```
> restart;
> ### This worksheet was written for Maple 16.01 Standard.
  ### May need tweaking for earlier versions of Maple or for Maple
  Classic.
  ### Last Revised 2012-10-01
  ### Report problems: contact@patricktoche.com
> ### Set display option
  mydisplayprecision:=3:
  interface(displayprecision=mydisplayprecision):
> ### Procedure to export plots

  MakePlot := proc(p::evaln, {[x,ext,extension]:=ps})
      local thename, theplace, opts:
      global N;
      thename := cat(convert(p,string),"_",convert(N,string),".",
  convert(x,string)):
      theplace := cat(currentdir(),kernelopts(dirsep),convert(N,
  string),kernelopts(dirsep)):
      if x = gif then
          opts := `color,portrait,noborder,transparent,height=512,
  width=512`: #default jpeg: height=360,width=480
      else
          #default gif : height=512,width=512
          opts := `color,portrait,noborder,transparent,height=360,
  width=480`:
      end if:
      plotsetup('x', 'plotoutput'=cat(theplace,thename),
  'plotoptions'=opts):
      print( plots:-display( eval(p), 'axesfont' = [ TIMES, 10 ],
  'labelfont' = [ TIMES, ROMAN, 10] ) ):
      plotsetup(default):
  end proc:

> ### Tractable Model Parameter Definitions
  ###   rho   : coefficient of relative risk aversion, CRRA
  ###   mu    : probability of job loss
  ###   R     : interest factor on financial wealth, i.e. R = 1+r
  ###   beta  : patience factor, i.e. inverse of discount factor
  ###   G     : growth factor of labor income
  ###   Gamma : Gamma = G/(1-mu)

> ############################### Incomplete
  ############################
  ### The Selection of Parameter Values is at the experimental
  stage ###
  ### Choices subject to change
   ###
  ### Not all figures have been tweaked or optimized
   ###
  ##################################################################
  ####

> ### Parameter values for ctdiscrete, fixing Gamma=1 (Zero Growth)
  ### To use this parameter configuration set N:=1;
```

```
parameters[1] := [ R = 103/100, beta = 100/110, Gamma = 1 ]:
    'parameters[1]' = evalf(%);
    'R*beta' = evalf(eval(R*beta,parameters[1]));
```

$$parameters_1 = [R = 1.03, \beta = 0.909, \Gamma = 1.]$$

$$R\,\beta = 0.936 \qquad (1)$$

```
> ### Parameter values for ctdiscrete, fixing G=1 (Zero Growth)
  ### To use this parameter configuration set N:=2;

  parameters[2] := [ R = 103/100, beta = 100/110, Gamma = 1/(1-mu)
  ]:
      'parameters[2]' = evalf(%);
      'R*beta' = evalf(eval(R*beta,parameters[2]));
```

$$parameters_2 = \left[R = 1.03, \beta = 0.909, \Gamma = \frac{1}{1-\mu}\right]$$

$$R\,\beta = 0.936 \qquad (2)$$

```
> ### Parameter values from cssUSsaving, 16 March 2012, section 5.2
  ### To use this parameter configuration set N:=3;
  ### R=1.04 and beta=0.975=10000/10256,e at annual frequency.
  ### R=1.01 and beta=1-0.0064=0.994, at quarterly frequency

  parameters[3] := [ R = 104/100, beta = 10000/10256, Gamma =
  101/100/(1-mu) ]:
      'parameters[3]' = evalf(%);
      'R*beta' = evalf(eval(R*beta,parameters[3]));
```

$$parameters_3 = \left[R = 1.04, \beta = 0.975, \Gamma = \frac{1.01}{1-\mu}\right]$$

$$R\,\beta = 1.01 \qquad (3)$$

```
> ### Parameter values, fixing Gamma=101/100 (Positive Growth)
  ### To use this parameter configuration set N:=4;

  parameters[4] := [ R = 103/100, beta = 100/110, Gamma = 101/100 ]
  :
      'parameters[4]' = evalf(%);
      'R*beta' = evalf(eval(R*beta,parameters[4]));
```

$$parameters_4 = [R = 1.03, \beta = 0.909, \Gamma = 1.01]$$

$$R\,\beta = 0.936 \qquad (4)$$

```
> ### Parameter values, fixing Gamma=101/100  (Positive Growth, R*
  beta=1)
  ### To use this parameter configuration set N:=5;

  parameters[5] := [ R = 103/100, beta = 100/103, Gamma = 101/100 ]
  :
      'parameters[5]' = evalf(%);
      'R*beta' = evalf(eval(R*beta,parameters[5]));
```

$$parameters_5 = [R = 1.03, \beta = 0.971, \Gamma = 1.01]$$

$$R\,\beta = 1. \tag{5}$$

```
> ### Set parameter values from the configurations above
  ### Select a value for N below, save, and  Edit -> Execute ->
  Worksheet

  N := 2:  # Parameter lists are numbered: N = 1,2,3...
      params := parameters[N]:
      'params' = evalf(params);
```

$$params = \left[R = 1.03,\ \beta = 0.909,\ \Gamma = \frac{1}{1-\mu}\right] \tag{6}$$

```
> ### Store selected individual parameters for convenience

  Rf := subs(params,R):
  betaf := subs(params,beta):
  Gammaf := subs(params,Gamma):
```

```
> ### Marginal propensity to consume in unemployment

  mpcu := (R,beta,rho) -> 1-(R*beta)^(1/rho)/R:
      'mpcu' = mpcu(R,beta,rho);
```

$$mpcu = 1 - \frac{(R\,\beta)^{\frac{1}{\rho}}}{R} \tag{7}$$

```
> ### Target wealth-income ratio

  m := (R,beta,Gamma,rho,mu) -> 1 + 1 / ( Gamma/R - 1 + mpcu(R,
  beta,rho) * ( 1 + ( ((R*beta)^(1/rho)/Gamma)^(-rho)-1 ) / mu )^
  (1/rho) ):
      'm' = m(R,beta,Gamma,rho,mu);
```

$$m = 1 + \frac{1}{\frac{\Gamma}{R} - 1 + \left(1 - \frac{(R\,\beta)^{\frac{1}{\rho}}}{R}\right)\left(1 + \frac{\left(\frac{(R\,\beta)^{\frac{1}{\rho}}}{\Gamma}\right)^{-\rho} - 1}{\mu}\right)^{\frac{1}{\rho}}} \tag{8}$$

```
> ### Target saving rate
  ### from pi/(1-pi)=rhs (c.f. equation in the text), we have pi=
  rhs/(1+rhs), so we have s=1-pi=1/(1+rhs)

  s := (R,beta,Gamma,rho,mu) -> 1 / (1 + mpcu(R,beta,rho)*(R/Gamma)
  *((((R*beta)^(1/rho)/Gamma)^(-rho)-(1-mu))/mu)^(1/rho) ):
      's' = s(R,beta,Gamma,rho,mu);
```

(9)

$$s = \frac{1}{1 + \frac{\left(1 - \frac{(R\beta)^{\frac{1}{\rho}}}{R}\right) R \left(\frac{\left(\frac{(R\beta)^{\frac{1}{\rho}}}{\Gamma} \right)^{-\rho} - 1 + \mu}{\mu} \right)^{\frac{1}{\rho}}}{\Gamma}} \quad (9)$$

```
> ### Create a list of values for rho

  rholist := [ seq(k, k = 1 .. 20) ]:
      'rho' = rholist[1..10];
```

$$\rho = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] \quad (10)$$

```
> ### Create a list of values for mu

  mulist := [ 0, seq(2^k/100, k = 0 .. 20) ]:
      'mu' = evalf(%)[1..10];
```

$$\mu = [0., 0.0100, 0.0200, 0.0400, 0.0800, 0.160, 0.320, 0.640, 1.28, 2.56] \quad (11)$$

```
> ### Check RIC and GIC Conditions

  RIC := (R,beta,rho) -> (R*beta)^(1/rho)/R:
  RICf := rho -> RIC(subs(params,R),subs(params,beta),rho):
  GIC := (R,beta,rho,Gamma) -> (R*beta)^(1/rho)/Gamma:
  GICf := (rho,mu) -> GIC(subs(params,R),subs(params,beta),rho,subs
  (params,Gamma)):

  ### Check the RIC
  Matrix([seq( [seq( is(RICf(rho)<1), mu=mulist[2..8])],rho=rholist
  [1..10])]):
      LinearAlgebra:-Transpose(%);

  ### Check the GIC
  Matrix([seq( [seq( is(GICf(rho,mu)<1), mu=mulist[2..8])],rho=
  rholist[1..10])]):
      LinearAlgebra:-Transpose(%);

  ### Check the strong GIC
  Matrix([seq( [seq( is(GICf(rho,mu)<(1-mu)^(-1/rho)), mu=mulist[2.
  .8])],rho=rholist[1..10])]):
      LinearAlgebra:-Transpose(%);
```

$$\begin{bmatrix} true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \end{bmatrix}$$

$$\begin{bmatrix} true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \end{bmatrix}$$

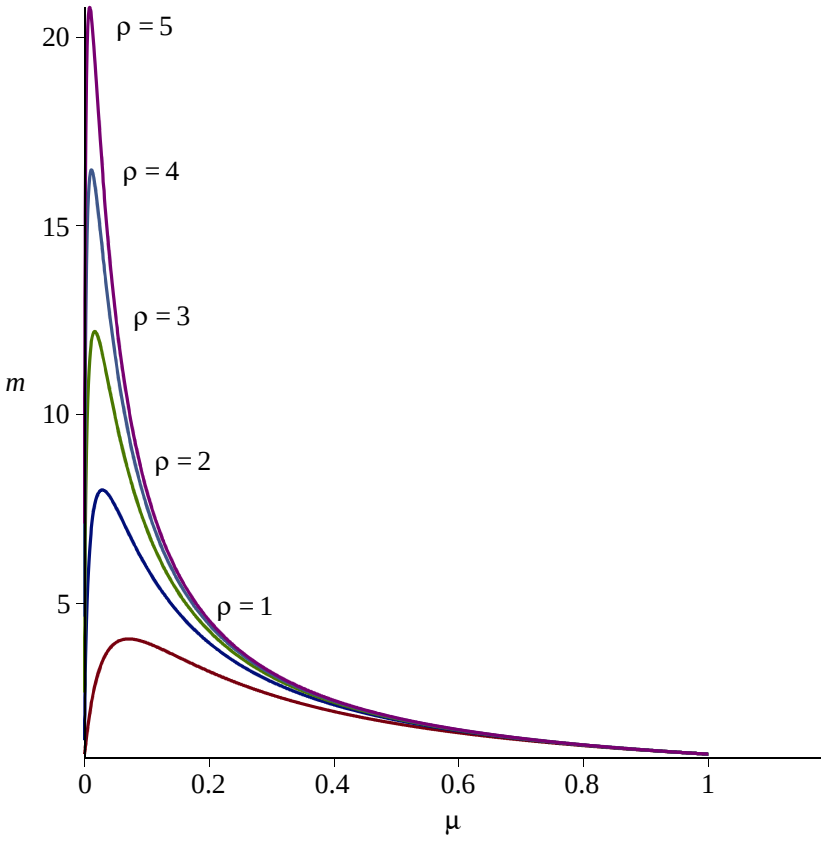
$$\begin{bmatrix} true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \\ true & true & true & true & true & true & true & true & true & true \end{bmatrix} \quad \textbf{(12)}$$

```
> ### Target wealth-income ratio for fixed values of R,Gamma,beta

  eval(m(R,beta,Gamma,rho,mu),params):
  mf := unapply(%,(rho,mu)):
  interface(displayprecision=3):
      'm' = evalf(mf(rho,mu));
  interface(displayprecision=mydisplayprecision):
```

$$m = 1 + \frac{1}{\frac{0.971}{1-\mu} - 1 + \left(1 - 0.971\, 0.936^{\frac{1}{\rho}}\right)\left(1 + \frac{\left(0.936^{\frac{1}{\rho}}\,(1-\mu)\right)^{-\rho} - 1}{\mu}\right)^{\frac{1}{\rho}}} \quad \textbf{(13)}$$

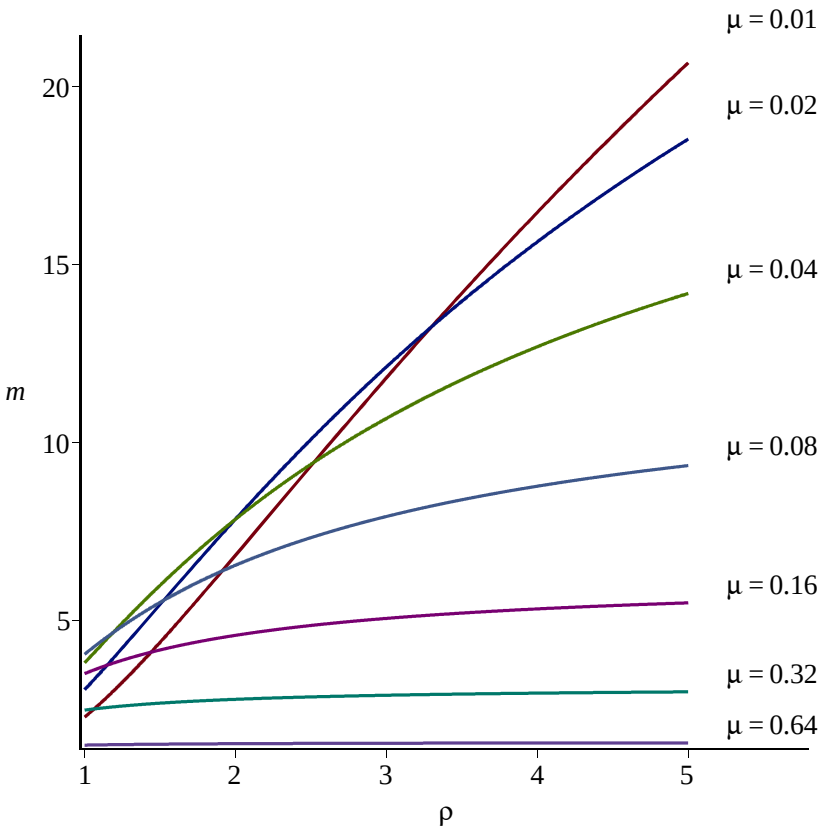
```
> ### Plot of m as rho and mu vary

  mTargetUrateVariesCRRAVaries := plots:-display( plot3d(mf(rho,
  mu), rho = 1..5, mu = 0..1)
      , 'axes' = normal
```

```
      , 'style' = surfacecontour
      , 'shading' = zhue
      , 'lightmodel' = light1
      , 'tickmarks' = [ 6, 6, 4 ]
      , 'labels' = [ rho, mu, 'm' ]
      , 'view' = [ 1 .. 5, 0 .. 1, default ]
      , 'orientation' = [ -10, 50 ]
    ) :  # % ;

> ### Animated plot of m as rho and mu vary

  mTargetUrateVariesCRRAVariesAnimation :=  plots:-display(
  mTargetUrateVariesCRRAVaries
      , 'viewpoint' = ["circleright", frames=200]
    ) : # % ;

> ### Set position of the plot labels, tweaked for stated parameter
  values

  if N=2 then
      xmu:=rho->0.2/rho:  ymu:=rho->1.4*mf(rho,xmu(rho)): # fix x-
  value, vary y-value
      xrho:=mu->5.2:      yrho:=mu->mf(xrho(mu),mu):  # fix x-
  value, vary y-value
  else
      xmu:=rho->1.05: ymu:=rho->mf(rho,xmu(rho)): # fix x-value,
  vary y-value
      xrho:=mu->5.2:  yrho:=mu->mf(xrho(mu),mu):  # fix x-value,
  vary y-value
  end if:

> ### Plot of m as mu varies for fixed values of rho

  plot_m_mu := plot( [ seq( mf(rho,mu) , rho=rholist[1..5] ) ]
      , mu = 0 .. 1
      , 'numpoints' = 1000
      , 'tickmarks' = [ 6, 6 ]
      , 'labels' = [ mu, 'm' ]
  #    , 'legend' = [ seq( 'rho' = k, k = rholist[1..5] ) ]
  #    , 'legendstyle' = [ 'font' = [TIMES,ROMAN,8], 'location' =
  bottom ]
      , 'view' = [ 0 .. 1.18, default ]
    ) :

  #### plot labels

  ptxt := seq( plots:-textplot([xmu(rho),ymu(rho),'typeset'('rho',
  " = ", rho)], 'align'={'above','right'}), rho=rholist[1..5]):

  mTargetCRRAFixedUrateVaries := plots:-display([plot_m_mu,ptxt]):
  %;
```




```
> ### Plot of m as rho varies for fixed values of mu

  interface(displayprecision=2):

  plot_m_rho := plot( [seq(mf(rho,mu),mu=mulist[2..8])]
      , rho = 1 .. 5
      , 'numpoints' = 1000
      , 'tickmarks' = [ 6, 6 ]
      , 'labels' = [ rho, 'm' ]
  #    , 'legend' = [ seq( 'mu' = evalf(k), k = mulist[2..8] ) ]
  #    , 'legendstyle' = [ 'font' = [TIMES,ROMAN,8], 'location' =
  bottom ]
      , 'view' = [ 1 .. 5.8, default ]
    ) :

  #### plot labels

  ptxt := seq( plots:-textplot([xrho(mu),yrho(mu),'typeset'('mu', "
  = ", evalf(mu))], 'align'={'above','right'}), mu=mulist[2..8]):

  mTargetUrateFixedCRRAVaries := plots:-display([plot_m_rho,ptxt]):
  %;
```

```
interface(displayprecision=mydisplayprecision):
```




```
> ### Table of target values m as rho and mu run through lists

  interface(displayprecision=6):
  mvalues := Matrix([seq( [seq(mf(rho,mu), rho=rholist[1..8])],mu=
  mulist[2..8])]):
  mvalues := ArrayTools:-Concatenate(2,Vector[column](evalf[2]
  (mulist[2..8])),mvalues):
  mvalues := ArrayTools:-Concatenate(1,Vector[row]([0,op(rholist[1.
  .8])]),mvalues):
      'mvalues' = evalf(%);
  interface(displayprecision=mydisplayprecision):
```

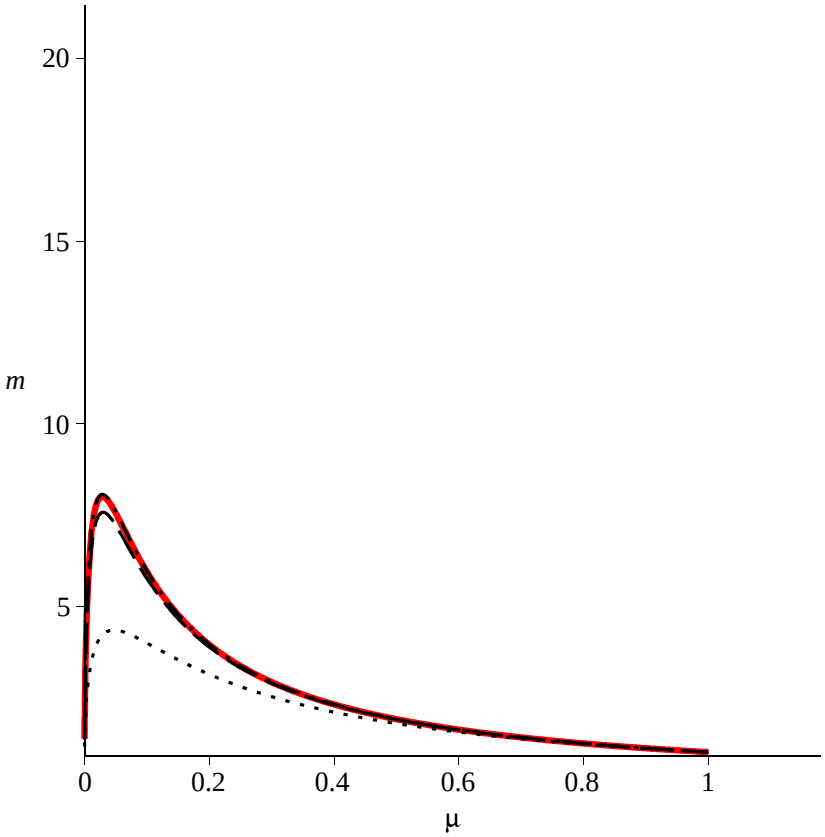
mvalues **(14)**

$$=\begin{bmatrix} 0. & 1. & 2. & 3. & 4. & 5. & 6. & 7. & 8. \\ 0.010 & 2.26986 & 6.82280 & 11.8125 & 16.4661 & 20.6719 & 24.4523 & 27.8566 & 30.9348 \\ 0.020 & 3.04257 & 7.84098 & 12.1220 & 15.6375 & 18.5287 & 20.9396 & 22.9800 & 24.7305 \\ 0.040 & 3.79466 & 7.83699 & 10.6723 & 12.6878 & 14.1862 & 15.3441 & 16.2671 & 17.0210 \\ 0.080 & 4.03984 & 6.54330 & 7.91435 & 8.76588 & 9.34568 & 9.76647 & 10.0863 & 10.3379 \\ 0.16 & 3.49435 & 4.57162 & 5.04806 & 5.31455 & 5.48467 & 5.60274 & 5.68952 & 5.75602 \\ 0.32 & 2.46897 & 2.77198 & 2.88641 & 2.94590 & 2.98218 & 3.00654 & 3.02398 & 3.03705 \\ 0.64 & 1.48376 & 1.52376 & 1.53726 & 1.54389 & 1.54780 & 1.55036 & 1.55217 & 1.55351 \end{bmatrix}$$

```
> ### Check of the accuracy of various approximations
  ### The plot shows that n>3 is needed for decent approximation

  Rho := 2: # Fix a value of rho = Rho

  mfn := (rho,mu,n) -> evalf[n](mf(rho,mu)):
      'mfn' = [mfn(Rho,mu,1),mfn(Rho,mu,2),mfn(Rho,mu,3),mfn(Rho,
  mu,4),mfn(Rho,mu,5)];

  plot_mff_mu := plot( mf(Rho,mu)
      , mu = 0 .. 1
      , 'numpoints' = 1000
      , 'color' = red
      , 'thickness' = 3
      , 'linestyle' = solid
    ) :

  plot_mfn_mu := n -> plot( mfn(Rho,mu,n)
      , mu = 0 .. 1
      , 'numpoints' = 1000
      , 'color' = black
      , 'thickness' = 1
      , 'linestyle' = n
    ) :


  ### plot labels
  xmu:=n->1.05: ymu:=n->mfn(Rho,1,n): # fix x-value, vary y-value
  ptxt := seq( plots:-textplot([xmu(n),ymu(n),'typeset'('n', " = ",
  n)], 'align'={'above','right'}), n=2..4):

  mTargetCRRAFixedUrateVariesApproximations :=
      plots:-display([plot_mff_mu,plot_mfn_mu(2),plot_mfn_mu(3),
  plot_mfn_mu(4),ptxt]
          , 'tickmarks' = [ 6, 6 ]
          , 'labels' = [ mu, 'm' ]
          , 'view' = [ 0 .. 1.18, default ]
        ) : %;
```

$$mfn = \left[1 + \frac{1}{\frac{1}{1-\mu} - 1 + 0.1\sqrt{1 + \frac{\frac{1}{(1-\mu)^2} - 1}{\mu}}}, 1 \right.$$
$$+ \frac{1}{\frac{0.97}{1-\mu} - 1 + 0.12\sqrt{1 + \frac{\frac{1.1}{(1-\mu)^2} - 1}{\mu}}}, 1$$
$$+ \frac{1}{\frac{0.971}{1-\mu} - 1 + 0.064\sqrt{1 + \frac{\frac{1.07}{(1-\mu)^2} - 1}{\mu}}}, 1$$
$$+ \frac{1}{\frac{0.971}{1-\mu} - 1 + 0.0600\sqrt{1 + \frac{\frac{1.07}{(1-\mu)^2} - 1}{\mu}}}$$
$$\left. + \frac{1}{\frac{0.971}{1-\mu} - 1 + 0.0606\sqrt{1 + \frac{\frac{1.07}{(1-\mu)^2} - 1}{\mu}}} \right]$$

Error, (in mf) numeric exception: division by zero




```
> ##########################
> ### Asymptotic values of m as risk-aversion rho becomes
  arbitrarily large

  asymptotic_m_mu := [seq(limit(mf(rho,mu),rho=infinity), mu=mulist
  [2..20])];
```

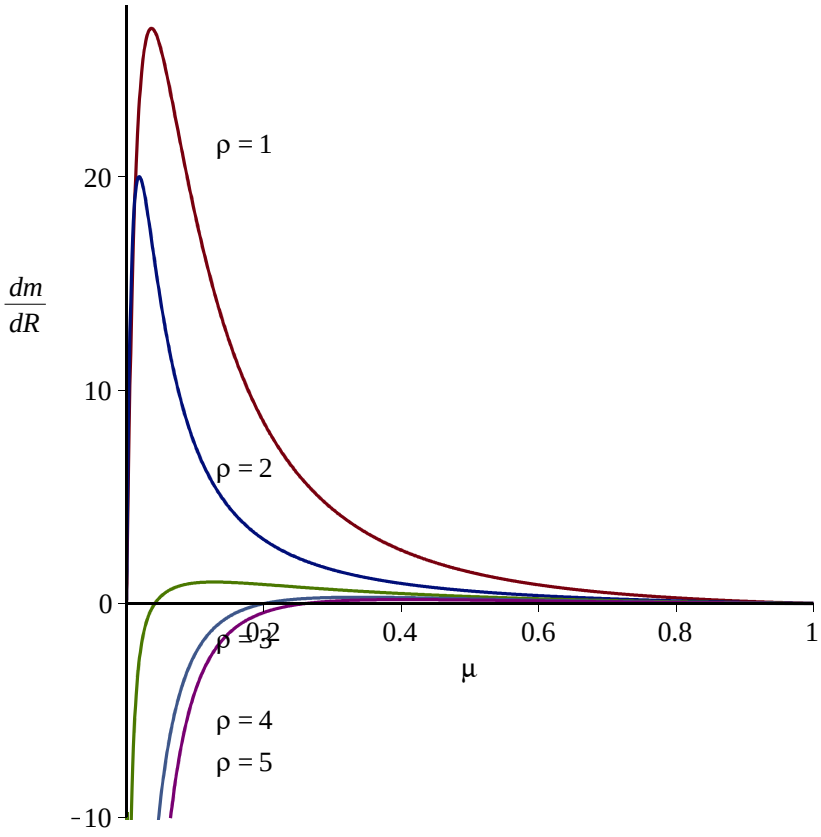
$$asymptotic\_m\_mu := \left[100, 50, 25, \frac{25}{2}, \frac{25}{4}, \frac{25}{8}, \frac{25}{16}, \frac{2479}{3200}, \frac{2383}{6400}, \frac{2191}{12800}, \frac{1807}{25600}, \frac{1039}{51200}, -\frac{497}{102400}, -\frac{3569}{204800}, -\frac{9713}{409600}, -\frac{22001}{819200}, -\frac{46577}{1638400}, -\frac{95729}{3276800}, -\frac{194033}{6553600}\right] \quad \textbf{(15)}$$

```
> ### Derivative of m with respect to R

  dm := (R,beta,Gamma,rho,mu) -> diff(m(R,beta,Gamma,rho,mu),R):
  eval(dm(R,beta,Gamma,rho,mu),params):
  dmf := unapply(%,(rho,mu)):
  interface(displayprecision=4):
      'dm' = evalf(dmf(rho,mu));
  interface(displayprecision=mydisplayprecision):
```

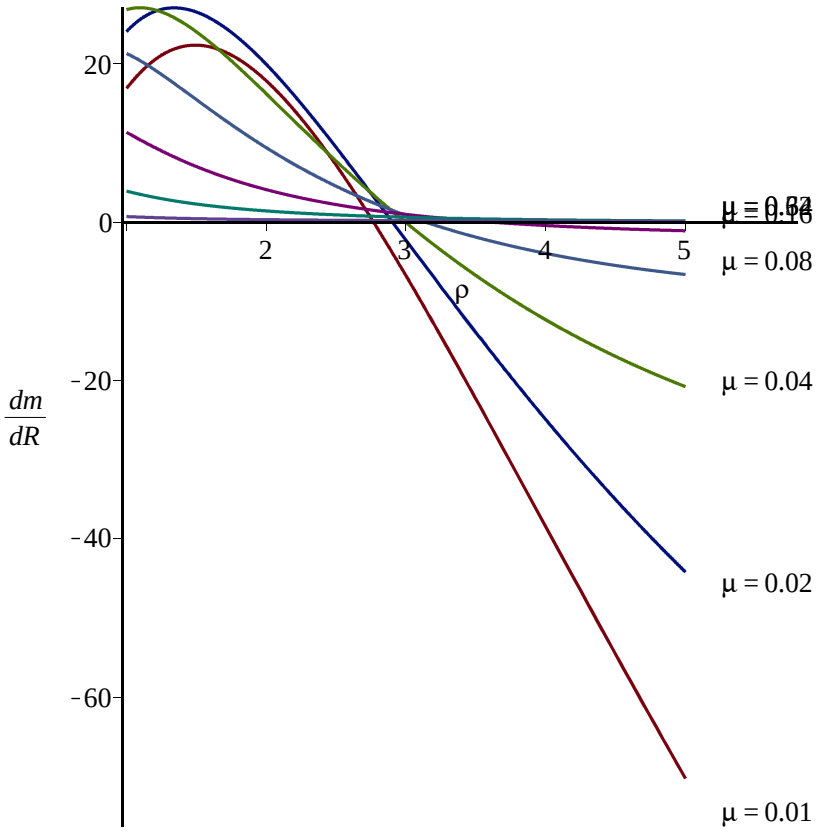
$$dm = -\left(-\frac{0.9426}{1-\mu} + \left(-\frac{0.9426\ 0.9364^{\frac{1}{\rho}}}{\rho} + 0.9426\ 0.9364^{\frac{1}{\rho}} \right) \left(1 + \frac{\left(0.9364^{\frac{1}{\rho}} (1-\mu) \right)^{-\rho} - 1}{\mu} \right)^{\frac{1}{\rho}} \right) \Bigg/ \left(-1 + \left(1 - 0.9709\ 0.9364^{\frac{1}{\rho}} \right) \left(1 + \frac{\left(0.9364^{\frac{1}{\rho}} (1-\mu) \right)^{-\rho} - 1}{\mu} \right)^{\frac{1}{\rho}} \right)^2 \right) \left(\frac{0.9709}{1-\mu} - \frac{1}{\rho\,\mu \left(1 + \frac{\left(0.9364^{\frac{1}{\rho}} (1-\mu) \right)^{-\rho} - 1}{\mu} \right)} \left(0.9709 \left(1 - 0.9709\ 0.9364^{\frac{1}{\rho}} \right) \left(1 + \frac{\left(0.9364^{\frac{1}{\rho}} (1-\mu) \right)^{-\rho} - 1}{\mu} \right)^{\frac{1}{\rho}} \left(0.9364^{\frac{1}{\rho}} (1-\mu) \right)^{-\rho} \right) \right) \tag{16}$$

```
> ### Set position of the plot labels, tweaked for stated parameter
  values

  if N=2 then
      xmu:=rho->0.12:  ymu:=rho->-4+1.6*dmf(rho,xmu(rho)): # fix x-
  value, vary y-value
      xrho:=mu->5.2:      yrho:=mu->dmf(xrho(mu),mu):  # fix x-
  value, vary y-value
  else
      xmu:=rho->1.05: ymu:=rho->dmf(rho,xmu(rho)): # fix x-value,
  vary y-value
      xrho:=mu->5.2:  yrho:=mu->dmf(xrho(mu),mu)+20:  # fix x-
```

```
value, vary y-value
end if:

> ### Plot of derivative of m with respect to R, for fixed values
of rho

plot_dmdR_mu := plot( [ seq( dmf(rho,mu) , rho=rholist[1..5] ) ]
    , mu = 0 .. 1
    , 'numpoints' = 1000
    , 'tickmarks' = [ 6, 6 ]
    , 'labels' = [ mu, 'dm/dR' ]
    , 'view' = [ 0 .. 1.18, default ]
  ) :

#### plot labels

ptxt := seq( plots:-textplot([xmu(rho),ymu(rho),'typeset'('rho',
" = ", rho)], 'align'={'above','right'}), rho=rholist[1..5]):

if N = 2 then
    theview := [ 0 .. 1, -10 .. 28 ] :
else
    theview := default :
end if:

mSlopeCRRAFixedUrateVaries := plots:-display( [plot_dmdR_mu,
ptxt], 'view' = theview ): %;
```




```
> ### Plot of derivative of m with respect to R, for fixed values
  of mu

  interface(displayprecision=2):

  plot_dmdR_rho := plot( [ seq( dmf(rho,mu) , mu=mulist[2..8] ) ]
      , rho = 1 .. 5
      , 'numpoints' = 1000
      , 'tickmarks' = [ 6, 6 ]
      , 'labels' = [ rho, 'dm/dR' ]
      , 'view' = [ 1 .. 5.8, default ]
    ) :

  #### plot labels

  ptxt := seq( plots:-textplot([xrho(mu),yrho(mu),'typeset'('mu', "
  = ", evalf(mu))], 'align'={'above','right'}), mu=mulist[2..8]):

  mSlopeUrateFixedCRRAVaries := plots:-display([plot_dmdR_rho,ptxt]
  ): %;

  interface(displayprecision=mydisplayprecision):
```




```
> ### Table of percentage change in target values m after 1% Change
  in After-Tax Interest Rate
  ### Mid-Point Formula

  interface(displayprecision=6):
  mchanges := Matrix([seq( [seq( 100*(m(Rf,betaf,Gammaf,rho,mu)-m
  (Rf-1/100,betaf,Gammaf,rho,mu))/((m(Rf,betaf,Gammaf,rho,mu)+m
  (Rf-1/100,betaf,Gammaf,rho,mu))/2) ,rho=rholist[1..8] )],mu=
  mulist[2..8])]):
  mchanges := ArrayTools:-Concatenate(2,Vector[column](evalf[2]
  (mulist[2..8])),mchanges):
  mchanges := ArrayTools:-Concatenate(1,Vector[row]([0,op(rholist
  [1..8])]),mchanges):
      'mchanges' = evalf(%);
  interface(displayprecision=mydisplayprecision):
```

$$mchanges = \begin{bmatrix} [0., 1., 2., 3., 4., 5., 6., 7., 8.], \\ [0.010, 6.85067, 2.12395, -1.10532, -2.88195, -3.94051, -4.60796, -5.04252, \\ -5.32907], \\ [0.020, 7.40539, 2.19271, -0.519852, -1.90832, -2.67932, -3.12683, -3.38958, \\ -3.54026], \end{bmatrix} \quad \textbf{(17)}$$

```
[0.040, 6.73005, 1.87739, −0.166010, −1.11329, −1.59088, −1.83904, −1.96471,
−2.02096],
[0.080, 5.09959, 1.36051, 0.0488566, −0.499400, −0.750834, −0.867911, −0.917986,
−0.932526],
[0.16, 3.18452, 0.876493, 0.177772, −0.0928640, −0.210104, −0.261919, −0.282727,
−0.287882],
[0.32, 1.57582, 0.507461, 0.211498, 0.0979433, 0.0464855, 0.0207802, 0.00713707,
−0.000378886],
[0.64, 0.476291, 0.202853, 0.123636, 0.0883697, 0.0687501, 0.0562822, 0.0476516,
0.0413177]]
> ##########################
> ### Target saving rate for fixed values of R,Gamma,beta

eval(s(R,beta,Gamma,rho,mu),params):
sf := unapply(%,(rho,mu)):
interface(displayprecision=4):
    's' = evalf(sf(rho,mu));
interface(displayprecision=mydisplayprecision):
```

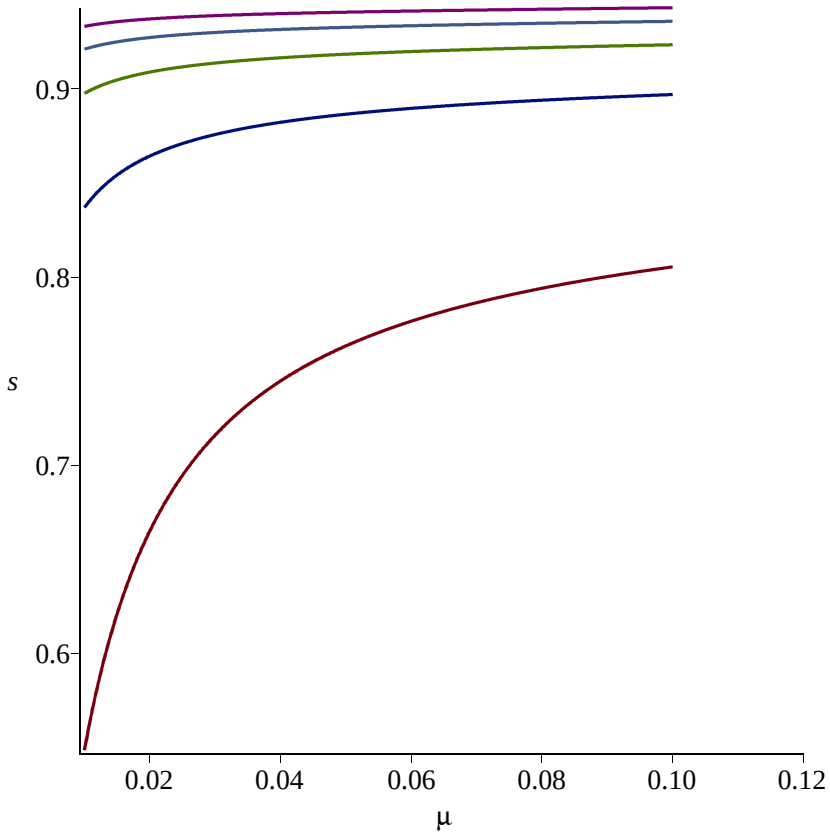
$$s = \frac{1}{1 + 1.030\left(1 - 0.9709\,0.9364^{\frac{1}{\rho}}\right)(1-\mu)\left(\frac{\left(0.9364^{\frac{1}{\rho}}(1-\mu)\right)^{-\rho} - 1 + \mu}{\mu}\right)^{\frac{1}{\rho}}} \qquad \textbf{(18)}$$

```
> ### Plot of s as rho and mu vary

sTargetUrateVariesCRRAVaries := plots:-display( plot3d(sf(rho,
mu), rho = 1..5, mu = 0..1)
    , 'axes' = normal
    , 'style' = surfacecontour
    , 'shading' = zhue
    , 'lightmodel' = light1
    , 'tickmarks' = [ 6, 6, 4 ]
    , 'labels' = [ rho, mu, 's' ]
    , 'view' = [ 1 .. 5, 0 .. 1, 0.5 .. 1 ]
    , 'orientation' = [ -10, 50 ]
  ) :

plot_s_rho_mu;
```

plot_s_rho_mu **(19)**

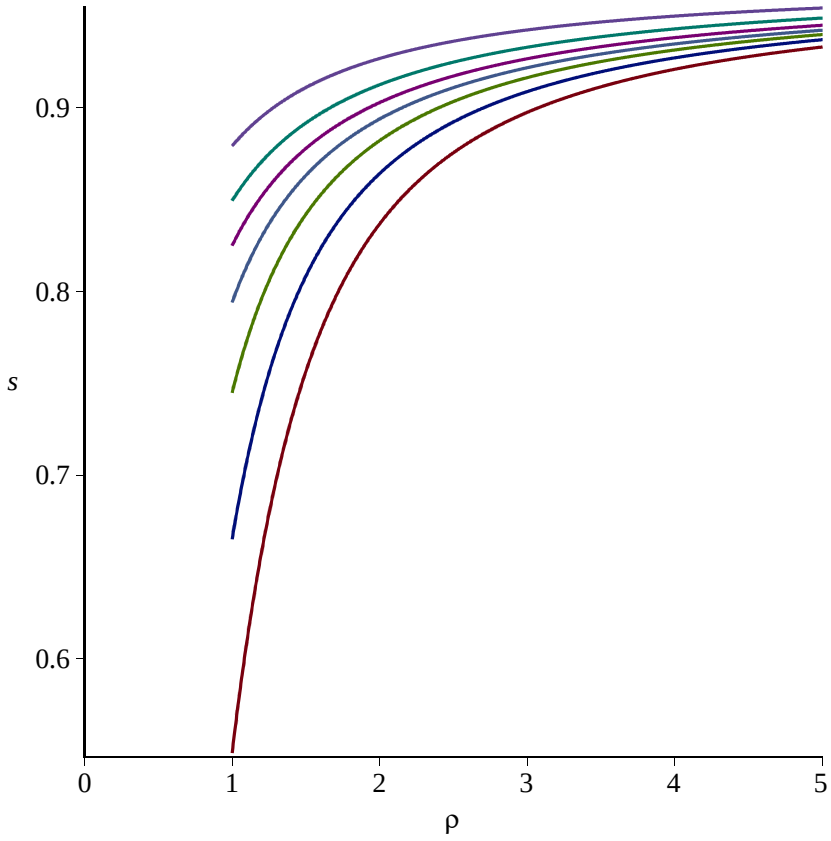
```
> ### Animated plot of m as rho and mu vary

sTargetUrateVariesCRRAVariesAnimation :=  plots:-display(
sTargetUrateVariesCRRAVaries
    , 'viewpoint' = ["circleright", frames=200]
  ) : # % ;

> ### Set position of the plot labels, tweaked for stated parameter
```

```
values

mumin := 0.01:
mumax := 0.1:
rhomin := 1:
rhomax := 5:

if N=2 then
    xmu:=rho->0.2/rho:      ymu:=rho->1.4*sf(rho,xmu(rho)): # fix
x-value, vary y-value
    xrho:=mu->1.05*rhomax:  yrho:=mu->sf(xrho(mu),mu):  # fix x-
value, vary y-value
elif N=4 or N=5 then
    xmu:=rho->1.05*mumax:   ymu:=rho->sf(rho,xmu(rho)): # fix x-
value, vary y-value
    xrho:=mu->1:            yrho:=mu->sf(xrho(mu),mu):  # fix x-
value, vary y-value
else
    xmu:=rho->1.05*mumax:   ymu:=rho->sf(rho,xmu(rho)): # fix x-
value, vary y-value
    xrho:=mu->1.05*rhomax:  yrho:=mu->sf(xrho(mu),mu):  # fix x-
value, vary y-value
end if:

> ### Plot of s as mu varies for fixed values of rho

plot_s_mu := plot( [ seq( sf(rho,mu) , rho=rholist[1..rhomax] ) ]
    , mu = mumin .. mumax
    , 'numpoints' = 1000
    , 'tickmarks' = [ 6, 6 ]
    , 'labels' = [ mu, 's' ]
#    , 'legend' = [ seq( 'rho' = k, k = rholist[rhomin..rhomax] )
]
#    , 'legendstyle' = [ 'font' = [TIMES,ROMAN,8], 'location' =
bottom ]
#    , 'view' = [ mumin .. 1.2*mumax, 0.85 .. max([seq(evalf(sf
(rho,mumax)),rho=rholist[rhomin..rhomax])]) ]
    , 'view' = [ mumin .. 1.2*mumax
        , min([seq(evalf(sf(rho,mumin)),rho=rholist[rhomin..
rhomax])]) .. max([seq(evalf(sf(rho,mumax)),rho=rholist[rhomin..
rhomax])]) ]
  ) :

#### plot labels

ptxt := seq( plots:-textplot([xmu(rho),ymu(rho),'typeset'('rho',
" = ", rho)], 'align'={'above','right'}), rho=rholist[rhomin..
rhomax]):

sTargetCRRAFixedUrateVaries := plots:-display([plot_s_mu,ptxt]):
%;
```




```
> ### Plot of s as rho varies for fixed values of mu

  interface(displayprecision=2):

  plot_s_rho := plot( [seq(sf(rho,mu),mu=mulist[2..8])]
      , rho = 1 .. 5
      , 'numpoints' = 1000
      , 'tickmarks' = [ 6, 6 ]
      , 'labels' = [ rho, 's' ]
  #    , 'legend' = [ seq( 'mu' = evalf(k), k = mulist[2..8] ) ]
  #    , 'legendstyle' = [ 'font' = [TIMES,ROMAN,8], 'location' =
  bottom ]
      , 'view' = [ 0 .. 5, default ]
    ) :

  #### plot labels

  if N=4 or N=5 then  # specifically tweaked for parameter values
  N=4
      ptxt := seq( plots:-textplot([xrho(mu)-0.9,yrho(mu),'typeset'
  ('mu', " = ", evalf(mu))], 'align'={'above','right'}), mu=mulist
  [2..8]):
  else
```

```
    ptxt := seq( plots:-textplot([xrho(mu),yrho(mu),'typeset'
('mu', " = ", evalf(mu))], 'align'={'above','right'}), mu=mulist
[2..8]):
end if:

sTargetUrateFixedCRRAVaries := plots:-display([plot_s_rho,ptxt]):
%;

interface(displayprecision=mydisplayprecision):
```




```
> ### Table of target values s as rho and mu run through lists

  interface(displayprecision=6):
  svalues := Matrix([seq( [seq(sf(rho,mu), rho=rholist[1..8])],mu=
  mulist[2..8])]):
  svalues := ArrayTools:-Concatenate(2,Vector[column](evalf[2]
  (mulist[2..8])),svalues):
  svalues := ArrayTools:-Concatenate(1,Vector[row]([0,op(rholist[1.
  .8])]),svalues):
      'svalues' = evalf(%);
  interface(displayprecision=mydisplayprecision):
```

svalues **(20)**

```
= [ [0., 1., 2., 3., 4., 5., 6., 7., 8. ],
[0.010, 0.548637, 0.836945, 0.897660, 0.921123, 0.933240, 0.940575, 0.945476, 0.948979
],
[0.020, 0.665079, 0.864340, 0.908961, 0.927334, 0.937220, 0.943376, 0.947577, 0.950628
],
[0.040, 0.744813, 0.882282, 0.916565, 0.931618, 0.940037, 0.945417, 0.949157, 0.951911
],
[0.080, 0.794075, 0.894018, 0.921958, 0.934911, 0.942379, 0.947244, 0.950670, 0.953217
],
[0.16, 0.825039, 0.902981, 0.926843, 0.938323, 0.945069, 0.949510, 0.952656, 0.955003 ],
[0.32, 0.849477, 0.912688, 0.933108, 0.943097, 0.948994, 0.952873, 0.955611, 0.957643 ],
[0.64, 0.879284, 0.926997, 0.942531, 0.950072, 0.954482, 0.957362, 0.959386, 0.960884 ] ]
```

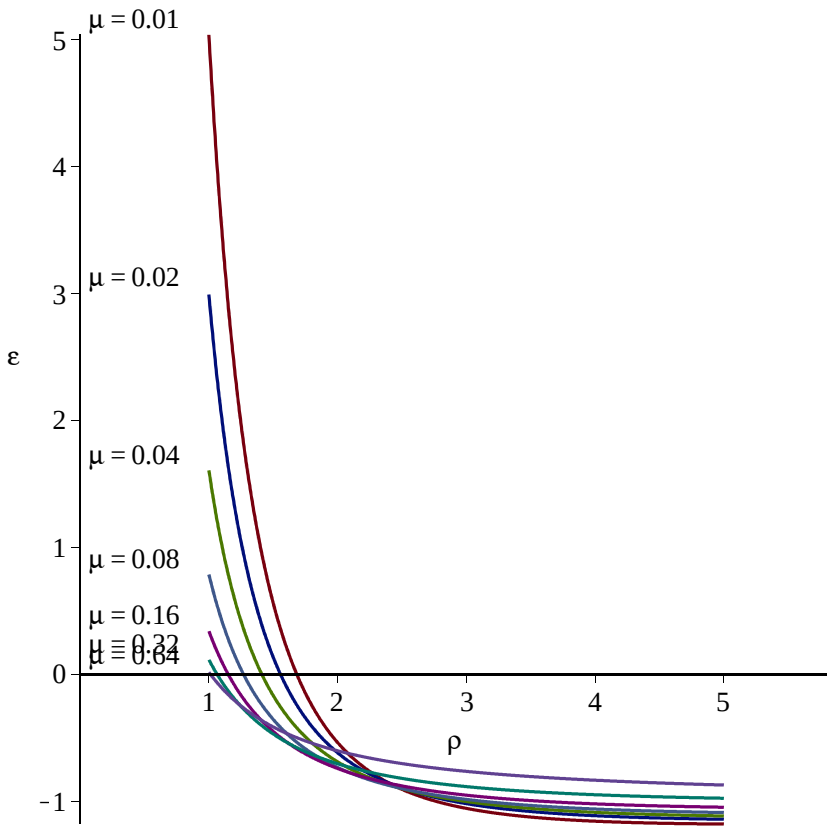
```
> ### Elasticity of s with respect to R

  ds := (R,beta,Gamma,rho,mu) -> diff(s(R,beta,Gamma,rho,mu),R):
  es :=  (R,beta,Gamma,rho,mu) -> R*ds(R,beta,Gamma,rho,mu)/s(R,
  beta,Gamma,rho,mu):
  eval(es(R,beta,Gamma,rho,mu),params):
  esf := unapply(%,(rho,mu)):
  interface(displayprecision=4):
      'es' = evalf(esf(rho,mu));
  interface(displayprecision=mydisplayprecision):
```

$$es = -\left(1.030\left(1.030\left(-\frac{0.9426\ 0.9364^{\frac{1}{\rho}}}{\rho}+0.9426\ 0.9364^{\frac{1}{\rho}}\right)(1-\mu)\left(\frac{\left(0.9364^{\frac{1}{\rho}}(1-\mu)\right)^{-\rho}-1+\mu}{\mu}\right)^{\frac{1}{\rho}}+\left(1-0.9709\ 0.9364^{\frac{1}{\rho}}\right)(1-\mu)\left(\frac{\left(0.9364^{\frac{1}{\rho}}(1-\mu)\right)^{-\rho}-1+\mu}{\mu}\right)^{\frac{1}{\rho}}\right.\right. \tag{21}$$

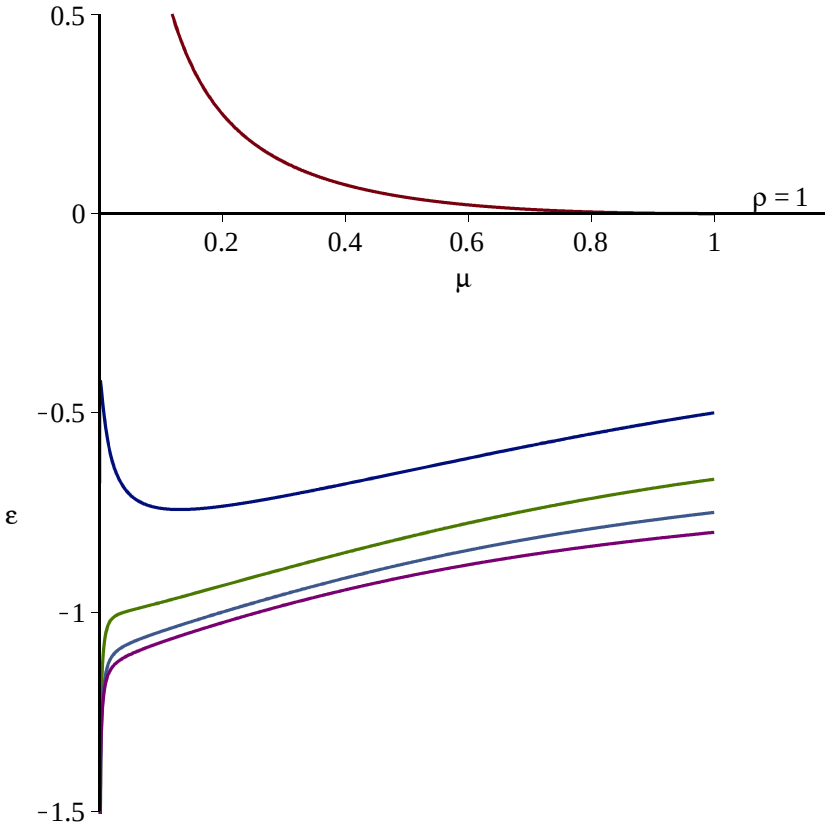
$$-\frac{1}{\rho\left(\left(0.9364^{\frac{1}{\rho}}\,(1-\mu)\right)^{-\rho}-1+\mu\right)}\left(\left(1-0.9709\,0.9364^{\frac{1}{\rho}}\right)(1-\mu)\left(\frac{\left(0.9364^{\frac{1}{\rho}}\,(1-\mu)\right)^{-\rho}-1+\mu}{\mu}\right)^{\frac{1}{\rho}}\left(0.9364^{\frac{1}{\rho}}\,(1-\mu)\right)^{-\rho}\right)\Bigg/\left(1+1.030\left(1-0.9709\,0.9364^{\frac{1}{\rho}}\right)(1-\mu)\left(\frac{\left(0.9364^{\frac{1}{\rho}}\,(1-\mu)\right)^{-\rho}-1+\mu}{\mu}\right)^{\frac{1}{\rho}}\right)$$

```
> ### Set position of the plot labels, tweaked for stated parameter
  values

  mumin := 1.0:
  mumax := 1.0:
  rhomin := 1:
  rhomax := 5:

  xmu:=rho->1.05*mumax:   ymu:=rho->esf(rho,xmu(rho)): # fix x-
  value, vary y-value
  xrho:=mu->mumin:        yrho:=mu->esf(xrho(mu),mu):  # fix x-
  value, vary y-value

> ### Plot of the elasticity of s with respect to R, for fixed
  values of mu

  interface(displayprecision=2):

  plot_es_rho := plot( [ seq( esf(rho,mu) , mu=mulist[2..8] ) ]
      , rho = 1 .. 5
      , 'numpoints' = 1000
      , 'tickmarks' = [ 6, 6 ]
      , 'labels' = [ rho, epsilon ]
      , 'view' = [ 0 .. 5.8, default ]
    ) :

  #### plot labels

  ptxt := seq( plots:-textplot([xrho(mu)-1,yrho(mu),'typeset'('mu',
  " = ", evalf(mu))], 'align'={'above','right'}), mu=mulist[2..8]):

  sElasticityUrateFixedCRRAVaries := plots:-display([plot_es_rho,
  ptxt]): %;
```

```
interface(displayprecision=mydisplayprecision):
```




```
> ### Plot of the elasticity of s with respect to R, for fixed
  values of rho

  plot_es_mu := plot( [ seq( esf(rho,mu) , rho=rholist[1..5] ) ]
      , mu = 0 .. 1
      , 'numpoints' = 1000
      , 'tickmarks' = [ 6, 6 ]
      , 'labels' = [ mu, epsilon ]
      , 'view' = [ 0 .. 1.18, default ]
    ) :

  #### plot labels

  ptxt := seq( plots:-textplot([xmu(rho),ymu(rho),'typeset'('rho',
  " = ", rho)], 'align'={'above','right'}), rho=rholist[1..5]):

  sElasticityCRRAFixedUrateVaries := plots:-display([plot_es_mu,
  ptxt], 'view' = [ default, -3/2 .. 1/2 ]): %;
```




```
> ### Table of elasticity of target saving rate s after 1% Change
  in After-Tax Interest Rate
  ### Mid-Point Formula

  interface(displayprecision=6):
  schanges := Matrix([seq( [seq( 100*(s(Rf,betaf,Gammaf,rho,mu)-s
  (Rf-1/100,betaf,Gammaf,rho,mu))/((s(Rf,betaf,Gammaf,rho,mu)+s
  (Rf-1/100,betaf,Gammaf,rho,mu))/2) ,rho=rholist[1..8] )],mu=
  mulist[2..8])]):
  schanges := ArrayTools:-Concatenate(2,Vector[column](evalf[2]
  (mulist[2..8])),schanges):
  schanges := ArrayTools:-Concatenate(1,Vector[row]([0,op(rholist
  [1..8])]),schanges):
      'schanges' = evalf(%);
  interface(displayprecision=mydisplayprecision):
```

$$schanges = \begin{bmatrix} [0., 1., 2., 3., 4., 5., 6., 7., 8.], \\ [0.010, 4.77172, -0.606254, -1.07722, -1.15913, -1.17185, -1.16748, -1.15858, \\ -1.14886], \\ [0.020, 2.86183, -0.651010, -1.02222, -1.10466, -1.12632, -1.12989, -1.12713, \\ -1.12209], \end{bmatrix} \quad \textbf{(22)}$$

```
    [0.040, 1.54797, −0.698035, −0.992757, −1.07029, −1.09523, −1.10257, −1.10296,
    −1.10041],
    [0.080, 0.760850, −0.728194, −0.968542, −1.03973, −1.06520, −1.07413, −1.07613,
    −1.07496],
    [0.16, 0.330220, −0.728825, −0.931647, −0.997120, −1.02240, −1.03242, −1.03579,
    −1.03603],
    [0.32, 0.111528, −0.688093, −0.863314, −0.925240, −0.952151, −0.965252,
    −0.972083, −0.975796],
    [0.64, 0.0161855, −0.587171, −0.745080, −0.812933, −0.849988, −0.873267,
    −0.889255, −0.900921]]
> ##########################
> ### Export Plots
  ### The best quality 2d plots are postscript, the best 3d plots
  are png
  ### figures are converted to pdf or png with epstopdf and
  imagemagick with batch file
> interface(displayprecision=2): # necessary to strip some trailing
  zeros
> MakePlot(mTargetUrateVariesCRRAVaries,'extension'=png); # 3d
  postscript plots buggy in Maple 16 and ugly in earlier versions
> MakePlot(mTargetUrateVariesCRRAVariesAnimation,'extension'=gif);
> MakePlot(mTargetCRRAFixedUrateVaries,'extension'=ps);
> MakePlot(mTargetUrateFixedCRRAVaries,'extension'=ps);
> MakePlot(mTargetCRRAFixedUrateVariesApproximations,'extension'=
  ps);
> MakePlot(mSlopeCRRAFixedUrateVaries,'extension'=ps);
> MakePlot(mSlopeUrateFixedCRRAVaries,'extension'=ps);
> MakePlot(sTargetUrateVariesCRRAVaries,'extension'=png); # 3d
  postscript plots buggy in Maple 16 and ugly in earlier versions
> MakePlot(sTargetUrateVariesCRRAVariesAnimation,'extension'=gif);
> MakePlot(sTargetCRRAFixedUrateVaries,'extension'=ps);
> MakePlot(sTargetUrateFixedCRRAVaries,'extension'=ps);
> MakePlot(sElasticityCRRAFixedUrateVaries,'extension'=ps);
> MakePlot(sElasticityUrateFixedCRRAVaries,'extension'=ps);
> ##########################
> ### Export Data to File
  theplace := cat(currentdir(),kernelopts(dirsep),convert(N,
  string),kernelopts(dirsep)):
  thedata := [ 'm'=m(R,beta,Gamma,rho,mu), 's'=s(R,beta,Gamma,rho,
  mu), 'parameters'=params ]:
> fd := fopen(cat(theplace,"ParametersAndFormulas_",convert(N,
  string),".txt"), WRITE):
  fprintf(fd, "%{c\n}a\n", <thedata>): fclose(fd):
> ExportMatrix(cat(theplace,"mvalues_mu_rho_",convert(N,string),".
  m")
      , evalf(mvalues), delimiter="&", format=rectangular, mode=
  ascii):
> ExportMatrix(cat(theplace,"mchanges_mu_rho_",convert(N,string),".
  m")
```

```
      , evalf(mchanges), delimiter="&", format=rectangular, mode=
ascii):
> ExportMatrix(cat(theplace,"svalues_mu_rho_",convert(N,string),".
m")
      , evalf(svalues), delimiter="&", format=rectangular, mode=
ascii):
> ExportMatrix(cat(theplace,"schanges_mu_rho_",convert(N,string),".
m")
      , evalf(schanges), delimiter="&", format=rectangular, mode=
ascii):
> interface(displayprecision=mydisplayprecision): # restore
preferences
```