

Nonparametric Identification of Dynamic Models with Unobserved State Variables: Supplemental Material

Yingyao Hu

Johns Hopkins University

Matthew Shum

California Institute of Technology

December 2009

Abstract

We provide some additional material pertaining to our paper Hu and Shum (2008). In section 1, we verify our assumptions for a dynamic discrete-choice model inspired by Rust's (1987) bus engine replacement model. Section 2 contains a comparison of our framework with that in Kasahara and Shimotsu (2009). Section 3 contains supplemental discussion related to Example 2 in the main paper and additional discussion of Assumption 2.

1 Additional example: dynamic discrete-choice model based on Rust (1987)

In addition to the two examples presented in the main paper, we present here a discussion of our assumptions in the context of a third example: Rust's (1987) bus-engine replacement model, augmented to allow for time-varying serially-correlated unobserved state variables. In this model, $W_t = (Y_t, M_t)$, where Y_t is the indicator that the bus engine was replaced in week t , and M_t is the mileage since the last engine replacement.

Let $S_t \equiv (M_t, X_t^*)$ denote the state variables in this model. The period utility from each choice is additive in a function of the state variables S_t , and a choice-specific non-persistent preference shock:

$$u_t = \begin{cases} u_0(S_t) + \epsilon_{0t} & \text{if } Y_t = 0 \\ u_1(S_t) + \epsilon_{1t} & \text{if } Y_t = 1 \end{cases}$$

where ϵ_{0t} and ϵ_{1t} are i.i.d. Type I Extreme Value shocks, which are independent over time,

and also independent of the state variables S_t .

The choice-specific utility functions are:

$$u_0(S_t) = -c(M_t); \quad u_1(S_t) = -RC. \quad (1)$$

In the above, $c(M_t)$ denotes the maintenance cost function, which is increasing in mileage M_t , and $0 < RC < +\infty$ denotes the cost of replacing the engine. We also assume that the maintenance cost function $c(\cdot)$ is bounded below and above: $c(0) = 0$; $\lim_{M \rightarrow +\infty} c(M) = \bar{c} < +\infty$. Mileage evolves as:

$$M_{t+1} = \begin{cases} M_t[1 + \exp(\eta_{t+1} + X_{t+1}^*)] & \text{if } Y_t = 0 \\ \exp(\eta_{t+1} + X_{t+1}^*) & \text{if } Y_t = 1 \end{cases}. \quad (2)$$

where $\eta_{t+1} \in \mathbb{R}$ follows an extreme value distribution with density $f_{\eta_{t+1}}(\eta) = \exp(\eta - e^\eta)$. This law of motion implies $f_{M_{t+1}|Y_t, M_t, X_t^*, X_{t+1}^*} = f_{M_{t+1}|Y_t, M_t, X_{t+1}^*}$. Hence, X_t^* affects the evolution of mileage, but not the agent's utilities. Furthermore, following Rust's assumptions, previous mileage M_{t-1} has no direct effect on current mileage M_t when the engine was replaced in the previous period ($Y_{t-1} = 1$).

X_t^* , the time-varying unobserved state variable, denotes the general physical condition (wear and tear) of the bus, which is unobserved by the econometrician but observed by the bus mechanics, and affects their decisions about replacing the bus engine. It evolves as an AR(1) process:

$$X_t^* = 0.8X_{t-1}^* + 0.2\nu_t. \quad (3)$$

$\nu_t \in \mathbb{R}$ is a standard normal shock, distributed independently over t . This law of motion implies $f_{X_t^*|Y_{t-1}, M_{t-1}, X_{t-1}^*} = f_{X_t^*|X_{t-1}^*}$. We also assume that $X_1^* \in \mathbb{R}$. Hence, $X_t^*|X_{t-1}^*$ is distributed with density determined by $f_{\nu_t}(\cdot)$.

In this stationary dynamic optimization model, the conditional choice probabilities take the multinomial logit form (for $Y_t = 0, 1$): $P(Y_t|S_t) = \exp(V_{Y_t}(S_t)) / \left[\sum_{y=0}^1 \exp(V_y(S_t)) \right]$ where $V_y(S_t)$ is the choice-specific value function in period t , defined recursively by $V_y(S_t) = u_y(S_t) + \beta E \left[\log \left\{ \sum_{y'=0}^1 \exp(V_{y'}(S_{t+1})) \right\} | Y_t = y, S_t \right]$.

The arguments we use here are very similar to those we used to verify the assumptions in Example 2 of the main paper. To avoid confusion, we repeat most of the details here. We verify the assumptions out of order, leaving Assumption 2 to the end. Since we focus here on the stationary case, without loss of generality we label the four observed periods of data W_t as $t = 1, 2, 3, 4$.

Assumption 1 is satisfied for this model. **Assumption 3** contains two restrictions on

the density $f_{W_3|W_2, X_3^*}$, which factors as

$$f_{W_3|W_2, X_3^*} = f_{Y_3|M_3, X_3^*} \cdot f_{M_3|Y_2, M_2, X_3^*}. \quad (4)$$

Assumption 3(i) requires that, for any w_3 , there exists $(w_2, \bar{w}_3, \bar{w}_2)$ such that the eigenvalues $k(w_3, \bar{w}_3, w_2, \bar{w}_2, x_3^*)$ are bounded between 0 and a constant C . The first term is the CCP $f_{Y_3|M_3, X_3^*}$, which is a logit probability. Because the per-period utilities, net of the ϵ 's, are bounded away from $-\infty$ and $+\infty$, the logit choice probabilities are also bounded from zero. Moreover, the CCP's are not a function of W_2 , so that Eq. (12) in the main text implies that the eigenvalues $k(w_3, \bar{w}_3, w_2, \bar{w}_2, x_3^*)$ in the spectral decomposition will not be a function of the CCP's.

The second term is the mileage law of motion $f_{M_3|Y_2, M_2, X_3^*}$ which, by assumption, is

$$\begin{aligned} & f_{M_3|Y_2, M_2, X_3^*}(m_3|y_2, m_2, x_3^*) \\ &= \frac{1}{m_3 - (1 - y_2)m_2} f_{\eta_3}(\ln(m_3 - (1 - y_2)m_2) - (1 - y_2) \ln m_2 - x_3^*) \end{aligned} \quad (5)$$

and f_{η_3} denotes the density of the extreme value distribution. For any $w_3 = (y_3, m_3)$, we consider

$$\begin{aligned} w_2 &= (y_2, m_2) = (0, m_2), \\ \bar{w}_2 &= (\bar{y}_2, \bar{m}_2) = (0, m_2 + \Delta), \\ \bar{w}_3 &= (\bar{y}_3, \bar{m}_3) = (y_3, m_3 - \Delta), \end{aligned}$$

which implies that the bus engine was not replaced in period 2. We may show that

$$k(w_3, \bar{w}_3, w_2, \bar{w}_2, x_3^*) = \exp\left(-e^{-x_3^*} \left[\frac{\Delta^2}{m_2 \bar{m}_2}\right]\right),$$

which is bounded between zero and one. Therefore, Assumption 3(i) holds. Furthermore, the equation above implies that the eigenvalue $k(w_3, \bar{w}_3, w_2, \bar{w}_2, x_3^*)$ is monotonic in x_3^* , which implies Assumption 3(ii).

Assumption 4 presumes a known functional G such that $G[f_{M_4|Y_3, M_3, X_3^*}(\cdot|y_3, m_3, x_3^*)]$ is monotonic in x_3^* , where we use $V_t = M_t$, for all periods t . Eqs. (3) and (2) imply that

$$M_4 = (1 - Y_3)M_3 + (M_3)^{1-Y_3} \exp(\eta_4 + 0.2\nu_4) \cdot \exp(0.8X_3^*). \quad (6)$$

Let C_{med} denote the median of the random variable $\exp(\eta_4 + 0.2\nu_4)$. Then

$$\text{med} \left[f_{M_4|Y_3, M_3, X_3^*}(\cdot | y_3, m_3, x_3^*) \right] = (1 - y_3)m_3 + (m_3)^{1-y_3} C_{med} \cdot \exp(0.8x_3^*)$$

which is monotonic in x_3^* . Hence, we can pin down $x_3^* = \text{med} \left[f_{M_4|Y_3, M_3, X_3^*}(\cdot | y_3, m_3, x_3^*) \right]$.

Assumption 2 contains three injectivity assumptions. For the V_t variables in Assumption 2, we use $V_t = M_t$, for all periods t .

For Assumption 2, it is sufficient to establish the injectivity of the operators L_{M_1, w_2, w_3, M_4} , $L_{M_4|w_3, X_3^*}$, and L_{M_1, w_2, M_3} for any (w_2, w_3) in the support. Applying Claim 2 from Example 2 of the main paper, it suffices to show the injectivity of L_{M_4, w_3, w_2, M_1} , $L_{M_4|w_3, X_3^*}$, and L_{M_3, w_2, M_1} . Assumption 1, along with the assumptions on the laws of motion, implies that

$$\begin{aligned} L_{M_4, w_3, w_2, M_1} &= L_{M_4|w_3, X_3^*} D_{w_3|w_2, X_3^*} L_{X_3^*, w_2, M_1} \\ &= L_{M_4|w_3, X_3^*} D_{w_3|w_2, X_3^*} L_{X_3^*|X_2^*} L_{X_2^*, w_2, M_1} \end{aligned} \quad (7)$$

$$L_{M_3, w_2, M_1} = L_{M_3|w_2, X_2^*} L_{X_2^*, w_2, M_1}. \quad (8)$$

Furthermore, we have $L_{M_4|w_3, X_3^*} = L_{M_4|w_3, X_4^*} L_{X_4^*|X_3^*}$.

Hence, the injectivity of L_{M_4, w_3, w_2, M_1} , $L_{M_4|w_3, X_3^*}$, and L_{M_3, w_2, M_1} is implied by the injectivity of $L_{M_4|w_3, X_4^*}$, $D_{w_3|w_2, X_3^*}$, $L_{X_3^*|X_2^*}$ and $L_{X_2^*, w_2, M_1}$.

(i) The diagonal operator $D_{w_3|w_2, X_3^*}$ has corresponding density function $f_{w_3|w_2, X_3^*} = f_{y_3|m_3, X_3^*} f_{m_3|m_2, X_3^*}$. In the discussion on Assumption 3(i) above, we have shown that both $f_{y_3|m_3, X_3^*}$ and $f_{m_3|m_2, X_3^*}$ are nonzero, for all values of (y_3, m_3, m_2, x_3^*) in the support. Therefore, the operator $D_{w_3|w_2, X_3^*}$ is injective.

(ii) For $L_{M_4|w_3, X_4^*}$, we use Eq. (2) whereby, for every (y_3, m_3) , M_4 is a convolution of X_4^* , i.e. $\log[M_4 - (1 - y_3)m_3] - (1 - y_3) \log m_3 = X_4^* + \eta_4$. As is well-known, as long as the characteristic function of η_4 has no real zeros, which is satisfied by the assumed extreme value distribution, the corresponding operator is injective.

(iii) Similarly, X_3^* is a convolution of X_2^* , i.e. $X_3^* = 0.8X_2^* + 0.2\nu_3$ (cf. Eq. (3)). Hence, $L_{X_3^*|w_2, X_2^*}$ is injective if the characteristic function of ν_3 has no real zeros, which is satisfied by the assumed normal distribution.

(iv) For the operator $L_{X_2^*, w_2, M_1}$, corresponding to the density $f_{X_2^*, w_2, M_1}$, the model assumptions do not allow us to establish injectivity directly. This is because this joint density confounds both the structural components (laws of motion) in the model and the initial condition $f_{X_1^*, M_1}$. However, as in Example 2 of the main paper, it turns out that some stochastic assumptions on the initial conditions (Y_1, M_1, X_1^*) ensure injectivity of $L_{X_2^*, w_2, M_1}$.

Claim 3: If (i) $f_{X_1^*, M_1} = f_{X_1^*} f_{M_1}$; and (ii) $Y_1 = 0$ with probability one, and is exogenous, then $L_{X_2^*, w_2, M_1}$ is injective.

Proof: The kernel of the operator $L_{X_2^*, w_2, M_1}$ is

$$\begin{aligned}
f_{X_2^*, w_2, M_1} &= \int \int f_{X_2^*, y_2, m_2, X_1^*, Y_1, M_1} dy_1 dx_1^* \\
&= f_{y_2 | m_2, X_2^*} \int \int f_{m_2 | X_2^*, M_1, Y_1} f_{X_2^* | X_1^*} f_{Y_1 | M_1, X_1^*} f_{X_1^*, M_1} dy_1 dx_1^* \\
&= f_{y_2 | m_2, X_2^*} f_{m_2 | X_2^*, M_1, Y_1=0} \int f_{X_2^* | X_1^*} f_{X_1^*, M_1} dx_1^* \\
&= f_{y_2 | m_2, X_2^*} f_{m_2 | X_2^*, M_1, Y_1=0} \left[\int f_{X_2^* | X_1^*} f_{X_1^*} dx_1^* \right] f_{M_1} \\
&= f_{y_2 | m_2, X_2^*} f_{X_2^*} f_{m_2 | X_2^*, M_1, Y_1=0} f_{M_1}
\end{aligned}$$

In the third line, we have utilized condition (ii), which implies $f_{Y_1 | M_1, X_1^*}(0 | m_1, x_1^*) = 1$, i.e., no engine is changed in the initial period of data. The fourth line applies the independence condition (i). The equivalent operator equation is

$$L_{X_2^*, w_2, M_1} = D_{y_2 | m_2, X_2^*} D_{X_2^*} L_{m_2 | X_2^*, M_1, Y_1=0} D_{M_1}.$$

The injectivity of $L_{X_2^*, w_2, M_1}$ then relies on that of the operator $L_{m_2 | X_2^*, M_1, Y_1=0}$ which, as Eq. (2) shows, is based on a convolution form. Using an argument identical to that used in the proof of Claim 1 in Appendix B of the main paper, we can show that $L_{m_2 | X_2^*, M_1, Y_1=0}$ is injective, which implies that $L_{X_2^*, w_2, M_1}$ is also injective. ■

Therefore, for this example, we have shown the injectivity of L_{M_4, w_3, w_2, M_1} , $L_{M_4 | w_3, X_3^*}$, and L_{M_3, w_2, M_1} . By applying Claim 2 in Example 2 of the main paper, we also obtain the injectivity of L_{M_1, w_2, w_3, M_4} and L_{M_1, w_2, M_3} , as required by Assumption 2(i) and 2(iii).

Without the assumption that the initial value Y_1 is exogenous, and that $Y_1 = 0$ with probability one (so that the engine is changed with zero probability in the initial period of data), the choice probability $f_{Y_1 | M_1, X_1^*}$ would also appear on the RHS of the preceding equations, and additional assumptions regarding this probability would be required to ensure injectivity. However, because these choice probabilities are endogenously determined, it is awkward to impose assumptions directly on it.

2 Additional comparison with Kasahara-Shimotsu (2009)

Here, we provide some additional details on the results in Kasahara and Shimotsu (2009) (KS), and show that KS's identification results are not applicable to the dynamic models with time-varying unobservables considered in our paper.

We start by summarizing KS's main results. Throughout, we state KS's results using our notation in this paper. Since KS assume that the unobserved heterogeneity X^* is time-invariant, we attach no t subscript to it.¹ Using the notation in our paper, the second equality of KS's Eq. (3) is:

$$f_{Y_1, M_1, \dots, Y_T, M_T} = \sum_{X^*} f_{X^*} f_{M_1, Y_1 | X^*} \prod_{t=2}^T f_{M_t | M_{t-1}, Y_{t-1}, \dots, M_1, Y_1, X^*} f_{Y_t | M_t, M_{t-1}, Y_{t-1}, X^*}.$$

In their baseline model (ie. their Assumption 1), they assume that the unobserved heterogeneity X^* does not affect the law of motion for the observed state variable M_t , and that Y_t is independent of (M_{t-1}, Y_{t-1}) conditional on M_t and X^* . This leads to

$$\frac{f_{Y_1, M_1, \dots, Y_T, M_T}}{\prod_{t=2}^T f_{M_t | M_{t-1}, Y_{t-1}}} = \sum_{X^*} f_{X^*} f_{M_1, Y_1 | X^*} \prod_{t=2}^T f_{Y_t | M_t, X^*}, \quad [\text{Eq. (9) in KS (2009)}]$$

which is Eq. (9) in KS. Notice that the LHS of the above is observed, and they demonstrate (in their Proposition 1) that the unknown densities on the RHS are identified from the observed quantity on the LHS for $T \geq 3$.

In section 3.2 of their paper, they consider a first-order Markovian model where the observed variables W_t can depend on W_{t-1} and X^* . They show that, by using $T \geq 6$ periods of data W_1, \dots, W_T , and fixing the values in the odd periods $w_1, w_3, w_5, \dots, w_{T-1}$,

¹The correspondence between KS's notation and ours is as follows:

$$\overbrace{\left[\begin{array}{c} a_t, x_t, s_t, m, \pi^m, Q^m(s_t | s_{t-1}) \\ P_t^m(a_t | x_t, x_{t-1}, a_{t-1}) \end{array} \right]}^{\text{KS (2009)}} \Leftrightarrow \overbrace{\left[\begin{array}{c} Y_t, M_t, W_t, X^*, f_{X^*}, f_{W_t | W_{t-1}, X^*} \\ f_{Y_t | M_t, M_{t-1}, Y_{t-1}, X^*} \end{array} \right]}^{\text{our notation}}.$$

one obtains

$$f_{w_1, W_2, w_3, W_4, \dots, w_{T-1}, W_T} = \sum_{X^*} f_{w_1, X^*} \left(\prod_{t=2,4,\dots}^{T-2} f_{w_{t+1}, W_t | w_{t-1}, X^*} \right) f_{W_T | w_{T-1}, X^*}, \quad [\text{Eq. (27) in KS (2009)}]$$

which is Eq. (27) in KS. As they note, Eq. (27) has the same “independent marginals” form as Eq. (9), so that their identification scheme also applies to first-order Markov process with time-invariant X^* for $T \geq 6$. This is their Proposition 6.

However, this scheme no longer works in the case where the latent variable X_t^* varies over time, even if X_t^* is discrete. To see this, we consider a joint first-order Markov process $\{W_t, X_t^*\}$ where both W_t and X_t^* vary over time, as in Example 1 in the main text of our paper. Analogously to Eq. (27) in KS, we may have

$$f_{w_1, W_2, w_3, W_4, \dots, w_{T-1}, W_T} = \sum_{X_{T-1}^*} \dots \sum_{X_5^*} \sum_{X_3^*} \sum_{X_1^*} f_{w_1, X_1^*} \left(\prod_{t=2,4,\dots}^{T-2} f_{w_{t+1}, X_{t+1}^*, W_t | w_{t-1}, X_{t-1}^*} \right) f_{W_T | w_{T-1}, X_{T-1}^*}.$$

Obviously, this takes a very different form than Eq. (27) above, because the components on the RHS involve values of the latent variable X_t^* in different periods. Hence, KS’s identification scheme does not apply here. Notice that using more periods of data only exacerbates the problem; the more periods of data one uses, the more latent variables X_t^* appear when X_t^* is time-varying.

In conclusion, the identification strategy in KS does not apply to models where X_t^* is time-varying, even if X_t^* is discrete. An important innovation of the present paper is that we provide nonparametric identification for dynamic models with time-varying unobserved variables.

3 Miscellaneous remarks

3.1 Remarks on dynamic investment models

For Example 2 in the main paper, we considered a general investment model in the framework of Doraszelski and Pakes (2007). There is a recent and growing empirical literature based on these types of dynamic models, including Collard-Wexler (2006), Ryan (2006), and Dunne, Klimer, Roberts, and Xu (2006). Pakes (2008, section 3) and Akerberg, Benkard, Berry, and Pakes (2007) discuss additional examples.

On the other hand, the productivity literature has by and large been based on the

“pure” investment model, typified by Olley and Pakes (1996) (OP). This model differs in an important way from the types of models considered in our paper. Namely, in OP, capital stock (corresponding to the M variable in Example 2) evolves deterministically, conditional on the previous period’s capital (M_{t-1}) and investment (Y_{t-1}). This feature violates two of our maintained assumptions (# 2,3), which require that M_t depend on X_t^* even conditional on (Y_{t-1}, M_{t-1}) . For this reason, in Example 2 in the main paper, we do not consider the “pure” investment model as in OP, but rather a generalized investment model in which M_t does not evolve deterministically.

3.2 Further discussion on Assumption 2

In this section we discuss how Assumption 2 is used to ensure the validity of two different ways for taking operator inverses. Consider two scenarios involving an operator equation

$$L_{R_1, r_2, R_4} = L_{R_1 | r_2, R_3} L_{r_2, R_3, R_4}. \quad (9)$$

In the first scenario, suppose we want to solve for L_{r_2, R_3, R_4} given L_{R_1, r_2, R_4} and $L_{R_1 | r_2, R_3}$. The assumption that $L_{R_1 | r_2, R_3}$ is one-to-one guarantees that we may have

$$L_{R_1 | r_2, R_3}^{-1} L_{R_1, r_2, R_4} = L_{r_2, R_3, R_4}. \quad (10)$$

As an example, Assumption 2(ii) guarantees that pre-multiplication by the inverse operator $L_{V_{t+1} | w_t, X_t^*}$ is valid, which is used in the equation following Eq. (9).

In the second scenario, suppose we need to solve for $L_{R_1 | r_2, R_3}$ given L_{R_1, r_2, R_4} and L_{r_2, R_3, R_4} in equation (9). We would need the operator L_{r_2, R_3, R_4} to be invertible as follows:

$$L_{R_1, r_2, R_4} L_{r_2, R_3, R_4}^{-1} = L_{R_1 | r_2, R_3}. \quad (11)$$

As proved in Lemma 1 in Hu and Schennach (2008), the sufficient condition for obtaining Eq. (11) from Eq. (9) is that the operator L_{R_4, R_3, r_2} is one-to-one.² (Notice that the operator L_{R_4, R_3, r_2} is from $L^p(\mathcal{R}_3)$ to $L^p(\mathcal{R}_4)$.)

Assumption 2(i) is an example of this. It is used to justify the post-multiplication by $L_{V_{t+1}, \bar{w}_t, w_{t-1}, V_{t-2}}^{-1}$ and $L_{V_{t+1}, w_t, \bar{w}_{t-1}, V_{t-2}}^{-1}$ in, respectively, Eqs. (9) and (10). Similarly, Assumption 2(iii) guarantees the validity of post-multiplication by $L_{V_t, w_{t-1}, V_{t-2}}^{-1}$, which is done in the second line in Eq. (29). Throughout this paper, we only post-multiply by

² A similar assumption is also used in Carroll, Chen, and Hu (2009).

the inverses of $L_{V_{t+1}, w_t, w_{t-1}, V_{t-2}}$ and $L_{V_t, w_{t-1}, V_{t-2}}$; all other cases of inverses involve pre-multiplication. For a more technical discussion, see Aubin (2000, sections 4.5-4.6).

References

- ACKERBERG, D., L. BENKARD, S. BERRY, AND A. PAKES (2007): “Econometric Tools for Analyzing Market Outcomes,” in *Handbook of Econometrics, Vol. 6A*, ed. by J. Heckman, and E. Leamer. North-Holland.
- AUBIN, J.-P. (2000): *Applied Functional Analysis*. Wiley-Interscience.
- CARROLL, R., X. CHEN, AND Y. HU (2009): “Identification and estimation of nonlinear models using two samples with nonclassical measurement errors,” *Journal of Nonparametric Statistics*, forthcoming.
- COLLARD-WEXLER, A. (2006): “Demand Fluctuations and Plant Turnover in the Ready-to-Mix Concrete Industry,” manuscript, New York University.
- DORASZELSKI, U., AND A. PAKES (2007): “A Framework for Dynamic Analysis in IO,” in *Handbook of Industrial Organization, Vol. 3*, ed. by M. Armstrong, and R. Porter, chap. 30. North-Holland.
- DUNNE, T., S. KLIMER, M. ROBERTS, AND D. XU (2006): “Entry and Exit in Geographic Markets,” manuscript, Penn State University.
- HU, Y., AND S. SCHENNACH (2008): “Instrumental variable treatment of nonclassical measurement error models,” *Econometrica*, 76, 195–216.
- HU, Y., AND M. SHUM (2008): “Nonparametric Identification of Dynamic Models with Unobserved State Variables,” Johns Hopkins University, Dept. of Economics working paper #543.
- KASAHARA, H., AND K. SHIMOTSU (2009): “Nonparametric Identification of Finite Mixture Models of Dynamic Discrete Choice,” *Econometrica*, 77, 135–175.
- OLLEY, S., AND A. PAKES (1996): “The Dynamics of Productivity in the Telecommunications Equipment Industry,” *Econometrica*, 64, 1263–1297.
- PAKES, A. (2008): “Theory and Empirical Work in Imperfectly Competitive Markets,” Fisher-Schultz Lecture at 2005 Econometric Society World Congress (London, England).
- RUST, J. (1987): “Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher,” *Econometrica*, 55, 999–1033.
- RYAN, S. (2006): “The Costs of Environmental Regulation in a Concentrated Industry,” manuscript, MIT.