

# The Rebelo AK Growth Model

Rebelo (1991) examines a model in which a social planner maximizes the discounted sum of utility in an economy with an  $AK$  production function:

$$\max_{\{C\}_0^\infty} \int_0^\infty u(C)e^{-\vartheta t} dt \quad (1)$$

subject to

$$\dot{K} = AK - C, \quad (2)$$

where we have assumed zero population growth and zero depreciation to make the analysis less cluttered.

This problem can be solved with the same Hamiltonian apparatus we used to solve the Ramsey/Cass-Koopmans model. In particular, with CRRA utility with risk aversion  $\rho$  one can show that optimal behavior requires

$$\dot{C}/C = \rho^{-1}(A - \vartheta). \quad (3)$$

Note that this equation comes about because the marginal product of capital in this model is always  $A = r$ , because  $f'(K) = A$ . Note further that according to this equation, the growth rate of consumption is always the same; unlike the Cass-Koopmans model with a normal production function, this model has no transitional dynamics.

We can also use (2) to obtain an expression for the steady-state growth rate:

$$\dot{K}/K = A - C/K. \quad (4)$$

If the model has a steady-state growth rate of  $\gamma = \dot{K}/K$ , this equation implies that

$$C/K = A - \gamma. \quad (5)$$

This is a model with a constant saving rate, because

$$S/AK = (AK - C)/AK \quad (6)$$

$$= 1 - A^{-1}(C/K) \quad (7)$$

$$= \gamma/A \quad (8)$$

$$\gamma = A(S/AK). \quad (9)$$

Thus, the steady-state growth rate in a Rebelo economy is directly proportional to the saving rate.

A further requirement for the Rebelo model to have a well-defined solution is that

$$\rho^{-1}(A - \vartheta) < A. \quad (10)$$

Recalling that  $A$  is effectively the real interest rate in this model, this equation can be interpreted as the ‘impatience’ condition that we imposed in the infinite horizon perfect foresight consumption model. In fact, the Rebelo  $AK$  model is essentially just a way of reinterpreting the perfect foresight infinite horizon consumption problem as a model for economic growth. The principal distinction is that we usually use the perfect foresight infinite horizon model to analyze circumstances where the agent has both labor and capital income, whereas the Rebelo model rules out labor income by assumption.

## References

- REBELO, SERGIO T. (1991): "Long-Run Policy Analysis and Long-Run Growth,"  
*Journal of Political Economy*, 99(3), 500–521.