

The Random Walk Model of Consumption

This handout derives the Hall (1978) random walk proposition for consumption. The consumption Euler equation when future consumption is uncertain takes the form¹

$$u'(c_t) = \beta R \mathbb{E}_t[u'(c_{t+1})]. \quad (1)$$

Suppose the utility function takes the quadratic form

$$u(c) = -(1/2)(\phi - c)^2 \quad (2)$$

where ϕ is the “bliss point” level of consumption.² Marginal utility is

$$u'(c) = (\phi - c) \quad (3)$$

and suppose further that $R\beta = 1$ so that (1) becomes

$$(\phi - c_t) = \mathbb{E}_t[(\phi - c_{t+1})] \quad (4)$$

$$\mathbb{E}_t[c_{t+1}] = c_t. \quad (5)$$

Defining the innovation to consumption as

$$\epsilon_{t+1} = c_{t+1} - c_t, \quad (6)$$

$$\equiv \Delta c_{t+1}, \quad (7)$$

the random walk proposition is simply that the expectation of consumption changes is zero:

$$\mathbb{E}_t[\Delta c_{t+1}] = 0. \quad (8)$$

This means that no information known to the consumer when the consumption choice c_t was made can have any predictive power for how consumption will change between period t and $t + 1$ (or for any date beyond $t + 1$).

¹See handout **Envelope** for the derivation of the Euler equation in the perfect foresight case; we will show later that the consequence of uncertainty is simply to insert the expectations operator.

²Assume that the consumer is sufficiently poor that it will be impossible for them ever to achieve consumption as large as ϕ .

References

HALL, ROBERT E. (1978): "Stochastic Implications of the Life Cycle–Permanent Income Hypothesis," *Journal of Political Economy*, 86(6), 971–987.