Sticky Expectations and Consumption Dynamics

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Consumption Dynamics: Macro vs Micro

Macro: Representative Agent Models

- Theory (With Separable Utility):
  - C responds instantly, completely to shock
  - Consequences of uncertainty are trivial
- Evidence: Consumption is too smooth (Campbell & Deaton, 1989)
- Solution: **"Habits" parameter** $\chi^{\textbf{Macro}} \approx 0.6 \sim 0.8$

$$\Delta \log C_{t+1} = \varsigma + \chi \Delta \log C_t + \epsilon$$

Micro: Heterogeneous Agent Models

- **Uninsurable risk is essential, changes everything**
- Var of micro income shocks much larger than of macro shocks:

$$\text{var}(\Delta \log p) \approx 100 \times \text{var}(\Delta \log P)$$

- Evidence: **"Habits" parameter** $\chi^{\textbf{Micro}} \approx 0.0 \sim 0.1$

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Persistence of Consumption Growth: Macro vs Micro

- New paper in EER, Havranek, Rusnak, and Sokolova (2017)
  Meta analysis of 597 estimates of χ
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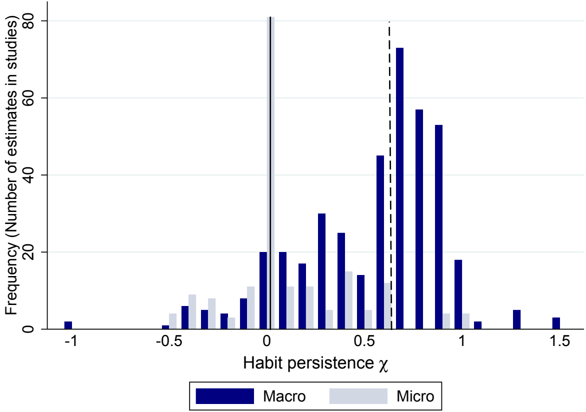

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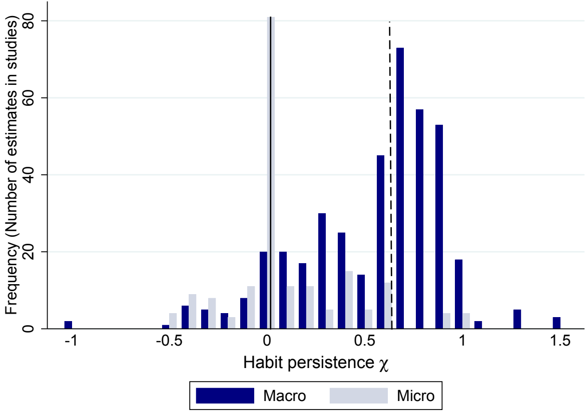

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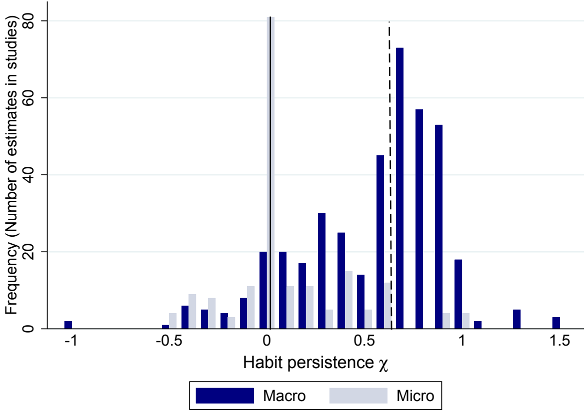

Claim: It's Not Habits, It's Inattention! (Macro not Micro)

Our Setup

- Income Has Idiosyncratic and Aggregate Components
- Idiosyncratic Component Is Perfectly Observed
- **Aggregate Component Is Stochastically Observed**
  - Updating à la Calvo (1983)

Not *ad hoc*

- Identical: Mankiw and Reis (2002), Carroll (2003)
- Similar: Reis (2006), Sims (2003), ...

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Why Macro Inattention Is Plausible

Idiosyncratic Variability Is $\sim 100\times$ Bigger

- If Same Specification Estimated on Micro vs Macro Data
- Pervasive Lesson of All Micro Data

Utility Cost of Inattention Small

- Micro: Critical (and Easy) To Notice You're Unemployed
- Macro: *Not* Critical To *Instantly* Notice If U $\uparrow$

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Quadratic Utility Frictionless Benchmark

Hall (1978) Random Walk

- Total Wealth (Human + Nonhuman):

$$o_{t+1} = (o_t - c_t)R + \zeta_{t+1}$$

- C Euler Equation:

$$u'(c_t) = R\beta\mathbb{E}_t[u'(c_{t+1})]$$

- $\Rightarrow$ Random Walk (for $R\beta = 1$):

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- Expected Wealth:

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Sticky Expectations—Individual c

- Consumer who happens to update at t and $t+n$

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\begin{aligned}
c_t &= (r/R)o_t \\
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\vdots & \quad \vdots \\
c_{t+n-1} &= c_t
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- Implies that $\Delta^n o_{t+n} \equiv o_{t+n} - o_t$ is white noise
- So **individual** c is RW across updating periods:

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Sticky Expectations—Aggregate C

- Pop normed to one, uniformly dist on [0, 1]: $C_t = \int_0^1 c_{t,i}\, di$
- **Calvo (1983)-Type Updating of Expectations:**
  - Probability $\Pi = 0.25$ (per quarter)
- Economy composed of many sticky-$\mathbb{E}$ consumers:

$$C_{t+1} = (1-\Pi)\underbrace{C^{\not\pi}_{t+1}}_{=C_t} + \Pi C^{\pi}_{t+1}$$

$$\Delta C_{t+1} \approx \underbrace{(1-\Pi)}_{\equiv\chi=0.75}\Delta C_t + \epsilon_{t+1}$$

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- Pop normed to one, uniformly dist on $[0,1]$: $\mathrm{C}_t = \int_0^1 \mathrm{c}_{t,i}\,\mathrm{d}i$
- **Calvo (1983)-Type Updating of Expectations:**
    - Probability $\Pi = 0.25$ (per quarter)
- Economy composed of many sticky-$\mathbb{E}$ consumers:

$$\begin{aligned} \mathrm{C}_{t+1} &= (1-\Pi)\underbrace{\mathrm{C}^{\not\pi}_{t+1}}_{=\mathrm{C}_t} + \Pi \mathrm{C}^{\pi}_{t+1} \\ \Delta \mathrm{C}_{t+1} &\approx \underbrace{(1-\Pi)}_{\equiv\chi=0.75}\Delta \mathrm{C}_t + \epsilon_{t+1} \end{aligned}$$

- **Substantial persistence ($\chi = 0.75$) in aggregate C growth**

One More Ingredient: Idiosyncratic Uncertainty . . .

- **Differences: Idiosyncratic vs Aggregate shocks**
  - Idiosyncratic shocks: Frictionless observation
    - I notice if I am fired, promoted, somebody steals my wallet
    - True RW with respect to these
  - Aggregate shocks: Sticky observation
    - May not instantly notice changes in aggregate productivity
- **Result:**
  - Idiosyncratic Δc: dominated by frictionless RW part
  - Aggregate ΔC: highly serially correlated
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Serious Models

Partial Equilibrium/Small Open Economy

- CRRA Utility
- Idiosyncratic Shocks Calibrated From Micro Data
- Aggregate Shocks Calibrated From Macro Data
- Markov Process (Discrete RW) for Aggr Income Growth
  - Handles changing growth 'eras'
- Liquidity Constraint
- Mildly Impatient Consumers

DSGE Heterogeneous Agents (HA) Model

- Same!

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Income Process

- Individual's labor productivity is

$$\boldsymbol{\ell}_{t,i} = \overbrace{\theta_{t,i}\Theta_t}^{\equiv \boldsymbol{\theta}_{t,i}} \overbrace{p_{t,i}P_t}^{\equiv \boldsymbol{p}_{t,i}}$$

- Idiosyncratic and aggregate p evolve according to

$$\begin{aligned} p_{t+1,i} &= p_{t,i}\psi_{t+1,i} \\ P_{t+1} &= \Phi_{t+1}P_t \ \Psi_{t+1} \end{aligned}$$

- **Φ is Markov 'underlying' aggregate pty growth**
  - Discrete (bounded) random walk
  - Calibrated to match postwar US pty growth variation
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Blanchard (1985) Mortality and Insurance

- Household survives from t to $t+1$ with probability $(1-\mathsf{D})$:

$$p_{t+1,i} = \begin{cases} 1 & \text{for newborns} \\ p_{t,i}\psi_{t+1,i} & \text{for survivors} \end{cases}$$

- Blanchardian scheme:

$$\mathrm{k}_{t+1,i} = \begin{cases} 0 & \text{if HH } i \text{ dies, is replaced by newborn} \\ \mathrm{a}_{t,i}/(1-\mathsf{D}) & \text{if household } i \text{ survives} \end{cases}$$

- Implies for aggregate:

$$\begin{aligned} \mathrm{K}_{t+1} &= \int_0^1 \left(\frac{1-\mathrm{d}_{t+1,i}}{1-\mathsf{D}}\right) \mathrm{a}_{t,i}\, \mathrm{d}i = \mathrm{A}_t \\ K_{t+1} &= A_t/(\Psi_{t+1}\Phi_{t+1}) \end{aligned}$$

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Resources

- Market resources:

$$\mathrm{m}_{t,i} = \underbrace{\mathrm{W_t}\boldsymbol{\ell}_{t,i}}_{\equiv \mathrm{y}_t} + \underbrace{\mathscr{R}_t}_{\daleth + \mathrm{r}_t} \mathrm{k}_{t,i}$$

- End-of-Period 'Assets'—Unspent resources:

$$\mathrm{a}_{t,i} = \mathrm{m}_{t,i} - \mathrm{c}_{t,i}$$

- Capital transition depends on prob of survival $1 - \mathrm{D}$:

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Frictionless Solution

- For exposition: Assume constant W and $\mathscr{R}$
- Normalize everything by $\boldsymbol{p}_{t,i} \equiv p_{t,i}P_t$, e.g. $m_{t,i} = \mathrm{m}_{t,i}/(p_{t,i}P_t)$
- $\mathrm{c}(m, \Phi)$ is the function that solves:

$$v(m_{t,i}, \Phi_t) = \max_{c} u(c) + \not{D}\beta\mathbb{E}_t\left[(\Phi_{t+1}\psi_{t+1,i})^{1-\rho} v(m_{t+1,i}, \Phi_{t+1})\right]$$

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Sticky Expectations about Aggregate Income

Calvo Updating of Perceptions of Aggregate Shocks

- *True* Permanent income: $P_{t+1} = \Phi_{t+1} P_t \Psi_{t+1}$
- Tilde ($\widetilde{P}$) denotes perceived variables
- Perception for consumer who has not updated for n periods:

$$\widetilde{P}_{t,i} = \mathbb{E}_{t-n}\left[P_t \middle| \Omega_{t-n}\right] = \Phi_{t-n}^{n} P_{t-n}$$

because Φ is random walk

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Sequence Within Period

1. Income shocks are realized and every individual sees her true y and m, i.e. $\mathrm{y}_{t,i} = \widetilde{\mathrm{y}}_{t,i}$ and $\mathrm{m}_{t,i} = \widetilde{\mathrm{m}}_{t,i}$ for all t and i
2. Updating shocks realized: i observes true P_t, Φ_t w/ prob Π; forms perceptions of her normalized market resources $\widetilde{m}_{t,i}$
3. Consumes based on her perception, using $\mathrm{c}(\widetilde{m}_{t,i}, \widetilde{\Phi}_{t,i})$

   Key Assumption:

   - People act as if their perceptions about aggregate state $\{\widetilde{P}_{t,i}, \widetilde{\Phi}_{t,i}\}$ are the true aggregate state $\{P_t, \Phi_t\}$

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Behavior under Sticky Expectations

- Normalized resources:
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  - $\widetilde{m}_{t,i} \equiv \mathrm{m}_{t,i}/(p_{t,i}\widetilde{P}_{t,i})$ is *perceived*
- **Usually $\widetilde{m}_{t,i} \neq m_{t,i}$ because P_t not perfectly observed**
  - in levels: $\widetilde{\mathrm{m}}_{t,i} = \mathrm{m}_{t,i}$; but normalized: $\widetilde{m}_{t,i} \neq m_{t,i}$
- Consumers behave according to frictionless consumption function
- But **based on** $\widetilde{m}_{t,i}$ (not $m_{t,i}$):

$$
\begin{aligned}
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\mathbf{c}_{t,i} &= \widetilde{c}_{t,i} \times p_{t,i}\widetilde{P}_{t,i}
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- Correctly perceive level of their own spending $\mathbf{c}_{t,i}$

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- Correctly perceive level of their own spending $\mathrm{c}_{t,i}$

Behavior under Sticky Expectations

- Normalized resources:
  - $m_{t,i} \equiv \mathrm{m}_{t,i} \big/ (p_{t,i} P_t)$ is *actual*
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- **Usually** $\widetilde{m}_{t,i} \neq m_{t,i}$ **because** P_t **not perfectly observed**
  - in levels: $\widetilde{\mathrm{m}}_{t,i} = \mathrm{m}_{t,i}$; but normalized: $\widetilde{m}_{t,i} \neq m_{t,i}$
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DSGE Heterogeneous Agents Model

- Idiosyncratic and aggregate shocks same as PE/SOE
- Endogenous W_t and $\mathscr{R}_t$
- Aggregate market resources M_t is a state variable

$$v(m_{t,i}, M_t, \Phi_t) = \max_{c} u(c) + \not{D}\beta\mathbb{E}_t\left[(\Phi_{t+1}\psi_{t+1,i})^{1-\rho} v(m_{t+1,i}, M_{t+1}, \Phi_{t+1})\right]$$

- Solved using Krusell and Smith (1998)
- Perception dynamics identical to sticky PE/SOE:

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Regressions on Simulated and Actual Data

Dynan (2000)/Sommer (2007) Specification:

$$\Delta \log \mathrm{C}_{t+1} \approx \varsigma + \chi \mathbb{E}[\Delta \log \mathrm{C}_t] + \eta \mathbb{E}[\Delta \log \mathrm{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

- **χ: Extent of habits**
  Data: Micro: $\chi^{\text{Micro}} = 0.1$ (EER 2017 paper)
  Macro: $\chi^{\text{Macro}} = 0.6$
- **η: Fraction of Y going to 'rule-of-thumb' C = Y types**
  Data: Micro: $0 < \eta^{\text{Micro}} < 1$ (Depends . . .)
  Macro: $\eta^{\text{Macro}} \approx 0.5$ (Campbell and Mankiw (1989))
- **α: Precautionary saving (micro) or IES (Macro)**
  Data: Micro: $\alpha^{\text{Micro}} < 0$ (Zeldes (1989))
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  [In GE r depends roughly linearly on A]

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Micro vs Macro: Theory and Empirics

$$\Delta \log \mathrm{C}_{t+1} \quad \approx \quad \varsigma + \chi \Delta \log \mathrm{C}_t + \eta \mathbb{E}_t[\Delta \log \mathrm{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

	χ	η	α
Micro (Separable)			
Theory	≈ 0	$0 < \eta < 1$	< 0
Data	≈ 0	$0 < \eta < 1$	< 0
Macro			
Theory: Separable	≈ 0	≈ 0	< 0
Theory: CampMan	≈ 0	≈ 0.5	< 0
Theory: Habits	≈ 0.75	≈ 0	< 0

Calibration I

		Macroeconomic Parameters
γ	0.36	Capital's Share of Income
δ	$1 - 0.94^{1/4}$	Depreciation Rate
σ^2_Θ	0.00001	Variance Aggregate Transitory Shocks
σ^2_Ψ	0.00004	Variance Aggregate Permanent Shocks
		Steady State of Perfect Foresight DSGE Model
		$(\sigma_\Psi = \sigma_\Theta = \sigma_\psi = \sigma_\theta = \wp = \mathrm{D} = 0,\ \Phi_t = 1)$
K/K^γ	12.0	SS Capital to Output Ratio
K	48.55	SS Capital to Labor Productivity Ratio $(= 12^{1/(1-\gamma)})$
W	2.59	SS Wage Rate $(= (1-\gamma)K^\gamma)$
r	0.03	SS Interest Rate $(= \gamma K^{\gamma-1})$
$\mathscr{R}$	1.015	SS Between-Period Return Factor $(= 1 - \delta + \mathrm{r})$

Calibration II

		Preference Parameters
ρ	2.	Coefficient of Relative Risk Aversion
β	0.970	Discount Factor (SOE Model)
Π	0.25	Probability of Updating Expectations (if Sticky)
		Idiosyncratic Shock Parameters
σ_θ^2	0.120	Variance Idiosyncratic Tran Shocks (=4$\times$ Annual)
σ_ψ^2	0.003	Variance Idiosyncratic Perm Shocks (=$\frac{1}{4}\times$ Annual)
$\wp$	0.050	Probability of Unemployment Spell
D	0.005	Probability of Mortality

Micro Regressions: Frictionless

$$\Delta \log c_{t+1,i} = \varsigma + \chi \Delta \log c_{t,i} + \eta \mathbb{E}_{t,i}[\Delta \log y_{t+1,i}] + \alpha \bar{a}_{t,i} + \epsilon_{t+1,i}$$

Model of Expectations	χ	η	α	$\bar{R}^2$
Frictionless				
	0.019			0.000
	(–)			
		0.011		0.004
		(–)		
			−0.190	0.010
			(–)	
	0.061	0.016	−0.183	0.017
	(–)	(–)	(–)	

Micro Regressions: Sticky

$$\Delta \log c_{t+1,i} = \varsigma + \chi \Delta \log c_{t,i} + \eta \mathbb{E}_{t,i}[\Delta \log y_{t+1,i}] + \alpha \bar{a}_{t,i} + \epsilon_{t+1,i}$$

Model of Expectations	χ	η	α	$\bar{R}^2$
Sticky				
	0.012			0.000
	(–)			
		0.011		0.004
		(–)		
			−0.191	0.010
			(–)	
	0.051	0.015	−0.185	0.016
	(–)	(–)	(–)	

Empirical Results for U.S.

$$\Delta \log C_{t+1} = \varsigma + \chi \Delta \log C_t + \eta \mathbb{E}_t[\Delta \log Y_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var Independent Variables			OLS or IV	2nd Stage $\bar{R}^2$	Hansen J p-val
Nondurables and Services					
$\Delta \log C_t^*$	$\Delta \log Y_{t+1}$	A_t			
0.468 (0.076)			OLS	0.216	
0.830 (0.098)			IV	0.278	0.439
	0.587 (0.110)		IV	0.203	0.319
		−0.17e−4 (5.71e−4)	IV	−0.005	0.181
0.618 (0.159)	0.305 (0.161)	−4.96e−4 (2.94e−4)	IV	0.304	0.825
Memo: For instruments $Z_t, \Delta \log C_t = Z_t \zeta$, $\bar{R}^2 = 0.358$					

Notes: Data source is NIPA, 1960Q1–2016Q. Robust standard errors are in parentheses. Instruments $\mathbf{Z}_t = \{\Delta \log C_{t-2}, \Delta \log C_{t-3}, \Delta \log Y_{t-2}, \Delta \log Y_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log C_{t-2}, \Delta_8 \log Y_{t-2}$, lags 2 and 3 of differenced Fed funds rate, lags 2 and 3 of the Michigan Index of Consumer Sentiment Expectations$\}$.

Small Open Economy: Sticky

$$\Delta \log C_{t+1} = \varsigma + \chi \Delta \log C_t + \eta \mathbb{E}_t[\Delta \log Y_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var Independent Variables			OLS or IV	2^{nd} Stage $\bar{R}^2$	Hansen J p-val
Sticky : $\Delta \log C^*_{t+1}$ (with measurement error $C^*_t = C_t \times \xi_t$);					
$\Delta \log C^*_t$	$\Delta \log Y_{t+1}$	A_t			
0.508			OLS	0.263	
(0.058)					
0.802			IV	0.260	0.554
(0.104)					
	0.859		IV	0.198	0.233
	(0.182)				
		−8.26e–4	IV	0.066	0.002
		(3.99e–4)			
0.660	0.192	0.60e–4	IV	0.261	0.546
(0.187)	(0.277)	(5.03e–4)			
Memo: For instruments Z_t, $\Delta \log C^*_t = Z_t \zeta$, $\bar{R}^2 = 0.260$; var(log(ξ_t)) = 5.99e–6					

Notes: Reported statistics are the average values for 100 samples of 200 simulated quarters each. Instruments

$\mathbf{Z}_t = \{\Delta \log C_{t-2}, \Delta \log C_{t-3}, \Delta \log Y_{t-2}, \Delta \log Y_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log C_{t-2}, \Delta_8 \log Y_{t-2}\}$.

Small Open Economy: Frictionless

$$\Delta \log C_{t+1} = \varsigma + \chi \Delta \log C_t + \eta \mathbb{E}_t[\Delta \log Y_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var Independent Variables			OLS or IV	2nd Stage $\bar{R}^2$	Hansen J p-val
Frictionless : $\Delta \log C^*_{t+1}$ (with measurement error $C^*_t = C_t \times \xi_t$);					
$\Delta \log C^*_t$	$\Delta \log Y_{t+1}$	A_t			
0.295 (0.066)			OLS	0.087	
0.660 (0.309)			IV	0.040	0.600
	0.457 (0.209)		IV	0.035	0.421
		−6.92e–4 (5.87e–4)	IV	0.026	0.365
0.420 (0.428)	0.258 (0.365)	0.45e–4 (9.51e–4)	IV	0.041	0.529
Memo: For instruments Z_t, $\Delta \log C^*_t = Z_t \zeta$, $\bar{R}^2 = 0.039$; $\text{var}(\log(\xi_t)) = 5.99\text{e–}6$					

Notes: Reported statistics are the average values for 100 samples of 200 simulated quarters each. Instruments

$\mathbf{Z}_t = \{\Delta \log C_{t-2}, \Delta \log C_{t-3}, \Delta \log Y_{t-2}, \Delta \log Y_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log C_{t-2}, \Delta_8 \log Y_{t-2}\}$.

Heterogeneous Agents DSGE: Sticky

$$\Delta \log C_{t+1} = \varsigma + \chi \Delta \log C_t + \eta \mathbb{E}_t[\Delta \log Y_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var Independent Variables			OLS or IV	2nd Stage $\bar{R}^2$	Hansen J p-val
Sticky : $\Delta \log C^*_{t+1}$ (with measurement error $C^*_t = C_t \times \xi_t$);					
$\Delta \log C^*_t$	$\Delta \log Y_{t+1}$	A_t			
0.467 (0.061)			OLS	0.223	
0.773 (0.108)			IV	0.230	0.542
	0.912 (0.245)		IV	0.145	0.187
		−0.97e–4 (0.56e–4)	IV	0.059	0.002
0.670 (0.181)	0.171 (0.363)	0.12e–4 (0.86e–4)	IV	0.231	0.551
Memo: For instruments Z_t, $\Delta \log C^*_t = Z_t \zeta$, $\bar{R}^2 = 0.232$; $\text{var}(\log(\xi_t)) = 4.16\text{e–}6$					

Notes: Reported statistics are the average values for 100 samples of 200 simulated quarters each. Instruments

$\mathbf{Z}_t = \{\Delta \log C_{t-2}, \Delta \log C_{t-3}, \Delta \log Y_{t-2}, \Delta \log Y_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log C_{t-2}, \Delta_8 \log Y_{t-2}\}$.

Heterogeneous Agents DSGE: Frictionless

$$\Delta \log \mathrm{C}_{t+1} = \varsigma + \chi \Delta \log \mathrm{C}_t + \eta \mathbb{E}_t[\Delta \log \mathrm{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var Independent Variables			OLS or IV	2nd Stage $\bar{R}^2$	Hansen J p-val
Frictionless : $\Delta \log \mathrm{C}^*_{t+1}$ (with measurement error $\mathrm{C}^*_t = \mathrm{C}_t \times \xi_t$);					
$\Delta \log \mathrm{C}^*_t$	$\Delta \log \mathrm{Y}_{t+1}$	A_t			
0.189			OLS	0.036	
(0.072)					
0.476			IV	0.020	0.556
(0.354)					
	0.368		IV	0.017	0.457
	(0.321)				
		−0.34e–4	IV	0.015	0.433
		(0.98e–4)			
0.289	0.214	0.01e–4	IV	0.020	0.531
(0.463)	(0.583)	(1.87e–4)			
Memo: For instruments Z_t, $\Delta \log \mathrm{C}^*_t = \mathrm{Z}_t \zeta$, $\bar{R}^2 = 0.023$; $\mathrm{var}(\log(\xi_t)) = 4.16\mathrm{e}{-6}$					

Notes: Reported statistics are the average values for 100 samples of 200 simulated quarters each. Instruments

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Utility Costs of Stickiness

- Simulate expected lifetime utility when market resources nonstochastically equal to W_t at birth under **frictionless**

$$\overline{v}_0 \equiv \mathbb{E}[v(W_t, \cdot)]$$

and **sticky expectations**: $\overline{\widetilde{v}}_0 \equiv \mathbb{E}[\widetilde{v}(W_t, \cdot)]$

- Expectations taken over state variables other than $m_{t,i}$
- Newborn's willingness to pay (as fraction of permanent income) to avoid having sticky expectations:

$$\omega = 1 - \left(\frac{\overline{\widetilde{v}}_0}{\overline{v}_0}\right)^{\frac{1}{1-\rho}}$$

- $\omega \approx 0.05\%$ **of permanent income**
$\omega_{SOE} = 4.82\text{e–}4$; $\omega_{HA-DSGE} = 4.51\text{e–}4$

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- Simulate expected lifetime utility when market resources nonstochastically equal to W_t at birth under **frictionless**

$$\overline{\mathrm{v}}_0 \equiv \mathbb{E}[\mathrm{v}(\mathrm{W}_t, \cdot)]$$

and **sticky expectations**: $\overline{\widetilde{\mathrm{v}}}_0 \equiv \mathbb{E}[\widetilde{\mathrm{v}}(\mathrm{W}_t, \cdot)]$

- Expectations taken over state variables other than $m_{t,i}$
- Newborn's willingness to pay (as fraction of permanent income) to avoid having sticky expectations:

$$\omega = 1 - \left(\frac{\overline{\widetilde{\mathrm{v}}}_0}{\overline{\mathrm{v}}_0} \right)^{\frac{1}{1-\rho}}$$

- $\omega \approx 0.05\%$ **of permanent income**
  $\omega_{SOE} = 4.82\text{e–}4$; $\omega_{HA-DSGE} = 4.51\text{e–}4$

Utility Costs of Stickiness

- Simulate expected lifetime utility when market resources nonstochastically equal to W_t at birth under **frictionless**

$$\overline{\mathrm{v}}_0 \equiv \mathbb{E}[\mathrm{v}(\mathrm{W}_t, \cdot)]$$

  and **sticky expectations**: $\overline{\widetilde{\mathrm{v}}}_0 \equiv \mathbb{E}[\widetilde{\mathrm{v}}(\mathrm{W}_t, \cdot)]$
- Expectations taken over state variables other than $m_{t,i}$
- Newborn's willingness to pay (as fraction of permanent income) to avoid having sticky expectations:

$$\omega = 1 - \left(\frac{\overline{\widetilde{\mathrm{v}}}_0}{\overline{\mathrm{v}}_0} \right)^{\frac{1}{1-\rho}}$$

- $\omega \approx 0.05\%$ **of permanent income**
  $\omega_{SOE} = 4.82\text{e–}4$; $\omega_{HA-DSGE} = 4.51\text{e–}4$

Conclusion

Model with 'Sticky Expectations' of aggregate variables can match both micro and macro consumption dynamics

$$\Delta \log \mathrm{C}_{t+1} \quad \approx \quad \varsigma + \chi \Delta \log \mathrm{C}_t + \eta \mathbb{E}_t[\Delta \log \mathrm{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

	χ	η	α
Micro			
Data	≈ 0	$0 < \eta < 1$	< 0
Theory: Habits	≈ 0.75	$0 < \eta < 1$	< 0
Theory: Sticky Expectations	≈ 0	$0 < \eta < 1$	< 0
Macro			
Data	≈ 0.75	≈ 0	< 0
Theory: Habits	≈ 0.75	≈ 0	< 0
Theory: Habits	≈ 0.75	≈ 0	< 0

References I

Abel, Andrew B. (1990): "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review*, 80(2), 38–42.

Alvarez, Fernando, Luigi Guiso, and Francesco Lippi (2012): "Durable Consumption and Asset Management with Transaction and Observation Costs," *American Economic Review*, 102(5), 2272–2300.

Blanchard, Olivier J. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93(2), 223–247.

Calvo, Guillermo A. (1983): "Staggered Contracts in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12(3), 383–98.

Campbell, John, and Angus Deaton (1989): "Why is Consumption So Smooth?," *The Review of Economic Studies*, 56(3), 357–373, http://www.jstor.org/stable/2297552.

Campbell, John Y., and N. Gregory Mankiw (1989): "Consumption, Income, and Interest Rates: Reinterpreting the Time-Series Evidence," in *NBER Macroeconomics Annual, 1989*, ed. by Olivier J. Blanchard, and Stanley Fischer, pp. 185–216. MIT Press, Cambridge, MA, http://www.nber.org/papers/w2924.pdf.

Carroll, Christopher D. (2003): "Macroeconomic Expectations of Households and Professional Forecasters," *Quarterly Journal of Economics*, 118(1), 269–298, http://econ.jhu.edu/people/ccarroll/epidemiologyQJE.pdf.

Chetty, Raj, and Adam Szeidl (2016): "Consumption Commitments and Habit Formation," *Econometrica*, 84, 855–890.

Christiano, Laurence J., Martin Eichenbaum, and Charles L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Coibion, Olivier, and Yuriy Gorodnichenko (2015): "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts," *American Economic Review*, 105(8), 2644–2678.

Constantinides, George M. (1990): "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, 98(3), 519–543.

References II

Dynan, Karen E. (2000): "Habit Formation in Consumer Preferences: Evidence from Panel Data," *American Economic Review*, 90(3), http://www.jstor.org/stable/117335.

Fuhrer, Jeffrey C. (2018): "Intrinsic Expectations Persistence: Evidence from Professional and Household Survey Expectations," Working Papers 18-9, Federal Reserve Bank of Boston.

Gabaix, Xavier (2014): "A Sparsity-Based Model of Bounded Rationality," *The Quarterly Journal of Economics*, 129(4), 1661–1710.

Hall, Robert E. (1978): "Stochastic Implications of the Life-Cycle/Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 96, 971–87, Available at http://www.stanford.edu/~rehall/Stochastic-JPE-Dec-1978.pdf.

Havranek, Tomas, Marek Rusnak, and Anna Sokolova (2017): "Habit Formation in Consumption: A Meta-Analysis," *European Economic Review*, 95(C), 142–167.

Krusell, Per, and Anthony A. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

Lucas, Robert E. (1973): "Some International Evidence on Output-Inflation Tradeoffs," *American Economic Review*, 63, 326–334.

Maćkowiak, Bartosz, and Mirko Wiederholt (2015): "Business Cycle Dynamics under Rational Inattention," *The Review of Economic Studies*, 82(4), 1502–1532.

Mankiw, N. Gregory, and Ricardo Reis (2002): "Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *Quarterly Journal of Economics*, 117(4), 1295–1328.

Muth, John F. (1960): "Optimal Properties of Exponentially Weighted Forecasts," *Journal of the American Statistical Association*, 55(290), 299–306.

Pischke, Jörn-Steffen (1995): "Individual Income, Incomplete Information, and Aggregate Consumption," *Econometrica*, 63(4), 805–40.

Reis, Ricardo (2006): "Inattentive Consumers," *Journal of Monetary Economics*, 53(8), 1761–1800.

References III

Rotemberg, Julio J., and Michael Woodford (1997): "An Optimization-Based Econometric Model for the Evaluation of Monetary Policy," in *NBER Macroeconomics Annual, 1997*, ed. by Benjamin S. Bernanke, and Julio J. Rotemberg, vol. 12, pp. 297–346. MIT Press, Cambridge, MA.

Sims, Christopher (2003): "Implications of Rational Inattention," *Journal of Monetary Economics*, 50(3), 665–690, available at http://ideas.repec.org/a/eee/moneco/v50y2003i3p665-690.html.

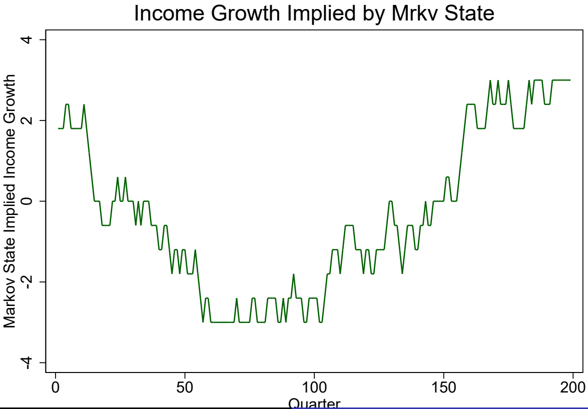
Sommer, Martin (2007): "Habit Formation and Aggregate Consumption Dynamics," *Advances in Macroeconomics*, 7(1), Article 21.

Zeldes, Stephen P. (1989): "Consumption and Liquidity Constraints: An Empirical Investigation," *Journal of Political Economy*, 97, 305–46, Available at http://www.jstor.org/stable/1831315.

Markov Process for Aggregate Productivity Growth Φ

$$\boldsymbol{\ell}_{t,i} = \theta_{t,i}\Theta p_{t,i}P_t, \quad p_{t+1,i} = p_{t,i}\psi_{t+1,i}, \quad P_{t+1} = \Phi_{t+1}P_t\Psi_{t+1}$$

- Φ_t follows bounded (discrete) RW
- 11 states; average persistence 2 quarters
- Flexible way to match actual pty growth data



Equilibrium

	SOE Model		HA-DSGE Model	
	Frictionless	Sticky	Frictionless	Sticky
Means				
A	7.49	7.43	56.85	56.72
C	2.71	2.71	3.44	3.44
Standard Deviations				
Aggregate Time Series ('Macro')				
$\log A$	0.332	0.321	0.276	0.272
$\Delta \log \mathrm{C}$	0.010	0.007	0.010	0.005
$\Delta \log \mathrm{Y}$	0.010	0.010	0.007	0.007
Individual Cross Sectional ('Micro')				
$\log \mathrm{a}$	0.926	0.927	1.015	1.014
$\log \mathrm{c}$	0.790	0.791	0.598	0.599
$\log p$	0.796	0.796	0.796	0.796
$\log \mathrm{y} \mid \mathrm{y} > 0$	0.863	0.863	0.863	0.863
$\Delta \log \mathrm{c}$	0.098	0.098	0.054	0.055
Cost of Stickiness	4.82e–4		4.51e–4	

Cost of Stickiness

Define (for given parameter values):

$v(\mathrm{W}_t, \cdot)$	Newborns' expected value for frictionless model
$\grave{v}(\mathrm{W}, \cdot)$	Newborns' expected value if $\sigma_\psi^2 = 0$
$\widetilde{v}(\mathrm{W}, \cdot)$	Newborns' expected value from sticky behavior

Fact suggested by theory (and confirmed numerically):

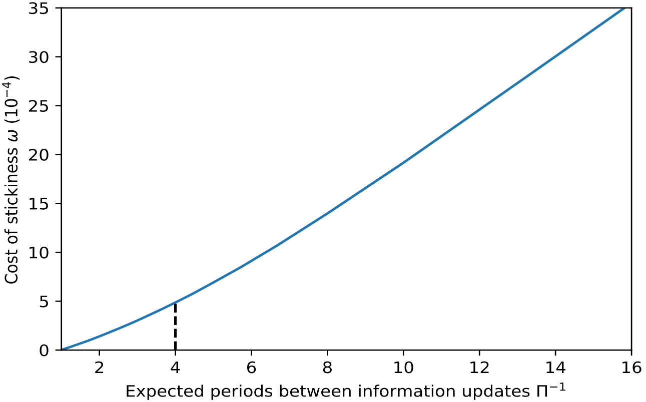
$$\mathrm{v}(\mathrm{W}_t, \cdot) \approx \grave{\mathrm{v}}(\mathrm{W}_t, \cdot) - \kappa\sigma_\Psi^2,$$

Guess (and verify) that:

$$\widetilde{\mathrm{v}}(\mathrm{W}_t, \cdot) \approx \grave{\mathrm{v}}(\mathrm{W}_t, \cdot) - (\kappa/\Pi)\sigma_\Psi^2. \tag{1}$$

Cost of Stickiness: ω and Π

Costs of stickiness ω and prob of aggr info updating Π



Notes: The figure shows how the utility costs of updating ω depend on the probability of updating of aggregate information Π in the SOE model.

Cost of Stickiness: Solution

Suppose utility cost of attention is $\iota\Pi$.

- If Newborns Pick Optimal Π, they solve

$$\max_{\Pi} \ \grave{\mathrm{v}}(W_t, \cdot) - (\kappa/\Pi)\sigma_{\Psi}^2 - \iota\Pi.$$

Solution:

$$\Pi = (\kappa/\iota)^{0.5}\sigma_{\Psi}.$$

Optimal Π characteristics:

- Increasing in κ ('importance' to value of perm shocks)
- Increasing in σ_{ψ} ('magnitude' of perm shocks)
- Decreasing as attention becomes more costly: $\iota \uparrow$

Is Muth–Lucas–Pischke Kalman Filter Equivalent?

No.

Muth (1960)–Lucas (1973)–Pischke (1995) Kalman filter

- All you can see is Y
  - Lucas: Can't distinguish agg. from idio.
  - Muth–Pischke: Can't distinguish tran from perm
- Here: Can see own circumstances perfectly
- Only the (tiny) aggregate part is hard to see
- **Signal extraction for aggregate Y_t gives too little persistence in ΔC_t: $\chi \approx 0.17$**

Muth–Pischke Perception Dynamics

- Optimal signal extraction problem (Kalman filter): Observe Y (aggregate income), estimate P, Θ
- Optimal estimate of P:

$$\hat{P}_{t+1} \quad = \quad \Pi \mathrm{Y}_{t+1} + (1 - \Pi)\hat{P}_t,$$

where for signal-to-noise ratio $\varphi = \sigma_\Psi / \sigma_\Theta$:

$$\Pi \quad = \quad \varphi\sqrt{1 + \varphi^2/4} - \varphi^2/2, \tag{2}$$

- But if we calibrate φ using observed macro data
  - $\Rightarrow \Delta \log \mathrm{C}_{t+1} \approx \mathbf{0.17}\ \Delta \log \mathrm{C}_t$
  - **Too little persistence!**