

The Hall-Jorgenson Model of Investment

Hall and Jorgenson (1967) consider the problem of a firm that produces output using capital k as its only input,

$$y = f(k) \quad (1)$$

and which obtains its capital k from a market in which a unit of capital can be rented for a unit of time at rate \varkappa_t .

In period t , the firm maximizes profit,

$$\max_{k_t} k_t^\alpha - \varkappa_t k_t$$

yielding first order conditions

$$f'(k_t) = \varkappa_t \quad (2)$$

$$\alpha k_t^{\alpha-1} = \varkappa_t \quad (3)$$

$$(y_t/k_t)\alpha = \varkappa_t \quad (4)$$

$$k_t = (y_t/\varkappa_t)\alpha. \quad (5)$$

What determines the cost of capital? In the simple case with no taxes and no capital market frictions of any kind, an investor must be indifferent between putting his money in the bank and earning interest at rate r , and buying a unit of capital, renting it out at rate \varkappa_t , and then reselling it the next period.

The price at which capital goods can be bought at date t is:

$$P_t \text{ — purchase price of one unit of capital,} \quad (6)$$

and in continuous time, the rate of change of P_t is \dot{P}_t . Assume that capital depreciates geometrically at rate δ . The net profit from the *continuous time* purchase-and-rent strategy is

$$\varkappa_t - \delta P_t + \dot{P}_t \text{ — Income from renting, minus loss from depreciation} \quad (7)$$

plus capital gain from the change in price of capital.

Thus, the no-arbitrage condition is

$$rP_t = \varkappa_t - \delta P_t + \dot{P}_t \quad (8)$$

$$(r + \delta)P_t = \varkappa_t + \dot{P}_t. \quad (9)$$

Now to simplify our lives we will assume constant capital goods prices, $\dot{P}_t = 0$. Thus, substituting the value for \varkappa_t from (9) into (5) we have:

$$k_t = \alpha y_t / \varkappa_t \quad (10)$$

$$= \alpha y_t / (r + \delta)P_t. \quad (11)$$

Now let's introduce taxes, defined as follows:

τ — corporate tax rate (≈ 0.34 in US)

ζ — investment tax credit (sometimes 10 percent, sometimes 0)

The net, discounted, after-tax price of capital to the firm is¹

$$\hat{P}_t = (1 - \zeta)P_t. \quad (12)$$

Now let's rewrite the arbitrage equation (9) taking account of taxes:

$$(r + \delta)\hat{P}_t = (1 - \tau)\varkappa_t + \dot{\hat{P}}_t. \quad (13)$$

If we simplify again by assuming that $\dot{\hat{P}}_t = 0$, we have

$$\varkappa_t = (r + \delta)P_t(1 - \zeta)/(1 - \tau). \quad (14)$$

Note that so far we have not derived a formula for investment - we have derived a formula for the *level* of the capital stock. But net investment is just the difference between the capital stock in periods t and $t - 1$. Thus, the Hall-Jorgenson model of gross investment is

$$i_{t-1} = k_t - k_{t-1} + \delta k_{t-1} \quad (15)$$

$$= \left(\Delta \frac{y_t}{\varkappa_t} \right) \alpha + \delta k_{t-1} \quad (16)$$

(where we neglect some minor complications having to do with the distinction between continuous and discrete time).

¹Assume that if the firm sells the capital, it must repay the ITC on a pro-rata basis; this prevents tax arbitrage opportunities.

References

HALL, ROBERT E., AND DALE JORGENSON (1967): "Tax Policy and Investment Behavior," *American Economic Review*, 57, Available at <http://www.stanford.edu/~rehall/Tax-Policy-AER-June-1967.pdf>.