

A Tractable Model of Precautionary Reserves, Net Foreign Assets, or Sovereign Wealth Funds

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Motivation

Three Hot Topics In International Macro:

- Huge Reserve Accumulation By Fast-Growing Developing Economies
 - China
- Surprising “Upstream” Capital Flows: Developing → Rich Countries
 - China – Following Japan, Korea, Taiwan, Singapore, Hong Kong
- Sovereign Wealth Funds
 - Middle East, China, Russia

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Connection?

Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
 - The Case of a Tractable TRACTABLE EU
 - The Natural Extension of the Ramsey Model
 - Shows Equiv. Relation between Precautionary, Other Motives
- Two applications
 - Economic Growth and Capital Flows
 - The Impact of Reducing Global Financial Imbalances

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Precautionary Motives Commonly Cited In All Three Cases

- Our Model of Precautionary Net Foreign Assets:
 - The Role of Precautionary Net Foreign Assets
 - The Impact of Precautionary Net Foreign Assets on Economic Growth
- Two applications
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- Calibration and Simulation
- Applications
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Overview

- Small Open Economy
- Balanced Growth Path With Population And Productivity Growth
- Accumulate Buffer Stock to Self-Insure Against Unemployment
- NFA: Aggregate Stock of Wealth Minus Domestic Capital Stock
- Closed-Form Solutions For Equilibrium

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Macroeconomic Assumptions

- Domestic output is produced with the Cobb-Douglas function:

$$Y_t = K_t^\alpha (z_t L_t)^{1-\alpha}, \quad (1)$$

- Labor productivity increases by G in every period,

$$z_{t+1} = G z_t. \quad (2)$$

- Capital perfectly mobile internationally,

$$\overbrace{\tau}^{\equiv 1-\delta} + \alpha \frac{Y_t}{K_t} = R, \quad (3)$$

- Capital-to-output ratio is constant and equal to,

$$\frac{K}{Y} = \frac{\alpha}{R - \tau}. \quad (4)$$

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People and Populations

- Each worker is part of a single 'generation' born at the same time
- Size of generation born at t : Ξ^t .
- Life Stages:
 - Employment
 - Unemployment/Retirement
 - Death
- Transitions to unemployment and death are Poisson processes
 - Flow probabilities ϑ and \mathcal{D} .
- Employed and Unemployed Populations:

$$\mathcal{E}_t = \frac{\Xi^{t+1}}{\Xi - \vartheta}$$

$$\mathcal{U}_t = \frac{\vartheta \Xi^{t+1}}{(\Xi - \mathcal{D})(\Xi - \vartheta)}$$

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Balanced Growth

- Capital and output grow at constant rates
- Real wage grows by factor G in every period.
- Main variable of interest = N_t , the aggregate net foreign assets of the economy at the beginning of period t .

$$N_t = B_t - K_t. \quad (5)$$

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The microeconomic consumer's problem

- Budget constraint of individual:

$$\frac{b_{t+1}}{R} + c_t = b_t + \overbrace{\xi_t l_t W_t}^{\text{labor income}}, \quad (6)$$

- Worker's labor supply l grows by a factor X per period over his lifetime,

$$l_t = X^t l_0, \quad (7)$$

- For consumer who remains employed, labor income grows by

$$\Gamma \equiv GX.$$

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The microeconomic consumer's problem

- Unemployment: Complete and permanent destruction of h
- CRRA felicity $u(\bullet) = \bullet^{1-\rho}/(1-\rho)$; geometric discounting at β
- Unemployed convert their wealth into annuities.
- Solution to the unemployed consumer's optimization problem,

$$c_t^u = \kappa^u b_t,$$

where κ is the marginal propensity to consume,

$$\kappa^u \equiv 1 - \beta \frac{(\beta R)^{1/\rho}}{R}.$$

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The microeconomic consumer's problem

- 'Growth impatience condition':

$$\mathbf{D}_\Gamma \equiv \frac{(\beta R)^{1/\rho}}{\Gamma} < 1$$

necessary for finite target ratio of wealth to income (Carroll (2011))

- Defining nonbold variables as, e.g., $c_t^e = \mathbf{c}_t^e / (W_t \ell_t)$, we get

$$b_{t+1}^e = (R/\Gamma) (b_t^e - c_t^e + 1). \quad (8)$$

$$c_{t+1}^e = \mathbf{D}_\Gamma \mathcal{U}^{1/\rho} c_t^e \left[1 - \mathcal{U} \left(\frac{\mathbf{D}_\Gamma c_t^e}{\kappa^u R/\Gamma (b_t^e - c_t^e + 1)} \right)^\rho \right]^{-1/\rho}. \quad (9)$$

- Saddle-point stable dynamics.

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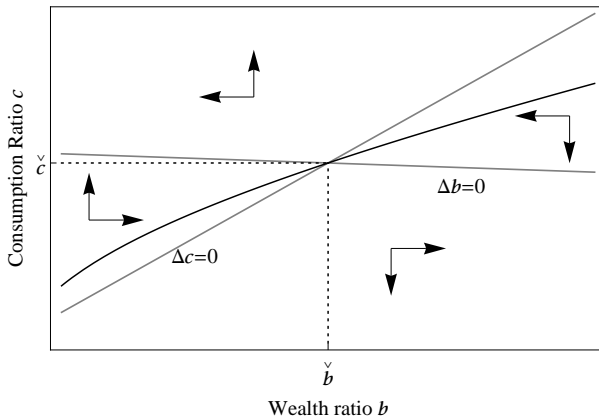
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Phase Diagram



The Growth Impatience Condition

- Target wealth-to-income ratio: impatience vs prudence.
- Closed-form solution for the target wealth-to-income ratio

$$\check{b} = \left[\frac{\Gamma}{R} - 1 + \kappa^u \left(1 + \frac{\mathbf{P}_{\Gamma}^{-\rho} - 1}{\mathcal{U}} \right)^{1/\rho} \right]^{-1}. \quad (10)$$

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$$\frac{\partial \check{b}}{\partial \mathcal{U}} > 0, \quad \frac{\partial \check{b}}{\partial \beta} > 0, \quad \frac{\partial \check{b}}{\partial \Gamma} < 0. \quad (11)$$

$$\frac{\partial \check{b}}{\partial \rho} > 0. \quad (12)$$

- The response of \check{b} to R is ambiguous.

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Foreign Assets

- Ratio of employed workers' wealth to output,

$$B_t^e = \frac{B_t^e}{Y_t} = (1 - \alpha) \left(1 - \underbrace{\frac{\delta X}{\Xi}}_{\equiv \Lambda} \right) \sum_{n=0}^{+\infty} \Lambda^n b_{t,t-n}^e, \quad (13)$$

where Λ is the factor by which the share of a generation in total labor supply shrinks every period.

- The Level of Unemployed Workers' Wealth is

$$B_{t+1}^u = R(1 - \kappa^u)B_t^u + \delta B_{t+1}^e. \quad (14)$$

Foreign Assets

- Ratio of employed workers' wealth to output,

$$B_t^e = \frac{B_t^e}{Y_t} = (1 - \alpha) \left(1 - \underbrace{\frac{\bar{w}X}{\Xi}}_{\equiv \Lambda} \right) \sum_{n=0}^{+\infty} \Lambda^n b_{t,t-n}^e, \quad (13)$$

where Λ is the factor by which the share of a generation in total labor supply shrinks every period.

- The Level of Unemployed Workers' Wealth is

$$B_{t+1}^u = R(1 - \kappa^u)B_t^u + \bar{w}B_{t+1}^e. \quad (14)$$

Foreign Assets (cont)

- Steady state ratio of net foreign assets to GDP

$$\frac{N}{Y} = \frac{\Xi G}{R} \left(1 + \frac{\Upsilon \Xi G}{\Xi G - \beta R^{1/\rho}} \right) \frac{B^e}{Y} - \Xi G \left(\frac{\alpha}{R - \tau} \right). \quad (15)$$

- Depends on Employed Workers' Target Savings

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'Stakes'

- Model with no stakes

$$B^e = \frac{B^e}{Y} = (1 - \alpha)(1 - \Lambda) \sum_{n=0}^{+\infty} \Lambda^n b^e(n). \quad (16)$$

- Model with stakes yielding a representative agent

$$\check{B} = \frac{B^e}{Y} = (1 - \alpha)\check{b}. \quad (17)$$

where

$$\check{b} = \left[\frac{\Gamma}{R} - \frac{1}{2 - \Lambda} + \kappa^u \left(1 + \frac{\mathbf{P}_\Gamma^{-\rho} - 1}{\mathcal{U}} \right)^{1/\rho} \right]^{-1} \quad (18)$$

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Calibration and Simulation

Table 1

α	δ	Ξ	G	R	β^{-1}	Φ	\bar{U}	ρ	d
0.3	0.06	1.01	1.04	1.04	1.04	1.01	0.025	2	0.05

- $N/Y = 0.17$ in the model with no stakes
- $N/Y = 0.79$ in the model with stakes

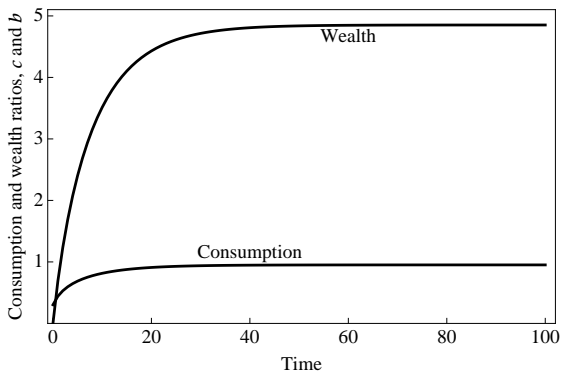
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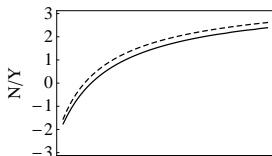
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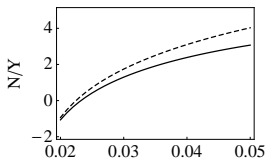
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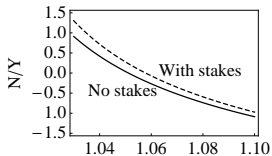
Sensitivity analysis



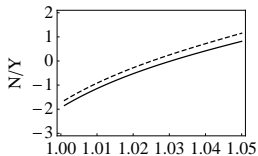
Risk aversion, ρ



Unemployment probability, U



Productivity growth, G



Interest factor, R

Social Insurance

- Many countries have social transfers to unemployed/retired
- New assumption: labor income tax on the employed in order to finance transfers to the unemployed.
- Unemployed receive transfer whose value is a multiple ς of the labor income that they would have received if they had remained employed.
- New formula for target wealth-to-income ratio. Going through the same steps as before, we get

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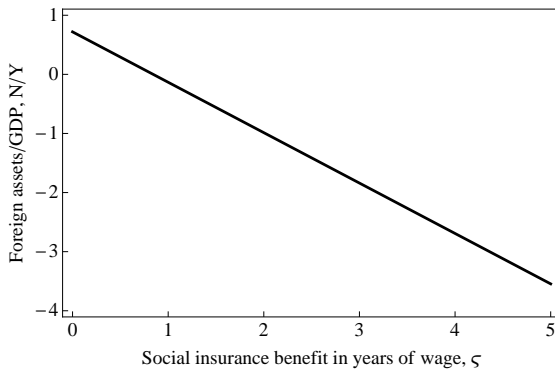
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- Data: Fast-Growing Countries *Export* Capital
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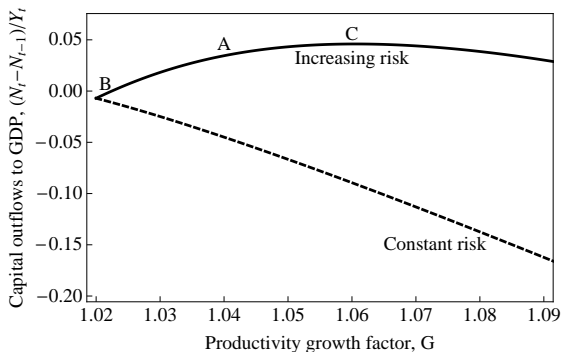
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Growth and capital flows



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- Small economy assumption not appropriate to study global savings glut or adjustment of global financial imbalances.
- Study steady state equilibria in two-country extension of the model.
- Global interest rate R endogenous

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- Two countries identical except for size ($h=20\%$, $f=80\%$) and level of social insurance ($\varsigma_h = 1.5$, $\varsigma_f = 0.75$).
- This implies

$$\frac{N_h}{Y_h} = -0.5 \quad (21)$$

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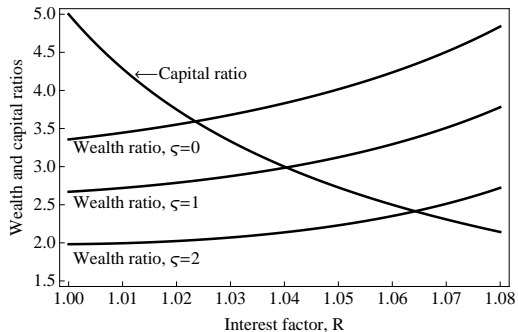
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