The Random Walk Model of Consumption

This handout derives the Hall (1978) random walk proposition for consumption.

The consumption Euler equation when future consumption is uncertain takes the form[1]

$$\mathrm{u}'(c_t) = \beta \mathsf{R}\, \mathbb{E}_t[\mathrm{u}'(c_{t+1})]. \tag{1}$$

Suppose the utility function is quadratic:

$$\mathrm{u}(c) = -(1/2)(\cancel{c} - c)^2 \tag{2}$$

where $\cancel{c}$ is the "bliss point" level of consumption.[2] Marginal utility is

$$\mathrm{u}'(c) = (\cancel{c} - c) \tag{3}$$

and suppose further that $\mathsf{R}\beta = 1$ so that (1) becomes

$$\begin{aligned} (\cancel{c} - c_t) &= \mathbb{E}_t[(\cancel{c} - c_{t+1})] \\ \mathbb{E}_t[c_{t+1}] &= c_t. \end{aligned} \tag{4}$$

Defining the innovation to consumption as

$$\begin{aligned} \epsilon_{t+1} &= c_{t+1} - c_t, \\ &\equiv \Delta c_{t+1}, \end{aligned} \tag{5}$$

the random walk proposition is simply that the expectation of consumption changes is zero:

$$\mathbb{E}_t[\Delta c_{t+1}] = 0. \tag{6}$$

This means that no information known to the consumer when the consumption choice c_t was made can have any predictive power for how consumption will change between period t and $t+1$ (or for any date beyond $t+1$).

References

Hall, Robert E. (1978): "Stochastic Implications of the Life-Cycle/Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 96, 971–87, Available at `http://www.stanford.edu/~rehall/Stochastic-JPE-Dec-1978.pdf`.

[1] See handout `Envelope` for the derivation of the Euler equation in the perfect foresight case; we will show later that the consequence of uncertainty is simply to insert the expectations operator.

[2] Assume that the consumer is sufficiently poor that it will be impossible for them ever to achieve consumption as large as $\cancel{c}$.