Consumption Under Perfect Foresight and CRRA Utility

1 The Problem

This handout solves the optimization problem of a consumer with perfect foresight who has intertemporally separable CRRA utility $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$ and discounts future utility geometrically by a factor β per period. The finite horizon formulas extend to the infinite horizon case by the imposition of intuitive 'impatience' and 'finite human wealth' conditions.

Formally, the consumer's problem in period t is to

$$\max \sum_{n=0}^{T-t} \beta^n \mathrm{u}(\boldsymbol{C}_{t+n}) \tag{1}$$

subject to the constraints

$$\boldsymbol{A}_t = \boldsymbol{M}_t - \boldsymbol{C}_t \tag{2}$$

$$\boldsymbol{B}_{t+1} = \boldsymbol{A}_t \mathsf{R} \tag{3}$$

$$\boldsymbol{M}_{t+1} = \boldsymbol{B}_{t+1} + \mathbf{P}_{t+1} \tag{4}$$

where $\mathbf{P}_{t+1}$ is 'permanent labor income,' which we will assume is growing steadily by a factor Γ from period to period:

$$\mathbf{P}_{t+1}/\mathbf{P}_t = \Gamma. \tag{5}$$

2 The Solution

Both market resources $\boldsymbol{M}_t$ and permanent income $\mathbf{P}_t$ are state variables in this problem. Bellman's equation is

$$\mathrm{V}_t(\boldsymbol{M}_t, \mathbf{P}_t) = \max_{\boldsymbol{C}_t} \left\{ \mathrm{u}(\boldsymbol{C}_t) + \beta \mathrm{V}_{t+1}(\overbrace{(\boldsymbol{M}_t - \boldsymbol{C}_t)\mathsf{R} + \mathbf{P}_{t+1}}^{=\boldsymbol{M}_{t+1}}, \mathbf{P}_{t+1}) \right\}. \tag{6}$$

The first order condition for this maximization is

$$\mathrm{u}'(\boldsymbol{C}_t) = \beta \left(\mathsf{R}\mathrm{V}^{\boldsymbol{M}}_{t+1}(\boldsymbol{M}_{t+1}, \mathbf{P}_{t+1}) - \overbrace{\frac{d\mathbf{P}_{t+1}}{d\boldsymbol{C}_t}}^{=0} \mathrm{V}^{\mathbf{P}}_{t+1}(\boldsymbol{M}_{t+1}, \mathbf{P}_{t+1}) \right), \tag{7}$$

and the Envelope theorem tells us that

$$\mathrm{V}_t^{\boldsymbol{M}}(\boldsymbol{M}_t, \mathbf{P}_t) = \mathsf{R}\beta \mathrm{V}_{t+1}^{\boldsymbol{M}}(\boldsymbol{M}_{t+1}, \mathbf{P}_{t+1}). \tag{8}$$

But note that the right hand sides of (7) and (8) are identical, so that

$$\mathrm{V}_t^{\boldsymbol{M}}(\boldsymbol{M}_t, \mathbf{P}_t) = \mathrm{u}'(\boldsymbol{C}_t) \tag{9}$$

and similar logic tells us that $\mathrm{V}_{t+1}^{\boldsymbol{M}}(\boldsymbol{M}_{t+1}, \mathbf{P}_{t+1}) = \mathrm{u}'(\boldsymbol{C}_{t+1})$, which (substituting u' for $\mathrm{V}^{\boldsymbol{M}}$ in (8)) gives us the Euler equation for consumption

$$\mathrm{u}'(\boldsymbol{C}_t) = \mathsf{R}\beta \mathrm{u}'(\boldsymbol{C}_{t+1}) \tag{10}$$

$$1 = \mathsf{R}\beta \left(\frac{\boldsymbol{C}_{t+1}}{\boldsymbol{C}_t} \right)^{-\rho} \tag{11}$$

$$\left(\frac{\boldsymbol{C}_{t+1}}{\boldsymbol{C}_t} \right) = (\mathsf{R}\beta)^{1/\rho}. \tag{12}$$

Thus, consumption grows in every period by a factor $(\mathsf{R}\beta)^{1/\rho}$.

The Intertemporal Budget Constraint tells us that the present discounted value of consumption must be equal to the PDV of total resources:

$$\mathbb{P}_t^T(\boldsymbol{C}) = \boldsymbol{B}_t + \mathbb{P}_t^T(\mathbf{P}). \tag{13}$$

Fact [FinSum] from MathFacts can be used to show that the PDV of labor income (also called 'human wealth' $\boldsymbol{H}_t$) is

$$\boldsymbol{H}_t = \mathbb{P}_t^T(\mathbf{P}) = \sum_{n=0}^{T-t} \mathsf{R}^{-n} \mathbf{P}_{t+n} \tag{14}$$

$$= \mathbf{P}_t \sum_{n=0}^{T-t} \mathsf{R}^{-n} \Gamma^n \tag{15}$$

$$= \mathbf{P}_t \sum_{n=0}^{T-t} (\Gamma/\mathsf{R})^n \tag{16}$$

$$= \mathbf{P}_t \left(\frac{1 - (\Gamma/\mathsf{R})^{T-t+1}}{1 - (\Gamma/\mathsf{R})} \right) \tag{17}$$

while the PDV of consumption is

$$\mathbb{P}_t^T(C) = \sum_{n=0}^{T-t} \mathsf{R}^{-n} \boldsymbol{C}_{t+n} \tag{18}$$

$$= \sum_{n=0}^{T-t} \mathsf{R}^{-n} \boldsymbol{C}_t ((\mathsf{R}\beta)^{1/\rho})^n \tag{19}$$

$$= \quad \boldsymbol{C}_t \sum_{n=0}^{T-t} [\mathsf{R}^{-1}(\mathsf{R}\beta)^{1/\rho}]^n \tag{20}$$

$$= \quad \boldsymbol{C}_t \left(\frac{1-[\mathsf{R}^{-1}(\mathsf{R}\beta)^{1/\rho}]^{T-t+1}}{1-[\mathsf{R}^{-1}(\mathsf{R}\beta)^{1/\rho}]} \right) \tag{21}$$

We can solve the model by combining (21) and (17) using (13):

$$\boldsymbol{C}_t \quad = \quad \left(\frac{1-[\mathsf{R}^{-1}(\mathsf{R}\beta)^{1/\rho}]}{1-[\mathsf{R}^{-1}(\mathsf{R}\beta)^{1/\rho}]^{T-t+1}} \right) \left[\boldsymbol{B}_t + \mathbf{P}_t \left(\frac{1-(\Gamma/\mathsf{R})^{T-t+1}}{1-(\Gamma/\mathsf{R})} \right) \right]. \tag{22}$$

In the infinite-horizon case ($T=\infty$), [`InfSum`] requires that for human wealth to be finite we need the condition

$$\Gamma/\mathsf{R} \quad < \quad 1 \tag{23}$$

$$\Gamma \quad < \quad \mathsf{R}. \tag{24}$$

Why is this? Because if income will grow faster than the interest rate forever, then the PDV of future income is infinite; with infinite human wealth, the problem has no well-defined solution. We henceforth call (24) the Finite Human Wealth (FHW) condition.

Similarly, in order for the PDV of consumption to be finite we must impose:

$$\underbrace{\left(\frac{(\mathsf{R}\beta)^{1/\rho}}{\mathsf{R}} \right)}_{\text{Þ}_{\mathsf{R}}} \quad < \quad 1. \tag{25}$$

where we will henceforth call Þ_{R} the 'return patience factor' whose log is the 'return patience rate' $\text{þ}_{\mathsf{r}} \approx \log \text{Þ}_{\mathsf{R}}$ and what (25) says is that the desired growth rate of consumption must be less than the interest rate in order for the model to have a well-defined solution. (Otherwise, for any finite initial $\boldsymbol{C}_t$ the PDV of future consumption is infinite). This condition therefore imposes a requirement that 'impatience' be greater than some minimum amount. (For more on the various definitions of impatience used in this handout, see Carroll (2011)).

If these conditions do hold, then the model has a well-defined infinite horizon solution, as can be seen by realizing that

$$\lim_{T\to\infty} (\Gamma/\mathsf{R})^{T-t+1} \quad = \quad 0 \tag{26}$$

$$\lim_{T\to\infty} (\mathsf{R}^{-1}(\mathsf{R}\beta)^{1/\rho})^{T-t+1} \quad = \quad 0. \tag{27}$$

Substituting these zeros into (22) yields

$$\boldsymbol{C}_t \quad = \quad \left(1-\mathsf{R}^{-1}(\mathsf{R}\beta)^{1/\rho}\right) \left[\boldsymbol{B}_t + \left(\frac{\mathbf{P}_t}{1-(\Gamma/\mathsf{R})} \right) \right] \tag{28}$$

$$= \quad \left(1-\mathsf{R}^{-1}(\mathsf{R}\beta)^{1/\rho}\right) [\boldsymbol{M}_t - \mathbf{P}_t + \boldsymbol{H}_t] \tag{29}$$

$$= \left(\frac{\mathsf{R} - (\mathsf{R}\beta)^{1/\rho}}{\mathsf{R}}\right) \boldsymbol{O}_t \tag{30}$$

where $\boldsymbol{O}_t$ is the consumer's 'total wealth,' the sum of human and nonhuman wealth.

Now consider the question 'What is the level of $\boldsymbol{C}_t$ that will leave total wealth intact, allowing the same value of consumption in period $t+1$ and forever after (that is, allowing $\boldsymbol{C}_{t+n} = \boldsymbol{C}_t \ \forall \ n > 0$)?'

The intuitive answer is that the wealth-preserving level of spending is exactly equal to the (properly conceived) interest earnings on one's total wealth. We call this the 'sustainable' level of consumption.

Because human wealth is exactly like any other kind of wealth in this framework, it is possible to work directly with the level of total wealth $\boldsymbol{O}$ to find the sustainable level of spending. Suppose we assume the consumer will spend fraction $\varkappa$ of total wealth in each period; the $\varkappa$ that leaves wealth intact will be given by $\varkappa$ in

$$\begin{aligned}
\boldsymbol{O}_{t+1} &= (\boldsymbol{O}_t - \boldsymbol{C}_t)\mathsf{R} && (31)\\
\bar{\boldsymbol{O}} &= (\bar{\boldsymbol{O}} - \varkappa\bar{\boldsymbol{O}})\mathsf{R} && (32)\\
1 &= (1 - \varkappa)\mathsf{R} && (33)\\
1/\mathsf{R} &= (1 - \varkappa) && (34)\\
\varkappa &= 1 - 1/\mathsf{R} && (35)\\
&= \left(\frac{\mathsf{R} - 1}{\mathsf{R}}\right) && (36)\\
&= \mathsf{r}/\mathsf{R}. && (37)
\end{aligned}$$

Thus, the consumer can spend only the interest earnings r on wealth, divided by the return factor R. (The division occurs because the requirement is to be able to spend the same amount *next* period, so you need to account for the time cost of that spending by dividing by R.) Note that the coefficient multiplying total wealth in (30) is also divided by R. Thus, whether the consumer is spending more than the sustainable amount, exactly the sustainable amount, or less than the sustainable amount depends upon whether the numerator in (30) is greater than, equal to, or less than r. The consumer will be 'absolutely impatient' if

$$\begin{aligned}
\mathsf{R} - (\mathsf{R}\beta)^{1/\rho} &> \mathsf{r} && (38)\\
1 - (\mathsf{R}\beta)^{1/\rho} &> 0 && (39)\\
1 &> (\mathsf{R}\beta)^{1/\rho}. && (40)
\end{aligned}$$

Now note that if $\mathsf{R}\beta = 1$ (which is to say, the interest rate is exactly equal to the time preference rate so that they offset each other), then $(\mathsf{R}\beta)^{1/\rho} = 1$ regardless of the value of ρ so that the consumer is poised on the knife-edge between patience and impatience. We refer to such a consumer as 'absolutely poised.' Similarly, we say that a consumer for whom $\text{Þ}_{\mathsf{R}} = 1$ is 'return poised.'[1]

Equation (28) can be simplified into something a bit easier to handle by making some approximations. If $\beta = 1/(1+\vartheta)$, then we can use facts [LogEps] and [ExpEps] to discover that

$$\begin{aligned}
\log(\mathsf{R}\beta)^{1/\rho}/\mathsf{R} &= (1/\rho)(\log \mathsf{R} + \log[1/(1+\vartheta)]) - \log \mathsf{R} && (41)\\
&= (1/\rho)(\log(1+r) + \log 1 - \log(1+\vartheta)) - \log \mathsf{R} && (42)\\
&\approx \rho^{-1}(\mathsf{r} - \vartheta) - \mathsf{r} && (43)\\
(\mathsf{R}\beta)^{1/\rho}/\mathsf{R} &\approx 1 + (\rho^{-1}(\mathsf{r} - \vartheta) - \mathsf{r}). && (44)
\end{aligned}$$

Substituting this into (29) gives

$$\boldsymbol{C}_t \approx \left(\mathsf{r} - \rho^{-1}(\mathsf{r} - \vartheta)\right) \boldsymbol{O}_t. \qquad (45)$$

From this we can see again that whether the consumer is return patient, return poised, or return impatient depends on the relationship between r and ϑ. Note also that if $\rho = \infty$ then the consumer is infinitely averse to changing the level of consumption, and so once again the consumer spends exactly the sustainable amount. (This consumer is 'absolutely poised' but 'return impatient').

Now a brief digression on what 'income' means in this model. Suppose for simplicity that the consumer had no capital assets $\boldsymbol{B}_t$, and suppose that income was expected to stay constant at level $\mathbf{P}_{t+n} = \mathbf{P} \ \forall \ n > 0$ forever. In this case human wealth would be:

$$\begin{aligned}
\boldsymbol{H}_t &= \mathbf{P} + \mathbf{P}/\mathsf{R} + \mathbf{P}/\mathsf{R}^2 + \ldots && (46)\\
&= \mathbf{P}(1 + 1/\mathsf{R} + 1/\mathsf{R}^2 + \ldots) && (47)\\
&= \mathbf{P}\left(\frac{1}{1 - 1/\mathsf{R}}\right) && (48)\\
&= \mathbf{P}\left(\frac{\mathsf{R}}{\mathsf{R} - 1}\right) && (49)\\
&= \mathbf{P}\left(\frac{\mathsf{R}}{\mathsf{r}}\right). && (50)
\end{aligned}$$

We found in equation (37) that the level of consumption that leaves 'wealth' $\boldsymbol{O}_t$ intact was

$$\begin{aligned}
\boldsymbol{C}_t &= \varkappa \boldsymbol{O}_t && (51)\\
&= \varkappa(\boldsymbol{B}_t + \boldsymbol{H}_t) && (52)
\end{aligned}$$

[1] 'Return impatience' guarantees a positive marginal propensity to consume; absolute impatience guarantees a falling level of consumption. If $\mathsf{r} > 0$, return impatience will hold even if the consumer is 'poised' with respect to absolute patience.

$$= \varkappa \mathbf{P}\left(\frac{\mathsf{R}}{\mathsf{r}}\right) \tag{53}$$

$$= \left(\frac{\mathsf{r}}{\mathsf{R}}\right)\mathbf{P}\left(\frac{\mathsf{R}}{\mathsf{r}}\right) \tag{54}$$

$$= \mathbf{P}. \tag{55}$$

So in this case, spending the 'interest income on human wealth' corresponds to spending exactly your labor income. This seems less mysterious if you think of income $\mathbf{P}_t$ as the 'return' on your human capital, which is an asset whose value is $\boldsymbol{H}_t$. If you 'capitalize' your stream of income using the interest factor R and then spend the interest income on the capitalized stream, it stands to reason that you are spending the flow of income from that source.

In this case we can rewrite (45) as

$$\boldsymbol{C}_t \approx \left(\mathsf{r} - \rho^{-1}(\mathsf{r} - \vartheta)\right)\left[\boldsymbol{B}_t + \mathbf{P}\left(\frac{\mathsf{R}}{\mathsf{r}}\right)\right]. \tag{56}$$

r appears three times in this equation, which correspond (in order) to the income effect, the substitution effect, and the human wealth effect. To see this, note that an increase in the first r reflects an increase in the payout rate on total wealth (set $\mathbf{P} = 0$ and refer to our formula above for $\varkappa$, realizing that for small r, $\mathsf{r}/\mathsf{R} \approx \mathsf{r}$.) That is, it simply reflects the consumption consequence of an increase in interest income – so it captures the 'income effect' of interest rates. The second term corresponds to the subsitution effect, as can be seen from its dependence on the intertemporal elasticity of substitution ρ^{-1}. Finally, the $\mathbf{P}\mathsf{R}/\mathsf{r}$ term clearly corresponds to human wealth, and therefore the sensitivity of consumption to r coming through this term corresponds to the human wealth *effect.*

3 Normalizing By P

This section shows that we can restate the whole problem by 'dividing through' by the level of permanent income before solving. Nonbold variables will hereafter be the normalized bold-letter equivalent, e.g. $C_t = \boldsymbol{C}_t/\mathbf{P}_t$, and note that if $\mathbf{P}_{t+1} = \Gamma \mathbf{P}_t \; \forall \; t$ then from the standpoint of time t we have that

$$\mathrm{u}(\boldsymbol{C}_{t+n}) = \frac{\boldsymbol{C}_{t+n}^{1-\rho}}{1-\rho} \tag{57}$$

$$= \frac{(C_{t+n}\mathbf{P}_{t+n})^{1-\rho}}{1-\rho} \tag{58}$$

$$= (\mathbf{P}_t\Gamma^n)^{1-\rho}\frac{C_{t+n}^{1-\rho}}{1-\rho} \tag{59}$$

which means that

$$\sum_{n=0}^{T-t} \beta^n \frac{\boldsymbol{C}_{t+n}^{1-\rho}}{1-\rho} = \mathbf{P}_t^{1-\rho} \sum_{n=0}^{T-t} (\Gamma^{1-\rho}\beta)^n \frac{C_{t+n}^{1-\rho}}{1-\rho}. \tag{60}$$

Furthermore, the accumulation equations can be rewritten by dividing both sides by $\mathbf{P}_{t+1}$:

$$\boldsymbol{B}_{t+1}/\mathbf{P}_{t+1} = \frac{(\boldsymbol{M}_t - \boldsymbol{C}_t)\mathsf{R}}{\mathbf{P}_{t+1}} \tag{61}$$

$$B_{t+1} = \left(\frac{(\boldsymbol{M}_t - \boldsymbol{C}_t)\mathsf{R}}{\mathbf{P}_t}\right)\left(\frac{\mathbf{P}_t}{\mathbf{P}_{t+1}}\right) \tag{62}$$

$$= (M_t - C_t)(\mathsf{R}/\Gamma) \tag{63}$$

$$\boldsymbol{M}_{t+1} = \boldsymbol{B}_{t+1} + \mathbf{P}_{t+1} \tag{64}$$

$$M_{t+1} = B_{t+1} + 1. \tag{65}$$

Now if we define $\beth \equiv \Gamma^{1-\rho}\beta$ and $\mathcal{R} \equiv \mathsf{R}/\Gamma$ it is clear that the original problem can be rewritten as:

$$\max \quad \mathbf{P}_t^{1-\rho} \sum_{n=0}^{T-t} \beth^n \mathrm{u}(C_{t+n}) \tag{66}$$

subject to the constraints

$$A_t = M_t - C_t \tag{67}$$

$$B_{t+1} = A_t \mathcal{R} \tag{68}$$

$$M_{t+1} = B_{t+1} + 1 \tag{69}$$

and we can go through the same steps as above to find that the solution is

$$C_t = (1 - \mathcal{R}^{-1}(\mathcal{R}\beth)^{1/\rho}) \left[B_t + \overbrace{\left(\frac{1}{1 - 1/\mathcal{R}}\right)}^{\equiv H} \right] \tag{70}$$

subject to the 'finite human wealth' condition

$$1 < \mathcal{R} \tag{71}$$

$$1 < \mathsf{R}/\Gamma \tag{72}$$

which is the same condition as above (24), and the 'return impatience condition'

$$(\mathcal{R}\beth)^{1/\rho} < \mathcal{R} \tag{73}$$

$$\left(\frac{\mathsf{R}}{\Gamma}\beta\Gamma^{1-\rho}\right)^{1/\rho} < \mathsf{R}/\Gamma \tag{74}$$

$$(\mathsf{R}\beta)^{1/\rho} < \mathsf{R} \tag{75}$$

which is the same as (25).

Now note that (70) can be rewritten

$$C_t = \left(\frac{\mathcal{R} - (\mathcal{R}\beth)^{1/\rho}}{\mathcal{R}}\right) O_t \tag{76}$$

where O_t is the consumer's total wealth-to-permanent-labor-income ratio.

As before, whether O is rising or falling depends upon the relationship between $\mathcal{R} - 1$ and $\mathcal{R} - (\mathcal{R}\beth)^{1/\rho}$. If we call a consumer who is drawing down his wealth-to-income ratio 'growth impatient,' the consumer will be growth impatient if

$$\mathcal{R} - (\mathcal{R}\beth)^{1/\rho} > \mathcal{R} - 1 \tag{77}$$

$$1 - (\mathcal{R}\beth)^{1/\rho} > 0 \tag{78}$$

$$1 > (\mathcal{R}\beth)^{1/\rho}. \tag{79}$$

Now substituting the definitions of $\mathcal{R}$ and $\beth$ we see that whether O is rising or falling depends on whether

$$1 > (\frac{\mathsf{R}}{\Gamma}\beta\Gamma^{1-\rho})^{1/\rho} \tag{80}$$

$$1 > (\mathsf{R}\beta\Gamma^{-\rho})^{1/\rho} \tag{81}$$

$$1 > \underbrace{\left(\frac{(\mathsf{R}\beta)^{1/\rho}}{\Gamma}\right)}_{\text{Þ}_\Gamma}. \tag{82}$$

where Þ_Γ is the 'growth patience factor' and we say that the consumer is 'growth impatient' if (82) holds.

Thus, whether the consumer is patient or impatient in the sense of building up or drawing down a wealth-to-income *ratio* depends on whether the growth rate of labor income is less than, equal to, or greater than the growth rate of consumption. Analogously to our earlier usages, a consumer for whom $\text{Þ}_\Gamma = 1$ would be 'growth poised.'

To get the intuition for this, consider the case of a consumer with no nonhuman wealth, $B_t = 0$. This consumer's absolute level of consumption will grow at $(\mathsf{R}\beta)^{1/\rho}$ and absolute level of income at Γ, but the PDV of future consumption and future income are equal. Thus, if income is growing faster than consumption but has the same PDV, consumption must be starting out at a level higher than income - which is to say, if the impatience condition holds, the consumer has a high level of consumption but slow consumption growth.

4 An Application

We can now apply the model to answer our first useful question: How large does the model imply the 'human wealth effect' is?

For simplicity, assume that $B_t = 0$. Then the original version of the formula tells us that the *level* of consumption will be given by:

$$\begin{aligned} \boldsymbol{C}_t &\approx \left[\mathsf{r} - \rho^{-1}(\mathsf{r} - \vartheta)\right] \left(\frac{\mathbf{P}}{1 - \Gamma/\mathsf{R}}\right) && (83) \\ &\approx \left[\mathsf{r} - \rho^{-1}(\mathsf{r} - \vartheta)\right] \left(\frac{\mathbf{P}}{\mathsf{r} - \gamma}\right). && (84) \end{aligned}$$

Now suppose we choose plausible values for $(\mathsf{r}, \vartheta, \gamma, \rho) = (0.04, 0.04, 0.02, 2)$. Then (84) becomes:

$$\begin{aligned} \boldsymbol{C}_t &\approx 0.04(\mathbf{P}/0.02) && (85) \\ &= 2\mathbf{P}. && (86) \end{aligned}$$

Now suppose the interest rate changes to $\mathsf{r} = 0.03$, while all other parameters remain the same. Then (84) becomes:

$$\begin{aligned} \boldsymbol{C}_t &\approx 0.035(\mathbf{P}/0.01) && (87) \\ &= 3.5\mathbf{P}. && (88) \end{aligned}$$

The point of this example is that for plausible parameter values, the human wealth effect is enormously stronger than the income and substitution effects, so that we should see large drops in consumption when interest rates rise and conversely strong gains when interest rates fall. This is a summary of the main point of the famous paper by Summers (1981); Summers derives formulas for an economy with overlapping generations of finite-lifetime consumers, but those complications do not change the basic message.

References

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Summers, Lawrence H. (1981): "Capital Taxation and Accumulation in a Life Cycle Growth Model," *American Economic Review*, 71(4), 533–544, http://ideas.repec.org/a/aea/aecrev/v71y1981i4p533-44.html.