Life-Cycle Portfolio Choice with Additive Habit Formation Preferences and Uninsurable Labor Income Risk*

Valery Polkovnichenko

University of Minnesota and The Federal Reserve Bank of Minneapolis

November 2004

Abstract

This paper explores the implications of the additive and endogenous habit formation preferences in the context of a life-cycle model of an investor who has stochastic uninsurable labor income. To solve the model, I analytically derive the habit–wealth feasibility constraints and show that they depend on the worst possible path of future labor income and on the habit strength, but not on the probability of the worst income. When there is only a slim chance of a severe income shock, the model implies much more conservative portfolios. The model also predicts that for some low to moderately wealthy households, the portfolio share allocated to stocks increases with wealth. Because of this feature, the model can generate more conservative portfolios for younger than for middle-aged households. One controversial finding is that for high values of the habit strength parameter, usually required for the resolution of asset pricing puzzles in general equilibrium, the life-cycle model predicts counterfactually high wealth accumulation.

*Comments are welcome at: polkovni@umn.edu or Finance Department, 3-122 CSOM, 312 19-th Avenue South, Minneapolis, MN 55455. Phone: (612) 624-6081. For comments, suggestions and helpful conversations I thank Jõao Cocco, Alex Michaelides, Annette Vissing-Jørgensen, Amir Yaron, Steve Zeldes, Stan Zin and workshop participants at LSE Financial Markets Group, 2004 AFA conference, 2004 WFA conference, 2003 Econometric Society winter meetings, 2003 Stanford Institute for Theoretical Economics (computational economics week), 2002 SED conference, and 2002 NBER Summer Institute (Capital Markets and the Economy group). I am also grateful to Bob McDonald (the editor) and an anonymous referee for many suggestions and comments which greatly improved the paper. I thank Donna Zerwitz for editorial assistance. Financial support from a McKnight Foundation Business and Economics Research Grant is gratefully acknowledged. The views expressed herein are those of the author and not necessarily of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. All errors and omissions are sole responsibility of the author.

Life-Cycle Portfolio Choice with Additive Habit Formation Preferences and Uninsurable Labor Income Risk

This revision: November 2004

Abstract

This paper explores the implications of the additive and endogenous habit formation preferences in the context of a life-cycle model of an investor who has stochastic uninsurable labor income. To solve the model, I analytically derive the habit–wealth feasibility constraints and show that they depend on the worst possible path of future labor income and on the habit strength, but not on the probability of the worst income. When there is only a slim chance of a severe income shock, the model implies much more conservative portfolios. The model also predicts that for some low to moderately wealthy households, the portfolio share allocated to stocks increases with wealth. Because of this feature, the model can generate more conservative portfolios for younger than for middle-aged households. One controversial finding is that for high values of the habit strength parameter, usually required for the resolution of asset pricing puzzles in general equilibrium, the life-cycle model predicts counterfactually high wealth accumulation.

1 Introduction

Additive habit formation preferences play an important role in the efforts to build a plausible general equilibrium model of asset prices.[1] Because tractable general equilibrium models with additive habit utility are confined to the representative agent setting, their implications for the behavior of individual agents are not well understood. In this paper, I consider additive habit utility in the context of a life-cycle portfolio choice problem. The analysis provides new insights about the determinants and dynamics of the portfolio allocation over the life cycle and views the results obtained in a representative agent setting with additive habit preferences from a new angle. In addition, the solution method used here should prove useful in a general equilibrium model with heterogeneous agents.

The life-cycle models, including the one studied here, traditionally are used to examine the determinants of individual portfolio allocation and its dynamics with age. Understanding how households choose their savings and exposure to risk is important in asset pricing, macroeconomics, and public finance. Existing life-cycle portfolio models have some implications which are at odds with empirical observations.[2] Portfolio choice of the younger households presents a particularly challenging and common problem because it represents a partial equilibrium counterpart to the equity premium puzzle. Standard life-cycle models, when reasonably parameterized, typically predict that young households should invest exclusively in equity. Not only is this prediction largely at odds with the data, but it also presents a challenge to the aggregate macro finance models which use the life-cycle model as a foundation. The present paper shows that an additive habit model implies that young investors should hold more conservative portfolios than the middle-aged investors; it also has other empirically plausible and distinct implications.

In the present paper the model is set in partial equilibrium and is concerned with life-cycle choice of consumption and portfolio allocation for an investor with additive and endogenous (internal) habit formation preferences. I assume a typical economic environment for a life-cycle model. The investor receives labor income, which is subject to uninsurable transitory and permanent shocks during the working life and is constant during retirement periods. There are two tradable securities available to the investor: a bond with constant return,

[1] A large body of literature explores additive habits in the models of asset pricing and aggregate consumption. See, for example, Sundaresan (1989), Constantinides (1990), Detemple and Zapatero (1991), Heaton (1995), Chapman (1998), Campbell and Cochrane (1999).

[2] For example, see a survey in Haliassos and Michaelides (2002). Later in the paper I summarize the main points of their study and compare life-cycle models based on habit and expected utility .

and a stock index with a stochastic return. Borrowing is constrained and short sales are not allowed. I assume that current habit is a fraction of consumption in the previous period and that exogenous variables follow a finite-state Markov process. Under these assumptions I analytically characterize the admissible wealth-habit region for every age and then solve the problem numerically within this region. The boundary of the admissible region depends on the *worst possible* future labor income path and on the habit strength parameter, but does not depend on the probability of the worst case income path. The optimal savings and portfolio choice policies depend on wealth accumulated in excess of the admissible boundary and on the current habit level.

One of the main distinct implications of the habit model is that portfolio policies, in general, are not monotone in wealth. There is a region from low to moderate wealth (in excess of the admissible boundary) where the portfolio share allocated to stocks *increases* with financial wealth. This policy function generates an empirically plausible positive relationship between wealth and portfolio allocation to equities for some households in the model, particularly the younger or not "too" wealthy households. Intuitively, when wealth is low, in order to insure that habit can be safely sustained under any scenario for income and asset returns, the households in the model resort to safer portfolios. As wealth builds, sustaining habits in adverse circumstances becomes less of a concern. Between moderate and high wealth (again, in excess of the admissible boundary), portfolio policies become qualitatively similar to those under expected utility: the portfolio share in stocks is *decreasing* with wealth. This effect is attributable to the similarity between human capital and bonds in the model, which implies that as financial wealth grows relative to human capital, households compensate by increasing their exposure to bonds.[3]

Because there is a range of low to medium wealth in which the portfolio share of stocks is increasing, the model implies interesting life-cycle dynamics in portfolio allocation. In the model, younger households have relatively more conservative portfolios than the middle-aged households: the young investors have not yet accumulated enough wealth to sustain consumption sufficiently above habit. This pattern of portfolio allocation between young and middle-aged is consistent with some empirical evidence, for example as in Heaton and Lucas (2000) or Faig and Shum (2002).[4] Later in the life cycle, portfolio allocation to stocks declines, partly because of the substitution between labor income and bonds and

[3]A detailed discussion of this effect is provided, for example, in Jagannathan and Kocherlakota (1996).

[4]Faig and Shum (2002) suggest yet another explanation for lower allocation to stocks for younger households. In their model, people save for acquiring illiquid assets, such as houses or private businesses, which introduces extra demand for safer liquid assets earlier in the life-cycle.

partly because of the need to maintain habit with lower wealth. In contrast, I find that an identically parameterized expected utility (CRRA) model does not generate conservative portfolios for the young, and that the decline of the portfolio share in stocks with age in the second half of the life cycle is less pronounced, even in simulations with low or zero income state. Comparative statics experiments also introduce the bequest motive which slows down wealth decumulation after retirement. Bequests have a more significant effect on portfolio choice and the wealth accumulation of the CRRA agents: without a bequest motive, they accumulate much less wealth than the habit agents.

Motivated by the structure of the feasibility constraints of the model, I consider calibrations which demonstrate the effect of the worst labor income outcome on portfolio allocation. In the benchmark case, I follow the standard assumptions and calibrate transitory income shocks to a symmetric distribution matching the variance in the data. In this calibration, the average portfolio allocation to stocks is at or near 100% for most of the life cycle. In this case, it takes relatively high habit strength, or a utility curvature coefficient, to generate conservative (interior) portfolios. In the alternative calibrations, I augment the transitory shock component by introducing a state in which income is low (or even zero) with a small probability calibrated from the distributions reported in Carroll (1992). The mere possibility of such a state generates a much more conservative portfolio allocation, especially for the young households with little wealth. For the older working households with sufficient wealth, low income states have little effect on portfolio choice, because their substantial wealth guarantees that consumption can be sustained comfortably above habit in all circumstances.[5]

In relation to general equilibrium asset pricing models with additive habit preferences, an important and controversial issue remains to be reconciled with the results obtained in the present paper. In a representative agent model, it is typically necessary to assume relatively high habit strength in order to resolve the asset pricing puzzles.[6] Higher habit strength implies higher risk aversion which is necessary to reconcile asset prices with smooth aggregate consumption. In the life-cycle model, high habit strength implies conservative portfolio choice, advancing the model towards a resolution of the equity premium puzzle in a partial equilibrium setting. However, high habit strength creates a tension, because the model predicts counterfactually high wealth accumulation throughout the life cycle,

[5]Low income states may have greater significance for the portfolio choice of wealthier middle-aged households if asset liquidity is modeled in more detail, so that part of savings is allocated to illiquid assets, for example, a house as in Yao and Zhang (2004) and Cocco (2004).

[6]Equivalently, in models with exogenous habit formation (e.g. Campbell and Cochrane (1999)), it is necessary to have a very low consumption surplus ratio (consumption-in-excess-of-habit relative to consumption).

well beyond what is observed in the data. This problem most likely reflects a failure of the representative agent assumption and should be investigated further in the context of a general equilibrium model with heterogeneous agents. One interesting question is whether it is possible to resolve the asset pricing puzzles and match stylized facts at the micro level at the same time.[7]

The analysis in this paper builds on several strands of the portfolio choice literature and combines labor income uncertainty, finite horizon, and additive habit formation preferences. Stochastic (and uninsurable) labor income has become a standard element in many recent studies which explore portfolio choice with CRRA or recursive preferences.[8] Models with infinitely lived agents, such as in Bodie, Merton and Samuelson (1996), Heaton and Lucas (1997), and Viceira (2001), show that introducing labor income implies more aggressive portfolio choice because human capital, despite its risk, resembles bonds more than stocks. Because of this effect, studies with finite horizon models – such as Jagannathan and Kocherlakota (1996), Cocco, Gomes, and Maenhout (2004), Gomes and Michaelides (2004) – find that portfolio share in stocks, in general, declines with age. Additive habit utility, on the other hand, implies that the typical young households find it optimal to invest more conservatively than middle-aged households. Several other studies focus on portfolio choice with habit formation preferences in various settings. Hindy, Huang, and Zhu (1997) use an infinite horizon complete markets model to investigate the impact of habits and consumption durability on portfolio allocation. Bodie, Detemple, Otruba, and Walter (2004), consider a finite-horizon model with complete markets and additive habit; they also incorporate labor supply flexibility and retirement decisions. A related model, but with constant labor income, is studied by Lupton (2001), who represents habit as "consumption liability" and simplifies the problem to obtain the same representation as in Merton (1971). Heaton and Lucas (1997) and Gomes and Michaelides (2003) include nontradable risky labor income in an infinite- and a finite-horizon setting respectively. The present paper advances the existing literature by considering both feasibility constraints and the effects of low/zero income realizations on portfolio allocation.

The rest of this paper is organized as follows. In section 2, I set up the model with

[7]Pijoan-Mas (2003) makes an important first step in this direction. He considers such an equilibrium model with the "catching-up-with-the-Joneses" ratio-habit preferences of Abel (1990). Most of the representative agent asset pricing models with habit utility require additive habit specification to achieve resolution of the asset pricing puzzles. And the additive habit specification awaits further analysis in a heterogeneous-agent general equilibrium model.

[8]Earlier studies consider consumption and savings with income uncertainty but do not study portfolio choice, for example, Deaton (1991), Carroll (1992), Gourinchas and Parker (2002).

additive habit formation utility and derive the feasibility constraints. The calibrations are described in section 3. The results of the simulations are discussed in section 4. In section 4.4, I compare the habit model with stylized facts and with CRRA utility models; section 4.5 presents the comparative statics analysis. Section 5 concludes. Appendix A includes a formal derivation of the feasibility constraints, and Appendix B describes the numerical method used to solve the model.

2 Life-cycle model

2.1 Household problem

There is one agent (household) in the model who lives a finite life with the deterministic horizon from t_0 to T and is characterized by the following habit formation utility function defined over the admissible consumption sequences $\{C_t\}_{t=t_0}^{T}$ s.t. $C_t > H_t \geq 0$:

$$U_{t_0} = E_{t_0}\left\{\sum_{t=t_0}^{T} \beta^{t-t_0}\frac{(C_t - H_t)^{1-\gamma}}{1-\gamma} + \beta^{T+1-t_0}B(W_{T+1})\right\}, \tag{1}$$

where $\beta < 1$ is the subjective discount factor, $\gamma > 0$ is the curvature parameter, H_t denotes time-t habit, and $B(W_{T+1})$ is a bequest function and W_{T+1} is bequested wealth. Consumption paths such that $C_t \leq H_t$ for some t in some state with nonzero probability are assigned infinitely negative utility. I assume that habit is internal and depends on one lag of consumption with the strength parameter δ:[9]

$$H_t = \delta C_{t-1}\ ,\ \ H_{t_0} = H^0. \tag{2}$$

The household receives stochastic labor income Y_t, which is subject to permanent (P_t) and transitory (η_t) shocks. Income is the product of these shocks and the deterministic age-dependent component $f(t)$:

$$\begin{aligned} Y_t &= f(t)P_t\eta_t \\ P_t &= P_{t-1}\kappa_t \end{aligned} \tag{3}$$

It is assumed that $\log(\kappa_t)$ and $\log(\eta_t)$ are i.i.d. normal with means such that their exponent has a mean of one. In the calibrations, it will be assumed that during the retirement years

[9]This particular functional form is assumed for simplicity. All qualitative results derived in this paper also apply in a more general case when current habit is a weighted average of past habit and lagged consumption, which is a more general case considered by Constantinides (1990) in continuous time.

Y_t becomes constant and equal to a fraction ϕ (replacement ratio) of the permanent income component just prior to retirement, i.e. $Y_t = \phi f(65)P_{65}$, for all $t > 65$. This simplifying assumption allows us to reduce the dimensionality of the problem in the post-retirement years.[10]

There are two securities available for saving: a risk-free bond and a risky stock. The bond pays constant interest rate, r_b, and the stock represents a market index and pays stochastic return $r_{s,t}$. Neither security can be sold short.[11] Let the purchases of the bond and stock at time t be denoted B_t and S_t respectively. The total cash on hand X_t available to the household at time t is defined as:

$$X_t = S_{t-1}(1 + r_{s,t}) + B_{t-1}(1 + r_b) + Y_t \tag{4}$$

At every period $t = t_0, \ldots, T$ the household chooses consumption and purchases of bond and stock subject to the following constraints:

$$\begin{aligned} X_t &= C_t + B_t + S_t \\ B_t &\geq 0 \ , \ S_t \geq 0 \\ C_t &> H_t \ , \ t = t_0, \ldots, T \end{aligned} \tag{5}$$

The first three constraints are familiar budget, borrowing, and short-sales constraints. The last system of inequality constraints on consumption is specific to the additive habit preferences. Consumption cannot fall below habit at any time, regardless of the random realizations of the exogenous variables. In the next section, and more formally in Appendix A, I discuss how these constraints may be rewritten to make their economic intuition more transparent.

The homotheticity of the utility function allows us to reduce the dimensionality of the problem by normalizing all level variables by their contemporaneous permanent income, P_t. Let the lower case letters denote normalized variables, one can then rewrite normalized cash on hand as follows:

$$\frac{X_t}{P_t} \equiv x_t = [s_{t-1}(1 + r_{s,t}) + b_{t-1}(1 + r_b)] \times \frac{P_{t-1}}{P_t} + y_t \tag{6}$$

[10] While there is less labor income risk (or none at all) after retirement, there are certainly other risks such as medical expenses and disability. However, computational burden and lack of empirical data prevent calibration of such risks and they are left out of the model.

[11] For stocks this is not a binding constraint, for bonds, it depends on the calibration. Such liquidity constraints are commonly assumed in the life-cycle literature to capture limited ability of households to contract on future labor income to obtain uncollateralized loans.

Habits are normalized, too, which results in the following equation for habit evolution:

$$\frac{H_t}{P_t} \equiv h_t = \delta \frac{C_{t-1}}{P_t} = \delta c_{t-1} \times \frac{P_{t-1}}{P_t} \tag{7}$$

After the normalization, the state vector is defined as $z_t = [h_t, x_t, t]'$. The household problem is to find the sequences of consumption $\{c_t(z_t)\}$, bondholdings $\{b_t(z_t)\}$ and stockholdings $\{s_t(z_t)\}$ as functions of the state z_t for $t = t_0, \ldots, T$ subject to the constraints (5) to maximize the utility in (1). For numerical solution, it is more convenient to write the household problem as a sequence of stochastic Bellman equations. Let $V_t(z_t)$ denote the value function at time t and state z_t. Then, the household problem can be written as this sequence of maximizations:

$$\begin{aligned} V_t(z_t) &= \max_{c_t, s_t, b_t} \frac{(c_t - h_t)^{1-\gamma}}{1-\gamma} + \beta E_t \left\{ V_{t+1}(z_{t+1}) \left(\frac{P_{t+1}}{P_t} \right)^{1-\gamma} \right\}, \\ &\quad \text{for } \; t = t_0, \ldots, T, \text{ s.t. (7), (6), and (5)} \\ V_{T+1} &\equiv \lambda \frac{\left(\frac{w_{T+1}}{\lambda} \right)^{1-\gamma}}{1-\gamma} \\ w_{T+1} &= s_T(1 + r_{s,T+1}) + b_T(1 + r_b) \end{aligned} \tag{8}$$

The bequest function V_{T+1} in (8) is similar to the one considered in recent studies of portfolio choice. Its functional form is maintained for compatibility with the existing literature.[12] The solutions to the stochastic Bellman equations (8) are computed on the grid in endogenous variables space, using bi-cubic spline approximations for the points that do not fall on the grid. Appendix B provides further details of the computational procedure.

2.2 Characterizing a feasible set of endogenous variables

The constraints on consumption and habit in (5) restrict the feasible pairs of the endogenous variables (X_t, H_t) and contain important information about the economic structure of the problem. It is also important to analyze these constraints to develop the intuition about the solution and to design an efficient numerical method. In Appendix A, I formally derive the restrictions on the admissible cash-habit pairs when the exogenous variables follow a finite-state Markov process. In this section, I describe the intuition of the derivation and the economic interpretation of the constraints.

[12] This function can be thought of as providing utility of bequest during λ periods. For analyses using similar bequest functions see, for example, Cocco *et. al.* (2004), Cagetti (2003), Gomes and Michaelides (2004), and Dynan *at. al.* (2004).

The constraints (5) imply that $X_t > H_t$ for all t. I.e. there must be sufficient total cash strictly exceeding the current habit. It should be noted that at any time, t, the constraints imply more than just $C_t > H_t$. It is not enough to restrict the household to consume today above today's habit. Consuming too much today may generate infeasible habits in the future. For example, consider a (budget-constraint-feasible) policy to consume all resources today and set $C_t = X_t > H_t$ $(t < T)$. If the following inequality holds:

$$\min_{\eta_{t+1},\kappa_{t+1}} Y_{t+1} \leq H_{t+1} = \delta X_t, \tag{9}$$

then the household does not have enough resources to consume above habit in the worst income state at time $t+1$. When (9) is satisfied with equality, the marginal utility is infinite. Thus, increasing consumption too much today may be inadmissible in the worst case scenario tomorrow.

To characterize the admissible endogenous variables, I start at time T and proceed backwards. At time T there is no need to look forward anymore and the only constraint is to have cash above habit. The admissible set for the endogenous variables is $X_T > H_T$. At time $T-1$, the household must have enough resources for the current habit. Thus, the admissible set is partly restricted by $X_{T-1} > H_{T-1}$. Another restriction emerges from the requirement that there should exist at least one feasible policy at $T-1$ such that in the next period, under any realizations of the exogenous variables, it guarantees cash above habit. Appendix A shows that this requirement imposes the following constraint on (X_{T-1}, H_{T-1}):

$$\frac{\underline{Y}_T}{1+r_b} + X_{T-1} - \left(1 + \frac{\delta}{1+r_b}\right) H_{T-1} > 0, \tag{10}$$

where $\underline{Y}_T$ is the lowest possible labor income at T. In deriving equation (10), I assume that the worst possible outcome for cash on hand is when labor income has the lowest possible value and stock return is negative. If habit and cash at time $T-1$ satisfy (10) with equality, the marginal utility is infinite. This is because even if consumption is set equal to habit at $T-1$, leaving the *maximum possible* savings for tomorrow, there won't be enough cash in the worst case at time T to consume above habit, no matter what portfolio is chosen at $T-1$. To see this, note that if $C_{T-1} = H_{T-1}$, then the household saves $X_{T-1} - H_{T-1}$. Assuming that in the worst state the stock return at T is negative, then to *maximize* cash on hand in the worst case the household should not buy any stock and should invest only in bonds. Thus, the maximum cash in tomorrow's worst case is given by $\underline{X}_T = (1+r_b)(X_{T-1} - H_{T-1}) + \underline{Y}_T$ and tomorrow's habit is $H_T = \delta H_{T-1}$. Note that we have $\underline{X}_T - H_T = 0$ if we assume (10)

holds with equality. Thus, even if the household saves the maximum possible and in the safest possible portfolio, there will not be enough resources in the worst case outcome to exceed the habit at time T.

Constraints similar to (10) are derived in Appendix A for a general time t. At every time t there are $T-t+1$ constraints indexed by $\tau = 0, \ldots, T-t$ as follows:

$$0 < \sum_{k=1}^{\tau} \frac{\underline{Y}_{t+k}}{(1+r_b)^k} + X_t - H_t \left(\sum_{k=0}^{\tau} \left(\frac{\delta}{1+r_b} \right)^k \right) \ , \ \tau = 0, \ldots, T-t \tag{11}$$

where the convention is to set the first sum equal to zero for $\tau = 0$. For future reference, it is convenient to adopt the following notation:

$$\begin{aligned} Y_{t,t+\tau} &= \sum_{k=1}^{\tau} \frac{\underline{Y}_{t+k}}{(1+r_b)^k} \ , \ Y_{t,t} \equiv 0 \\ \Delta_\tau &= \sum_{k=0}^{\tau} \left(\frac{\delta}{1+r_b} \right)^k = \frac{1 - \left(\frac{\delta}{1+r_b} \right)^{\tau+1}}{1 - \frac{\delta}{1+r_b}}. \end{aligned}$$

It is also convenient to normalize $Y_{t,t+\tau}$ by the time-t permanent income component and use the lower case notation. Note that $Y_{t,t+\tau}$ is a multiple of today's permanent income P_t for every τ and can be normalized as follows:

$$\begin{aligned} Y_{t,t+\tau} &\equiv \sum_{k=1}^{\tau} \frac{\underline{Y}_{t+k}}{(1+r_b)^k} = P_t \sum_{k=1}^{\tau} f(t+k) \times \frac{\underline{\eta}_{t+k} \Pi_{i=1}^{k} \underline{\kappa}_{t+i}}{(1+r_b)^k} \ , \ 1 \leq \tau \leq T-t+1 \ , \ Y_{t,t} \equiv 0 \\ y_{t,t+\tau} &\equiv \frac{Y_{t,t+\tau}}{P_t} = \sum_{k=1}^{\tau} f(t+k) \times \frac{\underline{\eta}_{t+k} \Pi_{i=1}^{k} \underline{\kappa}_{t+i}}{(1+r_b)^k} \ , \ 1 \leq \tau \leq T-t+1 \ , \ y_{t,t} \equiv 0 \end{aligned}$$

where $\underline{\kappa}_t$ and $\underline{\eta}_t$ denote, respectively, the minimum possible permanent and transitory shock values at time t.[13] Using normalized notation, the constraints in (11) can be written compactly as:

$$0 < y_{t,t+\tau} + x_t - h_t \Delta_\tau \ , \ \tau = 0, \ldots, T-t \tag{12}$$

The interpretation of inequalities in (12) is very intuitive. Note that $y_{t,t+\tau}$ is the time-t present value of the *minimum possible* labor income received between t and $t+\tau$. Thus, the first two terms in the equation above correspond to the minimum possible present value of available resources for a given time horizon τ. On the other hand, $h_t \Delta_\tau$ is the *infimum*

[13] Note that in the above expressions, the worst permanent income shocks "cumulate" while the transitory shocks do not. The above normalization is exact and does not rely on any particular values of $\underline{\kappa}$ and $\underline{\eta}$. Therefore, it is preserved in any Markov chain discretization of the labor income process.

of the present value of the future habits up to time $t+\tau$ when the current habit is h_t. To see this, note that if the household consumes $h_{t+\tau}+\epsilon$ in each period between t and $t+\tau$, where $\epsilon > 0$ is a very small number, then the limit, as ϵ goes to zero, of the present value of the habits generated by this consumption sequence is $h_t\Delta_\tau$. Thus, consumption cannot be decided based upon the budget constraint alone, and the household must consider what future habit it will generate. Note, too, that while the worst-case labor income enters the constraints, its probability does not: what matters is only that this probability is positive.

2.3 When do the constraints on the endogenous states matter?

Two recent papers, Heaton and Lucas (1997) and Gomes and Michaelides (2003), compute solutions to the additive habit formation models with labor income risk and endogenous additive habits but without referring to the constraints discussed here. Thus, it is instructive to examine under what conditions these constraints become important for the solution of the problem.

Any two consecutive constraints from (12) for τ and $\tau+1$ intersect at the point given by:

$$\begin{aligned} h^*_{t,\tau} &= \frac{y_{t+\tau+1}}{\delta^{\tau+1}} \\ x^*_{t,\tau} &= \frac{y_{t+\tau+1}}{\delta^{\tau+1}}\Delta_\tau - y_{t,t+\tau} \end{aligned}$$

The first intersection for $\tau = 0$ occurs at a point given by $x^*_{t,0} = h^*_{t,0} = \frac{y_{t+1}}{\delta}$. All subsequent intersections lie to the northwest of this point. If the lower bound on labor income is high, and/or the habit strength is low, this intersection point will be far away from the actual habit level occuring in simulations. In this case, the only relevant constraint to keep track of is $x_t > h_t$. To see this, consider a numerical example shown in Figure 1, where cash and habit are normalized by the expected permanent income component (denoted Ey) and the risk-free interest rate is assumed to be $r_b = 1\%$. The parameters for the example on panel A were chosen to be within the range of calibrations reported by the two papers mentioned above. On panel A, the strength parameter δ is 0.4 and the worst labor income outcome is equal to $0.7 \times Ey$. In this case, the first intersection of the constraints occurs at the point $(x^*_{t,0}, h^*_{t,0}) = (1.75, 1.75) = (0.7/0.4, 0.7/0.4)$. In simulations consumption (normalized by the permanent income Ey) is approximately given by $c_t \approx 1.0$.[14] Therefore, the habit is

[14]This is the approximate average across simulations of a stationary consumption distribution. See, for example, Heaton and Lucas (1997).

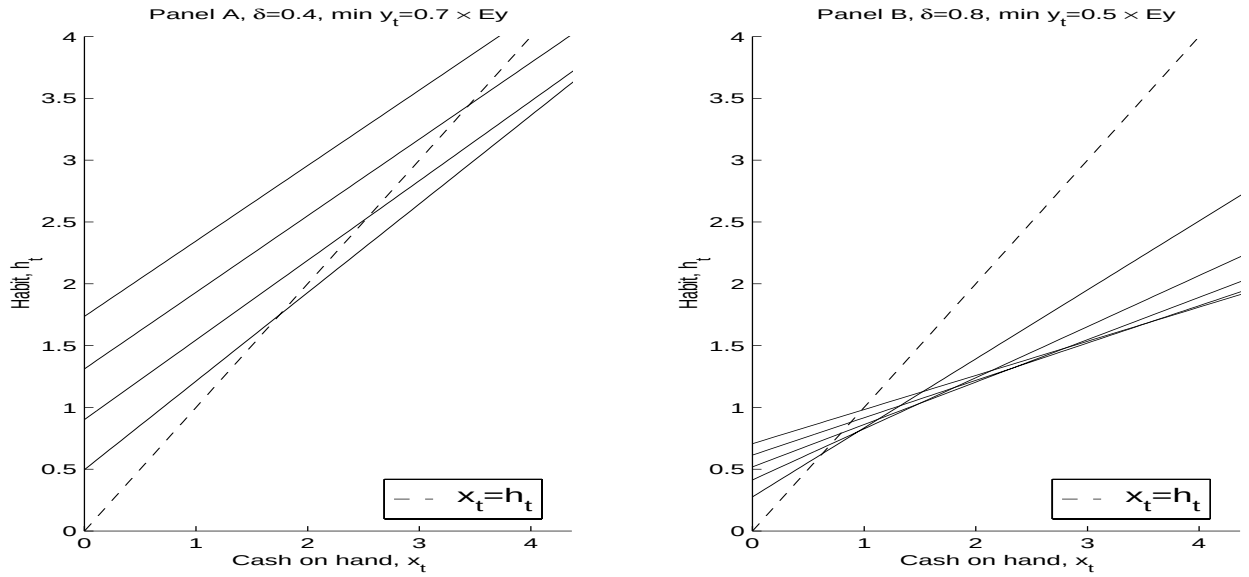


Figure 1: Constraints on endogenous state variables. The region below the low envelope of the lines defines admissible states.

approximately given by $h_t = 0.4 \times c_t = 0.4$. Thus, even the first intersection of the constraints on panel A at the point $(1.75, 1.75)$ is quite far away from the range of realizations of the endogenous variables. For such parameter values it is not necessary to pay attention to all the constraints in (12). Since the endogenous variables in the computations usually are confined to a rectangle $[x_l, x_h] \times [h_l, h_h]$, if h_h is well below the first intersection, the only relevant constraint in this rectangle would be $x_t > h_t$. As the lowest labor income realization falls, and/or the habit strength increases, more constraints will be within a given rectangle. Figure 1, panel B shows another example with a higher habit strength, $\delta = 0.8$, and a lower worst income, $0.5 \times Ey$. In this case, constraints move downward considerably and become tighter.

Since only some values of cash on hand and habit are admissible, to solve the problem one may restrict attention to the points in the state space to the right of the *lower envelope* of the constraints as shown in Figure 2. To rule out inadmissible states, it is convenient to transform cash on hand into cash on hand *in excess* of the minimum admissible level. This minimum level depends on the current habit. To proceed, it is convenient to define a variable τ^* as follows:

$$\tau^* = \arg \max_{\tau \in [0,\ldots,T-t+1]} \{h_t \Delta_\tau - y_{t,t+\tau}\} \tag{13}$$

In the above expression, τ^* simply identifies the constraint from (12) corresponding to the

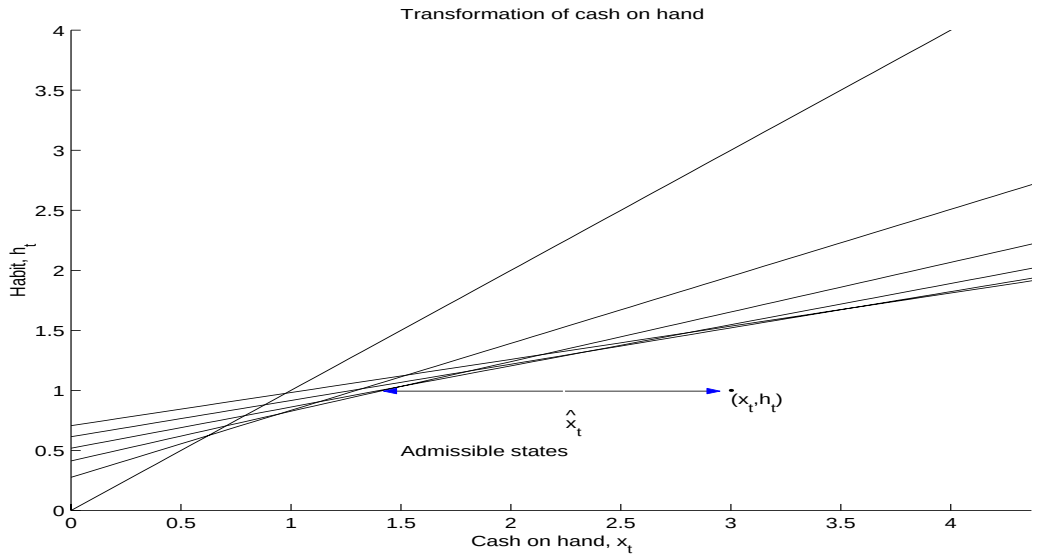


Figure 2: Cash on hand variable is transformed to cash on hand in excess of admissible boundary.

lower envelope segment at the level of habit h_t. This is the constraint closest to being binding. Geometrically, this corresponds to selecting the constraint line closest to the point (x_t, h_t) moving horizontally to the left from (x_t, h_t) until reaching the lower envelope of the constraints lines, as shown in Figure 2. Note that if $x_t = (h_t\Delta_{\tau^*} - y_{t,t+\tau^*})$, then the distance from (x_t, h_t) to the admissible boundary would be zero and the strict inequality constraint corresponding to τ^* in (12) would be violated. Thus, the amount $(h_t\Delta_{\tau^*} - y_{t,t+\tau^*})$ is equal to the minimum admissible cash on hand for a given habit. Using the definition of τ^*, the transformed cash on hand in excess of the minimum admissible level is defined as follows:

$$\hat{x}_t = x_t - (h_t\Delta_{\tau^*} - y_{t,t+\tau^*}) \ , \ \ \hat{x}_t > 0 \ , \tag{14}$$

Thus, $\hat{x}_t$ corresponds to the horizontal distance to the feasible boundary as shown by the arrows in Figure 2. Note that $\hat{x}_t > 0$ guarantees corresponding x_t to be feasible.[15] Besides automatically ruling out inadmissible endogenous states, there are other numerical advantages to using $\hat{x}_t$. Appendix B explains further how the transformed $\hat{x}_t$ is used in the

[15]This resembles Lupton's (2001) transformation to account for habit liability, but there are two important differences. First, the transformation here involves the lowest possible present value of future income. Second, the transformation does not substitute out the habit from the model. In the case of nonstochastic income considered by Lupton, the problem simplifies to one-variable state (as in Merton (1971)), so that wealth net of habit liability is a sufficient endogenous state. The simplification does not happen here because when income is stochastic, one still needs to track habit h_t to make sure that x_t satisfies feasibility constraints.

numerical procedure.

3 Calibration

Time period is assumed to be one year. The household is assumed to live from age 25 until age 75 and begins retirement after age 65. The age-income profile $f(t)$ is taken from Cocco, *et. al.*(2004). They estimate age-income profiles from the PSID data using cubic polynomials and they report the estimated coefficients for three cohorts divided by education. I use the age-income profile for high school graduates in the calibrations that follow. The utility function parameters are set as follows: $\beta = 0.95$ and $\gamma = 5$. The habit strength parameter δ is considered in the range of $[0, 0.8]$. In the baseline case, there is no bequest motive ($\lambda = 0$), primarily for the sake of transparency. In comparative static analysis, I relax this assumption, introduce bequest motive, and compare its impact on the behavior of habit versus expected utility investors.

From the earlier discussion, it is clear that the distribution of income shock is crucial for the feasible set of endogenous variables. Typically, income shock η_t is modeled as a finite-state Markov process with several symmetric outcomes around the mean of 1 matching the estimated volatility of income.[16] For the problem with habit formation, it is important to calibrate the worst realizations of labor income. I assume that shock η_t is i.i.d. and consider two different calibrations: symmetric and asymmetric.[17] Transitory shock η_t in the symmetric distribution has two values around the mean of 1 matching the standard deviation of 27%. Permanent income shocks are calibrated to have standard deviation of 10%.[18] In another calibration, I use Carroll's (1992) results: he estimates that at an annual frequency with a very small probability, labor income may fall substantially. To reflect that, I augment the symmetric transitory income shock calibration by introducing a third state

[16] See, for example, Storesletten, Telmer and Yaron (1998), Cocco *et. al.* (2004), Heaton and Lucas (1997).

[17] The assumptions of i.i.d. shocks to income and returns (next paragraph) are made primarily for computational convenience. These assumptions allow for computation of only one value function per period while assuming correlations increases computing time exponentially in the number of correlated variables. Nothing in the model relies on these assumptions and, if anything, the portfolio allocations implied by the model would be more conservative for positively correlated returns and income shocks. The available empirical evidence shows that income shocks and stock returns are not highly correlated. For example, Davis and Willen (1999) report correlations close to zero and statistically insignificant for most occupational and educational cohorts and similar empirical evidence is presented by Cocco *et. al* (2004).

[18] For consistency, I use shock volatilities from the same source as income profiles . Heaton and Lucas (1996, 1997) and Storesletten, *et. al.* (1998) estimate somewhat higher volatility of the transitory income shock.

with either zero or low income. The probability of this state is very low. I calibrate it using the histograms reported in Carroll's paper. For brevity, I only report two cases: zero income with 0.5% probability, and low income – equal to 10% of the permanent level – with 1.3% probability.[19]

Stock returns are calibrated to be an i.i.d. Markov process with two symmetric outcomes around the mean. The mean return is set at 6% and the standard deviation is 15%. Bond return is assumed to be constant 1%. While the equity premium here is 5%, which is lower than historic average value of 6%, one could argue for even a lower premium given that transactions costs of holding stocks and taxes are ignored in the model. Jagannathan, McGrattan and Scherbina (2001), and McGrattan and Prescott (2003), have argued that the equity premium is much lower than 6% after accounting for diversification costs, taxes and liquidity premium of the short-term government bonds.

For every calibration, I simulate a cross section of 10,000 households to compute the distributions of the variables of interest: wealth, consumption, and portfolio fraction invested in stocks. The initial values for habit for all calibrations is set to $0.5 \times f(t_0) \times P_{t_0}\delta$. Thus, the consumption just prior to entering working life is assumed to be half of the permanent labor income component in the first period of the model. Initial normalized habit will be $0.5 \times \delta$ as δ changes across calibrations. In all calibrations with symmetric income shocks the initial asset holdings are zero. For initial habit to be admissible in the calibrations with zero/low income state, the initial wealth is set to $0.8 \times f(t_0) \times P_{t_0}$, i.e., equal to 80% of the permanent income in the first period.[20]

4 Simulation results

4.1 Savings and portfolio policies

To better understand the intuition for the results obtained in simulations, it helps to examine optimal savings and portfolio policies. Figure 3 shows portfolio allocation and savings policies in the first period for the asymmetric income shock calibration with zero income. Qualitatively, the policies shown are similar to the other calibrations. There are a few

[19] Other low income states considered were between 15% and 30% of permanent income level. Qualitatively, the results are similar to those reported and are available upon request from the author.

[20] This initial wealth is chosen to be admissible for the initial habit level and to produce comparable consumption profiles in a chosen range of habit strengths. The alternative would be to set the initial habit at zero, so that the usual assumption of zero starting financial wealth is in the admissible range. However, zero initial habit implies unrealistically low consumption early in life for higher habit strength.

intuitive properties of the policies that are present throughout the life-cycle.

Figure 3, upper panel shows that the savings rate (out of discretionary resources, $x - h$) increases when the household is closer to the boundary of the admissible region (that is, $\hat{x}$ closer to zero). As $\hat{x}$ goes to zero, the savings rate approaches 1. This is because the expected marginal utility is higher near the admissible boundary and it approaches infinity. Holding $\hat{x}$ constant, the savings rate increases with habit because the same life-time resources will have to be used to sustain the higher habit.

The lower panel shows the portfolio fraction allocated to stocks. The allocation to stocks is decreasing approaching the admissible boundary and converges to zero as $\hat{x}$ goes to zero. This is because stock is a risky asset and its presence in the portfolio decreases the *lower bound* on future wealth. Therefore, when wealth is lower, the portfolio must become more conservative in order to guarantee that consumption stays above habit in the worst-case scenario. Holding $\hat{x}$ constant, the allocation to stocks declines with habit because higher habit is harder to maintain. So, portfolio choice is more influenced by the worst possible outcomes.

Compared to the expected and recursive utilities studied earlier in the literature, habit utility generates new implications for the relationship between wealth and portfolio share in stocks. This deserves a special discussion. In many life-cycle models of portfolio choice, including the one with habit utility, there is a well-known effect of substitutability between bond income and labor income. Because of this effect, households with substantial human capital but low financial capital (i.e. young households) have an incentive to tilt their portfolios toward stocks. On the other hand, households with relatively low human capital but high financial wealth (i.e. older households) would want to allocate their portfolios more conservatively. In the models with expected/recursive utility and moderate risk aversion, this effect counterfactually implies complete portfolio specialization in stocks for young (or not very wealthy) households.

While the same mechanism applies in the model with habit, there is a new effect in portfolio choice that helps to resolve the problem. Figure 4 shows two policy functions at ages 25 (first period) and 64 (one period before retirement), both evaluated for habit level $h = 0.3$.[21] The constraint equations at this habit (both for younger and older household) imply that the minimum admissible cash on hand is just under 0.73. Thus, the correspondence

[21]For $\delta = 0.6$ the habit of 0.3 corresponds to the initial habit at age 25. Recall that consumption prior to entering working life was assumed to be 0.5 times the initial permanent income level, so the initial normalized habit is $0.5 \times 0.6 = 0.3$. Portfolio policies for nearby habit levels are similar.

between $\hat{x}$ shown on the figure and cash on hand x is given by $x \approx \hat{x} + 0.73$ with values of $x \leq 0.73$ being inadmissible. The substitutability of human and financial capital is apparent in Figure 4: from medium to high wealth, a young investor's policy is to hold only stocks; for older investors, it is optimal to reduce exposure to stocks at higher wealth. However, near the minimum admissible level of cash on hand, both types of investors behave similarly and rapidly decrease their exposure to stocks. When cash on hand approaches the admissible boundary, both old and young investors must reduce their exposure to stocks: stocks aggravate the worst case scenario for cash on hand. Note that for the lower wealth and younger investor, the portfolio is more conservative than the for the older investor with the same wealth. This is because the young investor has to sustain the same habit under income uncertainty for many periods, while the older investor's income soon will become riskless. Therefore, for the younger who are not "too rich", the model predicts quite conservative portfolios. While this effect is present in all calibrations, it is particularly apparent in the simulations with asymmetric income shock distribution with low/zero income state.

4.2 Symmetric income shock distribution

Figure 5 shows the means of cross-sectional distributions of consumption, financial wealth, and portfolio share invested in stocks. The results are reported for three values of the habit strength parameter, $\delta = 0.8$, 0.6, and 0 (expected utility CRRA case). The effect of increasing δ is to increase wealth accumulation throughout the life cycle. Other things equal, a higher δ narrows the admissible region for endogenous states by tightening the feasibility constraints. Therefore, consistent with the properties of savings policies, a higher δ has the effect of increasing savings out of discretionary resources $(x_t - h_t)$ and leads to faster wealth accumulation.

As a result of higher savings under stronger habit, the consumption is lower early in life for higher values of δ. The consumption profiles are uneventful, except that two habit cases show a slight increase towards the end of life-cycle. This is driven by the anticipated change in the utility function in the last period, T, where the future habit does not matter anymore because the agent dies. Note that consumption for the CRRA case does not have this feature, and consumption is almost flat after retirement.[22]

The bottom panel of Figure 5 shows the portfolio allocation in stocks throughout the life-

[22]There is a very small growth of consumption in this case which is driven by the assumed relative values of asset returns and the subjective discount factor in the utility function, β. Consumption in the CRRA case does not decline because age of death is deterministic.

cycle. As expected from the earlier overview of the portfolio policies, higher habit strength reduces stockholdings. The figure shows a common problem of the life-cycle portfolio choice models: the predicted share invested in stocks is at, or close to, 100% throughout life for the CRRA case.[23] Essentially, this problem is a manifestation of the equity premium puzzle in a partial equilibrium setting. This effect is a well known feature of the portfolio choice models with non-financial income, and was shown first by Merton (1971) in a model where constant labor income substitutes for bonds in the financial portfolio.[24]

The predicted portfolio choice is not supported by the empirical evidence, though. For example, Poterba and Samwick (2002) and Ameriks and Zeldes (2000) summarize the data on portfolio allocation of U.S. households. Their findings suggest that, even conditional on nonzero stockholdings, household portfolios are far from 100% equity. Similar conclusions can be made from the findings of Heaton and Lucas (2000), and from a collection of international portfolio studies in Guiso, Haliassos and Jappelli (2002). As to the life-cycle shape of the portfolio share in equities, the evidence from all of these studies is mixed and depends on how the estimation procedures control for age, time or cohort effects. Later in the paper I will revisit the implications of habit and expected utility models in view of the empirical evidence.

Figure 5 shows that simulated portfolios are interior very early and later in life so that portfolio allocation is hump-shaped with a plateau. There are two effects at play in the portfolio choice of the older investors. First, as human capital decreases, households compensate by shifting their portfolio allocation towards bonds which were crowded out earlier by human capital. The second effect is specific to the habit model. As households decumulate assets but maintain consumption levels (hence the habit levels), the wealth must be invested more safely to insure that habit can be maintained in any circumstances. Younger households have not yet accumulated savings and their wealth is closer to the admissible boundary. However, their portfolios are not as conservative as those of the retired investors, because they own substantial human capital with a generous lower bound on realizations. Also, note that even for the higher levels of habit strength, the model is unable to generate conservative portfolios throughout the entire life-cycle. However, the calibrations attempted

[23]This prediction depends on utility parameters and correlation of stock returns and permanent income. For example, in Cocco *et. al.* (2004) portfolios become interior in the later stage of life-cycle for CRRA preferences with $\gamma = 10$ and correlation of permanent income and stock returns of 0.15.

[24]Merton did not consider a case with borrowing constraints. However, if one applies his result to a model in which labor income is constant but borrowing against it is not allowed, then the implied portfolio allocation would be constrained at 100% in equities earlier in the life-cycle.

so far do not exploit one of the key properties of the additive habit formation preferences: that the lower bound on labor income affects savings and portfolio choice.

One interesting and important observation from the simulations shown in Figure 5 is that, for high values of habit strength, $\delta = 0.8$, there is a substantial accumulation of wealth. While in the representative agent general equilibrium models the higher habit strength helps to achieve a low risk-free interest rate and a high equity premium, in partial equilibrium this presents a problem. For the utility assumed in the present model, the levels of habit strength typically required to resolve the asset pricing puzzles would be in a range above 0.8 (Constantinides (1990)).[25] For such strong habits, the life-cycle model predicts dramatic wealth accumulation relative to the cases of low habit strength and CRRA utility. Empirically, wealth accumulation is not even close to what the model predicts for high habit strength. For example, Gentry and Hubbard (2000) compute wealth-income ratios conditional on age and entrepreneurial status. For people without businesses, they find that median (mean) wealth-income ratios before retirement range from 0.3 (1.2) for people younger than 35 to 1.4 (2.9) for people between 35 and 55. While the model with low-to-moderate habit strength has plausible wealth-income ratios, the high-habit strength calibration ($\delta = 0.8$) has median and mean wealth-income ratios on the order of six to eight for the middle-aged households.[26] It is quite likely that this problem is due to a failure of the aggregation assumptions required by the representative agent models, and that one need not assume high habit strength when explicitly modeling heterogeneous individual consumption. Aggregate consumption is quite smooth, and for the resolution of asset pricing puzzles, the representative agent models require high risk aversion (habit strength).[27] To confront the problem of excessive wealth accumulation it would be necessary to construct a model with heterogeneous agents. Such a model would not aggregate analytically. Future research can determine whether heterogeneous-agents model can match asset returns and still have correct micro-

[25] Alternatively, for exogenous habit (Campbell and Cochrane (1999)), one must have consumption surplus ratio as low a 0.05. Even in the case of calibration for $\delta = 0.8$ the simulated consumption surplus ratio ($\frac{C_t - H_t}{C_t}$) is only about 0.3-0.25 for the larger part of the life cycle.

[26] The model in this paper is too stylized to formally investigate the determinants of cross section of life-cycle wealth accumulation, so this analysis is not attempted. Such an exercise would have to be primarily empirical and would require explicit modeling of household heterogeneity along factors such as education, demographic status, risk preferences, bequest motive, entrepreneurial/occupational status and real estate ownership.

[27] In the case with exogenous habit, as in Campbell and Cochrane (1999), the relative risk aversion is equal to the utility curvature coefficient divided by the excess consumption ratio, i.e. to $\gamma \frac{C_t}{C_t - H_t}$. In the case of endogenous habit, as in Constantinides (1990), the risk aversion is not expressed easily analytically, but the same intuition applies: the lower the excess consumption ratio, the higher the risk aversion.

level implications. Pijoan-Mas (2003) considers such a model with ratio-habit preferences. It seems feasible to combine his approach to solving the model with the techniques specific to the additive habit utility used in this paper.

4.3 Asymmetric income shock distribution

Typically, in the life-cycle literature it is assumed that uncertainty in the individual labor income stems from unemployment. The parameters for the labor income process usually are estimated from the panel data on earnings, and the empirical procedures use averages across individuals to obtain stable estimates.[28] Such averaging may underestimate the lower bound on nonfinancial income received during the periods of unemployment. There are also other risks and non-discretionary expenses which may significantly reduce household's disposable income. For example, out-of-pocket expenses on health care are especially likely to occur later in the life cycle. While public and private health insurance may cover such expenses, not all households have access to insurance and there are often substantial copays. Another major income shock is disability. For reasons of moral hazard, disability is only partially insurable. It may entail a permanent and significant reduction of income, even if disability benefits are awarded, which is far from guaranteed.

All of these risks reduce nonfinancial income in the worst case scenario. Therefore, they may have a significant impact in the model with habit formation preferences. While it is hard to calibrate the probabilities of specific events leading to major income shocks, the distribution of income constructed by Carroll (1992) using PSID data shows the evidence of significant income shocks faced by households. His distribution shows a non-negligible mass with zero or near zero income realizations. Carroll argues that these observations are unlikely to be driven simply by measurement error, because the households with zero or near zero income also report experiencing "bad" events, such as unemployment, injury, health problems, divorce, etc. In this section, I use the figures from Carroll (1992) and assume that income could experience an unusually bad state with a small probability. Several calibrations using Carroll's data were attempted; here I report the results of two of them. In one case, households may have zero income with probability equal to 0.5%; in the other case, the low income state is 10% of permanent income with a probability of 1.3%. These shocks

[28] Such calibrations of the labor income shock distribution parameters are abundant in the literature. See, for example, Heaton and Lucas (1996), Storesletten, Telmer and Yaron (1998), Cocco, *et. al.* (2004) and many others. One of the data sets commonly used for this purpose is the PSID (Panel Study of Income Dynamics).

are assumed to be uncorrelated with stock returns and may occur at any period during the working years.[29]

In the calibrations with low/zero income state, I use habit strength of 0.6 and compare it to the case of symmetric income shock considered earlier. I also compute a case with expected utility (CRRA) and a zero income state for comparison. A reminder: in these calibrations, the initial asset holdings are assumed to be equal to 80% of the initial permanent income. This is done to make sure that initial habit is admissible in all compared calibrations and is the same as in the earlier experiments. For the sake of brevity, I only discuss portfolio allocation profiles in these cases shown in Figure 6. Consumption and wealth accumulation are qualitatively similar to the cases presented earlier for symmetric income shock distribution.

Portfolio choice profiles for the cases of low and zero income are qualitatively similar and have a distinct hump-shape. Portfolios are somewhat more conservative when income can fall all the way to zero rather than when it is bounded at a minimum of 10% of permanent income. For the latter case, the feasibility constraints are relaxed which is increasing portfolio exposure to stocks (see discussion of Section 2.3 and Figures 1 and 2). When sufficient wealth is accumulated in mid-life, and when there are no income shocks at retirement, the profiles with the zero and the low-income state nearly coincide. In the case of habit utility with symmetric income shock, portfolios are aggressive earlier in life because of the extra "cushion" provided by labor income. In mid-life and retirement, portfolio choice is close to the two asymmetric distribution cases. This last implication of the model is likely to change if asset liquidity is considered in more detail, because liquid wealth is critical to maintaining consumption above habit under adverse circumstances. Liquid wealth may be low even for middle-aged households because the bulk of their assets may be tied up in housing which is not very liquid.

The case of expected utility (EU) is shown last. Because all calibrations in Figure 6 start with non-zero wealth, the EU investor is virtually unaffected by the possibility of zero income. In fact, because initial wealth is higher here than in the case shown earlier in Figure 5, the EU investor has more stocks early in life. Later in life and at retirement, EU investors reduce their exposure to stocks, but this reduction is not as strong as for habit investors.

[29]The empirical case for correlated bad transitory income shocks and stock returns is weak. Empirical evidence such as in Davis and Willen (1999) and in Cocco *et. al.* (2004) supports zero correlation of transitory income shocks and stock returns. If anything, these studies find only a small correlation between permanent income shocks and stock returns. Moreover, most of the events that potentially lead to bad transitory income shock, such as divorce, exceeding insurance limits, liability payments etc. (see Carroll (1992)), are unlikely to be correlated with stock market performance.

This calibration shows that there is a significant effect of introducing low labor income realizations in the model with habit formation. Even though the expected present value of labor income is virtually unchanged by the assumption of a zero/low income state, there is a substantial effect on portfolio choice. The main difference is that portfolios early in life are much more conservative. Expected utility preferences, under similar conditions, show more aggressive portfolios than habit preferences throughout the life cycle.

4.4 Further discussion: habits, expected utilities and stylized facts

In light of the empirical evidence, it is helpful to compare the habit model to the standard life-cycle models based on expected utility .[30] In a recent survey, Haliassos and Michaelides (2002) identify three main challenges to the existing portfolio choice models. The first challenge is the limited stock market participation of (predominantly) households with low wealth, an issue not addressed in the model presented here.[31] Second is the positive relationship between the equity portfolio share and wealth (or income) in the data, which is in contrast to the models in which the portfolio share of stocks declines with cash on hand. Third, the share of portfolio invested in stocks declines with age in the models, but the data suggests anything but this pattern: empirically, the share of stocks is either hump-shaped, increasing and then flattening out, or simply increasing, depending on the empirical procedure, dataset (country) and assets definitions.

As already mentioned, the present model can help to explain the positive relationship between portfolio equity share and wealth, as well as some aspects of the age dynamics of portfolio allocation to stocks. At the intersection of the second and third challenges just described is perhaps one of the most economically interesting implications of the habit model: the portfolio allocation of young households.[32] Empirically, young households who typically have lower wealth and a lower fraction of portfolios invested in equity, contribute to both the positive relationship between wealth and allocation to stocks and to the hump-shaped or increasing age profile of the equity portfolio share. In the expected utility models,

[30] The formal empirical tests are outside the scope of this paper, and the discussion here is intended only as a broad assessment of the model's predictions relative to the data and as a comparison to the predictions of the existing theoretical models with expected utility.

[31] Modeling limited participation usually requires fixed costs and is not attempted here to keep the focus on the properties related to habit formation utility. Gomes and Michaelides (2004) show that in a setting with relatively small fixed participation cost, Epstein-Zin utility, and heterogeneously risk averse and impatient agents, it is possible to match the observed limited stock market participation.

[32] More generally, the discussion here is also applicable to all other investors who have relatively high human capital compared to their financial wealth accumulated at any given age.

young investors usually specialize in equities because their human capital is not significantly correlated with stock returns and, for low risk aversion, a high equity premium makes stocks very attractive. Habit formation provides both a theoretical and an economic rationale for why the portfolios of young households may be conservative. In the usual economic interpretation of habit (standard of living), the desire to maintain one's standard of living dictates that when wealth is low, one must carefully manage resources so as not to fall behind the minimum acceptable level of consumption. Theoretically, this corresponds to higher risk aversion for the investor near the boundary of admissible resources. In general equilibrium asset pricing models, the variation of risk aversion creates time-varying risk premia. In partial equilibrium, the endogenous adjustment in risk aversion with wealth helps to generate empirically plausible portfolio choice patterns.

The empirical evidence mainly suggests that portfolio allocation to stocks is either increasing or hump-shaped with age, although the literature has not yet reached consensus. However, the comparative statics for portfolio allocation between young and middle-age households seem to be more coherent across studies. For the most liquid financial assets, Heaton and Lucas (2000) find that the share of equities is hump-shaped over a life time. Consistent with their results, Faig and Shum (2002) find that the fraction of *cash* in household liquid assets is U-shaped over a life time. However, Heaton and Lucas (2000) also demonstrate that equity allocation is sensitive to the measure of wealth and may be increasing with age.[33] Also, the findings of Ameriks and Zeldes (2001) suggest that all empirical analyses of equity allocation patterns over the life-cycle should be taken with a grain of salt. They show that there is fundamental lack of identification of the effects of time, age, and birth cohort on portfolio allocation because they are linearly dependent.[34] Also, controlling either for time or for cohort effects produces surprisingly different results. Controlling for time effect, the predicted portfolio share is slightly hump-shaped. When cohort effect is accounted for, the predicted share in stocks is increasing over the life-cycle. Several studies in a collection edited by Guiso, Haliassos and Jappelli (2002) examine individual portfolio allocation in select European countries (Germany, Italy, Netherlands, and the UK) and reconfirm most of the key findings from the U.S. data. International evidence suggests that, conditional on stock ownership, the portfolio allocation to stocks is increasing slightly with age, but this

[33]Additionally, tax status of the portfolio matters, e.g. tax deferred investments may have different allocations than taxable ones (see, for example, Poterba and Samwick (2002) and Gomes, Michaelides and Polkovnichenko (2004)).

[34]Specifically: age = calendar year − birth year. Calendar year is used to control for time effects and birth year for cohort effects.

effect is not always significant and is sensitive to the definition of total wealth. Both the U.S. and the international data are consistent with the view that wealthier or high income households, conditional on participation, invest more in stocks.

While the habit model compares favorably to its expected utility counterpart overall, it is not without deficiencies. The most important one was already mentioned and is related to the counterfactually high wealth accumulation observed for high habit strength. Ultimately, this issue can only be resolved in a general equilibrium model with heterogeneous agents. Another challenge is that the level of portfolio allocation to stocks is still somewhat high as compared to the actual data. This question will be addressed in the comparative statics by considering the model's predictions for a lower level of the equity premium. Finally, realistic wealth accumulation profiles require a bequest motive. This is also addressed in the next section.

4.5 Some comparative statics

In this section, I consider several perturbations of the model in order to understand the sensitivity of the results to the assumed parameters. I already discussed the comparative statics with respect to habit strength, δ. The effects of changing utility curvature, γ, are somewhat similar to habit strength and for the sake of brevity are not reported on a separate figure.[35] Lowering the utility curvature lowers the demand for precautionary savings.[36] As a result of lower precautionary savings, consumption profile shift upwards early in life and downward later in life. The portfolio choice profile shifts upward, indicating that households invest entirely or almost entirely in equities until the age of 65 with a minor decline afterwards. These comparative statics patterns occur in calibrations with or without a zero/low income state.

Previous simulations demonstrate that, without a bequest motive, the agents in any finite-horizon model rationally dissave all their wealth by the end of life, which contradicts the empirical evidence. Prior studies of life-cycle and portfolio choice show that bequests are crucial in matching wealth accumulation profiles (e.g. Cocco, *et. al.* (2004) and Gomes Michaelides (2004)). To explore the role of bequests in the habit formation model, and to contrast it to the expected utility-based model, I modify the problem to include a terminal bequest function in the Bellman equation (8). Cocco, Gomes and Maenhout (2004) report experiments with a bequest function identical to the one used here, and they calibrate bequest

[35]These results are available upon request from the author.

[36]The coefficient γ was set to 3 for this experiment.

strength, λ, in a range from 1 to 5. They find that only for higher λ's does the bequest motive has any significant effect. Motivated by their results I set the bequest parameter to 5.

Figure 7 reports wealth accumulation and portfolio choice for calibrations with and without bequest for habit formation utility with $\delta = 0.6$, and for CRRA utility. As expected, bequest motive increases savings. Also, later in life, wealth is not decumulated completely. Bequest motive has relatively more impact on wealth accumulation in the case of expected utility (EU). Habit utility investors must accumulate substantial wealth during working periods in order to guarantee that habit can be maintained safely. For them, bequest operates mostly by slowing down wealth decumulation after retirement. On the other hand, EU agents without a bequest motive do not maintain as significant wealth as the habit agents. For them, bequest motive increases savings more significantly during working life. The terminal wealth left after bequest is somewhat higher for expected utility agents. This is understandable, because habit at the end of life is non-zero. Other things equal, this increases the marginal utility of consumption for habit agents relative to the EU agents.

In both habit utility and EU cases, the effect of bequest is to decrease portfolio allocation to stocks by a small amount for a few years prior to retirement. After retirement, there is a slight shift back to stocks because retirement income is constant. There is a larger effect of bequest on portfolio allocation in the case of EU agents. This is related to the argument for wealth just presented. For EU agents, bequest increases wealth accumulation significantly, and higher financial wealth leads to more conservative portfolios. There is a similar but smaller effect in the case of habit utility, because habit agents accumulate substantial wealth with or without a bequest motive. These results suggest that issues other than the strength of the bequest motive may be important for portfolio allocation later in the life-cycle. These may include capital income taxes and health care expenses risks.

For the moderate habit strength, δ, required for realistic (not too high) wealth accumulation profiles, the level of portfolio share invested in stocks that is predicted by the habit formation model is still somewhat high.[37] Empirical studies cited earlier find that the share of wealth invested in stocks for market participants is in the range of 30-70%, while the model sometimes predicts a share close to 100%. In calibrations considered so far, bonds earn 1%, like Treasury bills, and holding a diversified portfolio of stocks is assumed to be costless. Because of these assumptions, equities may be unrealistically attractive in the model. In reality, household savings in risk-free assets usually are not only short term bonds. T-bills

[37]The same problem of increasing wealth accumulation prevents using a higher curvature coefficient, γ, to generate more conservative portfolio choice.

can be a proxy for cash, as well as for the most liquid transactions accounts, such as money market, checking and savings accounts. Other savings in safe assets usually are high grade corporate bonds or long-term government bonds or mutual funds that hold such bonds. The yields of these assets are typically higher than the yields on treasury bills. Also, it is costly to hold a diversified portfolio of stocks, either directly or through a mutual fund. Recognizing these issues, McGrattan and Prescott (2003) estimate the returns on equities and bonds that households would be able to earn after accounting for liquidity and transactions costs.[38] They argue that in the post-war U.S. data, the long-term bond real return was 4% and the equity index real return was slightly less than 5%, so the equity premium is just under one percent. For the comparative statics exercise here, I assume that the equity premium is 3% and raise the bond return to a constant 3%, keeping the expected stock return at 6%. This is motivated by the fact that transactions costs of holding equity funds were relatively high in the periods used by McGrattan and Prescott, and there was a considerable decline in mutual funds fees and expenses by the late 1980s and throughout the 1990s when most of the micro-level portfolio data was collected. The labor income process is assumed to have a zero income state, as in the previously reported calibrations. Figure 8 shows the portfolio allocation profiles from this calibration for habit and expected utility cases and, for comparison, the previously reported calibrations with a high equity premium. With lower equity premium, portfolios become more conservative and remain interior throughout life. Note that expected utility still predicts a higher portfolio allocation to stocks than habit utility. Also, for the younger investors aged between 25 and 35 the average EU portfolio allocation is still close to 100% in stock.

5 Conclusion

This paper solves a life-cycle model of consumption and portfolio choice with additive habit formation utility for a household with uninsurable risky labor income. I show that additive habit imposes feasibility constraints on the model which depend on the worst possible scenario for future labor income and on the habit strength parameter, but not on the probability of the worst possible labor income. As a consequence, even the slightest possibility of a "disastrously low" income implies more conservative portfolios and higher savings rates.

[38] Additionally, McGrattan and Prescott considered the effect of capital income taxation on the yields of various investments. In a related paper Jagannathan, McGrattan and Scherbina (2001) have also argued that the equity premium has become lower than a historic average computed by Mehra and Prescott (1985).

The model with habit formation compares favorably to several stylized facts about life-cycle portfolio allocation, particularly that it implies for some households a positive relationship between the portfolio share in stocks and wealth/income. Because of this relationship, the model also predicts relatively more conservative portfolios for younger investors with lower savings, which is consistent with the empirical facts. While a high curvature coefficient or high habit strength help bring the allocation to equities into the range observed in the data, they also imply counterfactually high wealth accumulation. This connection, between habit strength and savings rates, can be related to wealth distribution and asset market participation in a general equilibrium setting. It would be interesting to explore a model with habit formation and heterogeneous agents to see if it is capable of generating plausible results jointly at the aggregate and micro levels.

A Feasibility constraints on cash and habits

The best way to derive the constraints on endogenous variables is to begin from the last period, T, where there is only one constraint that $X_T > H_T$. Going back one period to $T-1$, we have to ensure that not only $X_{T-1} > H_{T-1}$, but also that there is at least one policy function such that there will be enough cash on hand for the last period no matter what happens to the exogenous variables (income shocks and stock return). We will look for the policy function that delivers maximum possible cash on hand under the worst possible shock realizations. If such a policy delivers cash on hand above the habit for the next period T, then a pair (X_{T-1}, H_{T-1}) is feasible. If even such a policy cannot guarantee that $X_T > H_T$, there clearly is no other policy that can do that in all realizations of the exogenous shocks. Also, strictly speaking, maximum cash on hand is not going to be achievable because the constraint on current consumption is strict ($C_{T-1} > H_{T-1}$). Thus we have to make sure that:

$$\sup_{S_{T-1}, B_{T-1}} \{ \min_{\eta_T, P_T, r_{s,T}} X_T \} > H_T \tag{A.1}$$

It is also clear that habit H_T cannot fall below δH_{T-1} because of the consumption restriction at $T-1$: $C_{T-1} > H_{T-1}$.

Because the exogenous shocks are assumed to be i.i.d., there is a state where, with nonzero probability, the stock return is negative and takes the lowest possible value while labor income shocks are lowest, too.[39] Thus, the policy that maximizes cash on hand under the worst case scenario is not going to call for buying any stocks because doing so only aggravates the worst case outcome. Therefore, we can restrict our attention to bonds only. To find the supremum in the left hand side of equation (A.1), suppose that the household consumes the amount: $C_{T-1} = H_{T-1} + \epsilon$. This consumption generates habit for time T given by $H_T = \delta(H_{T-1} + \epsilon)$. The amount left for savings is given by $(X_{T-1} - H_{T-1} - \epsilon)$, which, invested in bonds, gives the maximum possible cash on hand in the worst case at time T. Then feasibility requires that:

$$X_T = (1 + r_b)(X_{T-1} - H_{T-1} - \epsilon) + \underline{Y}_T > H_T = \delta(H_{T-1} + \epsilon),$$

where $\underline{Y}_T$ denotes the lowest possible income realization at T. A pair (X_{T-1}, H_{T-1}) is feasible if there is an ϵ such that the above inequality is satisfied. To find the lowest admissible cash

[39] Note that i.i.d. is not essential here. It is enough for the transition probability matrix to be such that in the states where the income shocks have the lowest possible value it is also possible for the stock return to be negative.

on hand X_{T-1} for a habit H_{T-1} we send ϵ to zero and obtain the constraint on time $T-1$ habit and cash:

$$(1+r_b)(X_{T-1}-H_{T-1})+\underline{Y}_T > \delta H_{T-1}$$

Rearranging this constraint in a more convenient present value form yields:

$$\frac{\underline{Y}_T}{1+r_b}+X_{T-1}-\left(1+\frac{\delta}{1+r_b}\right)H_{T-1}>0 \tag{A.2}$$

Consider going back one more period to $T-2$. For a pair (X_{T-1},H_{T-1}) to be feasible, it must be the case that $X_{T-2}>H_{T-2}$ and there should exist a portfolio policy that guarantees under any realizations of shocks that the next period's feasibility constraints will be satisfied. There are two feasibility constraints at time $T-1$. The first constraint is $X_{T-1}>H_{T-1}$. From the above analysis it is clear that to satisfy this constraint it must be the case that:

$$\frac{\underline{Y}_{T-1}}{1+r_b}+X_{T-2}-\left(1+\frac{\delta}{1+r_b}\right)H_{T-2}>0$$

Also, substituting into (A.2) the expressions $X_{T-1}=(1+r_b)(X_{T-2}-H_{T-2}-\epsilon)+\underline{Y}_{T-1}$ and $H_{T-1}=\delta(H_{T-2}+\epsilon)$ and taking the limit as ϵ goes to zero, after rearrangement, yields:

$$\left(\frac{\underline{Y}_{T-1}}{1+r_b}+\frac{\underline{Y}_T}{(1+r_b)^2}\right)+X_{T-2}-\left(1+\frac{\delta}{1+r_b}+\frac{\delta^2}{(1+r_b)^2}\right)H_{T-2}>0$$

The process of iterating backwards continues, each time substituting $X_{T-k}=(1+r_b)(X_{T-k-1}-H_{T-k-1}-\epsilon)+\underline{Y}_{T-k}$ and $H_{T-k}=\delta(H_{T-k-1}+\epsilon)$ into all constraints derived for the previous period and taking limit as ϵ goes to zero. Following this procedure one derives $T-t+1$ constraints for a generic time t. These constraints have the same familiar form as follows:

$$\sum_{k=1}^{\tau}\frac{\underline{Y}_{t+k}}{(1+r_b)^k}+X_t-\left(\sum_{k=0}^{\tau}\left(\frac{\delta}{1+r_b}\right)^k\right)H_t>0\ ,\ \ \tau=0,\ldots,T-t$$

The economic interpretation of these constraints is explained in detail in the main text of the paper.

B Numerical method

This section first describes the general steps of the numerical method and then explains how the transformed cash on hand is used in the numerical procedure. The model is solved recursively backwards starting from the last period. In the last period ($t = T$) the policy functions are trivial and without the bequest motive the agent consumes all available wealth ($c_T = x_T$). As a result, the value function is $V_T(\hat{x}_T, h_T) = \frac{(x_T(\hat{x}_T,h_T)-h_T)^{1-\gamma}}{1-\gamma}$.[40] For the problem with bequest, the solutions start at $T+1$ when all available wealth is bequested and $V_{T+1}(w_{T+1}) = B(w_{T+1})$. There are two control variables to be solved for in every period: the savings rate out of cash on hand in excess of the current habit and the fraction of savings allocated to stocks. For every age t prior to T, and for each point in the state space, the optimization is performed using a grid search over policy functions. I compute the value associated with each admissible savings rate and the share of savings invested in stocks. In deciding admissibility, the algorithm is using the transformed cash on hand. This procedure is explained below. From the Bellman equation, the value function is current utility plus the discounted expected continuation value ($\beta E_t V_{t+1}(.)$), computed using V_{t+1} from the previous iteration. I perform all of the numerical integrations using Gaussian quadrature to approximate the distributions of labor income shocks and the risky asset return. To evaluate the value function, for points which do not lie on the state space grid, I use bi-cubic spline interpolation. Once the value of all alternative admissible policies is computed, I select the policy associated with the maximum of the value function. The corresponding value function is stored as this period's value function ($V_t(.)$), and then used to solve the previous period's maximization problem. This process is iterated until $t = 1$.

As already mentioned, instead of using cash on hand as a state variable, I use a transformed variable $\hat{x}_t$ which is defined in (14) as follows:

$$\hat{x}_t = x_t - (h_t \Delta_{\tau^*} - y_{t,t+\tau^*})$$

Where τ^* is defined in (13) so that $(h_t \Delta_{\tau^*} - y_{t,t+\tau^*})$ represents the minimum feasible cash on hand for habit h_t (see Figure 2 and the discussion in Section 2.3 for more details). The state space is discretized so that $(\hat{x}_t, h_t)$ lie in a rectangle $[0.1, 20] \times [0, 4]$ and the grid has (uniform) steps in the range from 0.2 to 0.05 on $\hat{x}_t$ and from 0.5 to 0.1 on h_t with smaller steps chosen for calibrations with higher habit strength or utility curvature.

[40] In the last period the only constraint is $x_T > h_T$ and the transformation of cash on hand has a particularly simple form: $\hat{x}_T = x_T - h_T$. So, the value function is $V_T(\hat{x}_T, h_T) = \frac{\hat{x}_T^{1-\gamma}}{1-\gamma}$.

There is no need to explicitly define the evolution for $\hat{x}_t$ because it is not used in the computations. Only equation (6) and the transformation equation are used. The procedure is as follows. For a given grid point $(\hat{x}_t, h_t)$, I compute x_t corresponding to this point using the transformation equation. Then for every savings and portfolio policy, I use the equation (6) to compute x_{t+1} in all possible future exogenous states. Also, every savings policy corresponds to a given level of consumption today and allows me to compute future habit h_{t+1}. Using the obtained values of (x_{t+1}, h_{t+1}) for every exogenous state tomorrow, the algorithm transforms x_{t+1} into $\hat{x}_{t+1}$ and checks that $\hat{x}_{t+1} > 0$ in every possible exogenous state. This verifies the admissibility of the considered policy. Savings and portfolio policies which generate $\hat{x}_{t+1} \leq 0$ in any of the future states are ruled out because they imply infinitely negative utility. For feasible policies, the expected value function is computed by interpolating on the grid in $(\hat{x}_{t+1}, h_{t+1})$ coordinates using bi-cubic splines.

There are two advantages of using $\hat{x}_t$ instead of the usual x_t, both related to the computational properties of the problem. First, the transformation allows me to use standard fast interpolation routines for the rectangular regions rather than the slower routines for the irregularly shaped areas, as shown in Figure 2. Second, if needed the transformation can reduce the dimensionality of the problem. Usually one can reduce the number of grid points by using a nonuniform grid and by concentrating the grid points in the areas where the gradient of the value function is high. For this problem, the gradient of the value function is high everywhere in the vicinity of the lower envelope of the constraints shown in Figure 2. Thus, if one were to use an untransformed x_t for the computations, the grid on x_t would have to be dense throughout the chosen domain for x_t. This is because the area with high gradient of the value function depends jointly on cash and habit and stretches from low to high values of x_t along the lower envelope of the feasibility constraints. On the other hand, the transformation maps all points that are close to the admissible border to the points with the low values of $\hat{x}_t$, allowing for the grid on $\hat{x}_t$ to be more dense near zero and less dense away from zero. However, for the specific software used to solve this problem (MATLAB) it turned out that the interpolation algorithm is much faster when the grids have uniform rather than variable density (because uniform grids simplify the search for the location of interpolant on the grid). Thus, the main benefit of using the transformation in this paper comes from the fast interpolation over the rectangular areas for state variables.

References

Abel, A. B., 1990, "Asset Prices under Habit Formation and Catching Up with the Joneses," *American Economic Review*, 80, 38–42.

Ameriks, J., and S. Zeldes, 2001, "How Do Household Portfolio Shares Very With Age?," *working paper, Columbia University*.

Bodie, Z., J. B. Detemple, S. Otruba, and S. Walter, 2004, "Optimal Consumption portfolio Choices and Retirement Planning," *Journal of Economic Dynamics and Control*, 28(6), 1115–1148.

Cagetti, M., 2003, "Wealth Accumulation over the Life Cycle and Precautionary Savings," *Journal of Business and Economic Statistics*, 21(3), 339–53.

Campbell, J. Y., and J. H. Cochrane, 1999, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107, 205–51.

Carroll, C. D., 1992, "The Buffer-Stock Theory of Savings: Some Macroeconomic Evidence," *Brooking Papers on Economic Activity*, 2, 61–156.

Chapman, D., 1998, "Habit Formation and Aggregate Consumption," *Econometrica*, 66, 1223–30.

Cocco, J., 2004, "Portfolio Choice in the Presense of Housing," *The Review of Financial Studies*, forthcoming.

Cocco, J. F., F. J. Gomes, and P. J. Maenhout, 2004, "Consumption and portfolio choice over the life-cycle," *The Review of Financial Studies*, forthcoming.

Constantinides, G. M., 1990, "Habit Formation: A Resolution of the Equity Premium Puzzle," *Journal of Political Economy*, 98, 519–543.

Deaton, A., 1991, "Saving and liquidity constraints," *Econometrica*, 59, 1221–1248.

Detemple, J. B., and F. Zapatero, 1991, "Asset Prices in an Exchange Economy with Habit Formation," *Econometrica*, 59, 1633–57.

Dynan, K. E., J. Skinner, and S. P. Zeldes, 2004, "Do the Rich Save More?," *Journal of Political Economy*, 112(2), 397–444.

Faig, and Shum, 2002, "Portfolio Choice in the Presence of Personal Illiquid Projects," *Journal of Finance*.

Gentry, W. M., and G. R. Hubbard, 2001, "Entrepreneurship and Household Savings," *NBER working paper*.

Gomes, F., and A. Michaelides, 2003, "Portfolio Choice with Habit Formation: A Life-Cycle Model with Uninsurable Labor Income Risk," *Review of Economic Dynamics*, 6 (4), 729–66.

Gomes, F., and A. Michaelides, 2004, "Optimal Life-Cycle Asset Allocation: Understanding the Empirical Evidence," *Journal of Finance*, forthcoming.

Gomes, F., A. Michaelides, and V. Polkovnichenko, 2004, "Wealth Accumulation and Portfolio Choice with Taxable and Tax-Deferred Accounts," *Financial Markets Group, London School of Economics, working paper*.

Gourinchas, P., and J. Parker, 2002, "Consumption over the Life Cycle," *Econometrica*, 71(1), 47–89.

Guiso, L., M. Haliassos, and T. Japelli, 2002, "Household Portfolios: An International Comparison," in Luigi Guiso, Michael Haliassos, and Tullio Japelli (ed.), *Household Portfolios*. MIT Press.

Haliassos, M., and A. Michaelides, 2002, "Computing Solutions to Life Cycle Portfolio Choice Models," in Luigi Guiso, Michael Haliassos, and Tullio Japelli (ed.), *Household Portfolios*, Cambridge, MA, USA. MIT press.

Heaton, J., 1995, "An Empirical Inverstigation of Asset Pricing with Temporally Dependent Preference Specifications," *Econometrica*, 63, 681–717.

Heaton, J., and D. J. Lucas, 1996, "Evaluating the effects of incomplete markets on risk sharing and asset pricing," *Journal of Political Economy*, 104, 443–87.

Heaton, J., and D. J. Lucas, 1997, "Market frictions, savings behavior, and portfolio choice," *Macroeconomic Dynamics*, 1, 76–101.

Heaton, J., and D. J. Lucas, 2000, "Portfolio choice and asset prices: the importance of entrepreneurial risk," *Journal of Finance*, 55, 1163–98.

Hindy, A., C.-F. Huang, and S. H. Zhu, 1997, "Optimal Consumption and Portfolio Rules with Durability and Habit Formation," *Journal of Economic Dynamics and Control*, 21, 525–550.

Jagannathan, R., and N. R. Kocherlakota, 1996, "Why Should Older People Invest Less in Stocks Than Younger People," *Minneapolis Federal Reserve Quarterly Review*, 20, 11–23.

Jagannathan, R., E. McGrattan, and A. Scherbina, 2000, "The Declining US Equity Premium," *Quarterly Review, Federal Reserve Bank of Minneapolis*, Fall, 3–19.

Lupton, J. P., 2001, "The Habit Liability in Life-Cycle Consumption and Portfolio Choice," *working paper, University of Michigan*.

McGrattan, E., and E. Prescott, 2003, "Average Debt and Equity Returns: Puzzling?," *American Economic Review Papers and Proceedings*, 93, 392–97.

Mehra, R., and E. C. Prescott, 1985, "The equity premium: a puzzle," *Journal of Monetary Economics*, 15, 145–61.

Merton, R. C., 1971, "Optimum Consumption and Portfolio Choice Rules in a Continuous-Time Model," *Journal of Economic Theory*, 3, 373–413.

Pijoan-Mas, J., 2003, "Pricing Risk in Economies with Heterogeneous Agents and Incomplete Markets," *CEMFI working paper No. 0305*.

Poterba, J., and A. Samwick, 2002, "Taxation and Household Portfolio Composition: US Evidence from the 1980's and 1990's," *Journal of Public Economics*, 87, 5–38.

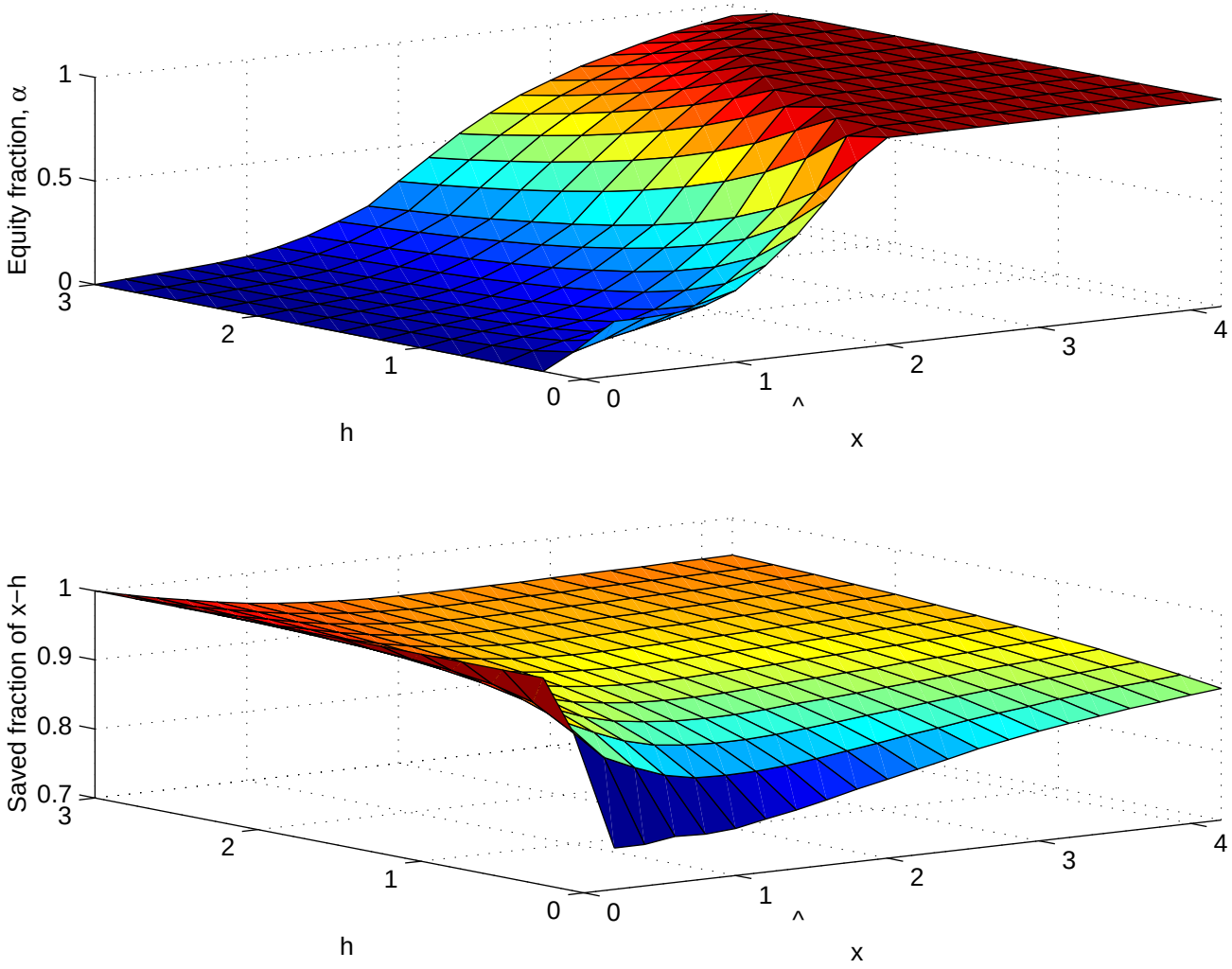
Storesletten, K., C. Telmer, and A. Yaron, 1998, "Persistent idiosyncratic shocks and incomplete markets," *manuscript, Carnegie Mellon University*.

Sundaresan, S. M., 1989, "Intertemporally Dependent Preferences and the Volatility of Consumption and Wealth," *The Review of Financial Studies*, 2, 73–89.

Viceira, L. M., 2001, "Optimal portfolio choice for long horizon investors with nontradable labor income," *Journal of Finance*, 55, 1163–98.

Yao, R., and H. H. Zhang, 2004, "Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints," *The Review of Financial Studies*, forthcoming.

Figure 3: Policy functions for asymmetric distribution of income shock (with zero income), $\gamma = 5$, $\delta = 0.6$ for the first period of the model. Top panel shows portfolio fraction invested in stock, bottom panel the fraction of available resources $x_1 - h_1$ to be saved. Transformed wealth $\hat{x}$ is defined in equation (14) as cash in excess of admissible boundary for a given habit h_t. Both transformed wealth and habit are normalized by the permanent income.



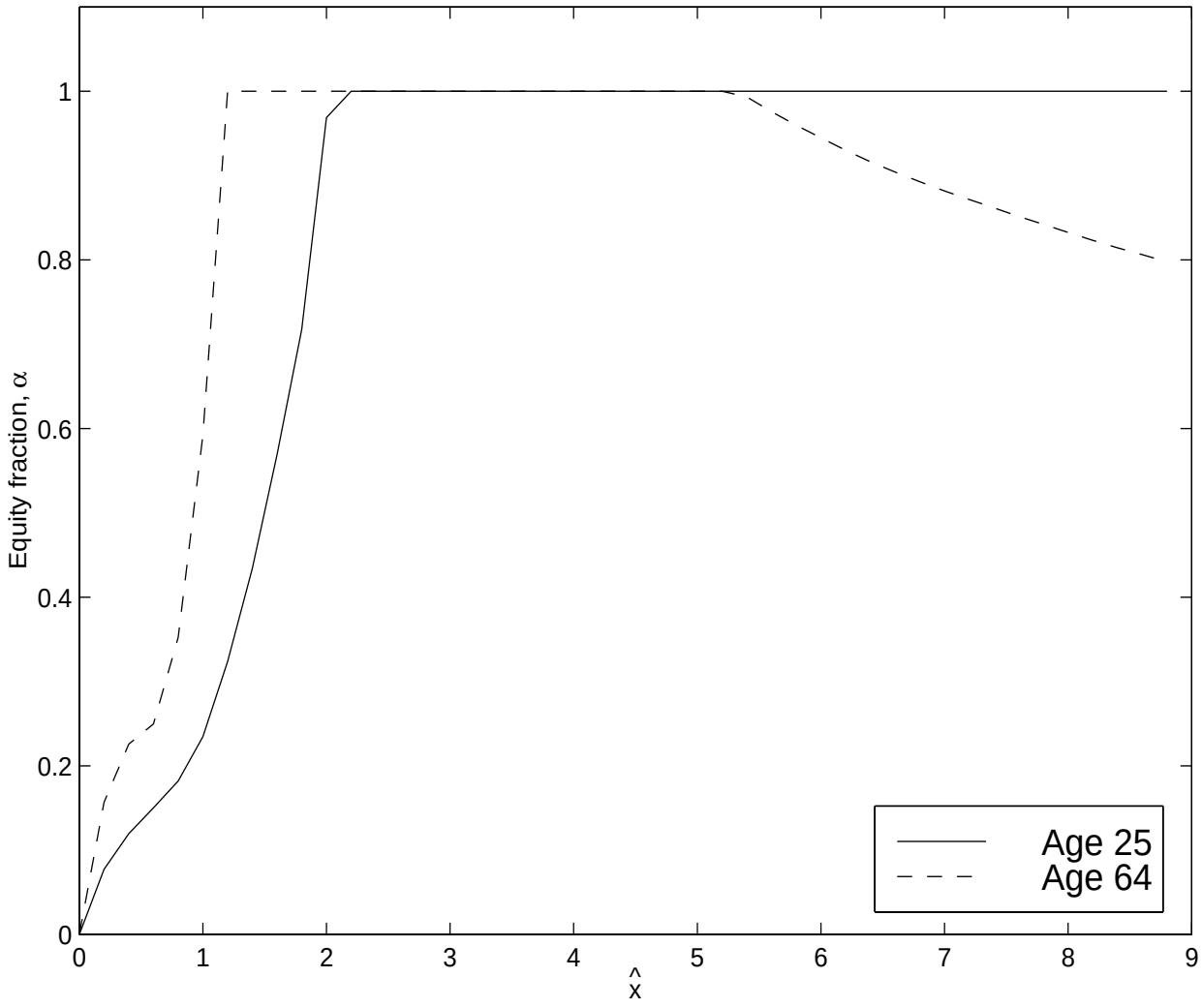


Figure 4: Portfolio policy functions for asymmetric income shock (zero income), $\gamma = 5$, $\delta = 0.6$ at ages 25 and 64 and evaluated at $h = 0.3$. For this level of habit the correspondence between cash on hand x and its transformed value $\hat{x}$ shown on the graph is given by: $x \approx \hat{x} + 0.73$.

Figure 5: Symmetric distribution of income shock for various values of δ, $\gamma = 5$. Financial wealth is the total holdings of bond and stock. Consumption, income and financial wealth are in thousands of dollars (inflation adjusted with 1992 as a base year).

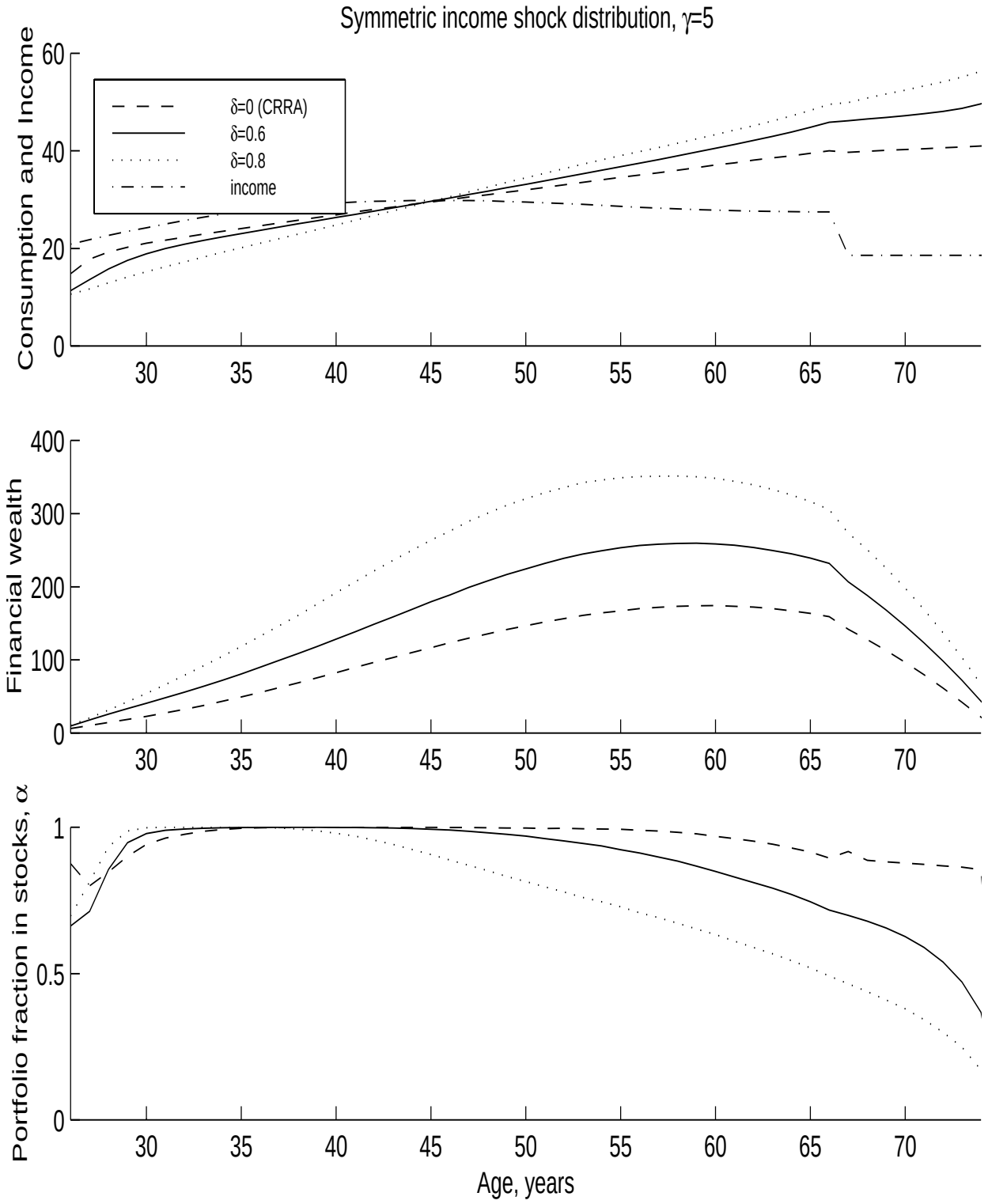


Figure 6: Portfolio choice with asymmetric distribution of income shock, $\gamma = 5$. Solid line ("Symmetric") corresponds to benchmark. Zero income $\eta = 0$ occurs with probability 0.5%. Low income $\eta = 0.1$ occurs with probability of 1.3%. "EU" denotes expected utility (CRRA) case with $\delta = 0$ and zero income state.

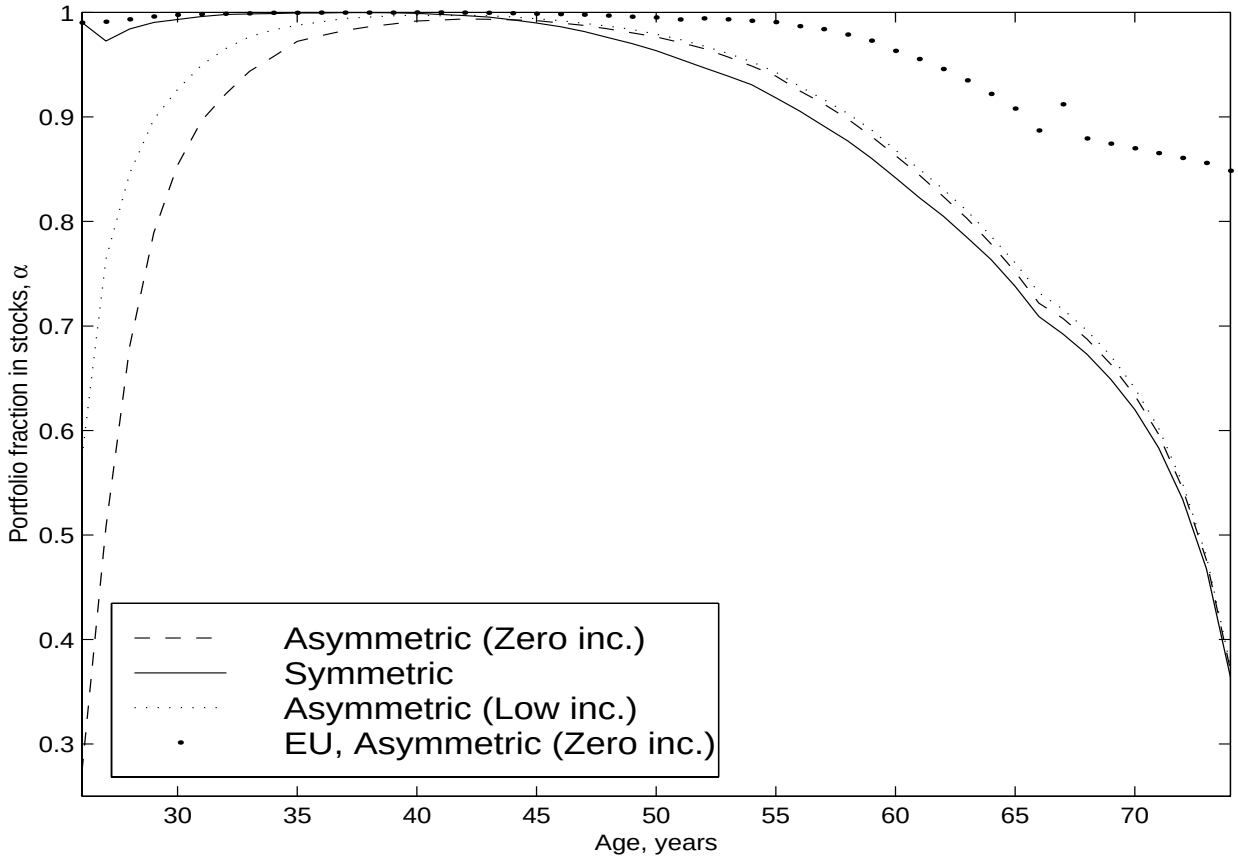


Figure 7: Bequest motive effect for asymmetric distribution of income shock for habit ($\delta = 0.6$) or EU cases (both with $\gamma = 5$). Financial wealth is the total holdings of bond and stock. Financial wealth is in thousands of dollars (inflation adjusted with 1992 as a base year). The bottom panel shows α, the fraction of financial wealth invested in stock.

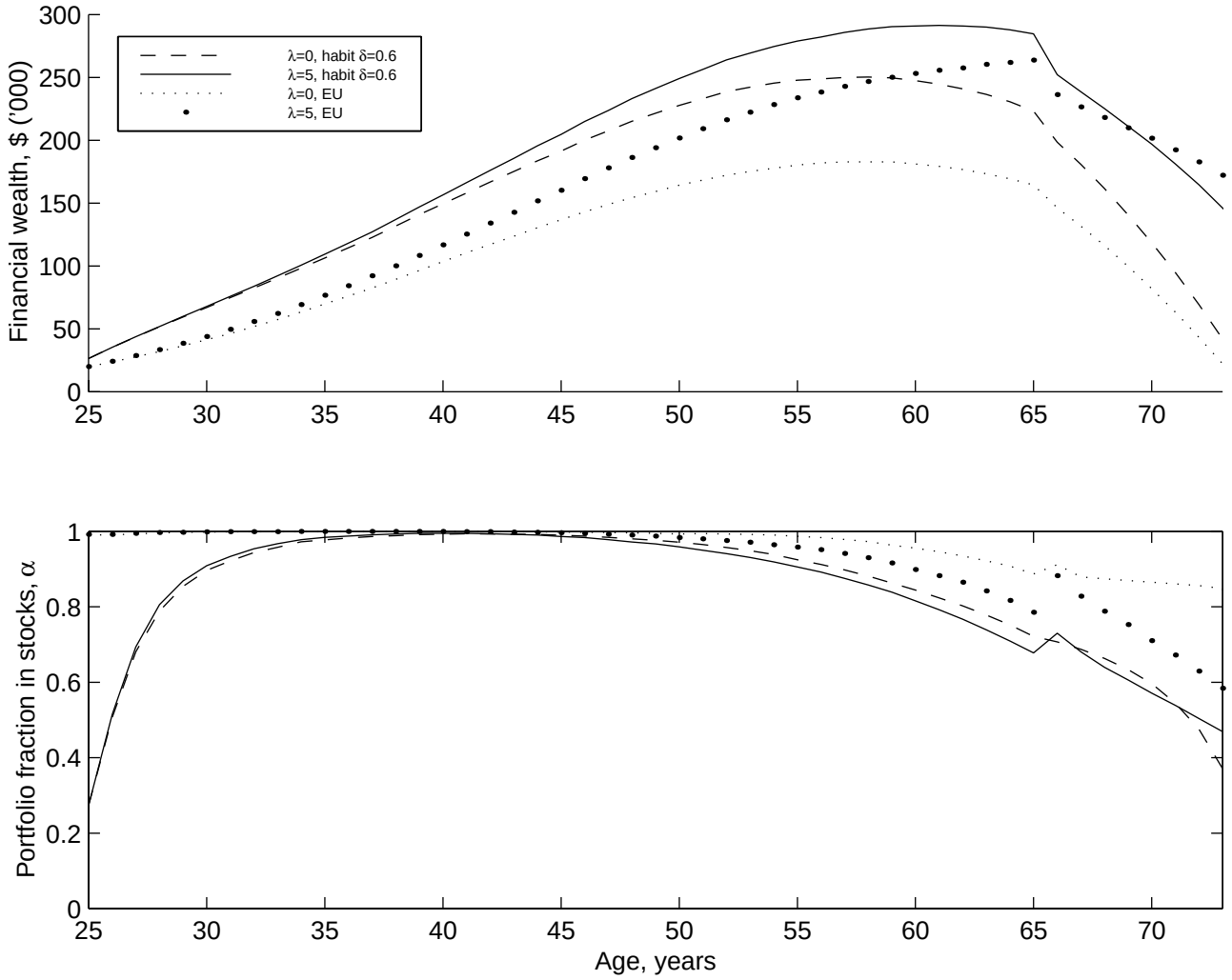


Figure 8: Asymmetric distribution of income shock, $\gamma = 5$, habit ($\delta = 0.6$) or EU for high and low equity premium.

