Buffer Stock Saving in a Krusell–Smith World

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Wealth Heterogeneity and Marginal Propensity to Consume

Consumption/(quarterly) permanent income ratio (left scale)

Histogram: empirical (SCF1998) density of $m_t/(p_t W_t)$ (right scale)
Consumption Modeling

Core since Friedman’s (1957) PIH:

- $c$ chosen optimally; want to smooth $c$ in light of $y$ fluctuations
- Single most important thing to get right is income dynamics!
- With smooth $c$, income dynamics drive everything!
  - Saving/dissaving: Depends on whether $\mathbb{E}[\Delta y] \uparrow$ or $\mathbb{E}[\Delta y] \downarrow$
  - Wealth distribution depends on integration of saving
- Cardinal sin: Assume crazy income dynamics
  - No end (‘match wealth distribution’) can justify this means
  - Throws out the defining core of the intellectual framework
Matching key micro facts may help understand macro ‘puzzles’ unresolvable in Rep Agent models.

Why might heterogeneity matter?

Concavity of the consumption function:
- Different $m \rightarrow$ HHs behave very differently
- $m$ affects
  - MPC
  - $L$ supply
  - response to financial change
The Idea

- Lots of people have cut their teeth on Krusell and Smith (1998) model
- **Our goal:** Bridge KS descr of macro and our descr of micro
- How does the model with realistic household income process improve on KS in matching the wealth distribution?
Friedman (1957): Permanent Income Hypothesis

\[ Y_t = P_t + T_t \]
\[ C_t = P_t \]

Progress since then

- **Micro data**: Friedman description of income shocks works well
- **Math**: Friedman’s words well describe optimal solution to dynamic stochastic optimization problem of impatient consumers with geometric discounting under CRRA utility with uninsurable idiosyncratic risk calibrated using these micro income dynamics (!)
Use the Benchmark KS model with Modifications

Modifications to Krusell and Smith (1998)

1. Serious income process
   - MaCurdy, Card, Abowd; Blundell, Low, Meghir, Pistaferri, . . .
2. Finite lifetimes (i.e., introduce Blanchard (1985) death, D)
Income Process

Idiosyncratic (household) income process is logarithmic Friedman:

\[ y_{t+1} = p_{t+1} \xi_{t+1} W \]
\[ p_{t+1} = p_t \psi_{t+1} \]

\( p_t \) = permanent income
\( \xi_t \) = transitory income
\( \psi_{t+1} \) = permanent shock
\( W \) = aggregate wage rate
Income Process

Modifications from Carroll (1992):
Trans income $\xi_t$ incorporates unemployment insurance:

$$
\xi_t = \mu \text{ with probability } u
$$
$$
= (1 - \tau) \bar{l} \theta_t \text{ with probability } 1 - u
$$

$\mu$ is UI when unemployed
$\tau$ is the rate of tax collected for the unemployment benefits
Model Without Aggr Uncertainty: Decision Problem

\[ v(m_{t,i}) = \max \{ c_{t,i} \} \ u(c_{t,i}) + \beta \mathbb{E}_t \left[ \psi_{t+1,i}^{1-\rho} v(m_{t+1,i}) \right] \]

s.t.

\[ a_{t,i} = m_{t,i} - c_{t,i} \]
\[ a_{t,i} \geq 0 \]
\[ k_{t+1,i} = \frac{a_{t,i}}{(\mathbb{D} \psi_{t+1,i})} \]
\[ m_{t+1,i} = (\overline{\gamma} + r)k_{t+1,i} + \xi_{t+1} \]
\[ r = \alpha a(K/\overline{I}L)^{\alpha-1} \]

Variables normalized by $p_t W$
What Happens After Death?

- You are replaced by a new agent whose permanent income is equal to the population mean.
- Prevents the population distribution of permanent income from spreading out.
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Ergodic Distribution of Permanent Income

Exists, if death eliminates permanent shocks:

$$\mathcal{D} \mathbb{E}[\psi^2] < 1.$$ 

Holds.

Population mean of $p^2$:

$$\mathbb{M}[p^2] = \left( \frac{D}{1 - \mathcal{D} \mathbb{E}[\psi^2]} \right)$$
Parameter Values

- \( \beta, \rho, \alpha, \delta, \tilde{l}, \mu \), and \( u \) taken from JEDC special volume
- Key new parameter values:

<table>
<thead>
<tr>
<th>Description</th>
<th>Param</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob of Death per Quarter</td>
<td>D</td>
<td>0.005</td>
<td>Life span of 50 years</td>
</tr>
<tr>
<td>Variance of Log ( \psi_t )</td>
<td>( \sigma_{\psi}^2 )</td>
<td>0.016/4</td>
<td>Carroll (1992); SCF</td>
</tr>
<tr>
<td>Variance of Log ( \theta_t )</td>
<td>( \sigma_{\theta}^2 )</td>
<td>0.010 ( \times 4 )</td>
<td>Carroll (1992)</td>
</tr>
</tbody>
</table>
### Annual Income, Earnings, or Wage Variances

<table>
<thead>
<tr>
<th></th>
<th>$\sigma^2_\psi$</th>
<th>$\sigma^2_\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our parameters</td>
<td>0.016</td>
<td>0.010</td>
</tr>
<tr>
<td>Carroll (1992)</td>
<td>0.016</td>
<td>0.010</td>
</tr>
<tr>
<td>Storesletten, Telmer, and Yaron (2004)</td>
<td>0.008–0.026</td>
<td>0.316</td>
</tr>
<tr>
<td>Meghir and Pistaferri (2004)*</td>
<td>0.031</td>
<td>0.032</td>
</tr>
<tr>
<td>Low, Meghir, and Pistaferri (2010)</td>
<td>0.011</td>
<td>—</td>
</tr>
<tr>
<td>Blundell, Pistaferri, and Preston (2008)*</td>
<td>0.010–0.030</td>
<td>0.029–0.055</td>
</tr>
<tr>
<td>Implied by KS-JEDC</td>
<td>0.000</td>
<td>0.038</td>
</tr>
<tr>
<td>Implied by Castaneda et al. (2003)</td>
<td>0.03</td>
<td>0.005</td>
</tr>
</tbody>
</table>

*Meghir and Pistaferri (2004) and Blundell, Pistaferri, and Preston (2008) assume that the transitory component is serially correlated (an MA process), and report the variance of a subelement of the transitory component. $\sigma^2_\xi$ for these articles are calculated using their MA estimates.
Cross-Sectional Variance of Income Processes and Data, \( \text{var}(\log y_{t+r,i} - \log y_{t,i}) \)

The data are based on DeBacker, Heim, Panousi, Ramnath, and Vidangos (2013), Figure IV(a) and were normalized so that the variance for \( r = 1 \), \( \text{var}(\log y_{t+1,i} - \log y_{t,i}) \) lie in the middle between the values for the KS and the FBS processes.
Our Models

Solve

1. Standard KS-JEDC
2. FBS, no aggregate uncertainty
3. FBS + KS aggregate uncertainty

Compare model-implied wealth distributions to data
Model(s) with KS Aggregate Shocks

Model with KS Aggregate Shocks: Assumptions

- Only two aggregate states (good or bad)
- Aggregate productivity $a_t = 1 \pm \triangle^a$
- Unemployment rate $u$ depends on the state ($u^g$ or $u^b$)

Parameter values for aggregate shocks from Krusell and Smith (1998)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\triangle^a$</td>
<td>0.01</td>
</tr>
<tr>
<td>$u^g$</td>
<td>0.04</td>
</tr>
<tr>
<td>$u^b$</td>
<td>0.10</td>
</tr>
<tr>
<td>Agg transition probability</td>
<td>0.125</td>
</tr>
</tbody>
</table>
Results: Wealth Distribution

- US data (SCF)
- KS–JEDC
- β–Point
- β–Dist

Percentile

0 25 50 75 100

F

0 0.25 0.5 0.75 1

Percentile

0 25 50 75 100

1
Results: Wealth Distribution

Proportion of Net Worth by Percentile in Models and the Data (in Percent)

<table>
<thead>
<tr>
<th>Percentile of Net Worth</th>
<th>KS-JEDC</th>
<th>Friedman/ Buffer Stock‡</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Our Solution</td>
<td>No Aggr Unc</td>
</tr>
<tr>
<td>Top 1%</td>
<td>$\sigma_\psi^2 = 0.01$</td>
<td>$\sigma_\psi^2 = 0.01$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_\theta^2 = 0.01$</td>
<td>$\sigma_\theta^2 = 0.01$</td>
</tr>
<tr>
<td>Top 10%</td>
<td>2.7</td>
<td>11.5</td>
</tr>
<tr>
<td></td>
<td>20.2</td>
<td>38.9</td>
</tr>
<tr>
<td>Top 20%</td>
<td>35.6</td>
<td>55.3</td>
</tr>
<tr>
<td>Top 40%</td>
<td>60.0</td>
<td>76.5</td>
</tr>
<tr>
<td>Top 60%</td>
<td>78.5</td>
<td>89.7</td>
</tr>
<tr>
<td>Top 80%</td>
<td>92.1</td>
<td>97.4</td>
</tr>
</tbody>
</table>
Conclusions

Micro-founded income process
- helps increase wealth inequality.
- simpler, faster, better in every way!


