

## Buffer Stock Saving in a Krusell–Smith World

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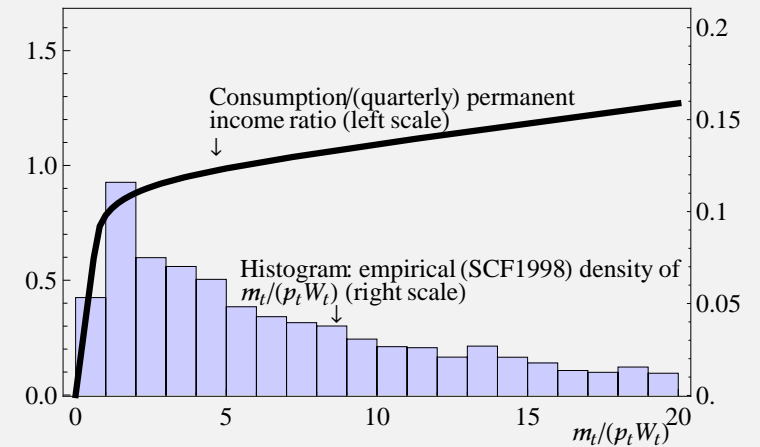
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## Wealth Heterogeneity and Marginal Propensity to Consume



## Consumption Modeling

Core since Friedman's (1957) PIH:

- ▶  $c$  chosen optimally;  
*want to smooth  $c$  in light of  $y$  fluctuations*
- ▶ Single most important thing to get right is **income dynamics!**
- ▶ With smooth  $c$ , income dynamics **drive everything!**
  - ▶ **Saving/dissaving:** Depends on whether  $\mathbb{E}[\Delta y] \uparrow$  or  $\mathbb{E}[\Delta y] \downarrow$
  - ▶ **Wealth distribution** depends on integration of saving
- ▶ **Cardinal sin:** Assume crazy income dynamics
  - ▶ No end ('match wealth distribution') can justify this means
  - ▶ Throws out the defining core of the intellectual framework

## Heterogeneity Matters

- ▶ Matching key micro facts may help understand macro 'puzzles' unresolvable in Rep Agent models
- ▶ Why might heterogeneity matter?
- ▶ **Concavity** of the consumption function:
  - ▶ Different  $m \rightarrow$  HHs behave very differently
  - ▶  $m$  affects
    - ▶ MPC
    - ▶  $L$  supply
    - ▶ *response to financial change*

## The Idea

- ▶ Lots of people have cut their teeth on Krusell and Smith (1998) model
- ▶ **Our goal:** Bridge KS descr of macro and our descr of micro
- ▶ How does the model with realistic household income process improve on KS in matching the wealth distribution?

## Friedman (1957): Permanent Income Hypothesis

$$Y_t = P_t + T_t$$
$$C_t = P_t$$

### Progress since then

- ▶ **Micro data:** Friedman description of income shocks works well
- ▶ **Math:** Friedman's words well describe optimal solution to dynamic stochastic optimization problem of impatient consumers with geometric discounting under CRRA utility with uninsurable idiosyncratic risk calibrated using these micro income dynamics (!)

## Use the Benchmark KS model with Modifications

### Modifications to Krusell and Smith (1998)

1. Serious **income process**
  - ▶ MaCurdy, Card, Abowd; Blundell, Low, Meghir, Pistaferri, ...
2. **Finite lifetimes** (i.e., introduce Blanchard (1985) death, D)

## Income Process

Idiosyncratic (household) income process is logarithmic Friedman:

$$y_{t+1} = p_{t+1}\xi_{t+1}W$$
$$p_{t+1} = p_t\psi_{t+1}$$

$p_t$  = permanent income

$\xi_t$  = transitory income

$\psi_{t+1}$  = permanent shock

$W$  = aggregate wage rate

## Income Process

Modifications from Carroll (1992):

Trans income  $\xi_t$  incorporates **unemployment insurance**:

$$\begin{aligned}\xi_t &= \mu \text{ with probability } u \\ &= (1 - \tau)\bar{l}\theta_t \text{ with probability } 1 - u\end{aligned}$$

$\mu$  is UI when unemployed

$\tau$  is the rate of tax collected for the unemployment benefits

## Model Without Aggr Uncertainty: Decision Problem

$$\begin{aligned}v(m_{t,i}) &= \max_{\{c_{t,i}\}} u(c_{t,i}) + \beta \mathbb{D}\mathbb{E}_t \left[ \psi_{t+1,i}^{1-\rho} v(m_{t+1,i}) \right] \\ &\text{s.t.} \\ a_{t,i} &= m_{t,i} - c_{t,i} \\ a_{t,i} &\geq 0 \\ k_{t+1,i} &= a_{t,i} / (\mathbb{D}\psi_{t+1,i}) \\ m_{t+1,i} &= (\mathbb{T} + r)k_{t+1,i} + \xi_{t+1} \\ r &= \alpha a(K/\bar{L})^{\alpha-1}\end{aligned}$$

Variables normalized by  $p_t W$

## What Happens After Death?

- ▶ You are replaced by a new agent whose permanent income is equal to the population mean
- ▶ Prevents the population distribution of permanent income from spreading out

## Ergodic Distribution of Permanent Income

Exists, if death eliminates permanent shocks:

$$\mathbb{D}\mathbb{E}[\psi^2] < 1.$$

**Holds.**

Population mean of  $p^2$ :

$$\mathbb{M}[p^2] = \left( \frac{\mathbb{D}}{1 - \mathbb{D}\mathbb{E}[\psi^2]} \right)$$

## Parameter Values

- ▶  $\beta, \rho, \alpha, \delta, \bar{l}, \mu$ , and  $u$  taken from JEDC special volume
- ▶ Key new parameter values:

Description	Param	Value	Source
Prob of Death per Quarter	D	0.005	Life span of 50 years
Variance of Log $\psi_t$	$\sigma_\psi^2$	0.016/4	Carroll (1992); SCF
Variance of Log $\theta_t$	$\sigma_\theta^2$	$0.010 \times 4$	Carroll (1992)

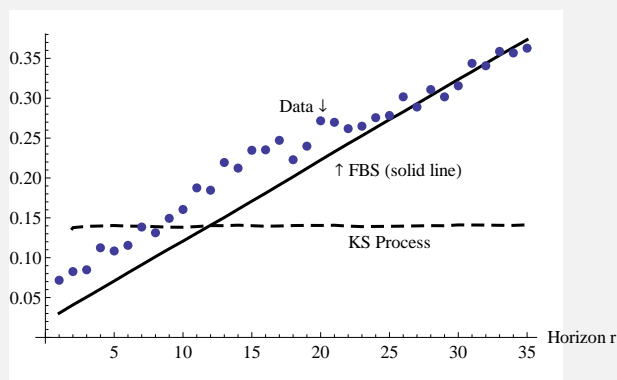
## Annual Income, Earnings, or Wage Variances

	$\sigma_\psi^2$	$\sigma_\xi^2$
Our parameters	0.016	0.010
Carroll (1992)	0.016	0.010
Storesletten, Telmer, and Yaron (2004)	0.008–0.026	0.316
Meghir and Pistaferri (2004)*	0.031	0.032
Low, Meghir, and Pistaferri (2010)	0.011	—
Blundell, Pistaferri, and Preston (2008)*	0.010–0.030	0.029–0.055
Implied by KS-JEDC	0.000	0.038
Implied by Castaneda et al. (2003)	0.03	0.005

\*Meghir and Pistaferri (2004) and Blundell, Pistaferri, and Preston (2008) assume that the transitory component is serially correlated (an MA process), and report the variance of a subelement of the transitory component.  $\sigma_\xi^2$  for these articles are calculated using their MA estimates.

## Cross-Sectional Variance of Income Processes and Data,

$$\text{var}(\log \mathbf{y}_{t+r,i} - \log \mathbf{y}_{t,i})$$



The data are based on DeBacker, Heim, Panousi, Ramnath, and Vidangos (2013), Figure IV(a) and were normalized so that the variance for  $r = 1$ ,  $\text{var}(\log \mathbf{y}_{t+1,i} - \log \mathbf{y}_{t,i})$  lie in the middle between the values for the KS and the FBS processes.

## Our Models

### Solve

1. Standard KS-JEDC
2. FBS, no aggregate uncertainty
3. FBS + KS aggregate uncertainty

Compare model-implied wealth distributions to data

## Model(s) with KS Aggregate Shocks

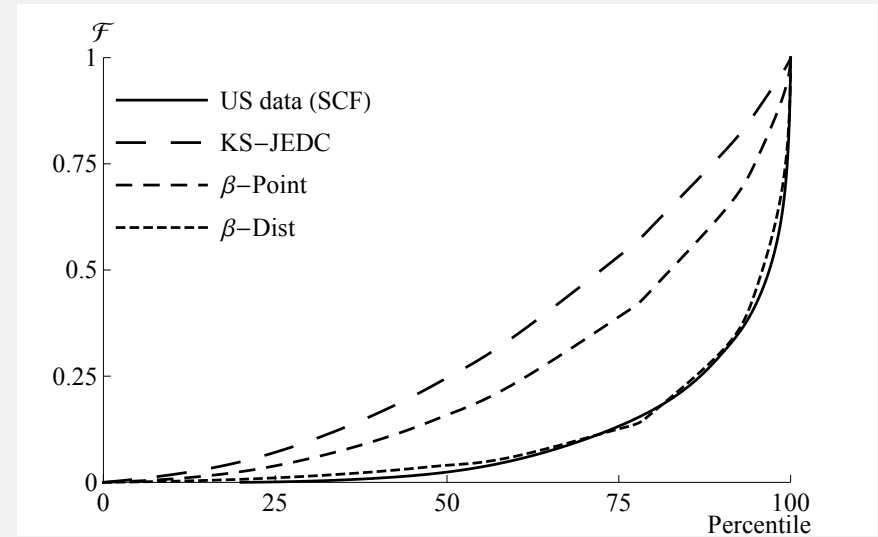
### Model with KS Aggregate Shocks: Assumptions

- ▶ Only two aggregate states (good or bad)
- ▶ Aggregate productivity  $a_t = 1 \pm \Delta^a$
- ▶ Unemployment rate  $u$  depends on the state ( $u^g$  or  $u^b$ )

Parameter values for aggregate shocks from Krusell and Smith (1998)

Parameter	Value
$\Delta^a$	0.01
$u^g$	0.04
$u^b$	0.10
Agg transition probability	0.125

## Results: Wealth Distribution



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Proportion of Net Worth by Percentile in Models and the Data (in Percent)

Percentile of Net Worth	Income Process					Data*
	KS-JEDC	Friedman/ Buffer Stock <sup>‡</sup>				
		No Aggr Unc	KS Aggr Unc			
	Our Solution	$\sigma_{\psi}^2 = 0.01$ $\sigma_{\theta}^2 = 0.01$	$\sigma_{\psi}^2 = 0.01$ $\sigma_{\theta}^2 = 0.01$	$\sigma_{\psi}^2 = 0.01$ $\sigma_{\theta}^2 = 0.15$	$\sigma_{\psi}^2 = 0.03$ $\sigma_{\theta}^2 = 0.01$	
Top 1%	2.7	11.5	9.1	8.8	15.0	33.9
Top 10%	20.2	38.9	35.9	35.3	44.8	69.7
Top 20%	35.6	55.3	52.4	51.9	60.0	82.9
Top 40%	60.0	76.5	74.1	74.0	78.4	94.7
Top 60%	78.5	89.7	88.2	88.2	89.8	99.0
Top 80%	92.1	97.4	96.8	96.9	97.0	100.2

## Conclusions

Micro-founded income process

- ▶ helps increase wealth inequality.
- ▶ simpler, faster, better in every way!

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