A Tractable Model of Precautionary Reserves, Net Foreign Assets, or Sovereign Wealth Funds

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Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
  - China
- Surprising "Upstream" Capital Flows: Developing → Rich Countries
  - China - Following Japan, Korea, Taiwan, Singapore, Hong Kong
- Sovereign Wealth Funds
  - Mainly Oil-Rich Countries

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Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
  - Tractable, Tractable! TRACTABLE!!!
  - The Natural Extension of the Ramsey Model
  - Shows Eqbm Relation Between Precautionary, Other Motives
- Two applications
  - Economic Growth and Capital Flows
  - Impact of Reducing Global Financial Imbalances

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  - "Real" microfoundations!
- Builds on Toche (2005)
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- Small Open Economy
- Balanced Growth Path With Population And Productivity Growth
- Accumulate Buffer Stock to Self-Insure Against Unemployment
- NFA: Aggregate Stock of Wealth Minus Domestic Capital Stock
- Closed-Form Solutions For Equilibrium

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Macroeconomic Assumptions

- Domestic output is produced with the Cobb-Douglas function:

$$\boldsymbol{P}_t = \boldsymbol{K}_t^{\alpha}(z_t \boldsymbol{L}_t)^{1-\alpha}, \tag{1}$$

- Labor productivity increases by G in every period,

$$z_{t+1} = \mathrm{G} z_t. \tag{2}$$

- Capital perfectly mobile internationally,

$$\overbrace{\urcorner}^{\equiv 1-\delta} + \alpha \frac{\boldsymbol{P}_t}{\boldsymbol{K}_t} = \mathrm{R}, \tag{3}$$

- Capital-to-output ratio is constant and equal to,

$$\frac{\boldsymbol{K}}{\boldsymbol{P}} = \frac{\alpha}{\mathrm{R} - \urcorner}. \tag{4}$$

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People and Populations

- Each worker is part of a single 'generation' born at the same time
- Size of generation born at t : Ξ^t.
- Life Stages:
  - Employment
  - Unemployment/Retirement
  - Death
- Transitions to unemployment and death are Poisson processes
  - Flow probabilities $\mathcal{U}$ and D.
- Employed and Unemployed Populations:

$$\mathcal{E}_t = \frac{\Xi^{t+1}}{\Xi - \mathcal{U}}$$

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Balanced Growth

- Capital and output grow at constant rates
- Real wage grows by factor G in every period.
- Main variable of interest= N_t, the aggregate net foreign assets of the economy at the beginning of period t.

$$N_t = S_t - K_t. \tag{5}$$

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- Budget constraint of individual:

$$\frac{\mathbf{s}_{t+1}}{\mathsf{R}} + \mathbf{c}_t = \mathbf{s}_t + \overbrace{\xi_t \ell_t \mathsf{W}_t}^{\text{labor income}} , \tag{6}$$

- Worker's labor supply ℓ grows by a factor X per period over his lifetime,

$$\ell_t = \mathsf{X}^t \ell_0, \tag{7}$$

- For consumer who remains employed, labor income grows by

$$\Gamma \equiv \mathsf{GX}.$$

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The microeconomic consumer's problem

- Unemployment: Complete and permanent destruction of h
- CRRA felicity $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$; geometric discounting at β
- Unemployed convert their wealth into annuities.
- Solution to the unemployed consumer's optimization problem,

$$c_t^u = \kappa^u s_t,$$

where κ is the marginal propensity to consume,

$$\kappa^u \equiv 1 - \not{D}\frac{(\beta \mathrm{R})^{1/\rho}}{\mathrm{R}}.$$

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- 'Growth impatience condition':

$$\Phi_\Gamma \equiv \frac{(\beta R)^{1/\rho}}{\Gamma} < 1$$

necessary for finite target ratio of wealth to income (Carroll (2011))

- Defining nonbold variables as, e.g., $c_t^e = \boldsymbol{c}_t^e/(W_t \ell_t)$, we get

$$s_{t+1}^e = (R/\Gamma)\,(s_t^e - c_t^e + 1)\,. \tag{8}$$

$$c_{t+1}^e = \Phi_\Gamma \mathcal{U}^{1/\rho} c_t^e \left[1 - \mathcal{U}\left(\frac{\Phi_\Gamma}{\kappa^u} \frac{c_t^e}{R/\Gamma(s_t^e - c_t^e + 1)}\right)^\rho\right]^{-1/\rho}. \tag{9}$$

- Saddle-point stable dynamics.

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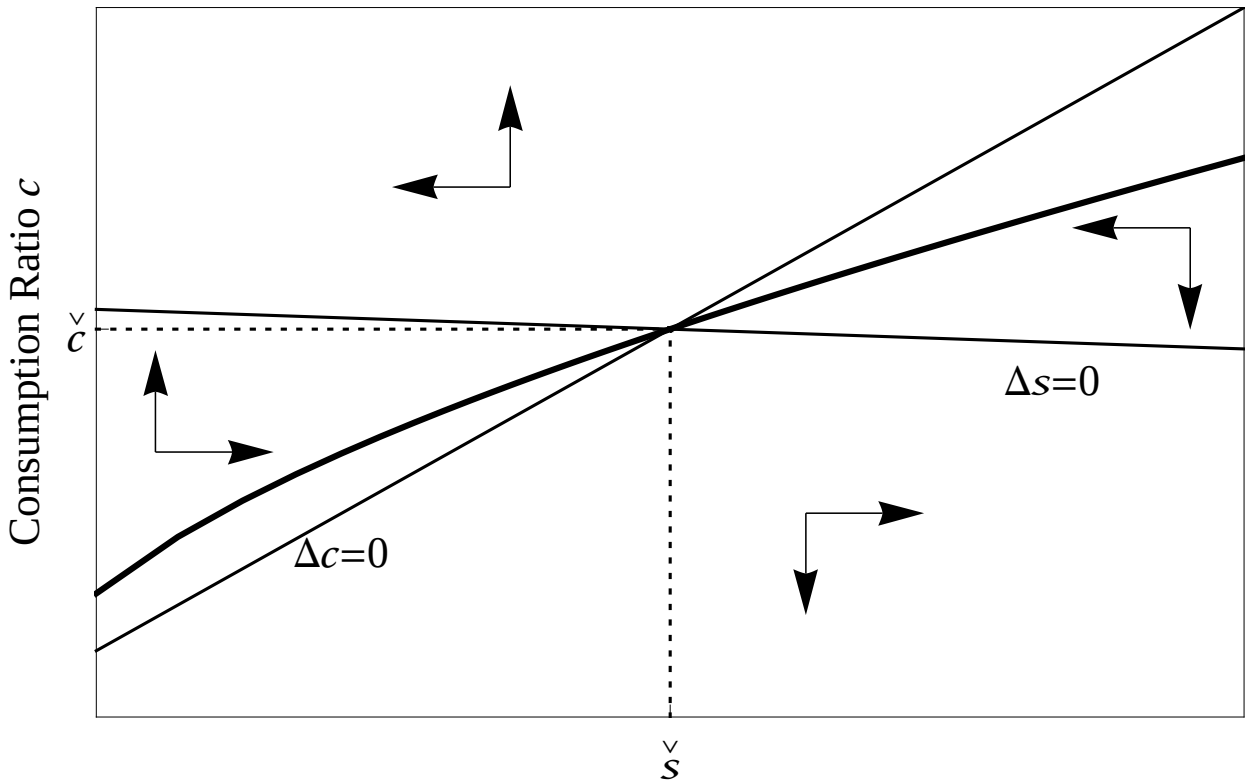
Phase Diagram



The Growth Impatience Condition

- Target wealth-to-income ratio: impatience vs prudence.
- Closed-form solution for the target wealth-to-income ratio

$$\check{s} = \left[\frac{\Gamma}{R} - 1 + \kappa^{u} \left(1 + \frac{\mathbb{P}_{\Gamma}^{-\rho} - 1}{\mho} \right)^{1/\rho} \right]^{-1}. \tag{10}$$

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$$\frac{\partial \check{s}}{\partial \mho} > 0, \frac{\partial \check{s}}{\partial \beta} > 0, \frac{\partial \check{s}}{\partial \Gamma} < 0. \tag{11}$$

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- The response of $\check{s}$ to R is ambiguous.

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$$\check{s} = \left[\frac{\Gamma}{R} - 1 + \kappa^{u} \left(1 + \frac{\mathbf{\Phi}_{\Gamma}^{-\rho} - 1}{\mho} \right)^{1/\rho} \right]^{-1} . \tag{10}$$

-

$$\frac{\partial \check{s}}{\partial \mho} > 0, \frac{\partial \check{s}}{\partial \beta} > 0, \frac{\partial \check{s}}{\partial \Gamma} < 0. \tag{11}$$

$$\frac{\partial \check{s}}{\partial \rho} > 0. \tag{12}$$

- The response of $\check{s}$ to R is ambiguous.

Foreign Assets

- Ratio of employed workers' wealth to output,

$$S_t^e = \frac{\boldsymbol{S}_t^e}{\boldsymbol{P}_t} = (1-\alpha)\left(1 - \underbrace{\frac{\cancel{\mho}\mathrm{X}}{\Xi}}_{\equiv\Lambda}\right)\sum_{n=0}^{+\infty}\Lambda^n s_{t,t-n}^e, \tag{13}$$

where Λ is the factor by which the share of a generation in total labor supply shrinks every period.

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Foreign Assets (cont)

- Steady state ratio of net foreign assets to GDP

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- Depends on Employed Workers' Target Savings

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'Stakes'

- Model with no stakes

$$S^e = \frac{S^e}{P} = (1-\alpha)(1-\Lambda)\sum_{n=0}^{+\infty}\Lambda^n s^e(n). \quad (16)$$

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$$\check{\check{S}} = \frac{S^e}{P} = (1-\alpha)\check{\check{s}}. \quad (17)$$

where

$$\check{\check{s}} = \left[\frac{\Gamma}{R} - \frac{1}{2-\Lambda} + \kappa^u\left(1 + \frac{\Phi_\Gamma^{-\rho} - 1}{\upsilon}\right)^{1/\rho}\right]^{-1} \quad (18)$$

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Calibration and Simulation

Table 1

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- $\boldsymbol{N/P} = 0.17$ in the model with no stakes
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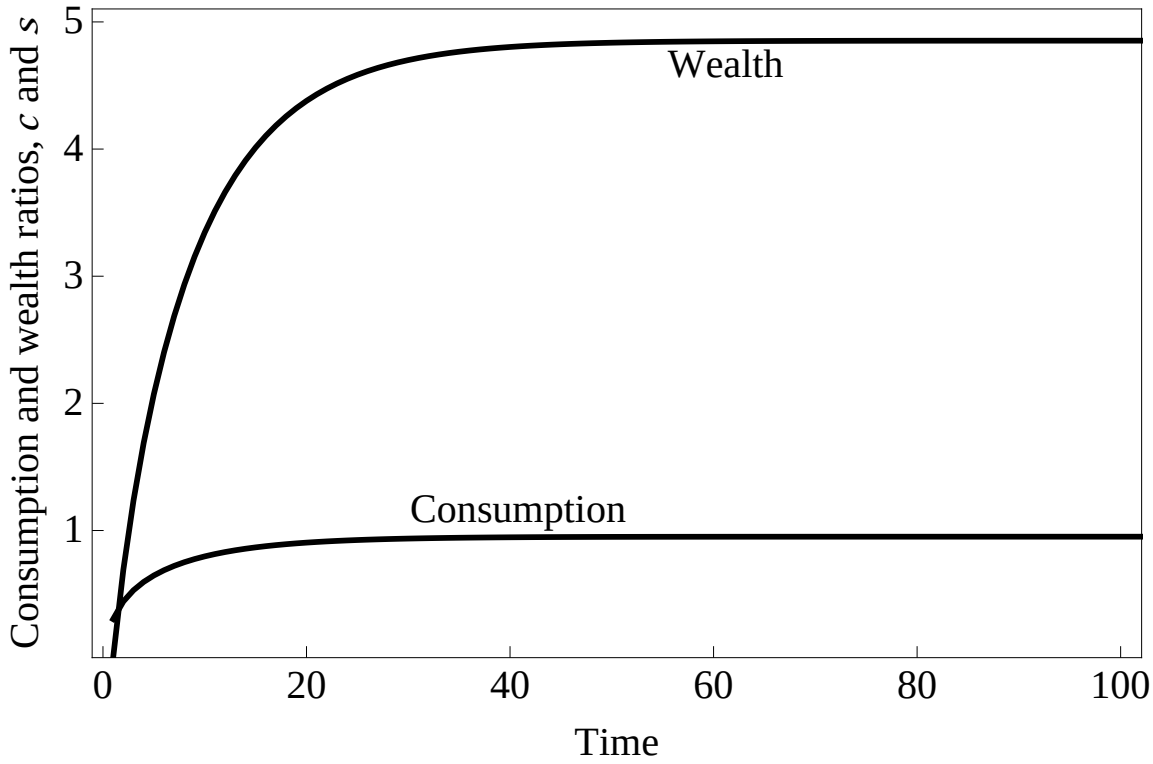
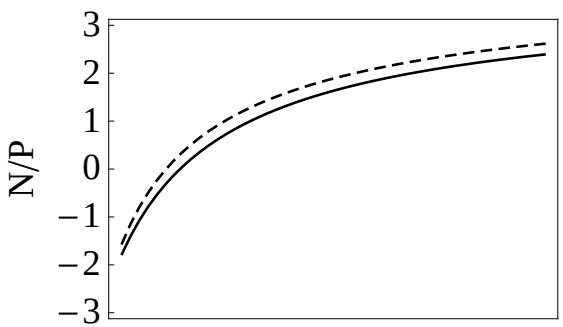
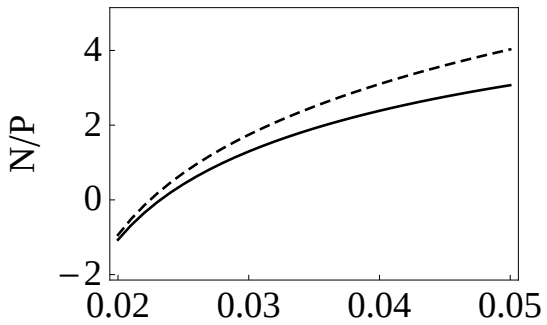

Consumption and wealth ratios, c and s
5
4
3
2
1
Wealth
Consumption
0
20
40
60
80
100
Time

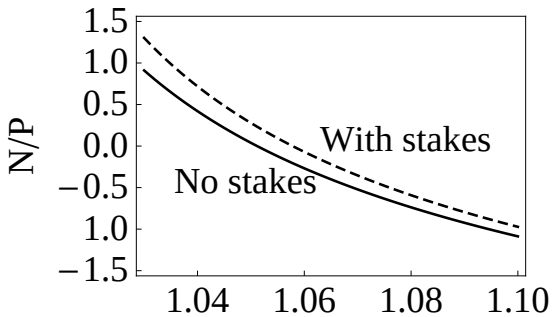
Sensitivity analysis



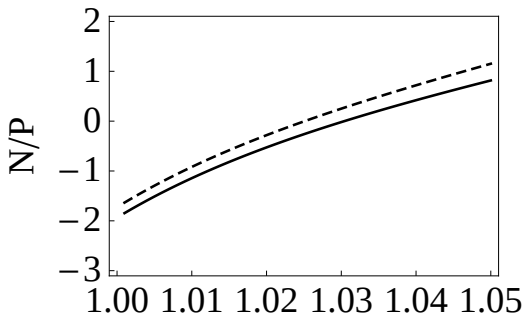
Risk aversion, ρ



Unemployment probability, $\mho$



Productivity growth, G



Interest factor, R

Social Insurance

- Many countries have social transfers to unemployed/retired
- New assumption: labor income tax on the employed in order to finance transfers to the unemployed.
- Unemployed receive transfer whose value is a multiple ς of the labor income that they would have received if they had remained employed.
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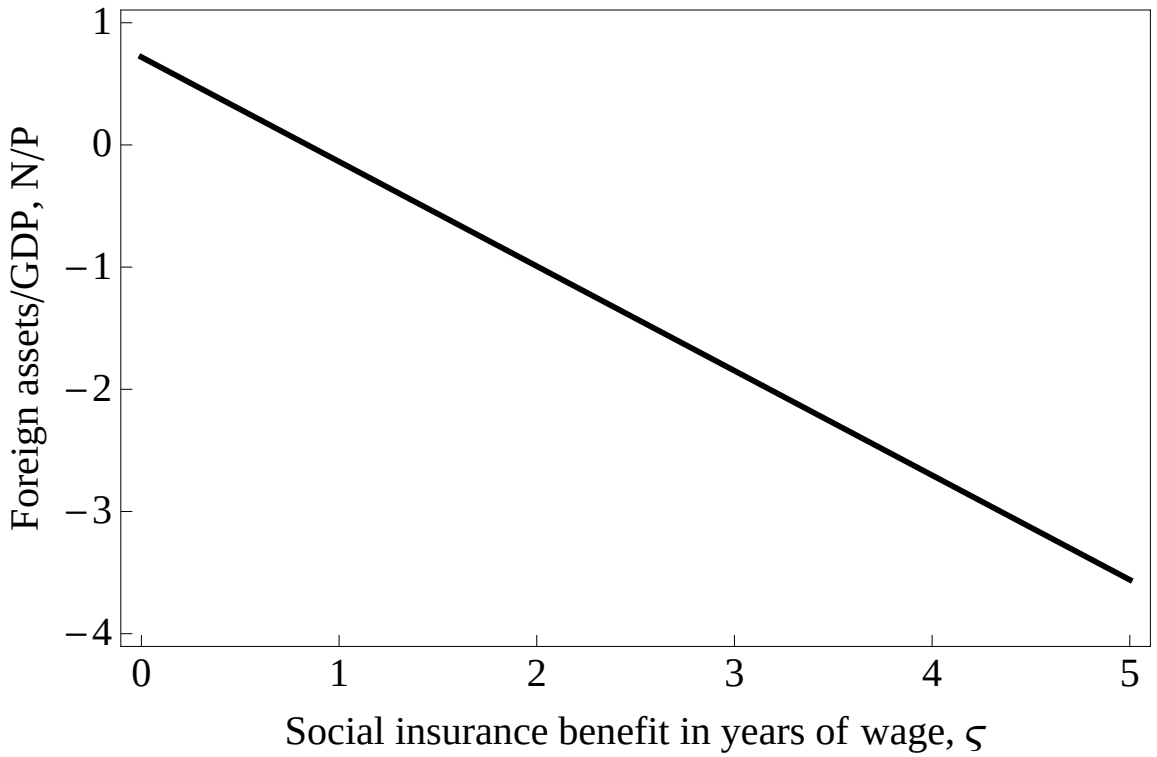

Growth And Saving

- Theory: Good Growth Prospects $\rightarrow$ Should Borrow to Invest
- Data: Fast-Growing Countries *Export* Capital
  - Carroll and Weil (1994); Loayza et al. (2000); Attanasio et al. (2000); Gourinchas and Jeanne, 2007, Prasad, Rajan and Subramanian (2007); Sandri (2008)
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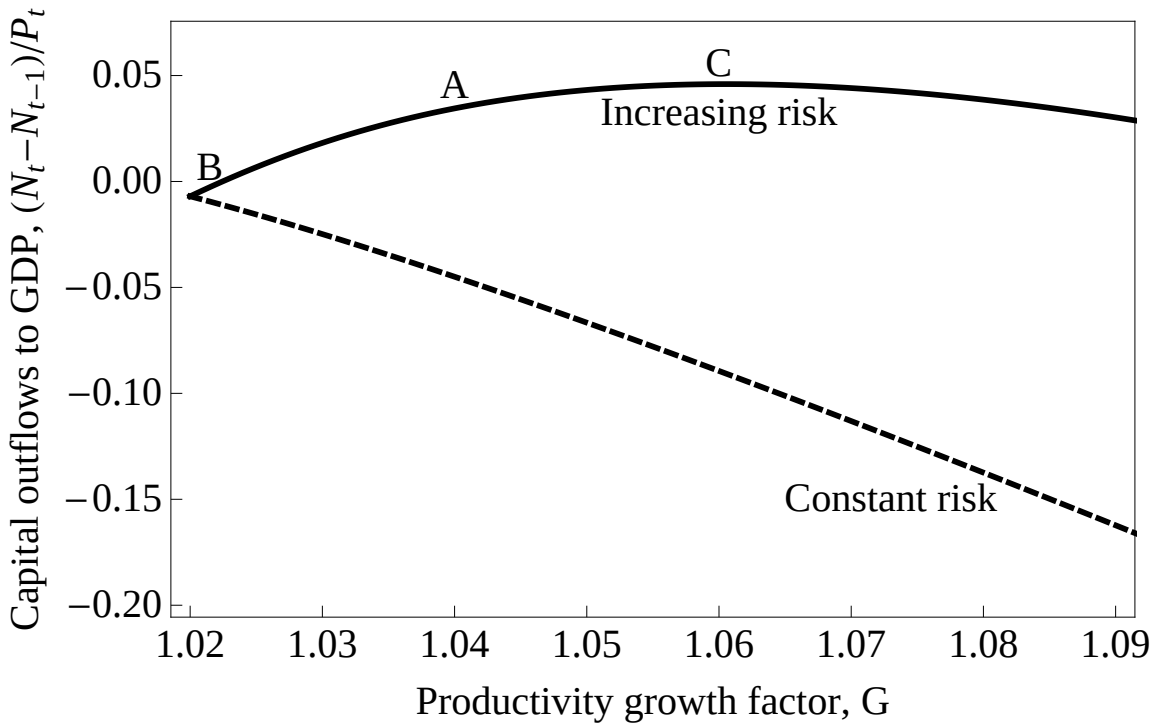
Growth and capital flows



World General Equilibrium

- Small economy assumption not appropriate to study global savings glut or adjustment of global financial imbalances.
- Study steady state equilibria in two-country extension of the model.
- Global interest rate R endogenous

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General Equilibrium

- Two countries identical except for size (h=20%, f=80%) and level of social insurance ($\varsigma_h = 1.5$, $\varsigma_f = 0.75$).
- This implies

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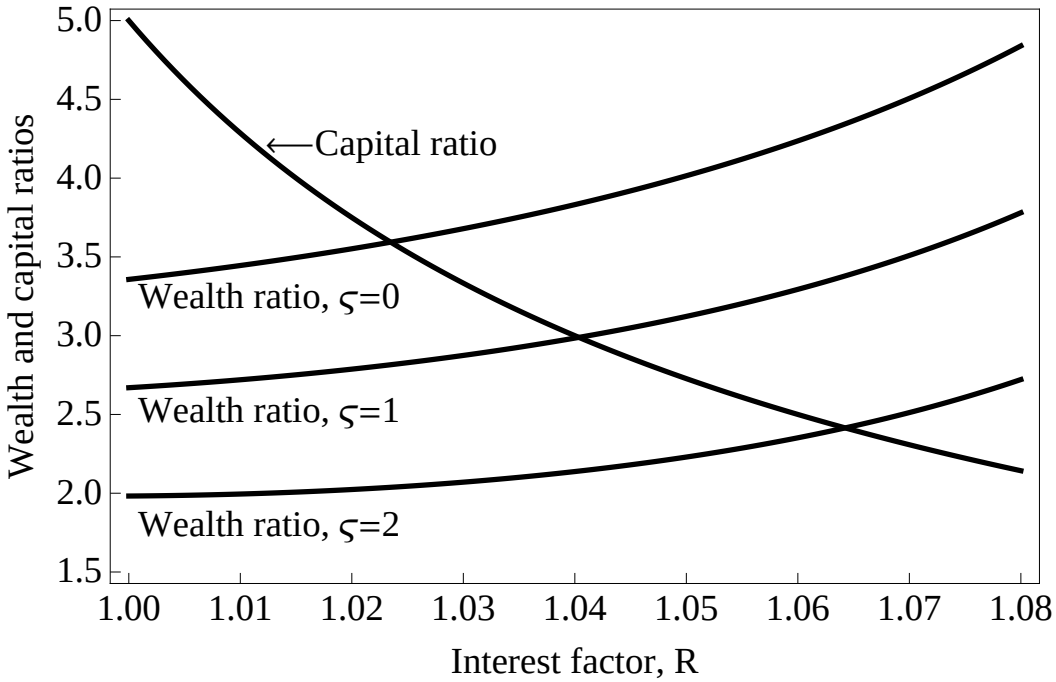

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