\end{aligned}$$

$A'\left(Q_j + \omega_1\right) - A'\left(Q_j\right)$), being the sum of negative terms and decaying positive terms, is negative.

Hence, if the state is ω_0, the first provider prescribes truthfully if and only if[18]

$$V\left(Q_j, v_i\right) \leq w\left(Q_j\right) + A\left(Q_j\right). \tag{6}$$

This condition may be written as

$$\begin{aligned} F\left(y^*\left(v_i\right)\right)&\left[\frac{1-c}{r}e^{-rQ_j}(1-e^{-r\omega_1}) + A\left(Q_j+\omega_i\right) - A\left(Q_j\right)\right] \\ &\leq \left(1 - F\left(y^*\left(v_i\right)\right)\right)v_i. \end{aligned} \tag{7}$$

Suppose that

$$\begin{aligned} F\left(y^*\left(v_i\right)\right)&\left[(1-c)\,e^{-r\bar{Q}_j}(1-e^{-r\omega_1})/r + A\left(\bar{Q}_j+\omega_i\right) - A\left(\bar{Q}_j\right)\right] \\ &< \left(1 - F\left(y^*\left(v_i\right)\right)\right)v_i \end{aligned}$$

and

$$F\left(y^*\left(v_i\right)\right)\left[(1-c)\,(1-e^{-r\omega_1})/r + A\left(\omega_i\right) - A\left(0\right)\right] > \left(1 - F\left(y^*\left(v_i\right)\right)\right)v_i$$

then, by the continuity and since, $A'\left(Q_j + \omega_1\right) - A'\left(Q_j\right) < 0$, the left-hand side of (7) is monotonic decreasing in Q_j, there is a unique $Q_j^*\left(v_i\right) \in (0, \bar{Q}_j]$ such that (7) holds with equality.

For all $Q_j < Q_j^*\left(v_i\right)$ the provider finds it profitable to prescribe ω_1 when the true state is ω_0. If $Q_j \geq Q_j^*\left(v_i\right)$ then truthful prescription is the provider's best response.

Conclusion: Ceteris paribus, the longer the supplier's queue and the more valuable the customer the more likely he is to prescribe truthfully.

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[18] Using (5), (6) may be written as follows:

$$F\left(y^*\left(v_i\right)\right)\left[w\left(Q_j+\omega_1\right) - w\left(Q_j\right) + A\left(Q_j+\omega_1\right) - A\left(Q_j\right)\right] < \left(1 - F\left(y^*\left(v_i\right)\right)\right)v_i.$$

But, upon integrating, (3) and (4) imply that

$$w\left(Q_j+\omega_1\right) - w\left(Q_j\right) = (1-c)\,\frac{-1}{r}\left[e^{-r\left(Q_j+\omega_1\right)} - e^{-r\left(Q_j+\omega_1\right)}\right].$$

Thus,

$$\begin{aligned} F\left(y^*\left(v_i\right)\right)&\left[w\left(Q_j+\omega_1\right) - w\left(Q_j\right) + A\left(Q_j+\omega_1\right) - A\left(Q_j\right)\right] \\ &= F\left(y^*\left(v_i\right)\right)\left[\frac{1-c}{r}e^{-rQ_j}(1-e^{-r\omega_1}) + A\left(Q_j+\omega_1\right) - A\left(Q_j\right)\right], \end{aligned}$$

which is the left-hand side of (7).

3 Equilibrium analysis

3.1 Stage game equilibrium

The analysis of the game invokes the concept of perfect Bayesian equilibrium. Formally, a *system of beliefs* η in extensive form game Γ_E is a specification of a probability $\eta(x) \in [0, 1]$ for each decision node x in Γ_E such that $\Sigma_{x\in H}\eta(x) = 1$, for all informations sets H. A strategy profile ζ in the extensive form game Γ_E is *sequentially rational at the information set* H *given the system of beliefs* η if player h, who moves at the information set H, maximizes his expected utility given the strategies of the other players. If the strategy profile satisfies this condition for all information sets H, then it is *sequentially rational given the system of beliefs* η.

A profile of strategies and system of beliefs (ζ, η) is a *perfect Bayesian equilibrium* (PBE) in the extensive form game Γ_E if (i) the strategy profile ζ is sequentially rational given the system of beliefs η and (ii) the system of beliefs η is derived from the strategy profile ζ using Bayes' rule whenever possible.[19]

I describe next the system of beliefs and the strategy profile of the extensive form stage game $\Gamma\left(\omega, Q_j, v_i\right)$.

The providers

Given that the customers' incomes are private information, the information sets of the first provider are $H_j^0 := \{(\omega_0, y, \theta) \mid (y, \theta) \in [0, \bar{y}] \times \Theta\}$ and $H_j^1 := \{(\omega_1, y, \theta) \mid (y, \theta) \in [0, \bar{y}] \times \Theta\}$. At each information set, the provider must choose between prescribing ω_0 and ω_1. At the information set H_j^1 the provider prescribes $p_1 = \omega_1$ with probability one. At the information set H_j^0, the provider prescribes $p_1 = \omega_0$ if the inequality (7) holds and $p_1 = \omega_1$, otherwise.

Denote by $Q_j^*(v_i)$ the solution to (7) with equality. Given the state ω_0, if $Q_j \geq Q_j^*(v_i)$ then the provider prescribes $p_1 = \omega_0$ and if $Q_j < Q_j^*(v_i)$, he prescribes $p_1 = \omega_1$. The probability of prescribing ω_1 when the state is ω_0 is $G_j\left(Q_j^*(v_i)\right)$.

The customers

Following her first visit, the customer $i \in C_j$ obtains a prescription p_1 on the basis of which she updates her beliefs about the states using Bayes' rule. Since the probability of $p_1 = \omega_1$ is 1 if the state is ω_1 and $G_j\left(Q_j^*(v_i)\right)$ if the state is ω_0, the customer's posterior probability on Ω conditional on the first provider prescribing $p_1 = \omega_1$ is:

$$\mu(\omega_1 \mid p_1 = \omega_1, v_i) = \frac{\mu(\omega_1)}{\mu(\omega_1) + \mu(\omega_0)\, G_j\left(Q_j^*(v_i)\right)} \tag{8}$$

and

$$\mu(\omega_0 \mid p_1 = \omega_1, v_i) = \frac{\mu(\omega_0)\, G_j\left(Q_j^*(v_i)\right)}{\mu(\omega_1) + \mu(\omega_0)\, G_j\left(Q_j^*(v_i)\right)} \tag{9}$$

[19] Note that the only off equilibrium path in this game is when the first provider prescribes ω_0 and the customer seeks a second opinion.

Note that $\mu(\omega_0 \mid p_1 = \omega_1, v_i) \leq \mu(\omega_0)$. The probabilities that the first provider prescribes $p_1 = \omega_0$ if the state is ω_1 is zero. Hence, by Bayes' rule,

$$\mu(\omega_1 \mid p_1 = \omega_0, v_i) = 0 \text{ and } \mu(\omega_0 \mid p_1 = \omega_0, v_i) = 1, \tag{10}$$

for all Q_j and v_i.

Customer's i system of beliefs is $\left(\mu, G_j, \mu(\cdot \mid p_1, v_i)\right)$. To delineate the customer's optimal strategy, we need to consider several possibilities. The least risk-averse customer is risk neutral and, consequently, she is indifferent between accepting the prescription $p_1 = \omega_1$ and seeking a second opinion if the search cost is zero and $D = \mu(\omega_0 \mid p_1 = \omega_1, v_i)\,\omega_1$. If $D \geq \mu(\omega_0 \mid p_1 = \omega_1, v_i)\,\omega_1$ then

$$u(y - \omega_1) > \bar{u}(y, v_i, \theta) \tag{11}$$

for all $(y, \theta) \in [0, \bar{y}] \times \Theta$. Consequently, regardless of her income, customer i accepts the first provider's prescription. In this case, the customer is captive and in equilibrium the supplier prescribes $p_1 = \omega_1$ regardless of the true state and his queue.[20]

Henceforth, I discuss the more interesting and relevant case in which $D < \mu(\omega_0 \mid p_1 = \omega_1, v_i)\,\omega_1$. In this case, for each $\theta \in \Theta$, there exist $y^*(v_i, \theta)$ such that $u(y^*(v_i, \theta) - \omega_1) = \bar{u}(y^*(v_i, \theta), v_i, \theta)$.[21] Given that the utility function displays decreasing absolute risk aversion customer i's optimal strategy is $\sigma_1^*(y, \theta, \omega_0) = 1$, for all $(y, \theta) \in [0, \bar{y}] \times \Theta$, $\sigma_1^*(y, \theta, \omega_1) = 1$ if $y \leq y^*(v_i, \theta)$ and $\sigma_1(y, \theta, \omega_1) = 0$ if $y > y^*(v_i, \theta)$. If the customer decides to seek a second opinion her strategy is $\sigma_2^*(y, \omega_1, \omega_0) = 1$ (i.e., the customer accepts the second provider's prescription) and $\sigma_2^*(y, \omega_1, \omega_1) = 0$ (i.e., loyalty makes the customer return to the first provider).

Definition 1 A *reservation-utility strategy* with reservation utility $u(y^*(v_i, \theta))$, consists of two mappings: $\sigma_1 : [0, \bar{y}] \times \Theta \times \Omega \to \{0, 1\}$ and $\sigma_2 : [0, \bar{y}] \times \Theta \times \Omega^2 \to \{0, 1\}$ and a function $u : [0, \bar{y}] \to \mathbb{R}_+$ such that:

(i) $\sigma_1(y, \theta, \omega_0) = 1$ for all $(y, \theta) \in [0, \bar{y}] \times \Theta$, $\sigma_1(y, \theta, \omega_1) = 1$ if $u(y - \omega_1) > u(y^*(v_i, \theta))$ and $\sigma_1(y, \theta, \omega_1) = 0$ otherwise.

(ii) $\sigma_2(y, \theta, \omega_1, \omega_0) = 1$ and $\sigma_2(y, \theta, \omega_1, \omega_1) = 0$, for all $y \in [0, \bar{y}]$ and $\theta \in \Theta$.

Proposition 1 *The customer's best-response strategy to the providers' pure prescriptions strategy profile* (p_1, p_2) *is a reservation-utility strategy with reservation utility* $u(y^*(v_i, \theta))$, *where the reservation incomes,* $y^*(v_i, \theta)$, $\theta \in \Theta$, *are the solution to* $u(y - \omega_1) = \bar{u}(y, v_i, \theta)$.

3.2 The provider's queue

The evolution of a provider's queue is driven by the random arrival of customers seeking maintenance or emergency service, their states, the provider's prescriptions,

[20] In this case, the prescription in non-informative. Hence, $\mu(\omega_0 \mid p_1 = \omega_1, v_i) = \mu(\omega_0)$ for all v_i.

[21] Here I assume, that D is sufficiently small that customers with lower income may still be inclined to bear the risk of seeking a second opinion.

and customers' responses. To trace the evolution of a provider's queue, let G_j denote the CDF of provider j's queue *hypothesized* by a customer $i \in C_j$. Because the customer does not know the state, ω, or the length, Q_j, of the provider's queue at the end of the preceding stage game, or how much time has passed since the preceding stage game ended, G_j is the unconditional expectations of $G_j\left(\hat{Q}_j \mid Q_j, \omega\right)$.

For a new customer to encounter a queue that is shorter or equal to Q', the preceding stage game must have ended with either $Q \leq Q'$ or $Q \geq \left[Q', \bar{Q}_j\right]$ and the time elapsed since the end of the preceding stage game during which no new customer arrives is $\tau \geq Q - Q'$. These events may obtain under three possible scenarios:

(a) The preceding stage game started with (ω_0, Q) and the provider prescribed $p_1 = \omega_0$ (which is accepted) or the provider prescribed $p_1 = \omega_1$, and the customer sought a second opinion and rejected the provider's prescription. The provider prescribes $p_1 = \omega_0$ if $Q \geq Q_j^*(v)$ and $p_1 = \omega_1$ otherwise, where v denotes the *loyalty value of the preceding customer*. The probability of this scenario is

$$\mu(\omega_0)\left[\int_0^{\bar{v}}\left(1 - G_j\left(Q_j^*(v)\right)F\left(y^*(v)\right)\right)dJ(v)\right.$$
$$\left.\left(\int_{Q'}^{\bar{Q}_j}\Phi_j\left(Q - Q'\right)dG_j(Q) + G_j\left(Q'\right)\right)\right]. \tag{12}$$

(b) The preceding stage game began with (ω_1, Q), in which case the provider prescribes $p_1 = \omega_1$ with probability one and the customer accepts it (immediately or after having sought a second opinion). The probability of this scenario is

$$\mu(\omega_1)\left[\int_{Q'-\omega_1}^{\bar{Q}_j}\Phi_j\left(Q + \omega_1 - Q'\right)dG_j(Q) + G_j\left(Q' - \omega_1\right)\right]. \tag{13}$$

Since $Q \geq 0$, if $Q' = 0$ then $\Phi_j\left(Q - Q'\right) = \Phi_j(Q) \geq \Phi_j\left(\bar{Q}_j\right)$. Hence, $G_j(0) > \mu(\omega_1)\Phi_j\left(\bar{Q}_j\right) > 0$.

(c) The preceding stage game began with (ω_0, Q), the provider prescribed $p_1 = \omega_1$, and it is accepted. The probability of this scenario is

$$\mu(\omega_0)\left[\int_0^{\bar{v}} G_j\left(Q_j^*(v)\right)F\left(y^*(v)\right)dJ(v)\right.$$
$$\left.\left(\int_{Q'-\omega_1}^{\bar{Q}_j}\Phi_j\left(Q + \omega_1 - Q'\right)dG_j(Q) + G_j\left(Q' - \omega_1\right)\right)\right]. \tag{14}$$

Let $\bar{\lambda}(y^*, Q^*(y^*)) := \mu(\omega_0)\int_0^{\bar{v}}\left(1 - G_j(Q^*(v))F(y^*(v))\right)dJ(v)$. Then the unconditional CDF of the length of the queue upon the arrival of the current customer being shorter or equal to Q', is

$$\hat{G}_j\left(Q'\right) = \bar{\lambda}\left(y^*, Q^*\left(y^*\right)\right)\left[\int_{Q'}^{\bar{Q}_j} \Phi_j\left(Q - Q'\right) dG_j\left(Q\right) + G_j\left(Q'\right)\right]$$
$$+ \left(1 - \bar{\lambda}\left(y^*, Q^*\left(y^*\right)\right)\right)\left[\int_{Q'-\omega_1}^{\bar{Q}_j} \Phi_j\left(Q + \omega_1 - Q'\right) dG_j\left(Q\right) + G_j\left(Q' - \omega_1\right)\right]. \tag{15}$$

Note that $\bar{\lambda}\left(y^*, Q^*\left(y^*\right)\right) < \mu\left(\omega_0\right)$ underscores the fact that the decisions of the customer and the provider shift the weight so that the prescription ω_1 is more likely than the probability of the state ω_1. This bias is a reflection of the provider's fraudulent behavior.

The probability of $Q' = 0$ is the probability of the event "the last stage game ends with a queue Q and the elapsed time since that end of that game during which no new customer arrives is equal to or exceeds Q." Formally, let $Q \geq 0$ denote the queue at the end of the last stage game. Then $G_j\left(0 \mid Q\right) = \Phi_j\left(Q\right)$ and the unconditional probability of $Q' = 0$ is

$$\hat{G}_j\left(0\right) := \int_0^{\bar{Q}_j} \Phi_j\left(Q\right)) dG_j\left(Q\right) > 0. \tag{16}$$

Lemma 1 *Let $\hat{G}_j$ be given by (15) then*

(a) *$\hat{G}_j$ has full support $\left[0, \bar{Q}_j\right]$ and an atom at zero.*
(b) *$\hat{G}_j$ is absolutely continuous with respect to the Lebesgue measure on $(0, \bar{Q}_j]$.*

Since $e^{-\tau}$ is monotonic decreasing and convex, the longer is Q, the smaller is the probability, $G_j\left(0 \mid Q\right)$, that the provider finds himself idle. Consequently, a longer queue reduces the provider's incentive to prescribe unnecessary service.

For every given $\left(y^*, Q^*\left(y^*\right)\right) \in [0, \bar{y}] \times \left[0, \bar{Q}_j\right]$, define a function $\Upsilon\left(\cdot \mid y^*, Q^*\left(y^*\right)\right) : \mathcal{G}_j \to \mathcal{G}_j$ by $\hat{G}_j = \Upsilon\left(G_j \mid y^*, Q^*\left(y^*\right)\right)$ in *(15)*.

Lemma 2 *For every $\left(y^*, Q^*\left(y^*\right)\right) \in [0, \bar{y}] \times \left[0, \bar{Q}_j\right]$, (a) $\Upsilon\left(\cdot \mid y^*, Q^*\left(y^*\right)\right)$ is continuous in y^* and $Q^*\left(y^*\right)$ and has fixed point and (b) If G_j^* is a fixed point of Υ then it is continuous in y^* and $Q^*\left(y^*\right)$.*

3.3 The value of the client relationship

As of any point in time, client relationships are of limited duration partly as a result of exogenous factors (e.g., the client's expected longevity, moving to a different location) and partly as outcomes of the equilibrium strategies of the provider and the customer (i.e., the relationship ends when the customer detects fraud). Consequently, their values varies over time reflecting the anticipated duration of the relationships, the provider's prescription of overtreatment in equilibrium and the customer's equilibrium search behavior.

Consider a customer $i \in C_j$ at the start of a stage game. Denote by Δ the expected time interval between routine checkups or maintenance service and let $k_i = 1, 2, ..$ be the anticipated number of visit including the current visit (i.e., Δk_i denote the expected duration of the client relationship as of the beginning of the current stage game). Let $v_i^{k_i}$ denote the expected present value of the future services to customer i. Then $v_i^{k_i}$ is given by recursive formula

$$
\begin{aligned}
v_i^{k_i} = &\left[(1-c)\,\omega_1 + e^{-r\Delta} v_i^{(k_i-1)}\right]\left[\mu(\omega_1) + \mu(\omega_0)\, G_j^*\left(Q_j^*\left(v_i^{k_i-1}\right)\right)\right.\\
&\left. F\left(y^*\left(v_i^{k_i-1}, \theta\right)\right)\right]\\
&+ \mu(\omega_0)\left(1 - G_j^*\left(Q_j^*\left(v_i^{k_i-1}\right)\right)\right) e^{-r\Delta} v_i^{(k_i-1)}.
\end{aligned} \tag{17}
$$

Note that $v_i^0 = 0$ and, since $G_j^*\left(Q_j^*(0)\right) = 1$, $v^1 = (1-c)\,\omega_1\big[\mu(\omega_1) + \mu(\omega_0)\, F\,(y^*(0, , \theta))\big]$.

The provider's and customer's decisions in the current stage game are captured by their strategies $Q_j^*\left(v_i^{k_i-1}\right)$ and $y^*\left(v_i^{k_i-1}, \theta\right)$, respectively. These decisions affect the evolution of the value of their relationships. For example, if the provider's queue happens to be longer than $Q_j^*\left(v_i^{k_i-1}\right)$ then he prescribes truthfully. Consequently, regardless of whether or not the customer seeks a second opinion, the customer remains loyal to her provider. In this case, Eq. (17) implies that $v_i^{k_i} = (1-c)\,\omega_1 \mu(\omega_1) + e^{-r\Delta} v_i^{(k_i-1)}$ or, equivalently, $v_i^{k_i-1} = \left[v_i^{k_i} - (1-c)\,\omega_1 \mu(\omega_1)\right] e^{r\Delta}$. The evolution of the queue under different configurations of the length of the provider's queue and the customer's income are depicted, implicitly, by (17).

3.4 Equilibrium

Theorem 1 *There exists a unique perfect Bayesian equilibrium in pure strategies of the stage game* $\Gamma\left(\omega, Q_j, v_i\right)$.

Truthful prescription is the second provider's dominant strategy. The customer always accept the prescription of the first (or second) provider when it is ω_0 and her best response to the providers' strategies is characterized by reservation incomes, $y^*(v_i, \theta)$, $\theta \in \Theta$ above which she seeks a second opinion whenever the prescription of the first provider is ω_1 and below which the prescription is accepted. The first provider's best response to the customer's reservation utilities strategy is characterized by reservation-queue length $Q_j^*(y^*, v_i)$ such that if the provider's actual queue exceeds it, he prescribes truthfully and if it is short of it, he prescribes unnecessary services. The corresponding steady-state service providers' queue-length cumulative distribution functions, $G_j^*(\cdot)$, $j = 1, ..., m$, are determined as part of the equilibrium.

3.5 Behavioral implications

The main behavioral implication of the analysis is that a certain level of fraud (i.e., recommendation of unnecessary treatment or service) is endemic to the competitive equilibrium in the markets considered here. The analysis also reveals certain characteristics of the fraudulent behavior. In particular, fraud is perpetrated by provider j against customer i when the provider's queue is shorter than $Q_j^* (y^*, v_i)$. Hence, given the customer's value, v_i and her reservation incomes, $y^* (v_i, \theta)$, $\theta \in \Theta$, the probability of fraudulent prescription, is $G_j^* \left(Q_j^* (y^*, v_i) \right)$. Providers are more likely to commit fraud when their queues are shorter, out of fear of finding themselves idle. Consequently, as the customers in the queue are being served and the queue depletes, the incentive to prescribe unnecessary preventive treatment to the next customer to arrive increases. If the waiting time before the arrival of the next customer happens to be long enough the queue becomes sufficiently short, or nonexistent, the temptation to overprescribe preventive treatment mounts and the likelihood that the provider defraud the client increases.

Since $Q_j^* (y^*, v_i)$ is a decreasing function of v_i, the likelihood that a provider prescribes unnecessary preventive services is smaller the higher is the value of keeping the customer's loyalty. The intuition of this claim is clear. The more valuable the customer, the less inclined the provider is to risk losing her. Consequently, valued customers are less likely to be defrauded. The client relationship helps mitigate the problem of fraudulent recommendation for preventive treatment.

Since the shorter the horizon the lower is the value of the customer's loyalty, the analysis suggests that fraud is more likely to be committed when the provider anticipates the client relationship to be of shorter duration. At the extreme, a customer who is *recognized as transient* (e.g., a motorist with out-of-state license plates stopping for mechanical service while on a trip) is much more likely to be warned that preventive (unnecessary) service is recommended to avoid mechanical breakdown down the road. To grasp this observation, note that if the value of the customer $v_i = 0$, which is the case when the customer is transient, fraudulent prescription is the provider's dominant strategy. Consequently, in equilibrium, a transient customer learns nothing from the prescription and must decide whether or not to accept the recommended service on the basis of her prior, μ. Seeking a second opinion yields no useful information either as the second provider, recognizing value of the client is zero, has no reason to prescribe differently.

Another implication of the analysis is that high-income, less risk-averse customers are more likely to seek a second opinion than low-income, more risk-averse, customers. Consequently, low-income customers are more likely to be defrauded and high-income customers are more likely to discover when fraud is committed and are more likely to change their providers than low-income customers.

The preceding observations are summarized in the following:

Proposition 2 *The equilibrium of the stage-game* $\Gamma \left(\omega, Q_j, v_i \right)$ *exhibits the following properties:*

(a) *Providers are more likely to commit fraud when their queues are shorter and the when the value of the customer's loyalty is smaller.*

(b) *Low-income customers are more likely to be prescribed unnecessary preventive treatment than high-income customers.*
(c) *High-income customer are more likely to seek second opinion and are more likely to change their providers than low-income customers.*

4 Concluding remarks

The model of this paper delineates a credence-good market with some special characteristics. In particular, it highlights the role of what Darby and Karni (1973) dubbed client relationships, which emerge when providers and customers interact repeatedly. A general conclusion of the analysis is that the more valuable is the customer and the more inclined she is to seek a second opinion the less fraud will be perpetrated. The fear of losing loyal customers deters fraud and mitigates the problem of prescribing unnecessary preventive treatments and services.

In the interest of tractability, the model of this paper includes some simplifying assumptions. The assumption that the state space is a doubleton means that excessive service can take only one value. This allows the second provider to determine whether fraud was committed and, consequently, induces him to prescribe truthfully. A richer state space would permit more levels of unnecessary service prescriptions. In this environment, the second provider may decide to prescribe unnecessary service, taking the chance that the first provider prescribed an even higher level of service and the customer accepts his prescription and joining his clientele. Awareness of this possibility may induce customers to seek more than two opinions, updating their beliefs according to the prescriptions they elicit. The customers' optimal stopping rule will still be characterized by reservation-utility strategy, and providers will still be more likely to recommend unnecessary services when their queue is short and when the value of the client loyalty is smaller. The conclusion that the second provider prescribed truthfully is, therefore, a special aspect of the model.

Given that the second provider prescribes truthfully why wouldn't the customers visit new providers to obtain truthful prescription in the first place? The answer to this question has to do with an important feature of the particular markets under consideration. The premise is that the preemptive treatment is recommended following regular maintenance or checkup visit. It is not the result of the customers looking for preemptive treatment per se. In other words, not being able to observe the state, the customer is unaware of looming problems and is not engaged in searching for preventive treatment. Consequently, unlike obtaining preventive treatment recommendation form her provider, which is free, seeking an opinion from unfamiliar provider the customer must incur a diagnostic cost. Moreover, as was mentioned above, truthful prescription is a special aspect of this model that is justified by tractability considerations. In a more realistic setting, in which there are more than two states, seeking the opinion of a new provider is not guaranteed to yield a truthful prescription.

The analysis of the equilibrium behavior in this model is based on the supposition the set of customers is partitioned to clienteles of the different providers and, therefore, when a provider sees an unfamiliar customer he infers that she is seeking a second opinion. In reality, new customers arrive on the market and the providers cannot distinguish a new customer and a customer seeking a second opinion. To analyze the equilibrium behavior in a model that admits new customers showing up would require the introduction of a new parameter—the proportion of newly arriving customers relative to that of the customers seeking second opinion. If the proportion of newly arriving customers is relatively small, prescribing truthfully is still the provider's best response even if his queue is short. To grasp this, suppose that the state is ω_0. Prescribing truthfully the provider would add the new customer to his clientele whether or not the customer seeks second opinion. Prescribing ω_1 when the actual state is ω_0 and given the small chance that the customer is new on the market, the risk of losing the perceived benefits from adding a second opinion seeker to his clientele is likely to outweighed the benefits of selling the extra service. By contrast, if the proportion of newly arriving customers is large relative to those seeking second opinion, if the provider's queue is short or nonexistence, overprescribing is tempting. The analysis in this case is no different form that of treatment of existing customers with the added feature mentioned above.

Another aspect of the model that deserves further attention is the assumption that the value of future services expected from a client is common knowledge in the stage game. This assumption implies that the expected duration and intensity of the client relationship is common knowledge in the stage game. If this is not the case, the parties must act on their perceptions of the customer's value, which may not be the same. The customer may try to impress the provider of her loyalty in order to incentivize him to prescribe honestly. These consideration suggests that another game may be played in which customers signal their values. Analysis of this aspect of the client relationship may reveal additional behavioral subtleties but is beyond the scope of this paper. I suspect, however, that these behavioral subtleties are of second-order significance relative to the main conclusions of this paper.

I assumed that all customers have the same utility function and that their heterogeneity is induced by their diverse incomes and, in each stage game, the search cost which is a realization of random variable. This assumption is intended to prevent the provider from identifying the customers who are likely to seek a second opinion. If the providers could identify such customers the nature of the game would change and no pure strategy equilibrium would exist.

5 Proofs

5.1 Proof of Lemma 1

(a) Any point $Q \in (0, \bar{Q}_j]$ can be reach if the preceding stage game ended with $Q + \tau$ and the elapsed time is $\tau \geq 0$. Since Φ has full support $\hat{G}_j$ has full if any point in $(0, \bar{Q}_j]$ may be reached after a finite sequence of stage games.

If the state is ω_1 then regardless of the state of his queue the provider prescribes $p_1 = \omega_1$, which is accepted with probability one. Because the elapsed time distribution Φ_j has full support, every $Q = (0, \bar{Q}_j]$ can be reach from $\bar{Q}_j$ if the elapsed time is $\bar{Q}_j - Q$. Since the probability of this event is $\Phi_j\left(\bar{Q}_j - Q\right) > 0$, suffices it to establish that $\bar{Q}_j$ can be reached with positive probability form $Q = 0$.

Consider the event $Q = 0$ and assume that during a period $\Delta < \omega_1$, every customer belonging to the clientele of j arrives in the state ω_1, (i.e., $\mid C_j \mid$ stage games are initiated during the period Δ). Then, by the end of the period the length of the provider's queue will be $\omega_1 \times \mid C_j \mid = \bar{Q}_j$. The probability of this event is $\left(\left(1 - \Phi_j(\Delta)\right)\mu(\omega_1)\right)^{|C_j|} > 0$. That the distribution of the provider's queue has an atom at 0 follows from (16).

(b) Given $Q \in \left[0, \bar{Q}_j\right]$, the queue at the start of the next stage game is $\hat{Q} - \tau$, where the transition from Q to $\hat{Q}$ is the outcome of the state, ω, the provider's prescription and the customer's decision, and τ is the elapsed time since the end of the preceding stage game. The transition probability is determined by the probability distribution, μ on Ω and the strategies of the provider and the customer. Thus, the resulting random variable is distributed according to a probability measure $\hat{G}_j$.

Define $\xi(\tau) = -\tau$, $\tau > 0$. Since the random variables $\hat{Q}$ and τ are stochastically independent, the sum $\hat{Q} + \xi(\tau)$ is distributed according to the convolution of $\hat{G}_j$ and Φ_j. Thus, by Feller (1971, Ch. V.4 Theorem 2),

$$\Pr\{\hat{Q} + \xi(\tau) \leq z\} := U(z) = \int_0^\infty \hat{G}_j(z - \xi(\tau))\, d\left(1 - \Phi_j(\tau)\right).$$

Because the Φ is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$, the measure of the random variable $\hat{Q} - \tau$ is absolutely continuous with respect to the Lebesgue measure on $\mathbb{R}$. Hence, it is non-atomic, except at 0. ▲

5.2 Proof of Lemma 2

Since $\left[0, \bar{Q}_j\right]$ is compact Polish space, by Prokhorov's theorem $\mathcal{G}_j$, the domain of $\Upsilon(\cdot \mid y^*, Q^*(y^*))$ is compact in the topology of weak convergence. Moreover, it is obviously convex.

Let (G_j^n) be a sequence in $\mathcal{G}_j$ that converges to G_j in the topology of weak convergence. Then, for all continuous real-valued functions f on $\left[0, \bar{Q}_j\right]$, $\lim_{n\to\infty} \int_0^{\bar{Q}_j} f dG_j^n = \int_0^{\bar{Q}_j} f dG_j$. Since Φ is continuous, by (15), for all $Q' \in [0, \bar{Q}_j]$

$$\begin{aligned}
&\lim_{n\to\infty} \Upsilon\left(G_j^n(Q')\right) \\
&\quad := \lim_{n\to\infty} \Bigg\{\bar{\lambda}\left(y^*, Q^*\left(y^*\right)\right)\left[\int_{Q'}^{\bar{Q}_j} \Phi_j\left(Q - Q'\right) dG_j^n(Q) + G_j^n\left(Q'\right)\right] \\
&\qquad + \left(1 - \bar{\lambda}\left(y^*, Q^*\left(y^*\right)\right)\right)\left[\int_{Q'-\omega_1}^{\bar{Q}_j} \Phi_j\left(Q + \omega_1 - Q'\right) dG_j^n(Q) + G_j^n\left(Q' - \omega_1\right)\right]\Bigg\} \\
&\quad = \bar{\lambda}\left(y^*, Q^*\left(y^*\right)\right)\left[\int_{Q'}^{\bar{Q}_j} \Phi_j\left(Q - Q'\right) dG_j(Q) + G_j\left(Q'\right)\right]
\end{aligned}$$

$$+\left(1-\bar{\lambda}\left(y^{*}, Q^{*}\left(y^{*}\right)\right)\right)\left[\int_{Q^{\prime}-\omega_{1}}^{\bar{Q}_{j}} \Phi_{j}\left(Q+\omega_{1}-Q^{\prime}\right) d G_{j}(Q)+G_{j}\left(Q^{\prime}-\omega_{1}\right)\right]$$
$$=\Upsilon\left(G_{j}\left(Q^{\prime}\right)\right). \tag{18}$$

Thus, $\Upsilon\left(\cdot \mid y^{*}, Q^{*}\left(y^{*}\right)\right)$ is continuous. That it is continuous in y^{*} and $Q^{*}\left(y^{*}\right)$ follows from the continuity of $\bar{\lambda}\left(y^{*}, Q^{*}\left(y^{*}\right)\right)$ in these variables. The conclusion that Υ has fixed point is implied by Brouwer's fixed point theorem. ▲

5.3 Proof of Theorem 1

Proof Given v_i, define a mapping $\Psi:[0, \bar{y}]^{2} \times\left[0, \bar{Q}_{j}\right] \times \mathcal{G}_{j} \rightarrow[0, \bar{y}]^{2} \times\left[0, \bar{Q}_{j}\right] \times \mathcal{G}_{j}$ by:

$$\Psi\left(y^{*}\left(v_{i}, \theta_{0}\right), y^{*}\left(v_{i}, \theta_{1}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right), G_{j}^{*}\left(\cdot \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)\right)$$
$$=\left(y^{**}\left(v_{i}, \theta_{0}\right), y^{**}\left(v_{i}, \theta_{1}\right), Q_{j}^{**}\left(y^{*}, v_{i}\right), G_{j}^{**}\left(\cdot \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)\right),$$

where $y^{**}\left(v_{i}, \theta_{0}\right)$ and $y^{**}\left(v_{i}, \theta_{1}\right)$ are the customer's reservation incomes given $Q_{j}^{*}\left(y^{*}, v_{i}\right)$ and $G_{j}^{*}\left(\cdot \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)$, $Q_{j}^{**}\left(y^{*}, v_{i}\right)$ is the provider's best response to $y^{*}\left(v_{i}\right)$ and $G_{j}^{*}\left(\cdot \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)$, and $G_{j}^{**}\left(\cdot \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)=\Upsilon\left(G_{j}^{*}\left(\cdot \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)\right)$.

Since each of the sets in the domain of Ψ is compact and convex, the product $[0, \bar{y}]^{2} \times\left[0, \bar{Q}_{j}\right] \times \mathcal{G}_{j}$ is compact (in the product topology) and convex.

By (8) $\mu\left(\cdot \mid p_{1}, v_{i}\right)$ is continuous in $G_{j}\left(Q_{j}^{*}\left(v_{i}\right)\right)$. Consequently, by their definitions and continuity of u, $y^{*}\left(v_{i}, \theta_{0}\right)$ and $y^{*}\left(v_{i}, \theta_{1}\right)$ are continuous in $G_{j}\left(Q_{j}^{*}\left(v_{i}\right)\right)$. By Lemma 1, G_{j} is continuous on $\left(0, \bar{Q}_{j}\right]$. Hence, $y^{*}\left(v_{i}, \theta_{0}\right)$ and $y^{*}\left(v_{i}, \theta_{1}\right)$, are continuous in $Q_{j.}^{*}$. Because F is continuous in y, it follows from (7) that $Q_{j}^{*}\left(y^{*}, v_{i}\right)$ is continuous in y^{*}. By Lemma 2, $G_{j}\left(\cdot \mid y, Q_{j}\right)$ is continuous in y and Q_{j}. Hence, Ψ is a continuous mapping. By Brouwer's fixed point theorem, it has fixed point $\left(y^{*}\left(v_{i}, \theta_{0}\right), y^{*}\left(v_{i}, \theta_{1}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right), G_{j}^{*}\left(\cdot \mid y^{*}, Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)\right)$, where

$$G_{j}^{*}\left(\cdot \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)=\Upsilon\left(G_{j}^{*}\left(\cdot \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)\right).$$

Consider next the system of beliefs $\left(\mu, G_{j}^{*}, \mu\left(\cdot \mid p_{1}, v_{i}\right)\right)$, where μ is the prior distribution on Ω, and $G_{j}^{*}\left(\cdot \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)$ is the fixed point of Υ given $y^{*}\left(v_{i}, \theta_{0}\right)$, $y^{*}\left(v_{i}, \theta_{1}\right)$ and $Q_{j}^{*}\left(y^{*}, v_{i}\right)$, and $\mu\left(\cdot \mid p_{1}, v_{i}\right)$ is given in (8), (9) and (13) with $G_{j}\left(Q_{j}^{*}\left(v_{i}\right)\right)=G_{j}^{*}\left(Q_{j}^{*}\left(v_{i}\right) \mid y^{*}\left(v_{i}\right), Q_{j}^{*}\left(y^{*}, v_{i}\right)\right)$.[22]

[22] Given that a customer seeks a second opinion only if the first provider prescribes ω_1, the beliefs of the second provider is that the probability of $p_1=\omega_1$ is 1, regardless of the true state.

Thus $\left(y^*\left(v_i,\theta_0\right), y^*\left(v_i,\theta_1\right), Q_j^*\left(y^*,v_i\right), G_j^*\left(\cdot \mid y^*\left(v_i\right), Q_j^*\left(y^*,v_i\right)\right)\right)$ are sequentially rational given the system of beliefs $\left(\mu, G_j^*, \mu\left(\cdot \mid p_1, v_i\right)\right)$, and the system of beliefs is derived from the strategy profile, using Bayes' rule. This completes the proof of existence.

To prove the uniqueness of the equilibrium I first establish the following result. Let y^* be defined by the solution to

$$u\left(y-\omega_1\right)=\mu u\left(y-x\right)+\left(1-\mu\right)u\left(y-z\right),$$

where $z>x$.

Claim The function u displays decreasing absolute risk aversion if and only if $dy^*/d\mu<0$.

Proof of Claim Differentiating we get:

$$\frac{dy^*}{d\mu}=\frac{u\left(y^*-x\right)-u\left(y^*-z\right)}{u'\left(y^*-\omega_1\right)-\mu u'\left(y^*-x\right)-\left(1-\mu\right)u'\left(y^*-z\right)}.$$

Because u is monotonic increasing and $z>x$ the numerator is positive. Hence, $dy^*/d\mu<0$ if and only if $u'\left(y^*-\omega_1\right)<\mu u'\left(y^*-x\right)+\left(1-\mu\right)u'\left(y^*-z\right)$.

The function u displays decreasing absolute risk aversion if and only if, for all y,

$$\frac{d}{dy}\left[-\frac{u''\left(y\right)}{u'\left(y\right)}\right]=-\frac{u'''\left(y\right)}{u'\left(y\right)}-\left(-\frac{u''\left(y\right)}{u'\left(y\right)}\right)^2<0.$$

Equivalently,

$$\frac{d}{dy}\left[-\frac{u''\left(y\right)}{u'\left(y\right)}\right]<0\text{ if and only if }-\frac{u'''\left(y\right)}{u''\left(y\right)}<-\frac{u''\left(y\right)}{u'\left(y\right)}.$$

By the theorem of Pratt (1964) the last inequality is equivalent to $u':=\varsigma\circ u$, where ς is strictly monotonic increasing concave function.

Define C^u and $C^{u'}$ by $u\left(C^u\right)=\mu u\left(y^*-x\right)+\left(1-\mu\right)u\left(y^*-z\right)$ and $u'\left(C^{u'}\right)=\mu u'\left(y^*-x\right)+\left(1-\mu\right)u'\left(y^*-z\right)$, respectively. Then, by Pratt's theorem, $C^u>C^{u'}$. But, by definition, $C^u=y^*-\omega_1$. Hence, $C^u>C^{u'}$ if and only if $u'\left(y^*-\omega_1\right)<\mu u'\left(y^*-x\right)+\left(1-\mu\right)u'\left(y^*-z\right)$. This complete the proof of the claim. ◇

To show that the equilibrium is unique suppose, by way of negation, that there exists another equilibrium $\left(y^{**}\left(v_i,\theta_0\right), y^{**}\left(v_i,\theta_1\right), Q_j^{**}\left(y^{**},v_i\right), G_j^{**}\left(\cdot \mid y^{**}\left(v_i\right), Q_j^{**}\left(v_i\right)\right)\right)$. Suppose that $G_j^{**}\left(Q_j^{**}\left(y^{**},v_i\right)\right)>G_j^*\left(Q_j^*\left(y^*,v_i\right)\right)$ then

$$\mu^*\left(\omega_0 \mid p_1=\omega_1,v_i\right)=\frac{\mu\left(\omega_0\right)G_j^*\left(Q_j^*\left(v_i\right)\right)}{\mu\left(\omega_1\right)+\mu\left(\omega_0\right)G_j^*\left(Q_j^*\left(v_i\right)\right)}$$

$$< \frac{\mu(\omega_0) G_j^{**}\left(Q_j^{**}(v_i)\right)}{\mu(\omega_1) + \mu(\omega_0) G_j^{**}\left(Q_j^{**}(v_i)\right)} = \mu^{**}(\omega_0 \mid p_1 = \omega_1, v_i).$$

By the claim, $y^{**}(v_i, \theta) < y^*(v_i, \theta)$, for $\theta \in \Theta$. Hence, $F(y^*(v_i)) > F(y^{**}(v_i))$. By (7), $Q_j^{**}(v_i)$ is given by $F(y^{**}(v_i)) e^{-rQ_j^{**}(v_i)}(1-c)\omega_1 = (1 - F(y^{**}(v_i))) v_i$. Thus, $F(y^*(v_i)) > F(y^{**}(v_i))$ implies that $Q_j^{**}(y^{**}, v_i) < Q_j^*(y^*, v_i)$. Hence,

$$\bar{\lambda}\left(y^*, Q^*\left(y^*\right)\right) = \mu(\omega_0) \int_0^{\bar{v}} \left(1 - G_j^*\left(Q^*(v)\right) F\left(y^*(v)\right)\right) dJ(v)$$
$$> \mu(\omega_0) \int_0^{\bar{v}} \left(1 - G_j^{**}\left(Q^{**}(v)\right) F\left(y^{**}(v)\right)\right) dJ(v) = \bar{\lambda}\left(y^{**}, Q^{**}\left(y^{**}\right)\right).$$

By (15), the last inequality implies that $G_j^{**}(Q) < G_j^*(Q)$, for all $Q \in \left[0, \bar{Q}_j\right]$. Thus, $Q_j^{**}(y^{**}, v_i) < Q_j^*(y^*, v_i)$ implies that $G_j^{**}\left(Q_j^{**}(y^{**}, v_i)\right) < G_j^*\left(Q_j^*(y^{**}, v_i)\right)$, a contradiction. Hence, there is no equilibrium such that $G_j^{**}\left(Q_j^{**}(y^{**}, v_i)\right) < G_j^*\left(Q_j^*(y^{**}, v_i)\right)$. By similar argument there is no equilibrium such that $G_j^{**}\left(Q_j^{**}(y^{**}, v_i)\right) > G_j^*\left(Q_j^*(y^*, v_i)\right)$.

Consider next the case $G_j^{**}\left(Q_j^{**}(y^{**}, v_i)\right) = G_j^*\left(Q_j^*(y^*, v_i)\right)$. Suppose that $y^{**} < y^*$ then $F(y^{**}(v)) < F(y^*(v))$, by the same argument as above, $Q_j^{**}(y^{**}, v_i) < Q_j^*(y^{**}, v_i)$. Hence, $\bar{\lambda}(y^*, Q^*(y^*)) > \bar{\lambda}(y^{**}, Q^{**}(y^{**}))$. By (15), $G_j^{**}(Q) < G_j^*(Q)$, for all $Q \in \left[0, \bar{Q}_j\right]$. Hence, $Q_j^{**}(y^{**}, v_i) < Q_j^*(y^{**}, v_i)$ implies that $G_j^{**}\left(Q_j^{**}(y^{**}, v_i)\right) < G_j^*\left(Q_j^*(y^*, v_i)\right)$. A contradiction. □

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