The Random Walk Model of Consumption

This handout derives the Hall (1978) random walk proposition for consumption. The consumption Euler equation when future consumption is uncertain takes the form

\[ u'(c_t) = \beta R E_t[u'(c_{t+1})]. \]  

Suppose the utility function is quadratic:

\[ u(c) = -(1/2)(\varphi - c)^2 \]

where \( \varphi \) is the “bliss point” level of consumption.\(^2\) Marginal utility is

\[ u'(c) = (\varphi - c) \]

and suppose further that \( R\beta = 1 \) so that (1) becomes

\[ (\varphi - c_t) = E_t[(\varphi - c_{t+1})] \]

\[ E_t[c_{t+1}] = c_t. \]  

Defining the innovation to consumption as

\[ \epsilon_{t+1} = c_{t+1} - c_t, \]

\[ \equiv \Delta c_{t+1}, \]

the random walk proposition is simply that the expectation of consumption changes is zero:

\[ E_t[\Delta c_{t+1}] = 0. \]

This means that no information known to the consumer when the consumption choice \( c_t \) was made can have any predictive power for how consumption will change between period \( t \) and \( t + 1 \) (or for any date beyond \( t + 1 \)).

References


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\(^1\)See handout *Envelope* for the derivation of the Euler equation in the perfect foresight case; we will show later that the consequence of uncertainty is simply to insert the expectations operator.

\(^2\)Assume that the consumer is sufficiently poor that it will be impossible for them ever to achieve consumption as large as \( \varphi \).