The Distribution of Wealth and the Marginal Propensity to Consume

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Our Claim: Heterogeneity Is Key To Modeling the MPC

The Question: How Large Is the MPC ($\equiv \kappa$)?

If households receive a surprise extra 1 unit of income, how much will be in aggregate spent over the next year?

Need to Consider:

- Households are heterogeneous
- Wealth is unevenly distributed
- C function is highly concave
- $\Rightarrow$ Distributional issues matter for aggregate C
  Giving 1 to the poor $\neq$ giving 1 to the rich
**Motivation**

- Model Without Aggregate Shock
- Two Specifications of Aggregate Shock
- Matching Net Worth vs Liquid Assets
- Life Cycle Model
- References

**The MPC**

**Theory and Evidence**

Friedman (1957)

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**Consumption Concavity and Wealth Heterogeneity**

- Consumption/((quarterly) permanent income ratio (left scale)
- Histogram: empirical (SCF1998) density of $m_t/(p_t W_t)$ (right scale)

Carroll, Slacalek, Tokuoka and White

Wealth and MPC
Why Worry About the MPC ($\equiv \kappa$)?

Nobody trying to make a forecast in 2008–2010 would ask:

- Big ‘stimulus’ tax cuts
- Keynesian *multipliers* should be big in liquidity trap
- Crude Keynesianism: Transitory tax cut multiplier is $1/(1 - \kappa) - 1$
  - If $\kappa = 0.75$ then multiplier is $4 - 1 = 3$
  - Some micro estimates of $\kappa$ are this large
  - If $\kappa = 0.05$ then multiplier is only $\approx 0.05$
  - This is about the size of $\kappa$ in RBC models
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  - If $\kappa = 0.05$ then multiplier is only $\approx 0.05$
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To-Do List

1. Calibrate realistic income process
2. Match empirical wealth distribution
3. Back out optimal C and MPC out of transitory income
4. Is MPC in line with empirical estimates?

Our Question:

Does a model that matches micro facts about income dynamics and wealth distribution give different (and more plausible) answers than KS to macroeconomic questions (say, about the response of consumption to fiscal ‘stimulus’)?
Friedman (1957): Permanent Income Hypothesis

\[
Y_t = P_t + T_t \\
C_t = P_t
\]

Progress since then

- **Micro data**: Friedman description of income shocks works well
- **Math**: Friedman’s words well describe optimal solution to dynamic stochastic optimization problem of impatient consumers with geometric discounting under CRRA utility with uninsurable idiosyncratic risk calibrated using these micro income dynamics (!)
Our (Micro) Income Process

Idiosyncratic (household) income process is logarithmic Friedman:

\[ y_{t+1} = p_{t+1} \xi_{t+1} W \]
\[ p_{t+1} = p_t \psi_{t+1} \]

- \( p_t = \) permanent income
- \( \xi_t = \) transitory income
- \( \psi_{t+1} = \) permanent shock
- \( W = \) aggregate wage rate
Further Details of Income Process

Modifications from Carroll (1992)

Transitory income $\xi_t$ incorporates unemployment insurance:

$$\xi_t = \mu \text{ with probability } u$$

$$= (1 - \tau) \bar{\theta}_t \text{ with probability } 1 - u$$

$\mu$ is UI when unemployed

$\tau$ is the rate of tax collected for the unemployment benefits
Model Without Aggr Uncertainty: Decision Problem

\[ v(m_t) = \max \{ c_t \} u + \beta \mathbb{E}_t \left[ \psi_{t+1}^{1-\rho} v(m_{t+1}) \right] \]

s.t.

\[ a_t = m_t - c_t \]
\[ a_t \geq 0 \]
\[ k_{t+1} = a_t / (\psi_t^\rho) \]
\[ m_{t+1} = (1 + r)k_{t+1} + \xi_{t+1} \]
\[ r = \alpha Z(K/\bar{L})^{\alpha-1} \]

(State and control variables normalized by \( p_tW \))
What Happens After Death?

- You are replaced by a new agent whose permanent income is equal to the population mean.
- Prevents the population distribution of permanent income from spreading out.
What Happens After Death?

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- Prevents the population distribution of permanent income from spreading out.
Ergodic Distribution of Permanent Income

Exists, if death eliminates permanent shocks:

\[ \mathcal{D} \mathbb{E} [ \psi^2 ] < 1. \]

Holds.

Population mean of \( p^2 \):

\[ \mathbb{M} [ p^2 ] = \frac{D}{1 - \mathcal{D} \mathbb{E} [ \psi^2 ]} \]
Parameter Values

- $\beta$, $\rho$, $\alpha$, $\delta$, $\ell$, $\mu$, and $u$ taken from JEDC special volume
- Key new parameter values:

<table>
<thead>
<tr>
<th>Description</th>
<th>Param</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob of Death per Quarter</td>
<td>$D$</td>
<td>0.00625</td>
<td>Life span of 40 years</td>
</tr>
<tr>
<td>Variance of Log $\psi_t$</td>
<td>$\sigma^2_{\psi}$</td>
<td>0.016/4</td>
<td>Carroll (1992); SCF</td>
</tr>
<tr>
<td>Variance of Log $\theta_t$</td>
<td>$\sigma^2_{\theta}$</td>
<td>0.010 × 4</td>
<td>DeBacker et al. (2013)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Carroll (1992)</td>
</tr>
</tbody>
</table>
**Annual Income, Earnings, or Wage Variances**

<table>
<thead>
<tr>
<th></th>
<th>$\sigma^2_\psi$</th>
<th>$\sigma^2_\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Our parameters</strong></td>
<td>0.016</td>
<td>0.010</td>
</tr>
<tr>
<td><strong>Carroll (1992)</strong></td>
<td>0.016</td>
<td>0.010</td>
</tr>
<tr>
<td>Storesletten, Telmer, and Yaron (2004)</td>
<td>0.008–0.026</td>
<td>0.316</td>
</tr>
<tr>
<td>Meghir and Pistaferri (2004)*</td>
<td>0.031</td>
<td>0.032</td>
</tr>
<tr>
<td>Low, Meghir, and Pistaferri (2010)</td>
<td>0.011</td>
<td>—</td>
</tr>
<tr>
<td>Blundell, Pistaferri, and Preston (2008)*</td>
<td>0.010–0.030</td>
<td>0.029–0.055</td>
</tr>
<tr>
<td>DeBacker, Heim, Panousi, Ramnath, and Vidangos (2013)</td>
<td>0.007–0.010</td>
<td>0.15–0.20</td>
</tr>
<tr>
<td><strong>Implied by KS-JEDC</strong></td>
<td>0.000</td>
<td>0.038</td>
</tr>
<tr>
<td><strong>Implied by Castaneda et al. (2003)</strong></td>
<td>0.03</td>
<td>0.005</td>
</tr>
</tbody>
</table>

*Meghir and Pistaferri (2004) and Blundell, Pistaferri, and Preston (2008) assume that the transitory component is serially correlated (an MA process), and report the variance of a subelement of the transitory component. $\sigma^2_\xi$ for these articles are calculated using their MA estimates.*
Typology of Our Models—Four Dimensions

1. **Discount Factor $\beta$**
   - ‘$\beta$-Point’ model: Single discount factor
   - ‘$\beta$-Dist’ model: Uniformly distributed discount factor

2. **Aggregate Shocks**
   - (No)
   - Krusell–Smith
   - Friedman/Buffer Stock

3. **Empirical Wealth Variable to Match**
   - Net Worth
   - Liquid Financial Assets

4. **Life Cycle**
   - Perpetual Youth (a la Blanchard)
   - Overlapping Generations

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Wealth and MPC
**Dimension 1: Estimation of $\beta$-Point and $\beta$-Dist**

**'β-Point' model**

- 'Estimate' single $\beta$ by matching the capital–output ratio

**'β-Dist' model—Heterogenous Impatience**

- Assume uniformly distributed $\beta$ across households
- Estimate the band $[\hat{\beta} - \nabla, \hat{\beta} + \nabla]$ by minimizing distance between model ($w$) and data ($\omega$) net worth held by the top 20, 40, 60, 80%

$$\min_{\{\beta, \nabla\}} \sum_{i=20,40,60,80} (w_i - \omega_i)^2,$$

s.t. aggregate net worth–output ratio matches the steady-state value from the perfect foresight model
Results: Wealth Distribution

Carroll, Slacalek, Tokuoka and White

Wealth and MPC
## Results: Wealth Distribution

<table>
<thead>
<tr>
<th>Top Weights</th>
<th>Friedman/Buffer Stock</th>
<th>KS-JEDC</th>
<th>KS-Orig (\Diamond)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\beta)-Point (</td>
<td>\beta)-Dist</td>
<td>\beta)-Point</td>
</tr>
<tr>
<td>Top 1%</td>
<td>11.5</td>
<td>27.9</td>
<td>2.7</td>
</tr>
<tr>
<td>Top 20%</td>
<td>55.3</td>
<td>83.6</td>
<td>35.6</td>
</tr>
<tr>
<td>Top 40%</td>
<td>76.5</td>
<td>94.1</td>
<td>60.</td>
</tr>
<tr>
<td>Top 60%</td>
<td>89.7</td>
<td>97.7</td>
<td>78.5</td>
</tr>
<tr>
<td>Top 80%</td>
<td>97.4</td>
<td>99.4</td>
<td>92.1</td>
</tr>
</tbody>
</table>

Notes:
- \(\beta\)-Point: \(\beta\)-Uniformly, \(\beta\)-Distributed
- \(\Diamond\): \(\beta\)-Point, \(\Diamond\)-Hetero
- \(\ast\): (\(\beta\), \(\nabla\)) = (0.9876, 0.0060)
- Bold points are targeted.
- \(\nabla\): `\(\beta\)= 0.9899.`
Marginal Propensity to Consume & Net Worth

Histogram: empirical density of net worth (right scale)

Representative agent's net worth →

Most Patient (left scale) ↑

Identical Patience (left scale) ↓

Most Impatient (left scale) ↓

Parameter Values
Annual Income Variances
Our Strategy
Results: Marginal Propensity to Consume

Carroll, Slacalek, Tokuoka and White
Wealth and MPC
## Results: MPC (in Annual Terms)

<table>
<thead>
<tr>
<th>Micro Income Process</th>
<th>Friedman/Buffer Stock</th>
<th>KS-JEDC</th>
<th>Our solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>β-Point</td>
<td>β-Dist</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall average</td>
<td>0.1</td>
<td>0.23</td>
<td>0.05</td>
</tr>
<tr>
<td>By wealth/permanent income ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.07</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>Top 20%</td>
<td>0.07</td>
<td>0.06</td>
<td>0.04</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.07</td>
<td>0.08</td>
<td>0.04</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.07</td>
<td>0.12</td>
<td>0.04</td>
</tr>
<tr>
<td>Bottom 1/2</td>
<td>0.13</td>
<td>0.35</td>
<td>0.05</td>
</tr>
<tr>
<td>By employment status</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.09</td>
<td>0.2</td>
<td>0.05</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.22</td>
<td>0.54</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Notes: Annual MPC is calculated by $1 - (1 - \text{quarterly MPC})^4$. 

Carroll, Slacalek, Tokuoka and White: Wealth and MPC
### Estimates of MPC in the Data: $\sim 0.2–0.6$

<table>
<thead>
<tr>
<th>Authors</th>
<th>Nondurables</th>
<th>Durables</th>
<th>Total PCE</th>
<th>Horizon</th>
<th>Event/Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blundell et al. (2008b)$^\ddagger$</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td>Estimation Sample: 1980–92</td>
</tr>
<tr>
<td>Coronado et al. (2005)</td>
<td></td>
<td>0.36</td>
<td>1 Year</td>
<td>2003 Tax Cut</td>
<td></td>
</tr>
<tr>
<td>Hausman (2012)</td>
<td>0.6–0.75</td>
<td></td>
<td>1 Year</td>
<td>1936 Veterans’ Bonus</td>
<td></td>
</tr>
<tr>
<td>Johnson et al. (2009) $^\ddagger$</td>
<td>~ 0.25</td>
<td></td>
<td>3 Months</td>
<td>2003 Child Tax Credit</td>
<td></td>
</tr>
<tr>
<td>Lusardi (1996)$^\ddagger$</td>
<td>0.2–0.5</td>
<td></td>
<td>3 Months</td>
<td>Estimation Sample: 1980–87</td>
<td></td>
</tr>
<tr>
<td>Parker (1999)</td>
<td>0.2</td>
<td></td>
<td>3 Months</td>
<td>Estimation Sample: 1980–93</td>
<td></td>
</tr>
<tr>
<td>Parker et al. (2011)</td>
<td>0.12–0.30</td>
<td>0.50–0.90</td>
<td>3 Months</td>
<td>2008 Economic Stimulus</td>
<td></td>
</tr>
<tr>
<td>Sahm et al. (2009)</td>
<td>~ 1/3</td>
<td></td>
<td>1 Year</td>
<td>2008 Economic Stimulus</td>
<td></td>
</tr>
<tr>
<td>Shapiro and Slemrod (2009)</td>
<td>~ 1/3</td>
<td></td>
<td>1 Year</td>
<td>2008 Economic Stimulus</td>
<td></td>
</tr>
<tr>
<td>Souleles (1999)</td>
<td>0.045–0.09</td>
<td>0.29–0.54</td>
<td>3 Months</td>
<td>Estimation Sample: 1980–91</td>
<td></td>
</tr>
<tr>
<td>Souleles (2002)</td>
<td>0.6–0.9</td>
<td></td>
<td>1 Year</td>
<td>The Reagan Tax Cuts of the Early 1980s</td>
<td></td>
</tr>
</tbody>
</table>

Notes: $^\ddagger$: elasticity.
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1. **Discount Factor $\beta$**
   - 'β-Point' model: Single discount factor
   - 'β-Dist' model: Uniformly distributed discount factor

2. **Aggregate Shocks**
   - (No)
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   - Friedman/Buffer Stock

3. **Empirical Wealth Variable to Match**
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4. **Life Cycle**
   - Perpetual Youth (à la Blanchard)
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Carroll, Slacalek, Tokuoka and White
Wealth and MPC
Dimension 2.a: Adding KS Aggregate Shocks

Model with KS Aggregate Shocks: Assumptions

- Only two aggregate states (good or bad)
- Aggregate productivity $Z_t = 1 \pm \Delta Z$
- Unemployment rate $u$ depends on the state ($u^g$ or $u^b$)

Parameter values for aggregate shocks from Krusell and Smith (1998)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta Z$</td>
<td>0.01</td>
</tr>
<tr>
<td>$u^g$</td>
<td>0.04</td>
</tr>
<tr>
<td>$u^b$</td>
<td>0.10</td>
</tr>
<tr>
<td>Agg transition probability</td>
<td>0.125</td>
</tr>
</tbody>
</table>
Motivation:
More plausible and tractable aggregate process, also simpler
Eliminates ‘good’ and ‘bad’ aggregate state

Aggregate production function: \( K_t^\alpha (L_t)^{1-\alpha} \)
- \( L_t = P_t \Xi_t \)
- \( P_t \) is aggregate permanent productivity
- \( P_{t+1} = P_t \Psi_{t+1} \)
- \( \Xi_t \) is the aggregate transitory shock.

Parameter values estimated from U.S. data:

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance of Log ( \Psi_t )</td>
<td>( \sigma^2_{\Psi} )</td>
<td>0.00004</td>
</tr>
<tr>
<td>Variance of Log ( \Xi_t )</td>
<td>( \sigma^2_{\Xi} )</td>
<td>0.00001</td>
</tr>
</tbody>
</table>
Dimension 2.b: Adding FBS Aggregate Shocks

Friedman/Buffer Stock Shocks

- **Motivation:**
  More plausible and tractable aggregate process, also simpler
- Eliminates ‘good’ and ‘bad’ aggregate state
- Aggregate production function: \( K_t^\alpha (L_t)^{1-\alpha} \)
  - \( L_t = P_t \Xi_t \)
  - \( P_t \) is aggregate permanent productivity
  - \( P_{t+1} = P_t \Psi_{t+1} \)
  - \( \Xi_t \) is the aggregate transitory shock.
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</tr>
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<tbody>
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<td>Variance of Log ( \Psi_t )</td>
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<td>0.00004</td>
</tr>
<tr>
<td>Variance of Log ( \Xi_t )</td>
<td>( \sigma_{\Xi}^2 )</td>
<td>0.00001</td>
</tr>
</tbody>
</table>
Results

**Our/FBS model**

- A few times faster than solving KS model
- The results are similar to those under KS aggregate shocks
## Results: MPC Over the Business Cycle

<table>
<thead>
<tr>
<th>Model: $\beta$-Dist</th>
<th>Krusell–Smith (KS)</th>
<th>Friedman/Buffer Stock (FBS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Base</td>
<td>Recssn</td>
</tr>
<tr>
<td>Overall average</td>
<td>0.23</td>
<td>0.25</td>
</tr>
<tr>
<td>By wealth/permanent income ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Top 10%</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Top 20%</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>Top 50%</td>
<td>0.09</td>
<td>0.10</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Bottom 50%</td>
<td>0.35</td>
<td>0.38</td>
</tr>
<tr>
<td>By employment status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.54</td>
<td>0.56</td>
</tr>
</tbody>
</table>
Results: MPC Over the Business Cycle

**Krusell–Smith**
- Aggregate and idiosyncratic shocks positively correlated
- Higher MPC during recessions, especially for the unemployed

**Friedman/Buffer Stock**
- Shocks uncorrelated
- MPC essentially doesn’t vary over BC
Motivation

Model Without Aggregate Shock

Two Specifications of Aggregate Shock

Matching Net Worth vs Liquid Assets

Life Cycle Model

References

Typology of Our Models—Four Dimensions

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Krusell–Smith

Permanent/Transitory Aggregate Shocks

Carroll, Slacalek, Tokuoka and White

Wealth and MPC
Dimension 3: Matching Net Worth vs. Liquid Financial (and Retirement) Assets

- **Motivation**
  - Model Without Aggregate Shock
  - Two Specifications of Aggregate Shock
  - Matching Net Worth vs Liquid Assets
- **References**
  - Carroll, Slacalek, Tokuoka and White
  - Wealth and MPC

![Histogram: empirical density of net worth](left scale)

- Most impatient (left scale)
- Most patient (left scale)

- Histogram: empirical density of liquid financial asset + retirement assets (right scale)

- Histogram: empirical density of net worth (right scale)

- $f$: $\sum_{t=0}^{20} c_t f c_t m_t$
Buffer stock saving driven by accumulation of liquidity
May make more sense to match liquid (and retirement) assets (Hall (2011), Kaplan and Violante (2011))
Aggregate MPC Increases Substantially: 0.23 $\uparrow$ 0.43

<table>
<thead>
<tr>
<th>$\beta$-Dist</th>
<th>Net Worth</th>
<th>Liq Fin and Ret Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall average</td>
<td>0.23</td>
<td>0.43</td>
</tr>
<tr>
<td>By wealth/permanent income ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.05</td>
<td>0.12</td>
</tr>
<tr>
<td>Top 20%</td>
<td>0.06</td>
<td>0.13</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.08</td>
<td>0.2</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.12</td>
<td>0.28</td>
</tr>
<tr>
<td>Bottom 1/2</td>
<td>0.35</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Notes: Annual MPC is calculated by $1 - (1 - \text{quarterly MPC})^{4}$. 

Carroll, Slacalek, Tokuoka and White
Wealth and MPC
Wealth heterogeneity translates into heterogeneity in MPCs.
Motivation
Model Without Aggregate Shock
Two Specifications of Aggregate Shock
Matching Net Worth vs Liquid Assets
Life Cycle Model
References

Typology of Our Models—Four Dimensions

1. **Discount Factor $\beta$**
   - ‘$\beta$-Point’ model: Single discount factor
   - ‘$\beta$-Dist’ model: Uniformly distributed discount factor

2. **Aggregate Shocks**
   - (No)
   - Krusell–Smith
   - Friedman/Buffer Stock

3. **Empirical Wealth Variable to Match**
   - Net Worth
   - Liquid Financial Assets

4. **Life Cycle**
   - Perpetual Youth (a la Blanchard)
   - Overlapping Generations
Dimension 4: Overlapping Generations

Realistic Life-Cycle Model

- Three education levels: \( e \in \{D, HS, C\} \)
- Age/education-specific income profiles

\[
\begin{align*}
\bar{y}_t &= \xi_t \bar{p}_t = (1 - \tau) \theta_t \bar{p}_t, \\
\bar{p}_t &= \psi_t \psi_{es} \bar{p}_{t-1}
\end{align*}
\]

- Age-specific variances of income shocks
- Transitory unemployment shock with prob \( u \)
- Household-specific mortality \( D_{es} \)
Household Decision Problem

\[ v_{es}(m_t) = \max_{c_t} u \]
Population growth $N$, technological progress $\Gamma$

**Tax rate** to finance social security and unemployment benefits:

$\tau = \tau_{SS} + \tau_U$

$\tau_{SS} = \frac{\sum_{e \in \{D, HS, C\}} \theta e \bar{p}_{e0} \sum_{t=164}^{384} \left( ((1+\Gamma)(1+N))^{-t} \prod_{s=0}^{t} (\psi_{es} \bar{D}_{es}) \right)}{\sum_{e \in \{D, HS, C\}} \theta e \bar{p}_{e0} \sum_{t=0}^{163} \left( ((1+\Gamma)(1+N))^{-t} \prod_{s=0}^{t} (\psi_{es} \bar{D}_{es}) \right)}$

$\tau_U = u \mu$

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## Calibration

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient of relative risk aversion</td>
<td>$\rho$</td>
<td>1</td>
</tr>
<tr>
<td>Effective interest rate</td>
<td>$(r - \delta)$</td>
<td>0.01</td>
</tr>
<tr>
<td>Population growth rate</td>
<td>$N$</td>
<td>0.0025</td>
</tr>
<tr>
<td>Technological growth rate</td>
<td>$\Gamma$</td>
<td>0.0037</td>
</tr>
<tr>
<td>Rate of high school dropouts</td>
<td>$\theta_D$</td>
<td>0.11</td>
</tr>
<tr>
<td>Rate of high school graduates</td>
<td>$\theta_{HS}$</td>
<td>0.55</td>
</tr>
<tr>
<td>Rate of college graduates</td>
<td>$\theta_C$</td>
<td>0.34</td>
</tr>
<tr>
<td>Average initial permanent income, dropout</td>
<td>$\bar{p}_{D0}$</td>
<td>5000</td>
</tr>
<tr>
<td>Average initial permanent income, high school</td>
<td>$\bar{p}_{HS0}$</td>
<td>7500</td>
</tr>
<tr>
<td>Average initial permanent income, college</td>
<td>$\bar{p}_{C0}$</td>
<td>12000</td>
</tr>
<tr>
<td>Unemployment insurance payment</td>
<td>$\mu$</td>
<td>0.15</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>$u$</td>
<td>0.07</td>
</tr>
<tr>
<td>Labor income tax rate</td>
<td>$\tau$</td>
<td>0.0942</td>
</tr>
</tbody>
</table>

Carroll, Slacalek, Tokuoka and White
Results: Wealth Distribution

Carroll, Slacalek, Tokuoka and White
Wealth and MPC
## Results: MPC (in Annual Terms)

<table>
<thead>
<tr>
<th>Wealth Measure</th>
<th>Micro Income Process</th>
<th>Life-Cycle Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>KS-JEDC</td>
<td>FBS</td>
</tr>
<tr>
<td></td>
<td>Our solution</td>
<td>β-Dist</td>
</tr>
<tr>
<td>NW</td>
<td>0.05</td>
<td>0.23</td>
</tr>
<tr>
<td>Overall average</td>
<td></td>
<td></td>
</tr>
<tr>
<td>By wealth/permanent income ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Top 20%</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.04</td>
<td>0.12</td>
</tr>
<tr>
<td>Bottom 1/2</td>
<td>0.05</td>
<td>0.35</td>
</tr>
<tr>
<td>By employment status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.05</td>
<td>0.2</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.06</td>
<td>0.54</td>
</tr>
</tbody>
</table>

Notes: Annual MPC is calculated by $1 - (1 - \text{quarterly MPC})^4$. 

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Wealth and MPC
Results: MPC by Age

- Initial drop in MPC: Build-up of buffer stock
- Rise while rapid income growth, fall before retirement, then increasing mortality risk

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Wealth and MPC
Conclusions

- Definition of “serious” microfoundations: Model that matches
  - Income Dynamics
  - Wealth Distribution
- The model produces more plausible implications about:
  - Aggregate MPC
  - Distribution of MPC Across Households
- Version with more plausible aggregate specification is simpler, faster, better in every way!
References


