

# Sticky Expectations and Consumption Dynamics

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# Consumption Dynamics: Macro vs Micro

## Macro: Representative Agent Models

- Theory (With Separable Utility):
  - C responds instantly, completely to shock
  - Consequences of uncertainty are trivial
- Evidence: Consumption is too smooth (Campbell & Deaton, 1989)
- Solution: **"Habits" parameter**  $\chi^{\text{Macro}} \approx 0.6 \sim 0.8$   
$$\Delta \log C_{t+1} = \varsigma + \chi \Delta \log C_t + \epsilon$$

## Micro: Heterogeneous Agent Models

- **Uninsurable risk is essential, changes everything**
- Var of micro income shocks much larger than of macro shocks:  
$$\text{var}(\Delta \log p) \approx 100 \times \text{var}(\Delta \log P)$$
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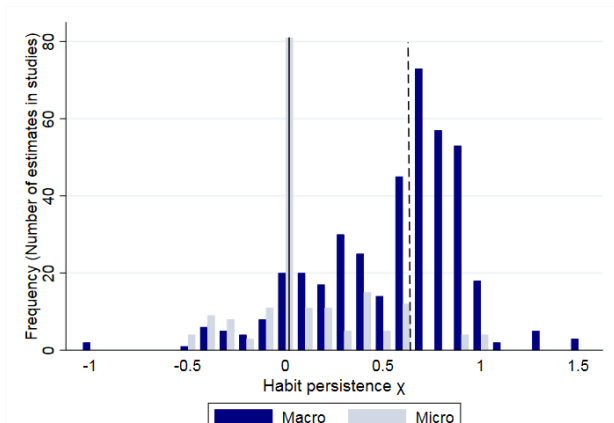
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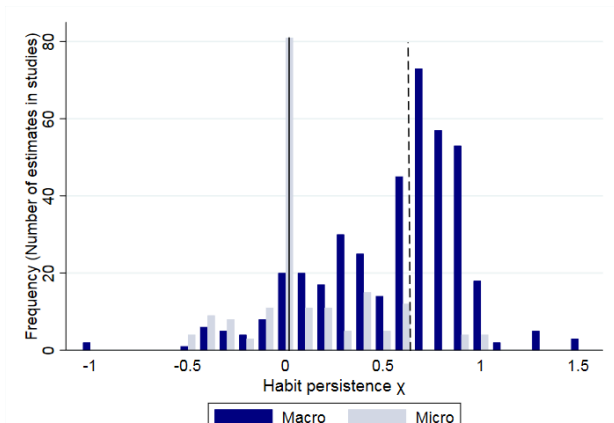
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Meta analysis of 597 estimates of  $\chi$
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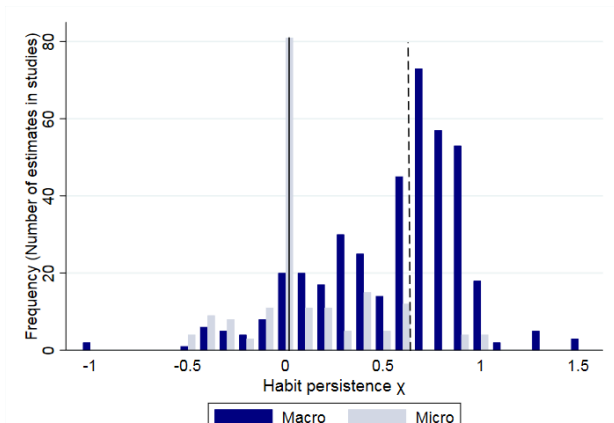
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# Claim: It's Not Habits, It's Inattention! (Macro not Micro)

## Our Setup

- Income Has Idiosyncratic and Aggregate Components
- Idiosyncratic Component Is Perfectly Observed
- **Aggregate Component Is Stochastically Observed**
  - Updating à la Calvo (1983)

### Not *ad hoc*

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# Why Macro Inattention Is Plausible

## Idiosyncratic Variability Is $\sim 100\times$ Bigger

- If Same Specification Estimated on Micro vs Macro Data
- Pervasive Lesson of All Micro Data

## Utility Cost of Inattention Small

- Micro: Critical (and Easy) To Notice You're Unemployed
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# Quadratic Utility Frictionless Benchmark

## Hall (1978) Random Walk

- Total Wealth (Human + Nonhuman):

$$\mathbf{o}_{t+1} = (\mathbf{o}_t - \mathbf{c}_t)R + \zeta_{t+1}$$

- C Euler Equation:

$$u'(\mathbf{c}_t) = R\beta\mathbb{E}_t[u'(\mathbf{c}_{t+1})]$$

- $\Rightarrow$  Random Walk (for  $R\beta = 1$ ):

$$\Delta\mathbf{c}_{t+1} = \epsilon_{t+1}$$

- Expected Wealth:

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# Sticky Expectations—Individual $\mathbf{c}$

- Consumer who happens to update at  $t$  and  $t + n$

$$\begin{aligned}\mathbf{c}_t &= (r/R)\mathbf{o}_t \\ \mathbf{c}_{t+1} &= (r/R)\tilde{\mathbf{o}}_{t+1} = (r/R)\mathbf{o}_t = \mathbf{c}_t \\ &\vdots \\ \mathbf{c}_{t+n-1} &= \mathbf{c}_t\end{aligned}$$

- Implies that  $\Delta^n \mathbf{o}_{t+n} \equiv \mathbf{o}_{t+n} - \mathbf{o}_t$  is white noise
- So **individual**  $\mathbf{c}$  is RW across updating periods:

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# Sticky Expectations—Aggregate $C$

- Pop normed to one, uniformly dist on  $[0, 1]$ :  $\mathbf{C}_t = \int_0^1 \mathbf{c}_{t,i} di$
- **Calvo (1983)-Type Updating of Expectations:**
  - Probability  $\Pi = 0.25$  (per quarter)
- Economy composed of many sticky- $\mathbb{E}$  consumers:

$$\mathbf{C}_{t+1} = (1 - \Pi) \underbrace{\mathbf{C}_{t+1}^{\neq}}_{=\mathbf{C}_t} + \Pi \mathbf{C}_{t+1}^{\pi}$$

$$\Delta \mathbf{C}_{t+1} \approx \underbrace{(1 - \Pi)}_{\equiv \chi = 0.75} \Delta \mathbf{C}_t + \epsilon_{t+1}$$

- **Substantial persistence ( $\chi = 0.75$ ) in aggregate  $C$  growth**

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  - Probability  $\Pi = 0.25$  (per quarter)
- Economy composed of many sticky- $\mathbb{E}$  consumers:

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# One More Ingredient: Idiosyncratic Uncertainty ...

- **Differences: Idiosyncratic vs Aggregate shocks**

- **Idiosyncratic shocks:** Frictionless observation
  - I notice if I am fired, promoted, somebody steals my wallet
  - True RW with respect to these
- **Aggregate shocks:** Sticky observation
  - May not instantly notice changes in aggregate productivity

- **Result:**

- **Idiosyncratic  $\Delta c$ :** dominated by frictionless RW part
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# Serious Models

## Partial Equilibrium/Small Open Economy

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- Idiosyncratic Shocks Calibrated From Micro Data
- Aggregate Shocks Calibrated From Macro Data
- Markov Process (Discrete RW) for Aggr Income Growth
  - Handles changing growth 'eras'
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## DSGE Heterogeneous Agents (HA) Model

- Same!



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# Income Process

- Individual's labor productivity is

$$l_{t,i} = \underbrace{\theta_{t,i}}_{\Theta_t} \underbrace{p_{t,i}}_{P_t}$$

- Idiosyncratic and aggregate  $p$  evolve according to

$$\begin{aligned} p_{t+1,i} &= p_{t,i} \psi_{t+1,i} \\ P_{t+1} &= \Phi_{t+1} P_t \Psi_{t+1} \end{aligned}$$

- $\Phi$  is Markov 'underlying' aggregate pty growth
  - Discrete (bounded) random walk
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# Blanchard (1985) Mortality and Insurance

- Household survives from  $t$  to  $t + 1$  with probability  $(1 - D)$ :

$$p_{t+1,i} = \begin{cases} 1 & \text{for newborns} \\ p_{t,i}\psi_{t+1,i} & \text{for survivors} \end{cases}$$

- Blanchardian scheme:

$$k_{t+1,i} = \begin{cases} 0 & \text{if HH } i \text{ dies, is replaced by newborn} \\ a_{t,i}/(1 - D) & \text{if household } i \text{ survives} \end{cases}$$

- Implies for aggregate:

$$K_{t+1} = \int_0^1 \left( \frac{1 - d_{t+1,i}}{1 - D} \right) a_{t,i} di = A_t$$
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# Resources

- Market resources:

$$\mathbf{m}_{t,i} = \underbrace{W_t \ell_{t,i}}_{\equiv \mathbf{y}_t} + \underbrace{R_t}_{1+r_t} \mathbf{k}_{t,i}$$

- End-of-Period 'Assets'—Unspent resources:

$$\mathbf{a}_{t,i} = \mathbf{m}_{t,i} - \mathbf{c}_{t,i}$$

- Capital transition depends on prob of survival  $1 - D$ :

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# Frictionless Solution

- For exposition: Assume constant  $W$  and  $\mathcal{R}$

- Normalize everything by  $p_{t,i} \equiv p_{t,i}P_t$ , e.g.

$$m_{t,i} = \mathbf{m}_{t,i} / (p_{t,i}P_t)$$

- $c(m, \Phi)$  is the function that solves:

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- Level of consumption:

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# Sticky Expectations about Aggregate Income

## Calvo Updating of Perceptions of Aggregate Shocks

- True Permanent income:  $P_{t+1} = \Phi_{t+1} P_t \Psi_{t+1}$
- Tilde ( $\tilde{P}$ ) denotes perceived variables
- Perception for consumer who has not updated for  $n$  periods:

$$\tilde{P}_{t,j} = \mathbb{E}_{t-n} [P_t | \Omega_{t-n}] = \Phi_{t-n}^n P_{t-n}$$

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# Sticky Expectations about Aggregate Income

## Sequence Within Period

- 1 Income shocks are realized and every individual sees her true  $\mathbf{y}$  and  $\mathbf{m}$ , i.e.  $\mathbf{y}_{t,i} = \tilde{\mathbf{y}}_{t,i}$  and  $\mathbf{m}_{t,i} = \tilde{\mathbf{m}}_{t,i}$  for all  $t$  and  $i$
- 2 Updating shocks realized:  $i$  observes true  $P_t, \Phi_t$  w/ prob  $\Pi$ ; forms perceptions of her normalized market resources  $\tilde{m}_{t,i}$
- 3 Consumes based on her perception, using  $c(\tilde{m}_{t,i}, \tilde{\Phi}_{t,i})$

### Key Assumption:

- People act as if their perceptions about aggregate state  $\{\tilde{P}_{t,i}, \tilde{\Phi}_{t,i}\}$  are the true aggregate state  $\{P_t, \Phi_t\}$

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- 2 Updating shocks realized:  $i$  observes true  $P_t, \Phi_t$  w/ prob  $\Pi$ ; forms perceptions of her normalized market resources  $\tilde{m}_{t,i}$
- 3 Consumes based on her perception, using  $c(\tilde{m}_{t,i}, \tilde{\Phi}_{t,i})$

### Key Assumption:

- People act as if their perceptions about aggregate state  $\{\tilde{P}_{t,i}, \tilde{\Phi}_{t,i}\}$  are the true aggregate state  $\{P_t, \Phi_t\}$



# Behavior under Sticky Expectations

- **Normalized resources:**

- $m_{t,i} \equiv \mathbf{m}_{t,i}/(p_{t,i}P_t)$  is *actual*

- $\tilde{m}_{t,i} \equiv \mathbf{m}_{t,i}/(p_{t,i}\tilde{P}_{t,i})$  is *perceived*

- **Usually  $\tilde{m}_{t,i} \neq m_{t,i}$  because  $P_t$  not perfectly observed**

- in levels:  $\tilde{\mathbf{m}}_{t,i} = \mathbf{m}_{t,i}$ ; but normalized:  $\tilde{m}_{t,i} \neq m_{t,i}$

- Consumers behave according to frictionless consumption function

- But **based on  $\tilde{m}_{t,i}$**  (not  $m_{t,i}$ ):

$$\tilde{c}_{t,i} = c(\tilde{m}_{t,i}, \tilde{\Phi}_{t,i})$$

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# DSGE Heterogeneous Agents Model

- Idiosyncratic and aggregate shocks same as PE/SOE
- Endogenous  $W_t$  and  $\mathcal{R}_t$
- Aggregate market resources  $M_t$  is a state variable

$$v(m_{t,i}, M_t, \Phi_t) = \max_c u(c) + \beta \mathbb{E}_t [(\Phi_{t+1} \psi_{t+1,i})^{1-\rho} v(m_{t+1,i}, M_{t+1}, \Phi_{t+1})]$$

- Solved using Krusell and Smith (1998)
- Perception dynamics identical to sticky PE/SOE:

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# Regressions on Simulated and Actual Data

## Dynan (2000)/Sommer (2007) Specification:

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \mathbb{E}[\Delta \log \mathbf{C}_t] + \eta \mathbb{E}[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

- $\chi$ : **Extent of habits**

Data: Micro:  $\chi^{\text{Micro}} = 0.1$  (EER 2017 paper)

Macro:  $\chi^{\text{Macro}} = 0.6$

- $\eta$ : **Fraction of  $\mathbf{Y}$  going to 'rule-of-thumb'  $\mathbf{C} = \mathbf{Y}$  types**

Data: Micro:  $0 < \eta^{\text{Micro}} < 1$  (Depends ...)

Macro:  $\eta^{\text{Macro}} \approx 0.5$  (Campbell and Mankiw (1989))

- $\alpha$ : **Precautionary saving (micro) or IES (Macro)**

Data: Micro:  $\alpha^{\text{Micro}} < 0$  (Zeldes (1989))

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# Micro vs Macro: Theory and Empirics

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

	$\chi$	$\eta$	$\alpha$
<b>Micro (Separable)</b>			
Theory	$\approx 0$	$0 < \eta < 1$	$< 0$
Data	$\approx 0$	$0 < \eta < 1$	$< 0$
<b>Macro</b>			
Theory: Separable	$\approx 0$	$\approx 0$	$< 0$
Theory: CampMan	$\approx 0$	$\approx 0.5$	$< 0$
Theory: Habits	$\approx 0.75$	$\approx 0$	$< 0$

## Calibration I

**Macroeconomic Parameters**

$\gamma$	0.36	Capital's Share of Income
$\tau$	$0.94^{1/4}$	Depreciation Factor
$\sigma_{\Theta}^2$	0.00001	Variance Aggregate Transitory Shocks
$\sigma_{\Psi}^2$	0.00004	Variance Aggregate Permanent Shocks

**Steady State of Perfect Foresight DSGE Model**

$$(\sigma_{\Psi} = \sigma_{\Theta} = \sigma_{\psi} = \sigma_{\theta} = \varrho = D = 0, \Phi_t = 1)$$

$\check{K}/\check{K}^{\gamma}$	12.0	SS Capital to Output Ratio
$\check{K}$	48.55	SS Capital to Labor Productivity Ratio ( $= 12^{1/(1-\gamma)}$ )
$\check{W}$	2.59	SS Wage Rate ( $= (1 - \gamma)\check{K}^{\gamma}$ )
$\check{r}$	0.03	SS Interest Rate ( $= \gamma\check{K}^{\gamma-1}$ )
$\check{R}$	1.015	SS Between-Period Return Factor ( $= \tau + \check{r}$ )

## Calibration II

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**Preference Parameters**

$\rho$	2.	Coefficient of Relative Risk Aversion
$\beta_{SOE}$	0.970	SOE Discount Factor
$\beta_{DSGE}$	0.986	HA-DSGE Discount Factor ( $= \check{\mathcal{R}}^{-1}$ )
$\Pi$	0.25	Probability of Updating Expectations (if Sticky)

---

**Idiosyncratic Shock Parameters**

$\sigma_{\theta}^2$	0.120	Variance Idiosyncratic Tran Shocks ( $=4 \times$ Annual)
$\sigma_{\psi}^2$	0.003	Variance Idiosyncratic Perm Shocks ( $=\frac{1}{4} \times$ Annual)
$\wp$	0.050	Probability of Unemployment Spell
D	0.005	Probability of Mortality

---

# Micro Regressions: Frictionless

$$\Delta \log \mathbf{c}_{t+1,i} = \varsigma + \chi \Delta \log \mathbf{c}_{t,i} + \eta \mathbb{E}_{t,i}[\Delta \log \mathbf{y}_{t+1,i}] + \alpha \bar{a}_{t,i} + \epsilon_{t+1,i}$$

Model of Expectations	$\chi$	$\eta$	$\alpha$	$\bar{R}^2$
Frictionless	0.019 (-)			0.000
		0.011 (-)		0.004
			-0.190 (-)	0.010
	0.061 (-)	0.016 (-)	-0.183 (-)	0.017

# Micro Regressions: Sticky

$$\Delta \log \mathbf{c}_{t+1,i} = \varsigma + \chi \Delta \log \mathbf{c}_{t,i} + \eta \mathbb{E}_{t,i}[\Delta \log \mathbf{y}_{t+1,i}] + \alpha \bar{a}_{t,i} + \epsilon_{t+1,i}.$$

Model of Expectations	$\chi$	$\eta$	$\alpha$	$\bar{R}^2$
Sticky	0.012 (-)			0.000
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	0.051 (-)	0.015 (-)	-0.185 (-)	0.016

## Empirical Results for U.S.

$$\Delta \log \mathbf{C}_{t+1} = \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var			OLS or IV	2 <sup>nd</sup> Stage $\bar{R}^2$	KP $p$ -val Hansen J $p$ -val
Independent Variables					
Nondurables and Services					
$\Delta \log \mathbf{C}_t^*$	$\Delta \log \mathbf{Y}_{t+1}$	$A_t$			
0.468*** (0.076)			OLS	0.216	
0.830*** (0.098)			IV	0.278	0.222 0.439
	0.587*** (0.110)		IV	0.203	0.263 0.319
		-0.17e-4 (5.71e-4)	IV	-0.005	0.081 0.181
0.618*** (0.159)	0.305* (0.161)	-4.96e-4* (2.94e-4)	IV	0.304	0.415 0.825
Memo: For instruments $\mathbf{Z}_t$ , $\Delta \log \mathbf{C}_t = \mathbf{Z}_t \zeta$ , $\bar{R}^2 = 0.358$					

**Notes:** Data source is NIPA, 1960Q1–2016Q. Robust standard errors are in parentheses. Instruments  $\mathbf{Z}_t = \{\Delta \log \mathbf{C}_{t-2}, \Delta \log \mathbf{C}_{t-3}, \Delta \log \mathbf{Y}_{t-2}, \Delta \log \mathbf{Y}_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log \mathbf{C}_{t-2}, \Delta_8 \log \mathbf{Y}_{t-2}, \text{lags 2 and 3 of differenced Fed funds rate, lags 2 and 3 of the Michigan Index of Consumer Sentiment Expectations}\}$ .

## Small Open Economy: Sticky

$$\Delta \log \mathbf{C}_{t+1} = \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var			OLS or IV	2 <sup>nd</sup> Stage $\bar{R}^2$	KP $p$ -val Hansen J $p$ -val
Independent Variables					
Sticky : $\Delta \log \mathbf{C}_{t+1}^*$ (with measurement error $\mathbf{C}_t^* = \mathbf{C}_t \times \xi_t$ );					
$\Delta \log \mathbf{C}_t^*$	$\Delta \log \mathbf{Y}_{t+1}$	$A_t$			
0.508*** (0.058)			OLS	0.263	
0.802*** (0.104)			IV	0.260	0.000 0.554
	0.859*** (0.182)		IV	0.198	0.060 0.233
		-8.26e-4** (3.99e-4)	IV	0.066	0.000 0.002
0.660*** (0.187)	0.192 (0.277)	0.60e-4 (5.03e-4)	IV	0.261	0.359 0.546

Memo: For instruments  $\mathbf{Z}_t$ ,  $\Delta \log \mathbf{C}_t^* = \mathbf{Z}_t \zeta$ ,  $\bar{R}^2 = 0.260$ ;  $\text{var}(\log(\xi_t)) = 5.99e-6$

**Notes:** Reported statistics are the average values for 100 samples of 200 simulated quarters each. Bullets indicate that the average sample coefficient divided by average sample standard error is outside of the inner 90%, 95%, and 99% of the standard normal distribution. Instruments  $\mathbf{Z}_t = \{\Delta \log \mathbf{C}_{t-2}, \Delta \log \mathbf{C}_{t-3}, \Delta \log \mathbf{Y}_{t-2}, \Delta \log \mathbf{Y}_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log \mathbf{C}_{t-2}, \Delta_8 \log \mathbf{Y}_{t-2}\}$ .

## Small Open Economy: Frictionless

$$\Delta \log \mathbf{C}_{t+1} = \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var	OLS	2 <sup>nd</sup> Stage	KP $p$ -val
Independent Variables	or IV	$\bar{R}^2$	Hansen J $p$ -val
Frictionless : $\Delta \log \mathbf{C}_{t+1}^*$ (with measurement error $\mathbf{C}_t^* = \mathbf{C}_t \times \xi_t$ );			
$\Delta \log \mathbf{C}_t^*$	$\Delta \log \mathbf{Y}_{t+1}$	$A_t$	
0.295*** (0.066)			
			OLS
			0.087
0.660** (0.309)			IV
			0.040
			0.237
			0.600
	0.457** (0.209)		IV
			0.035
			0.035
			0.059
			0.421
		-6.92e-4 (5.87e-4)	IV
			0.026
			0.026
			0.000
			0.365
0.420 (0.428)	0.258 (0.365)	0.45e-4 (9.51e-4)	IV
			0.041
			0.041
			0.516
			0.529
Memo: For instruments $\mathbf{Z}_t$ , $\Delta \log \mathbf{C}_t^* = \mathbf{Z}_t \zeta$ , $\bar{R}^2 = 0.039$ ; $\text{var}(\log(\xi_t)) = 5.99e-6$			

**Notes:** Reported statistics are the average values for 100 samples of 200 simulated quarters each. Bullets indicate that the average sample coefficient divided by average sample standard error is outside of the inner 90%, 95%, and 99% of the standard normal distribution. Instruments  $\mathbf{Z}_t = \{\Delta \log \mathbf{C}_{t-2}, \Delta \log \mathbf{C}_{t-3}, \Delta \log \mathbf{Y}_{t-2}, \Delta \log \mathbf{Y}_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log \mathbf{C}_{t-2}, \Delta_8 \log \mathbf{Y}_{t-2}\}$ .



## Heterogeneous Agents DSGE: Sticky

$$\Delta \log \mathbf{C}_{t+1} = \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var	OLS	2 <sup>nd</sup> Stage	KP $p$ -val
Independent Variables	or IV	$\bar{R}^2$	Hansen J $p$ -val
Sticky : $\Delta \log \mathbf{C}_{t+1}^*$ (with measurement error $\mathbf{C}_t^* = \mathbf{C}_t \times \xi_t$ );			
$\Delta \log \mathbf{C}_t^*$	$\Delta \log \mathbf{Y}_{t+1}$	$A_t$	
0.467*** (0.061)			OLS
			0.223
0.773*** (0.108)			IV
			0.230
			0.000
	0.912*** (0.245)		IV
			0.145
			0.105
		-0.97e-4* (0.56e-4)	IV
			0.059
			0.000
			0.002
0.670*** (0.181)	0.171 (0.363)	0.12e-4 (0.86e-4)	IV
			0.231
			0.460
			0.551
Memo: For instruments $\mathbf{Z}_t$ , $\Delta \log \mathbf{C}_t^* = \mathbf{Z}_t \zeta$ , $\bar{R}^2 = 0.232$ ; $\text{var}(\log(\xi_t)) = 4.16e-6$			

**Notes:** Reported statistics are the average values for 100 samples of 200 simulated quarters each. Bullets indicate that the average sample coefficient divided by average sample standard error is outside of the inner 90%, 95%, and 99% of the standard normal distribution. Instruments  $\mathbf{Z}_t = \{\Delta \log \mathbf{C}_{t-2}, \Delta \log \mathbf{C}_{t-3}, \Delta \log \mathbf{Y}_{t-2}, \Delta \log \mathbf{Y}_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log \mathbf{C}_{t-2}, \Delta_8 \log \mathbf{Y}_{t-2}\}$ .

## Heterogeneous Agents DSGE: Frictionless

$$\Delta \log \mathbf{C}_{t+1} = \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

Expectations : Dep Var	OLS	2 <sup>nd</sup> Stage	KP $p$ -val
Independent Variables	or IV	$\bar{R}^2$	Hansen J $p$ -val
Frictionless : $\Delta \log \mathbf{C}_{t+1}^*$ (with measurement error $\mathbf{C}_t^* = \mathbf{C}_t \times \xi_t$ );			
$\Delta \log \mathbf{C}_t^*$	$\Delta \log \mathbf{Y}_{t+1}$	$A_t$	
0.189*** (0.072)			
	0.476 (0.354)		
		0.368 (0.321)	
		-0.34e-4 (0.98e-4)	
0.289 (0.463)	0.214 (0.583)	0.01e-4 (1.87e-4)	
Memo: For instruments $\mathbf{Z}_t$ , $\Delta \log \mathbf{C}_t^* = \mathbf{Z}_t \zeta$ , $\bar{R}^2 = 0.023$ ; $\text{var}(\log(\xi_t)) = 4.16e-6$			

**Notes:** Reported statistics are the average values for 100 samples of 200 simulated quarters each. Bullets indicate that the average sample coefficient divided by average sample standard error is outside of the inner 90%, 95%, and 99% of the standard normal distribution. Instruments  $\mathbf{Z}_t = \{\Delta \log \mathbf{C}_{t-2}, \Delta \log \mathbf{C}_{t-3}, \Delta \log \mathbf{Y}_{t-2}, \Delta \log \mathbf{Y}_{t-3}, A_{t-2}, A_{t-3}, \Delta_8 \log \mathbf{C}_{t-2}, \Delta_8 \log \mathbf{Y}_{t-2}\}$ .

# Utility Costs of Stickiness

- Simulate expected lifetime utility when market resources nonstochastically equal to  $W_t$  at birth under **frictionless**

$$\bar{v}_0 \equiv \mathbb{E}[v(W_t, \cdot)]$$

and **sticky expectations**:  $\tilde{v}_0 \equiv \mathbb{E}[\tilde{v}(W_t, \cdot)]$

- Expectations taken over state variables other than  $m_{t,i}$
- Newborn's willingness to pay (as fraction of permanent income) to avoid having sticky expectations:

$$\omega = 1 - \left( \frac{\tilde{v}_0}{\bar{v}_0} \right)^{\frac{1}{1-\rho}}$$

- $\omega \approx 0.05\%$  of permanent income  
 $\omega_{SOE} = 4.82e-4$ ;  $\omega_{HA-DSGE} = 4.51e-4$

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 $\omega_{SOE} = 4.82e-4$ ;  $\omega_{HA-DSGE} = 4.51e-4$

# Conclusion

**Model with 'Sticky Expectations' of aggregate variables can match both micro and macro consumption dynamics**

$$\Delta \log \mathbf{C}_{t+1} \approx \varsigma + \chi \Delta \log \mathbf{C}_t + \eta \mathbb{E}_t[\Delta \log \mathbf{Y}_{t+1}] + \alpha A_t + \epsilon_{t+1}$$

	$\chi$	$\eta$	$\alpha$
<b>Micro</b>			
Data	$\approx 0$	$0 < \eta < 1$	$< 0$
Theory: Habits	$\approx 0.75$	$0 < \eta < 1$	$< 0$
Theory: Sticky Expectations	$\approx 0$	$0 < \eta < 1$	$< 0$
<b>Macro</b>			
Data	$\approx 0.75$	$\approx 0$	$< 0$
Theory: Habits	$\approx 0.75$	$\approx 0$	$< 0$
Theory: Habits	$\approx 0.75$	$\approx 0$	$< 0$

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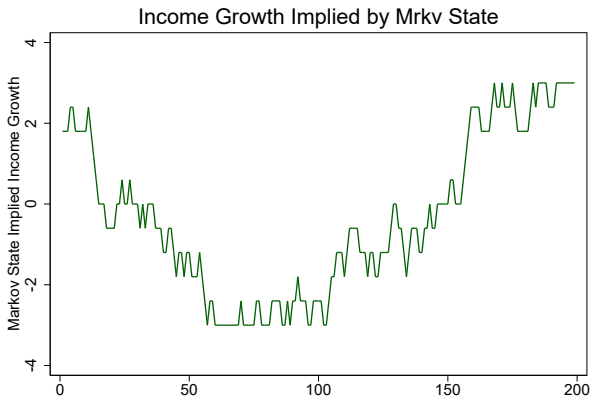
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# Markov Process for Aggregate Productivity Growth $\Phi$

$$\ell_{t,i} = \theta_{t,i} \Theta p_{t,i} P_t, \quad p_{t+1,i} = p_{t,i} \psi_{t+1,i}, \quad P_{t+1} = \Phi_{t+1} P_t \Psi_{t+1}$$

- $\Phi_t$  follows bounded (discrete) RW
- 11 states; average persistence 2 quarters
- Flexible way to match actual pty growth data



## Equilibrium

	SOE Model		HA-DSGE Model	
	Frictionless	Sticky	Frictionless	Sticky
Means				
A	7.49	7.43	56.85	56.72
C	2.71	2.71	3.44	3.44
Standard Deviations				
Aggregate Time Series ('Macro')				
log A	0.332	0.321	0.276	0.272
$\Delta \log \mathbf{C}$	0.010	0.007	0.010	0.005
$\Delta \log \mathbf{Y}$	0.010	0.010	0.007	0.007
Individual Cross Sectional ('Micro')				
log a	0.926	0.927	1.015	1.014
log c	0.790	0.791	0.598	0.599
log p	0.796	0.796	0.796	0.796
log $\mathbf{y}   \mathbf{y} > 0$	0.863	0.863	0.863	0.863
$\Delta \log \mathbf{c}$	0.098	0.098	0.054	0.055
Cost of Stickiness				
	4.82e-4		4.51e-4	

# Cost of Stickiness

Define (for given parameter values):

- $v(W_t, \cdot)$  Newborns' expected value for frictionless model
- $\dot{v}(W, \cdot)$  Newborns' expected value if  $\sigma_\psi^2 = 0$
- $\tilde{v}(W, \cdot)$  Newborns' expected value from sticky behavior

Fact suggested by theory (and confirmed numerically):

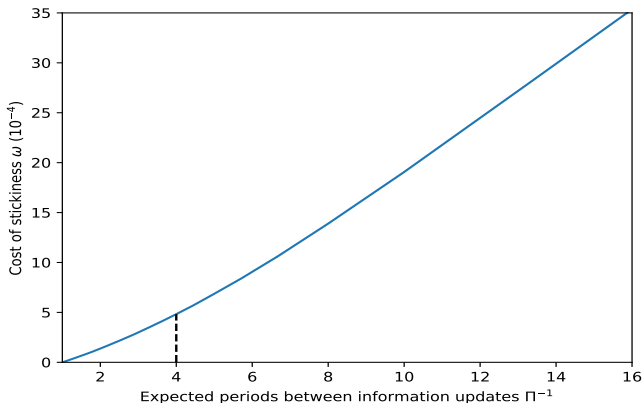
$$v(W_t, \cdot) \approx \dot{v}(W_t, \cdot) - \kappa \sigma_\Psi^2, \quad (1)$$

Guess (and verify) that:

$$\tilde{v}(W_t, \cdot) \approx \dot{v}(W_t, \cdot) - (\kappa/\Pi) \sigma_\Psi^2. \quad (2)$$

# Cost of Stickiness: $\omega$ and $\Pi$

Costs of stickiness  $\omega$  and prob of aggr info updating  $\Pi$



Notes: The figure shows how the utility costs of updating  $\omega$  depend on the probability of updating of aggregate information  $\Pi$  in the SOE model.

# Cost of Stickiness: Solution

Suppose utility cost of attention is  $\iota\Pi$ .

- If Newborns Pick Optimal  $\Pi$ , they solve

$$\max_{\Pi} \dot{v}(W_t, \cdot) - (\kappa/\Pi)\sigma_{\Psi}^2 - \iota\Pi. \quad (3)$$

Solution:

$$\Pi = (\kappa/\iota)^{0.5}\sigma_{\Psi}. \quad (4)$$

Optimal  $\Pi$  characteristics:

- Increasing in  $\kappa$  ('importance' to value of perm shocks)
- Increasing in  $\sigma_{\psi}$  ('magnitude' of perm shocks)
- Decreasing as attention becomes more costly:  $\iota \uparrow$

# Is Muth–Lucas–Pischke Kalman Filter Equivalent?

**No.**

Muth (1960)–Lucas (1973)–Pischke (1995) Kalman filter

- All you can see is  $Y$ 
  - Lucas: Can't distinguish agg. from idio.
  - Muth–Pischke: Can't distinguish tran from perm
- Here: Can see own circumstances perfectly
- Only the (tiny) aggregate part is hard to see
- **Signal extraction for aggregate  $Y_t$  gives too little persistence in  $\Delta C_t$ :  $\chi \approx 0.17$**



# Muth–Pischke Perception Dynamics

- Optimal signal extraction problem (Kalman filter):  
Observe  $\mathbf{Y}$  (aggregate income), estimate  $P$ ,  $\Theta$
- Optimal estimate of  $P$ :

$$\hat{P}_{t+1} = \Pi \mathbf{Y}_{t+1} + (1 - \Pi) \hat{P}_t,$$

where for signal-to-noise ratio  $\varphi = \sigma_{\Psi} / \sigma_{\Theta}$ :

$$\Pi = \varphi \sqrt{1 + \varphi^2 / 4} - \varphi^2 / 2, \quad (5)$$

- But if we calibrate  $\varphi$  using observed macro data
  - $\Rightarrow \Delta \log \mathbf{C}_{t+1} \approx \mathbf{0.17} \Delta \log \mathbf{C}_t$
  - **Too little persistence!**