Portfolio Choice and Wealth Accumulation with Taxable and Tax-Deferred Accounts*

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Abstract

We calibrate a life-cycle model with uninsurable labor income risk and borrowing constraints to match portfolio allocation and wealth accumulation profiles of direct and indirect stockholders in both taxable and tax-deferred accounts. Tax-deferred accounts generate an increase in wealth accumulation that is larger for wealthier households. Furthermore, while the cost of following a fixed contribution rate over the life cycle is small, the optimal rate can differ substantially across households, and the welfare losses from choosing the wrong one can be substantial. Finally, the welfare gain from having access to a tax-deferred account ranges from less than 0.1% to 11.5%, depending on the preference parameters.

JEL Classification: G11.

Key Words: Portfolio Choice, Tax-Deferred Accounts, Retirement Savings, Liquidity Constraints, Uninsurable Labor Income Risk.

1 Introduction

Individual tax-deferred retirement accounts in the US pension system have become considerably more significant during the last two decades. As of December 2002, according to the Investment Companies Institute, defined contribution plans accounted for over $4.5 trillion, or 45% of all retirement assets.[1] As more households rely on defined contribution plans to finance their retirement expenditures, understanding the influence of tax-deferred accounts on portfolio choice and wealth accumulation over the life cycle becomes increasingly important. In this paper we solve a life-cycle model with both a taxable account (TA) and a tax-deferred account (TDA) which replicates several stylized facts on wealth accumulation and portfolio choice. We then use this model to evaluate the impact of TDAs on welfare and savings and to assess the cost of deviations from the optimal TDA contribution.

We model the behavior of two important household groups, namely *indirect* and *direct* stockholders. Indirect stockholders (IS) own stocks only in their tax-deferred accounts, while direct stockholders (DS) hold equities in their taxable accounts (and may also own stocks in their TDAs). This distinction is motivated by empirical evidence from the Survey of Consumer Finances (SCF), which shows that these two groups have different wealth accumulation profiles (in both TDA and TA) and portfolio allocations. In particular, IS accumulate less total wealth than DS, especially in their TAs. In the model we capture this by assuming that the IS have lower risk aversion and lower elasticity of intertemporal substitution than the DS. To match the wealth accumulation profiles, it is also important to realize that a significant fraction of stockholders do not have a TDA. Moreover, for those

[1] These are accounts with individually-directed asset allocation such as IRA, Keogh plans, 401(k), 403(b), 457.

that do, most have only had one for a relatively short time. Therefore, we use the 2001 SCF to compute a distribution of number of years since first ever access to the TDA, and include this as an input in the model. This allows us to match the total and separate account (TA and TDA) wealth accumulation of both groups.

The portfolio allocation implied by the model is consistent with the data. For IS the only relevant asset allocation decision is in their TDA. In the model, they allocate almost all of their TDA wealth to stocks, since this account offers them the only possibility of investing in the stock market. We observe a similar pattern in the data. Since future labor income (human capital) and bonds are close substitutes (see Jagannathan and Kocherlakota (1996) or Heaton and Lucas (1997)), young households invest a larger fraction of their financial wealth in stocks. This pattern occurs for both DS and IS, in both accounts. As retirement approaches, the present value of future labor income decreases and households respond by reducing their exposure to the stock market.

We also compare the portfolio allocations of DS across the two accounts. As shown by several authors (e.g. Black (1980), Tepper (1981), Dammon Spatt and Zhang (2004)) when the capital gains tax is lower than interest income tax (as is the case in the US), households should invest their stocks primarily in the TA, and their bonds primarily in the TDA, such that a mixed portfolio of bonds and stocks can exist at most in one account. However, in the data the average portfolio allocation includes both bonds and stocks in both accounts. Interestingly, in our model we also obtain mixed portfolios in both accounts. This occurs for two reasons. First, the presence of uninsurable labor income risk creates a demand for bonds in the TA which partially counteracts the previously mentioned tax-arbitrage argument. Second, households that have only recently had access to a TDA, or

households without a TDA, will optimally invest a significant fraction of their TA wealth in bonds. Therefore, even when most individual households follow the tax arbitrage rule, the cross-sectional means (in the data and in the model) exhibit mixed portfolios in both accounts.

We next use the model to evaluate the impact on wealth accumulation from introducing a TDA.[2] Here we are exclusively measuring the impact of tax incentives in a partial equilibrium setting, ignoring general equilibrium considerations. Our results can be summarized as follows. First, across all age groups, there is an increase in total wealth accumulation. Second, the percentage increase in wealth is a positive function of both risk aversion and elasticity of intertemporal substitution. Therefore, wealthier households increase wealth accumulation more, so that introducing TDAs contributes to an increase in the cross-sectional wealth inequality. Third, even for the same preference parameters, these wealth increases are slightly smaller for IS. Since IS tend to be less wealthy than DS, this also leads to an increase in wealth inequality.

In our model, the optimal contribution rate varies over the life-cycle. During the first stage of the life-cycle, when households are still liquidity constrained, these contributions are significantly smaller. Households with higher risk aversion and/or higher elasticity of intertemporal substitution (EIS) contribute more to their TDAs, while investors with low risk aversion and low EIS finance all of their retirement savings in a small number of years. We also identify strong incentives stemming from employer-matching schemes and show how investors are willing to distort their (otherwise) optimal life-cycle savings profile to benefit

[2]This analysis is related to a long-standing debate in public economics on whether TDAs represent new savings or substitute for other savings. See for example, the recent survey by Bernheim (2002).

from them. However, some households find it optimal to give up (part of) the matching if it forces them to save too much early in life. This offers an explanation for the observation that a significant fraction of participants in defined contribution plans do not take full advantage of the employer match (see Choi *et. al.* (2001)).

We next evaluate the potential welfare cost associated with fixed contribution rates. We measure welfare as percentage changes in certainty equivalent life-time consumption levels. Our main results are the following. First, the cost of following a fixed contribution rate is not large. Namely, for all households, the minimum welfare loss is less than 0.5%. Second, this fixed contribution rate must be household-specific. A simple "one-size-fits-all" rule does not work: for some households the best (fixed) contribution rate is around 0.5%, while for others it is closer to 3%. Moreover, the welfare losses from choosing the wrong value can be substantial. Third, if we impose a fixed contribution rate, it is better to set it significantly below the average contribution computed from the optimal decisions. This is due to liquidity constraints. For young households the marginal utility of current consumption is extremely high, and therefore a large transfer of resources for retirement is suboptimal.

Finally, we use our model to measure the welfare gain from having access to a TDA. For households with low risk aversion and low EIS these benefits are less than 0.1%, since they do not accumulate much wealth. However, investors with a risk aversion of five enjoy large welfare gains (above 10%). Naturally, these gains are higher for IS (i.e. otherwise non-stockholders), since the TDA offers them access to the stock market.

This paper is part of a growing literature on portfolio choice with taxable and tax-deferred accounts. Dammon, Spatt and Zhang (2004) and Garlappi and Huang (2003) model endogenous capital gain realizations and are primarily concerned with normative aspects of

asset location and allocation. Our paper focuses on the impact of undiversifiable labor income risk and life cycle savings and our model is aimed at explaining empirical patterns in both portfolio choice and wealth accumulation. Amromin (2002) also incorporates non-financial income but in a stylized model and, as the above papers, only focuses on optimal asset location.[3] In addition we also match separately the behavior of direct and indirect stockholders.[4]

The paper is organized as follows. Section 2 describes the empirical evidence on wealth accumulation and asset allocation. We present the model and calibration in section 3. Section 4 discusses the results for the baseline case and compares them with the data. Section 5 discusses the optimal contribution rates and the impact of TDAs on wealth accumulation and welfare. Section 6 concludes. In Appendix A we describe the construction of variables from SCF data and in Appendix B we outline the numerical procedure used to solve the model.

[3]In Amromin (2002) each period corresponds to 5 years and labor income uncertainty is modeled exclusively as an unemployment shock. In our paper we have a realistically calibrated labor income profile, with both permanent and transitory shocks. We separately model the decisions of direct and indirect stockholders and we use Epstein-Zin (1989) preferences that enable us to disentangle risk aversion from the elasticity of intertemporal substitution. The combination of these three features is crucial for matching wealth accumulation and portfolio choice simultaneously.

[4]Additional research on the optimal asset allocation behavior in the presence of taxes and/or tax-deferred accounts includes Elton and Gruber (1978), Constantinides (1983 and 1984), Balcer and Judd (1987), Dybvig and Koo (1996), Huang (2001), Garlappi, Naik and Slive (2001), Dammon *et. al.* (2001a), Gallmeyer, Kaniel and Tompaidis (2002), and DeMiguel and Uppal (2003), among others.

2 Empirical Evidence

2.1 The Data

We first investigate stock ownership and TDA participation among the US population using the Survey of Consumer Finances database for 1989, 1992, 1995, 1998, and 2001.[5] We divide households into two groups based on stock ownership: (*i*) *direct* stockholders (DS) who own taxable stocks/equity funds (note that these households also may own equities in the TDA); (*ii*) *indirect* stockholders (IS) who have equity *only* in the TDA (employer stock and/or funds with equity investments). We define TDAs as account-type pension plans, i.e. defined contribution plans where participants accumulate balances.

The distinction between direct and indirect stockholders is important for our subsequent theoretical analysis and is motivated by the following considerations. We present empirical evidence that stock market participation through the TAs and TDAs is undertaken by different types of households. Buying stocks and mutual funds "on your own" in the TA requires a certain degree of financial sophistication as well as sufficient funds to justify paying transaction costs. This intuition is usually captured in the limited stock market participation literature by using a fixed cost that represents a combination of explicit and implicit hurdles such as brokerage fees, information acquisition about various type of accounts and investment opportunities, more complicated tax filing and time spent on setting up, rebalancing, and monitoring the investment.[6] In the context of the TDAs it is hard to justify such frictions, especially in the case of employer-provided retirement plans. TDAs simplify access

[5]We do not use the first survey in 1983 because it does not have information about the type of mutual funds (equity vs. fixed income) in household portfolios. In computing the reported statistics we use all SCF imputations with corresponding population weights.

[6]For a model using fixed participation cost, see for example, Gomes and Michaelides (2004).

to capital markets by providing a uniform, simple and virtually costless vehicle in which to invest one's savings for retirement. Thus, we would expect households that participate in the stock market *only* through the TDAs to have different financial characteristics from those that hold stocks directly.

In the data we find evidence consistent with the above argument and demonstrate three specific points. First, we show that there was considerable growth in stock market and TDA participation. Both direct and indirect stockholders contributed significantly to this growth. Second, we find that financial wealth and its distribution across taxable and tax-deferred accounts are substantially different for direct and indirect stockholders. Finally, we construct age profiles of wealth accumulation and portfolio allocation for direct and indirect stockholders and again find them to be different.

2.2 Stock market participation and tax-deferred accounts

We begin by considering trends in TDA ownership and stock market participation. Table 1 shows population dynamics for the two types of stockholders between 1989 and 2001. The table reports both the number of households and the fraction of households in the population assigned to each type.[7] Note that there has been considerable growth in the combined number of stockholders (direct and indirect), rising from 32.2 million in 1989 to 56.3 million in 2001. As a fraction of the population, stockholders have increased considerably as well from 34.6% in 1989 to 52.9% in 2001. The table shows that this growth has been primarily concentrated with the *DS* with TDA and the *IS*. Note too, that the fraction of direct stockholders without a TDA declined slightly over the period from 7.3% to 7% of the population. Between direct

[7]Remember that, by definition, there are no households who are indirect stockholders and do not have a TDA, so the corresponding cells in the table are always marked with an empty sign "–".

and indirect stockholders the growth has been slightly higher among direct stockholders. The number of stockholders with TDAs has increased considerably over the same period from 25.5 million (27.3%) in 1989 to 48.9 million (45.9%) in 2001.[8] We conclude from Table 1 that the increase in stock market participation in the US has been primarily driven by households with access to TDAs and that both direct and indirect stockholders have contributed significantly to this process.

2.3 Balances in taxable and tax-deferred accounts

We now compare financial wealth between direct and indirect stockholders. In measuring financial wealth we include relatively liquid financial assets and provide a detailed description of this variable in Appendix A. Table 2 reports typical balances (medians) by cohort in the TAs, TDAs and the sum of the two. The DS are significantly wealthier than the IS with the difference in wealth widening over time.

The difference in the TA balances across the two groups is of special interest. IS have very small liquid balances in the TAs in all years. This helps justify our assumption to exogenously impose a non-participation constraint on IS in the model. We will show that such low *liquid balances* would have generated non-participation *endogenously* for a relatively small fixed cost. We do not endogenize this constraint however, because our model aims to match *total* wealth accumulation, which includes less liquid housing wealth. Our model does not distinguish asset liquidity due to its complexity, and we therefore have to make the participation constraint for IS exogenous.

Looking at a value-weighted basis, we compute the distribution of the aggregate TDA

[8] These figures capture the majority of households with TDAs. There is a small group (3-6%) of households who have TDAs but are not stockholders.

balances. In 2001, the aggregate TDA wealth is split as 24.4% for the *IS* and 71.6% for the *DS* group.[9] Thus, IS are significant not only by number of households and fraction of population, but also by aggregate TDA wealth.

2.4 Wealth accumulation and portfolio allocation over the life cycle

We construct age profiles for wealth and portfolio allocations according to the following age groups: (*i*) 35 and under; (*ii*) 36-50; (*iii*) 51-65; (*iv*) over 65. Using the 2001 SCF we construct Table 3 which shows separately for the TAs and TDAs the cross-sectional medians of wealth-income ratios and means/medians of the fractions of financial assets invested in stocks.[10] Here we define TA wealth as the sum of the financial wealth used previously and net housing equity. Since one of our goals is to match wealth accumulation over the life cycle this definition better reflects total household savings. This definition is consistent with the one in Hubbard, Skinner and Zeldes (1995) and in Heaton and Lucas (2000). To construct wealth-income ratios we use all *non-financial* income which includes wages and salaries, proprietor's income, and various sources of government aid. Appendix A provides further details.

Panel A of Table 3 shows wealth-income ratios for each age group. Wealth-income ratios increase with age. After 65, wealth-income ratios "jump up" because by that age many people retire and non-financial income (the denominator in the ratio) declines substantially.

[9]The remaining small fraction (4%) is held by the nonstockholders with TDAs. We do not consider this group here.

[10]We use only the most recent SCF to maximize the number of households in the sample who had access to the TDAs during their working life. We report median rather than mean wealth-income ratios because the wealth distribution is heavily skewed. For portfolio shares, on the other hand, we report both means and medians because this variable is not as skewed as wealth.

Throughout the life cycle, DS have considerably higher taxable wealth compared to IS: taxable wealth differs by a factor of about $2.5 - 5.5$ across the two types in various age groups. In contrast, in the TDAs, wealth-income ratios are closer, probably due to caps on contributions to TDAs. Further, the historical access to TDAs has not been uniform across the age cohorts. Tax-deferred retirement plans have only become widespread during the 1980's. For many older households, therefore, these plans were not available until they were close to retirement. This becomes important when matching simulations of the model with the data.

The age profiles of portfolio allocation to stocks (shown in panel B) are generally declining with age and are somewhat flatter in the TDA. Note that IS have more aggressive allocations in their TDAs compared to the DS. Comparing the allocation in TAs and TDAs for DS, we find that TAs have more conservative portfolios. For portfolio allocations we also report medians, which are consistently higher than the means. In retirement, DS' portfolios become conservative while the TDA allocation to stock for IS remains flat.

2.5 Summary of empirical findings

Overall, the data indicates that households participating in the stock market exclusively through TDAs are on average poorer than those holding stocks in their TAs. In this sense, TDAs facilitate access to capital markets. We find that significant growth of TDAs and stock market participation in the 1980's and 1990's can be attributed equally to direct and indirect stockholders. The financial wealth of direct and indirect stockholders is split differently across TAs and TDAs. DS have more significant savings in the TAs while IS have most savings concentrated in the TDAs. When we add housing to measure total household

savings, the wealth-income ratios profiles increase with age. Moreover DS have higher wealth to income ratios than IS at any point in the life cycle. The TDA wealth to income ratios are similar in the two types throughout working life. Portfolio fractions invested in stocks are generally declining with age, but are relatively flat for IS in the TDAs.

In the remaining sections of the paper we construct a theoretical model and calibrate it to replicate some stylized facts about wealth accumulation and portfolio allocation over the life cycle for direct and indirect stockholders. We then use the model to perform several normative experiments. In particular, we evaluate the effect on welfare, savings and portfolio choice from allowing access to TDAs and from following fixed contribution policies.

3 Model

We first model direct stockholders (DS) and indirect stockholders (IS) separately. As previously stated, the second group cannot invest in stocks in their taxable accounts. We do not endogenize this constraint, but rationalize it in the context of our results. We assume that the two groups have different preferences that, in turn, determine their different wealth accumulation profiles. We then continue with a welfare analysis, in which we consider several different household groups with differences in preferences and investment opportunities.

3.1 Preferences

Time is discrete and t denotes adult age which, following the typical convention in the life-cycle literature, corresponds to effective age minus 19. Each period corresponds to one year and agents live for a maximum of $T = 81$ periods (age 100). The probability that a consumer/investor is alive at time $(t+1)$ conditional on being alive at time t is denoted by p_t

($p_0 = 1$ and $p_T = 0$). Households have Epstein-Zin utility functions (Epstein and Zin (1989)) defined over a single non-durable consumption good. Letting C_t and W_t denote respectively consumption and the sum of cash-on-hand with the TDA wealth at time t, the household's preferences are given by the following recursion:

$$V_t = \{(1-\beta p_t)C_t^{1-1/\psi} + \beta\left(E_t\left[p_t[V_{t+1}^{1-\rho}] + (1-p_t)b\frac{(W_{t+1}/b)^{1-\rho}}{1-\rho}\right]\right)^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}} \quad (1)$$

where ρ is the coefficient of relative risk aversion, ψ determines the elasticity of intertemporal substitution and β is the discount factor. The continuation payoff in the event of death (that occurs with probability $1-p_t$) is the utility of bequest. The strength of the bequest motive is determined by the parameter b. The terminal condition for the recursion (1) is

$$V_T = b\frac{(W_T/b)^{1-\rho}}{1-\rho} \quad (2)$$

The components of wealth are defined later in the paper. DS and IS will differ in their values of ρ and ψ.

3.2 Labor income process

The labor income process before retirement follows the standard specification in the life-cycle literature and is given by

$$Y_t = P_t U_t \quad (3)$$

$$P_t = \exp(f(t, Z_t))P_{t-1}N_t \quad (4)$$

where $f(t, Z_t)$ is a deterministic function of age and household characteristics Z_t, P_t is a permanent component, and U_t a transitory component. We assume that $\ln U_t$ and $\ln N_t$ are independent and identically distributed with variances σ_u^2 and σ_n^2 respectively. The log of P_t evolves as a random walk with a deterministic drift $f(t, Z_t)$.

For simplicity, retirement is assumed to be exogenous and deterministic, with all households retiring in time period K, corresponding to age 65 ($K = 46$). In retirement ($t > K$) the investor receives a constant pension given by $Y_t = \lambda P_K$, where λ is the replacement ratio (a scalar between zero and one). This specification facilitates the solution of the model, as it does not require the introduction of an additional state variable.

3.3 Financial assets and taxation

The investment opportunity set is constant with two financial assets: one riskless (bonds or cash) and one risky (stocks). There are no transaction costs and we do not allow for short sales. Given that asset returns are taxed and taxes are paid on nominal returns, we assume a constant inflation rate (π). The riskless asset yields a constant real return (r^b). The nominal return is taxed at a rate τ_d, which is also assumed to be the tax rate on both labor income and dividends. The after-tax, real return on the riskless asset is therefore given by

$$\widetilde{r}^b = \frac{1 + [(r^b + 1)(1 + \pi) - 1](1 - \tau_d)}{(1 + \pi)} - 1 \tag{5}$$

The real return on the risky asset is given by

$$r_t^s - r^b = \mu^s + \varepsilon_t^s \tag{6}$$

where μ^s is the average, real, before-tax equity premium, and ε_t^s follows an i.i.d. $N(0, \sigma_\varepsilon^2)$, potentially correlated with the labor income shocks ($\ln N_t$ and $\ln U_t$). The random real gross stock return (r_t^s) is comprised of a constant nominal dividend yield (d) and a stochastic nominal capital gain (g_t), deflated by the inflation rate:

$$r_t^s = \frac{1 + g_t + d}{1 + \pi} - 1 \tag{7}$$

These two components are taxable at different rates. More specifically, nominal capital gains are taxed at the rate τ_g, whereas nominal dividends are taxed at the rate τ_d. For simplicity, we assume that all income is taxed at the source.[11] The after-tax real return on the risky asset is given by

$$\widetilde{r}_t^s = \frac{1 + g_t(1-\tau_g) + d(1-\tau_d)}{1+\pi} - 1 \tag{8}$$

3.4 Budget constraint and wealth dynamics

Securities can be kept in two accounts: a tax-deferred retirement account (TDA) and a regular taxable account (TA). In the TA, there is no deferral of dividend and interest income taxes and all taxes are assumed to be paid at the source. In the retirement account no taxes are withheld and the investor is free to rebalance her portfolio without creating a tax liability. Throughout working life the investor contributes to the TDA a fraction k_t of before-tax earnings. We set the maximum contribution rate equal to 10% and we do not allow for early withdrawals from TDAs prior to retirement.[12] After retirement, the investor faces a minimum withdrawal rate equal to the inverse of her life expectancy. She pays taxes on the withdrawals at the income tax rate (τ_d).

[11] Capital gains in the US are only taxed upon realization. Modeling this feature requires two additional state variables and substantially complicates the model. For detailed studies of the impact of tax-timing on the optimal portfolio allocation behavior in both taxable and tax-deferred accounts, see Dammon, Spatt and Zhang (2001b, 2004) and Garlappi and Huang (2003).

[12] In reality withdrawals are allowed after paying a penalty of 10% of the amount withdrawn. We have restricted contributions to be positive mainly for computational reasons. Some calibrations were recomputed allowing for early withdrawals ($k_t < 0$) with 10% penalty and we found that the main *quantitative* implications remained unchanged (the results are available on request). We cap the contribution rate at 10% mostly for computational reasons as well, and because this is consistent with the data, as reported by Choi *et. al* (2001).

3.4.1 Direct stockholders

We first consider the working life period ($t < K$). Let α_t^r and α_t^τ denote the share of wealth invested in stocks in the retirement and taxable accounts, respectively. The wealth dynamics equations for the taxable account (W_t^τ) and for the retirement or tax-deferred account (W_t^r) are given by:

$$W_{t+1}^\tau = [\alpha_t^\tau(1+\widetilde{r}_{t+1}^s) + (1-\alpha_t^\tau)(1+\widetilde{r}^b)](W_t^\tau - C_t - k_t Y_t(1-\tau_d)) + (1-\tau_d)Y_{t+1} \quad (9)$$

$$W_{t+1}^r = [\alpha_t^r(1+r_{t+1}^s) + (1+r^b)(1-\alpha_t^r)](W_t^r + k_t^* Y_t) \quad (10)$$

where k_t^* includes an employer matching contribution, up to a maximum (k_e), and is thus given by

$$k_t^* = \min(2k_t, k_t + k_e) \quad (11)$$

After retirement ($t \geq K$) these equations change. Let w_t denote the withdrawal from the retirement account and let h_t denote the investor's life expectancy. The equations become:

$$W_{t+1}^\tau = [\alpha_t^\tau(1+\widetilde{r}_{t+1}^s) + (1-\alpha_t^\tau)(1+\widetilde{r}^b)](W_t^\tau - C_t + (1-\tau_d)w_t) + (1-\tau_d)Y_{t+1} \quad (12)$$

$$W_{t+1}^r = [\alpha_t^r(1+r_{t+1}^s) + (1+r^b)(1-\alpha_t^r)](W_t^r - w_t) \quad (13)$$

subject to the constraint

$$w_t \geqslant \frac{1}{h_t} W_t^r \quad (14)$$

which imposes a minimum withdrawal rate.

We also impose the following borrowing constraints

$$\alpha_t^\tau \in [0,1], W_{t+1}^\tau \geqslant 0 \quad (15)$$

$$\alpha_t^r \in [0,1], W_{t+1}^r \geqslant 0 \quad (16)$$

$$k_t \in [0, 0.1] \quad (17)$$

At the time of death, all funds are withdrawn from the TDA and are paid to the beneficiary together with the remaining cash on hand balance in the TA.[13]

3.4.2 Indirect stockholders

Naturally, IS are subject to the same taxation rules and face the same return processes. The only difference is that they can only invest in stocks in their tax-deferred retirement accounts. Therefore, when solving their dynamic programming problem, we only have to replace equations (9) and (12) with

$$W_{t+1}^{\tau} = (1 + \widetilde{r}^{b})(W_t^{\tau} - C_t - k_t Y_t(1 - \tau_d)) + (1 - \tau_d)Y_{t+1} \tag{18}$$

and

$$W_{t+1}^{\tau} = (1 + \widetilde{r}^{b})(W_t^{\tau} - C_t + (1 - \tau_d)w_t) + (1 - \tau_d)Y_{t+1} \tag{19}$$

respectively, and eliminate α^{τ} as a choice variable.

3.5 Calibration

3.5.1 Preference parameters

We start by considering two groups of agents. The first group has low risk aversion ($\rho = 2.5$) and low elasticity of intertemporal substitution ($\psi = 0.1$). This constitutes our IS population. The DS have higher risk aversion ($\rho = 5$) and higher elasticity of intertemporal substitution ($\psi = 0.3$). In both cases we set the discount factor (β) equal to 0.97 and the importance of the bequest motive (b) equal to 2.5. We use the mortality tables of the National Center for Health Statistics to parameterize the conditional survival probabilities. In our welfare analysis we consider a larger range of values for the preference parameters.

[13] Estate taxation would not be an active feature in the model and we omit it entirely. Estates are subject to generous exemptions ($650,000), nearly 20 times the typical peak labor income of high school graduates. Therefore, it is (almost) never replicated in our simulations.

3.5.2 Labor income process

The deterministic labor income profile reflects the hump shape of earnings over the life-cycle. The corresponding parameter values and the standard deviations of the idiosyncratic shocks ($\sigma_u = 25\%$ and $\sigma_n = 10\%$) are taken from Cocco, Gomes and Maenhout (2004). It is common practice to estimate different labor income profiles for different education groups (college graduates, high-school graduates, households without a high-school degree). In our paper we only report the results obtained with the parameters estimated from the sub-sample of high-school graduates, as the implied results for the other two groups are very similar. The replacement ratio (λ) is set at 50% of net earnings which is slightly lower than the one estimated by Cocco et al. (2004), since households with TDAs have lower average state pension replacement ratios.

3.5.3 Asset returns, taxes and contributions

The real bond return r^b is set at 2%. For the stock return process we consider a mean equity premium (μ^s) equal to 4% and a standard deviation (σ_{ε^s}) of 20%. We consider 4% as opposed to the historical 6% to take into account transaction costs (e.g. mutual fund and brokerage fees). The nominal dividend yield d is set at 3.2%.[14] The proportional tax on dividends and labor income (τ_d) is 25% and the tax on capital gains, τ_g, is 20%. We consider a labor income tax of 25% to reflect the average income tax of the typical household in our sample.[15] Inflation rate is set at 3.15% corresponding to the average from CRSP data from 1926 to 1999. We set the cap on employer matching (k_e) equal to 3%, based on two sources of information. First, Mitchell (2000) reports that, for the plans with some degree

[14] We have scaled down the historical dividend yield by the same factor as the equity return.

[15] Solving the model without proportional taxation would require one additional state variable.

of matching, the mode of employer contributions is 6%. Second, numbers from the SCF indicate that approximately only 50% of TDAs have employer-matching.

The correlation between stock returns (ε_t) and permanent labor income shocks ($\ln N_t$) is set equal to 0.15, based on the results from Campbell et al. (2001), while the correlation with transitory labor income shocks ($\ln U_t$) is set equal to 0.

3.5.4 Distribution of years since first access to TDA

According to the data, even among stockholders, a significant fraction of households does not have a tax-deferred account. Moreover, even within those that have a TDA, most of them have only had one for a small number of years. Therefore, to replicate the average wealth accumulation of households aged 50 to 65, for example, we cannot assume that all of them have had access to a TDA since age 20.

From the 2001 SCF we have information on the number of years each household has contributed to an employer-provided retirement plans, including the years of contributions in any previous plans provided that these have been rolled over to start the current plan. Thus, this variable provides us with a lower bound on the number of years since the household has had access to a TDA.[16] We can now use this variable to compute a distribution of number of years since households first had access to the TDA (θ) and use this as an input for the model.

For simplicity, we consider three groups with different values of θ. Table 4 reports the different groups and their corresponding population weights, both in the SCF and in the model. The values of θ used in the model correspond roughly to the mean of each interval

[16]This constitutes a lower bound both because some households might not have rolled over previous accounts or they had started non-employer provided accounts, such as IRA's, earlier.

in the data, and produce an average θ of 7.25, which is higher than in the data (6.26).[17]

Finally we set the percentage of DS without a TDA to 25% to approximate the corresponding figure from the SCF. Therefore, for the DS population, the relative weights of the three groups with TDA ($\theta = 3$, 8 and 15) have to be re-scaled by a factor of 0.75.

4 Results for the baseline model

We initially solve a version of the model that captures the main features of the data presented in section 2.[18] In particular, we want to replicate the wealth accumulation profiles and asset allocation decisions of both DS and IS. We show that differences in preferences (risk aversion and elasticity of intertemporal substitution) can explain most of the behavioral differences between these two groups.

4.1 Direct stockholders

We model DS as having a stronger incentive for wealth accumulation. In a life-cycle model with uninsurable labor income risk and liquidity constraints, households save for two reasons. When they are young they save mostly for precautionary motives, while later on they also save for retirement. More risk-averse households care more about background risk, while those with higher elasticity of intertemporal substitution care more about retirement savings.[19] Therefore, we model DS as having higher risk aversion ($\rho = 5$) and higher elasticity of intertemporal substitution ($\psi = 0.3$). This combination delivers higher wealth accumu-

[17] We set the calibrated average TDA participation years higher than in the data because, as mentioned earlier, the SCF counterpart is biased downwards.

[18] The numerical procedures are presented in Appendix B.

[19] The relationship between the EIS and savings depends on the difference between the expected rate of return on the investor's portfolio and the mortality-adjusted discount rate. For this specific combination of parameter values, higher EIS increases savings.

lation, consistent with the empirical observations in section 2. As previously mentioned, we have four groups of direct stockholders, each with different values of θ (years since first access to TDA) and different population weights. More precisely, $\theta = 0$, 3, 8 and 15, with weights 25%, 37.5%, 18.75% and 18.75%, respectively.

4.1.1 Policy functions

We highlight the qualitative properties of the solutions by presenting and discussing the policy functions for DS who had access to their TDA from the start of working life.[20] Figure 1 shows the consumption and portfolio policy functions at ages 25 and 90. For younger households, the consumption function is strongly increasing in taxable wealth and almost flat in TDA wealth. This is because they cannot access the latter. This result remains unchanged even when withdrawals from the TDA are allowed, at a 10% penalty (not reported).[21] Retirees can freely withdraw funds from their TDA, and as such their consumption is an increasing function of both TA and TDA wealth.

For a given level of TA and TDA wealth, the portfolio allocation to stocks of an older household is more conservative in both accounts. As TA (TDA) wealth increases, the portfolios in TA (TDA) become more conservative, unless constraints bind. These age and wealth effects on portfolio allocation are driven by the well-documented close substitutability between bonds and human capital (see Jagannathan and Kocherlakota (1996) or Heaton and Lucas (1997)). Because of this substitutability, as financial wealth (in any account) grows, or as the household becomes older, human capital wealth becomes relatively less important

[20] Qualitatively, the policies are similar for cases when TDA is accessible later in life and for IS and are available upon request.

[21] As mentioned above, we have solved the model allowing for early withdrawals with 10% penalty, and the main conclusions were unaffected. The corresponding results are available upon request.

and the household invests more in bonds to compensate for the lower exposure to human capital.

There are also across-account interactions in portfolio allocation when wealth in one account affects the portfolio allocation in the other account. Consider first the effect that TA wealth exerts on TDA portfolio allocation. As TA wealth increases, TDA portfolios become more conservative. Due to the human capital effect just discussed, as TA wealth grows, the optimal exposure to stocks becomes lower. It is tax-efficient to reduce the exposure to stocks through the TDA because bond income is more heavily taxed. This effect is more pronounced for older households because they have relatively small human capital and therefore the same changes in financial wealth have a more dramatic effect on the balance of human and financial capital.

Another cross-account effect is that of TDA wealth on the TA allocation to stocks. Note that for the young household, the allocation in TA is virtually unaffected by the level of TDA wealth. This is due to the illiquidity of TDA wealth early in life and is similar to the effect observed on the consumption function. For the older household, higher TDA wealth leads to higher stock allocation in TA. Because it is tax efficient to allocate TDA wealth to bonds (a higher taxed security) the household compensates increases in TDA wealth by reallocating TA wealth to stocks.

4.1.2 Wealth accumulation and retirement savings

Table 5 shows the median wealth to income ratios in the model and the data for different age groups. Panel A shows the ratios for DS and panel B for IS. The results from the model are obtained by simulating 20,000 individual life histories and taking the medians across

households and across age groups. We report medians instead of means since the wealth distribution exhibits significant skewness.

Young households face a high expected future labor income, against which they are not allowed to borrow. Therefore, they prefer to consume most of their income, while saving is done mostly for precautionary reasons. As a result, at this life-cycle stage most of the wealth accumulation occurs in the TA. Saving for retirement starts later on in the life-cycle, and as a result the contribution rates to the TDA start to increase. This is shown in Table 6 which reports the average contribution rate to the TDA for the different subgroups. Despite the increasing contribution rates, Table 5 shows that most wealth is still accumulated in the TA. The data also shows this seemingly counterintuitive pattern. This occurs because within each age group, there are households that have only had access to the TDA for a small number of years, and therefore they do not have much wealth in their TDA. This shows the importance of accurately calibrating the distribution of θ.

Overall, our results are broadly consistent with their empirical counterparts (columns 3-4 in Table 5), with the exception of retirement period. Standard life-cycle models usually predict that households should decrease their asset balances quickly during retirement, while in the data this is not the case. Hubbard, Skinner and Zeldes (1995) match the wealth profiles during retirement much better by introducing uninsurable health shocks in a life-cycle model of consumption and savings. We have not included these shocks in our model for computational reasons, and because we are mainly concerned with matching wealth accumulation prior to retirement, and computing the optimal contributions and asset allocations implied by this behavior. Within our model, we could instead have assumed a stronger bequest motive, but that would affect the level of wealth accumulation at retirement age and therefore

the optimal contributions to the TDA.

4.1.3 Portfolio allocations

We now look at the asset allocation decisions in the two accounts. Table 7 shows the average share of wealth invested in stocks in the TA, the TDA, and in total wealth and compares them with the corresponding numbers from the 2001 SCF. As previously discussed, future labor income and bonds are close substitutes. Therefore young households invest a larger component of their financial wealth in stocks: the implicit riskless asset in the form of human capital is still quite high. As retirement approaches, the present value of future labor income decreases. Households respond by reducing their exposure to the stock market. This pattern is visible in the two accounts, both in the model and in the data.

Next we compare the portfolio allocations across the two accounts. Black (1980), Tepper (1981) and Dammon *et. al.* (2004) show that when the capital gains tax is lower than interest income tax (as is the case in the US, and in our model), it is more tax-efficient to receive capital gains income in the taxable account and interest income in the tax-deferred account. As a result, a direct tax-arbitrage argument implies that households should invest their stocks primarily in the TA, and their bonds primarily in the TDA, implying that a mixed portfolio of stocks and bonds should exist at most in one account. However, in the data, the average portfolio allocation includes both bonds and stocks in both accounts (as previously shown by Poterba and Samwick (2002) Ameriks and Zeldes (2001) and Bergstresser and Poterba (2002)).

In our model we obtain mixed portfolios in both accounts for two reasons. First, the presence of uninsurable labor income risk creates a demand for bonds in the TA. Second,

households that have only recently had access to the TDA, or households without a TDA ($\theta = 0$), will optimally have a significant fraction of their TA wealth invested in bonds. Since the results in Table 7 correspond to cross-sectional averages across groups with different values of θ (as in the data), the cross-sectional mean will show mixed portfolios in both accounts. We should point out that we obtain a higher fraction of equities in the TA than in the TDA (tax-arbitrage effect), while in the data this pattern is reversed. Nevertheless, the differences in the average allocation across both accounts are small both in the model and in the data.

4.2 Indirect stockholders

We now consider the indirect stockholders (IS). In the data these households do not accumulate as much wealth as the DS, and we capture this by assuming that they have lower risk aversion ($\rho = 2.5$) and lower elasticity of intertemporal substitution (EIS, $\psi = 0.1$). Naturally, all households in this category have access to TDA (otherwise they would be non-stockholders). The distribution of θ, therefore, does not include 0 and we now only have three groups: $\theta = 3$, 8 and 15, with weights 50%, 25% and 25%, respectively.

By definition these households do not invest in equities in their TA, and we impose this participation constraint exogenously. At the end of this section we discuss this restriction and how it could be endogenized.

4.2.1 Wealth accumulation and retirement savings

In panel B of Table 5 we compare the model's median wealth to income ratios with that in the data. As usual we consider four different age groups. Since IS are prevented from investing in equities in their TA we might expect most wealth accumulation to occur in the

TDA. However, comparing these results with those obtained for DS (panel A of the same table), we find that wealth accumulation is now smaller in both accounts. IS have lower risk aversion and lower EIS than DS. Consequently, they care less about both background risk and retirement savings. This is also visible in panel B of Table 6 which reports the optimal simulated contribution rates for IS. Although IS have a stronger incentive to use their TDAs because they cannot otherwise invest in equities, they still contribute less than DS (panel A of the same table). Again, this is driven by the differences in preferences across the two groups. The reduction in wealth accumulation is stronger in the TA because of the participation constraint, as we would expect. Nevertheless, the median cross-sectional wealth to income ratios are higher in the TA. Similarly to DS, this is due to the fact that some households have only recently had access to their TDAs, both in the model and the data.

4.2.2 Portfolio allocations and participation decision

The only relevant portfolio decision for IS is the asset allocation in their TDA (see Table 8). In the model, these households invest almost all of their TDA wealth in stocks. We observe similar behavior in the data, although not as strong, especially when considering means rather than medians.

As previously mentioned, we do not endogenize the non-participation restriction of IS. Instead we assume that they face a prohibitive stock market entry cost.[22] This assumption can be rationalized in the context of our results, if we include a distinction between liquid and non-liquid wealth. In our model we do not make this distinction for computational reasons.

[22]This may include explicit or implicit costs associated with equity investments such as direct costs of maintaining investment accounts, informational or opportunity costs of figuring where to open an account, what securities to buy, how to implement a more complicated tax filing, etc.

However, as previously shown (see table 2), IS hold very little financial wealth in their TAs. Therefore, a small cost of stock market participation would be sufficient to prevent these households from investing in equities.

We use our model to quantify the required cost in such a context. For this purpose, we solve an extended version of our current model, with an explicit fixed cost of participation in the stock market (F). This cost is expressed as a percentage of the household's current permanent income.[23] We consider values of F between one and five percent. In table 9 we report the lowest wealth to income ratio for which (some) households would choose to pay the fixed cost and invest in stocks in their TA. For each group we report the minimum threshold across all ages, when the participation decisions are evaluated at the level of TDA wealth reported in table 3. In every case, the level of wealth required to generate some participation is significantly higher than the median taxable financial wealth in the corresponding population in 2001 SCF (column 5), except during the retirement period where this only happened with $F = 5\%$. This provides supportive evidence for our hypothesis: adding a distinction between financial and non-financial wealth to our current model would allow us to endogenize the non-participation decision of indirect stockholders with a moderate participation cost.

5 The impact of tax-deferred accounts

In this section we study the impact of the TDA along different dimensions. First, we take the existence of the TDAs as given and compute the optimal life-cycle profile of contribution

[23]This formulation offers several advantages: First, it simplifies the solution as it does not require the introduction of an additional state variable. Second, it is consistent with the interpretation of the fixed cost as an opportunity cost of time. Third, it has been previously used in the literature (e.g. Gomes and Michaelides (2004)) which makes it easier to interpret our results.

rates for different types of investors along with the cost of following suboptimal contribution rules. Second, we want to determine how much additional wealth accumulation is generated by the presence of a TDA. Do these accounts only crowd-out wealth accumulation in the TA, or does the preferential tax treatment effectively induce extra savings? Finally, we compute the welfare gain from having access to a TDA for different groups of the population.

For this analysis we consider four sets of parameter values. More precisely, in addition to our two previous cases ($\rho = 2.5/\psi = 0.1$ and $\rho = 5/\psi = 0.3$), we also study the case of an investor that saves less than our benchmark IS ($\rho = 1.5/\psi = 0.1$), and the case of an investor that saves more than our benchmark DS ($\rho = 5/\psi = 0.5$). Moreover, for the purposes of comparability and completeness, we consider these four combinations for both DS and IS. The welfare measures are computed as the change in certainty equivalent life-time consumption, evaluated at age 20.

5.1 Contribution rates

We solve our baseline model for a household that has full access to a TDA over the life-cycle. In the first case, the investor can choose the optimal contribution rate (k_t) every year, and then we solve the model for several fixed contribution rates. In all cases, we have kept the cap for the employer matching (k_e) equal to 3% as in the baseline model. We only report results for DS since the conclusions are essentially the same as for IS.[24]

5.1.1 Optimal contribution over the life cycle

The average optimal contribution rate for each set of parameters for different age groups are shown in table 10. In all cases contributions are smaller during the first life-cycle stage,

[24]The results for IS are available upon request.

when households are still liquidity constrained. As expected, households with higher risk aversion and/or higher EIS contribute more to their TDAs. Investors with very low risk aversion and very low EIS ($\rho = 1.5$ and $\psi = 0.1$) contribute close to nothing until age 59, when they finance their retirement savings over a small number of years.

The average contribution rate among investors with the strongest savings incentive ($\rho = 5$) exceeds 3% from early on. This is clearly driven by the household's desire to take advantage of the employer-matching scheme.[25] In the absence of employer matching these households would have lower contribution rates early in life, when they are more liquidity constrained. This illustrates the strong incentives offered by the employer-matching schemes, and how investors are willing to distort their (otherwise) optimal life-cycle savings profile to benefit from them. However, as we can see from observing the behavior of the other two groups, some households find it optimal to give up (part of) the matching if it forces them to save much more than they would like.

5.1.2 Cost of suboptimal contribution rates

It is clear from table 10 that the optimal contribution rate over the life-cycle is not constant. However, investment advisors often recommend fixed contributions (e.g. ones that fully utilize employer matching) and investors often follow simple rules of thumb when determining their contribution rates. We now, therefore, evaluate the potential welfare cost associated with these suboptimal decisions. We have considered five different fixed contribution rates, $k = 0.5\%$, 1%, 2%, 3%, and 5%. The results are shown in Table 11. In the last column of the table we report the average optimal contribution for each group.

[25] When almost all households are trying to take full advantage of the employer matching, the average must be above 3%.

The main results from this analysis are the following. First, the cost of following a fixed contribution rate is small. For all households, the minimum welfare loss is less than 0.5%. Second, a simple "one-size-fits-all" rule does not work. If we set $k = 1\%$, households in the first two groups have modest welfare losses (less than 1%) while the others lose up to 5.60%. If instead we minimize the welfare cost of the more risk-averse households by setting $k = 3\%$, households with $\rho = 1.5$ face a welfare loss of 2.75%. Third, if we impose a fixed k, it is better to set it below the average k computed from the optimal decisions. For example, for the second group, the average k is 1.53%. The welfare loss from $k = 1\%$ is smaller than that from $k = 2\%$ (0.86% versus 1.72%). In fact, the welfare cost of setting $k = 0.5\%$ is even lower than both (0.43%). This striking result is explained by the liquidity constraints. For young households the marginal utility of current consumption is extremely high, such that transferring significant resources for retirement is highly suboptimal. This result is also visible for the $\rho = 5$ households. Their average optimal k is above 5%, yet they still prefer $k = 3\%$ to $k = 5\%$ as a fixed contribution.

5.2 Wealth accumulation and welfare

In this section we measure the impact of TDAs, both in terms of wealth accumulation and welfare. We do this by solving a version of the model without TDAs, and comparing the optimal allocations and value functions with the ones obtained in the model with TDAs. For the purpose of this analysis it is important to exclude the employer matching feature, since these matching contributions lead to an increase in wealth accumulation and welfare.

5.2.1 Wealth accumulation

An important policy debate about the impact of TDAs asks the following question: do these accounts only crowd-out wealth accumulation in the TA, or does the preferential tax treatment effectively induce extra savings, and if so by how much? (see, for example, the survey in Bernheim (2002)). In our model, in the absence of tax benefits, the optimal investment in the TDA would be zero, since the TDA would be dominated by the TA which offers liquidity before retirement. Therefore, we are measuring the impact of this tax incentive on household-level wealth accumulation.

We first discuss the case of DS. Table 12, panel A, shows the percentage average increase in total wealth accumulation (i.e. combining TDA with TA) for different age groups and different combinations of preference parameters. Across all age groups, this increase is a positive function of both risk aversion and elasticity of intertemporal substitution. For example, households in the first group ($\rho = 1.5$ and $\psi = 0.1$) are almost unaffected by the presence of a TDA, while for the investors with $\rho = 5$ and $\psi = 0.5$, wealth accumulation more than doubles during the last pre-retirement years. Therefore, even in percentage terms, wealthier households increase wealth accumulation more than their less wealthy counterparts. As a result, the introduction of TDAs contributes to increase the cross-sectional wealth inequality.

Figure 2 plots the life-cycle consumption profiles of two groups ($\rho = 5/\psi = 0.3$, and $\rho = 2.5/\psi = 0.1$), both with and without the TDA. During the first stage of the life-cycle, the less-risk averse households essentially follow the same consumption pattern. The $\rho = 5$ households, however, respond to the TDA by increasing their total savings. This

explains the results in table 12, panel A. From approximately age 45 onwards, both types gradually increase their consumption (relative to the no-TDA situation). The less risk-averse households have accumulated more wealth for retirement with a similar savings rate, because of the tax-benefits.[26] The more risk-averse households gain both from the tax benefits and from their extra savings. Consequently they enjoy a more substantial increase in consumption.

Now we turn our attention to the IS. For this group, in addition to the tax benefits, the TDA offers them access to stock market investment. Table 12, panel B, shows the percentage average increase in total wealth accumulation for different age groups and different combinations of preference parameters. As expected, we obtain a similar pattern to the one observed for DS (panel A of the same table): households with higher risk aversion and/or higher EIS increase their wealth accumulation more in response to the TDA introduction. Perhaps surprisingly, when comparing the panels A and B of table 12, we see that the wealth increases are slightly smaller for IS than for DS. This shows that the income effect from having access to stock market investments more than compensates the corresponding substitution effect. When prevented from investing in stocks, these households were forced to save substantially early in life to finance their retirement expenditures. With the TDA providing stock investment opportunities, they can now obtain a much wealth accumulation at age 65 with less savings.[27]

We have seen that, for given preference parameters, DS will increase their wealth accumulation slightly more than non-stockholders (now IS). In addition, the $\rho = 5$ households

[26]Naturally, they cannot access this wealth until retirement, but they instead accumulate less wealth in their taxable accounts from age 45 onwards.

[27]Naturally, total wealth accumulation also increases because of the tax benefit effect.

are more likely to have already invested in stocks. Therefore, the TDAs will typically have a stronger impact on the wealth accumulation of the households that are already stockholders. This strengthens our result that TDAs contribute to increase the cross-sectional wealth inequality.

5.2.2 Welfare

Now we measure the welfare gain from having access to the TDA. Table 13 reports the percentage increases in (life-time) consumption certainty equivalents resulting from the introduction of the TDA, for the different subgroups that we have considered.[28]

We start with the DS again. As expected from our previous results, for DS with low risk aversion ($\rho = 1.5$ or $\rho = 2.5$) the benefit is very small (less than 0.1%). These households do not save much over the life-cycle, and therefore do not gain much by having a TDA. On the other hand, more risk averse investors enjoy welfare gains of almost 10%. The results for IS are qualitatively similar but the benefits are predictably larger (up to 11.2%), since the TDA offers them access to the stock market. It is interesting to note that even for the IS, the welfare gain is very small for the less risk-averse households. For the $\rho = 2.5$ investors the benefit doubles compared to the DS, but it is still only 0.2%. For those with $\rho = 1.5$ the gain remains essentially constant at 0.04%. These investors save very little, and even the higher return from investing in the stock market does not offer them a sufficient incentive to change that.

[28]These numbers are not directly comparable with the (previously shown) costs from following a fixed k rule. In particular, the welfare loss associated with a hypothetical $k = 0$ in table 11 would be higher than the cost reported in this section. This is because the benchmark in this section does not include the employer matching, while the benchmark for the fixed k results did (and fixed values of k that are lower than 3% effectively didn't exhaust the employer match).

6 Concluding remarks

We have analyzed a model of life-cycle consumption and portfolio allocation with a taxable and a tax-deferred account in the presence of uninsurable income risk and borrowing constraints. The model is able to match wealth accumulation and portfolio choice for direct and indirect stockholders conditional on age and account tax status. Our findings indicate that TDAs increase savings and that this effect is stronger for wealthier households. We also compute the optimal contributions into TDAs over the life cycle and find that it may be optimal not to contribute even in the presence of employer matching.

This model has several natural extensions, which can be implemented in future research. First, the tax rate in our model is flat while in reality many countries adopt a progressive schedule. Modeling this feature would likely increase the benefits from the TDAs as retirees tend to face lower tax rates than workers. Second, capital gains realizations in the TA could be explicitly considered, as in Dammon *et. al.* (2001b, 2004) and Garlappi and Huang (2003). This would incorporate the value of the tax-timing option and its interaction with labor income risk. Finally, the general equilibrium implications of TDA introduction in a heterogeneous agents model remain to be explored.

Appendix A The Survey of Consumer Finances Data

The SCF is probably the most comprehensive source of data on U.S. household assets. The SCF uses a two-part sampling strategy to obtain a sufficiently large and unbiased sample of wealthier households (the rich sample is chosen randomly using tax reports). To enhance the reliability of the data, the SCF makes weighting adjustments for survey non-respondents; we used these weights to compute the values reported in the tables. The specific names in the codebook and the acronyms used by the net worth program supplied by the SCF are used below.

We construct a measure of non-financial income to match the process for Y_{it} (earnings) in the model. Non-financial income is defined as the sum of wages and salaries (X5702), business/practice/farm income (X5704), rent and royalties (X5714), unemployment or worker's compensation (X5716), child support and alimony (X5718), food stamps and welfare income (X5720), Social Security or other pensions, annuities, or other disability or retirement programs (X5722) and other income (X5724).

We next construct measures of bonds and stocks held in the taxable and tax-deferred accounts. Bonds in the TA are made up of SAVING and MMA (savings and money market accounts), CDS (certificates of deposit), GBMUTF (government bond mutual funds), OBMUTF (other bond mutual funds), BOND (corporate bonds), SAVBND (saving bonds), TFBMUTF (tax free bond mutual funds) and COMUTF (combination mutual funds), for which we assume that half is allocated to bonds. We also include annuities (ANNUIT) and trusts (TRUSTS) that are allocated to bonds.[29] We subtract non-credit card and non-

[29]For annuities, trusts, mutual funds, pension funds and IRA and KEOGH plans in which the allocation between bonds and stocks is mixed, we assume an equal allocation between bonds and stocks.

residential real estate debt (variables ODEBT and OTHLOC which include unsecured loans and loans secured by pensions).

Stocks in the taxable account consist of STOCKS (directly held stocks), STMUTF (stock mutual funds), half of COMUTF (combination mutual funds), annuities (ANNUIT) and trusts (TRUSTS) that are allocated to stocks. Bonds (stocks) in the TDA include bonds (stocks) from IRA/KEOGH plans, bonds (stocks) in other future pensions (FUTPEN) and bonds (stocks) in account-type retirement plans for which we have information on asset allocation. For example, for the first pension plan of the respondent we require that variable X4216 is less than or equal to 18 (various account-type plans) and X4324 equals $1, 2, 3$ or -7 (known asset allocation), and for all the other pension plans of the respondent and spouse the requirement is the same. The shares of wealth invested in stocks in each account are constructed by averaging across the shares of individual households.

To construct total taxable wealth, we add bonds and stocks in the taxable accounts, checking and call accounts (CHECKING plus CALL) and subtract revolving credit card debt (CCBAL). We also add home equity (HOMEEQ) and non-residential home equity (NNRESRE). Home equity is defined as the value of the home less the amount still owed on the first and second/third mortgages and the amount owed on home equity lines of credit. This wealth definition is consistent with both the definition in Hubbard, Skinner and Zeldes (1995) and Heaton and Lucas (2000). To construct total TDA wealth we add bonds and stocks in the TDAs.

Appendix B Numerical Solution

We exploit the scale-independence of the maximization problem and rewrite all variables as ratios to the permanent component of labor income (P_t). The laws of motion and the value function can then be rewritten in terms of these normalized variables, and we use lower case letters to denote them (for instance, $w_t^\tau \equiv \frac{W_t^\tau}{P_t}$). This normalization allows us to reduce the number of state variables to three: liquid wealth in the taxable account, accumulated wealth in the tax-deferred account and age. We discretize the state-space along the two continuous state variables.

We solve the model recursively backwards starting from the last period. In the last period ($t = T$) the policy functions are trivial and the value function corresponds to the bequest function.[30] We need to solve for four control variables in every year: the fraction of taxable wealth being saved, the fraction of the taxable portfolio allocated to stocks, the fraction of retirement wealth allocated to stocks and the contribution rate. For every age t prior to T, and for each point in the state space, we optimize using grid search. From the Bellman equation the optimal decisions are given as current utility plus the discounted expected continuation value ($E_t V_{t+1}(.)$), which we can compute since we have just obtained V_{t+1}. We perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the labor income process and the risky asset returns. We evaluate the value function, for points which do not lie on the state space grid, using a bi-cubic spline interpolation along the two wealth dimensions. We also use a more dense set of grid points for low values of wealth for the two accounts since the consumption function

[30]Note that in $T-1$ the life expectancy is exactly one year and the agent is required to withdraw all retirement funds and invest them in the taxable account so that $w_T^R = 0$.

exhibits a kink at the points where liquidity constraints are no longer binding. Once we have computed the value of each alternative we pick the maximum, thus obtaining the policy rules for the current period. Substituting these decision rules in the Bellman equation, we obtain this period's value function ($V_t(.)$), which is then used to solve the previous period's maximization problem. This process is iterated until $t = 1$.

References

Ameriks, John, and Stephen Zeldes, 2001, How do household portfolio shares vary with age?, *Working paper, Columbia University.*

Amromin, Gene, 2002, Portfolio allocation choices in taxable and tax-deferred accounts: An empirical analysis of tax-efficiency, *working paper, The Board of Governors of the Federal Reserve.*

Balcer, Yves, and Kenneth Judd, 1987, Effects of capital gains taxation on life-cycle investment and portfolio management, *Journal of Finance* 42, 743–761.

Bergstresser, Daniel, and James M. Poterba, 2002, Asset allocation and asset location: Household evidence from the survey of consumer finances, *working paper, MIT.*

Bernheim, Douglas B., 2002, Taxation and saving, in Alan J. Auerbach, and Martin Feldstein, ed.: *Handbook of Public Economics*, vol. 3 . pp. 1173–1249 (Esevier Science B.V.: Amsterdam, The Netherlands).

Black, Fischer, 1980, The tax consequences of long-run pension policy, *Financial Analysts Journal* July-August, 21–28.

Campbell, John, Joao Cocco, Francisco Gomes, and Pascal Maenhout, 2001, Investing retirement wealth: A life-cycle model, in *Risk Aspects of Social Security Reform* (University of Chicago Press).

Choi, James J., David Laibson, Brigitte C. Madrian, and Andrew Metrick, 2001, Defined contribution pensions: Plan rules, participants choices, and the path of least resistance, *NBER Working Paper Series 8655*.

Cocco, João F., Francisco J. Gomes, and Pascal J. Maenhout, 2004, Consumption and portfolio choice over the life-cycle, *The Review of Financial Studies* forthcoming.

Constantinides, George, 1983, Capital market equilibrium with personal taxes, *Econometrica* 51, 611–636.

——— , 1984, Optimal stock trading with personal taxes, *Journal of Financial Economics* 13, 65–89.

Dammon, Robert M., Chester S. Spatt, and Harold Zhang, 2001a, Diversification and capital gains taxes with multiple risky assets, *working paper, Carnegie Mellon University*.

——— , 2001b, Optimal consumption and investment with capital gains taxes, *The Review of Financial Studies* 14(3), 583–616.

——— , 2004, Optimal asset location and allocation with taxable and tax-deferred investing, *Journal of Finance* 59 (3).

DeMiguel, Angel, and Raman Uppal, 2003, Portfolio investment with the exact tax basis via nonlinear programming, *working paper, London Business School*.

Dybvig, Philip, and H. Koo, 1996, Investment with taxes, *working paper, Washington University*.

Elton, J., and M. Gruber, 1978, Taxes and portfolio composition, *Journal of Financial Economics* 6, 399–410.

Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Gallmeyer, Michael, Ron Kaniel, and Stathis Tompaidis, 2002, Tax management strategies with multiple risky assets, *working paper, Carnegie Mellon University and UT Austin.*

Garlappi, Lorenzo, and Jennifer Huang, 2003, Taxable and tax-deferred investing with portfolio constraints, *working paper, University of Texas at Austin.*

Garlappi, Lorenzo, Vasant Naik, and Joshua Slive, 2001, Portfolio selection with multiple assets and capital gains taxes, *working paper, UT Austin and University of British Columbia.*

Gomes, Francisco, and Alexander Michaelides, 2004, Optimal life-cycle asset allocation: Understanding the empirical evidence, *Journal of Finance* forthcoming.

Heaton, John, and Deborah J. Lucas, 1997, Market frictions, savings behavior, and portfolio choice, *Macroeconomic Dynamics* 1, 76–101.

——— , 2000, Portfolio choice and asset prices: the importance of entrepreneurial risk, *Journal of Finance* 55, 1163–98.

Huang, Jennifer, 2001, Taxable or tax-deferred account? portfolio decision with multiple investment goals, *working paper, UT Austin.*

Hubbard, Glenn, Jonathan Skinner, and Stephen Zeldes, 1995, Precautionary saving and social insurance, *Journal of Political Economy* 103, 360–399.

Jagannathan, Ravi, and Narayana R. Kocherlakota, 1996, Why should older people invest less in stocks than younger people, *Minneapolis Federal Reserve Quarterly Review* 20, 11–23.

Mitchell, Olivia, 2000, New trends in pension benefit and retirement provisions, *working paper, The Wharton School, University of Pennsylvania*.

Poterba, James, and Andrew Samwick, 2002, Taxation and household portfolio composition: US evidence from the 1980's and 1990's, *Journal of Public Economics* 87, 5–38.

Tepper, Irwin, 1981, Taxation and corporate pension policy, *Journal of Finance* 36, 1–13.

Figure 1: Consumption and portfolio policy functions for direct stockholders at ages 25 and 90. TA and TDA wealth and consumption axes are scaled by the permanent income component.

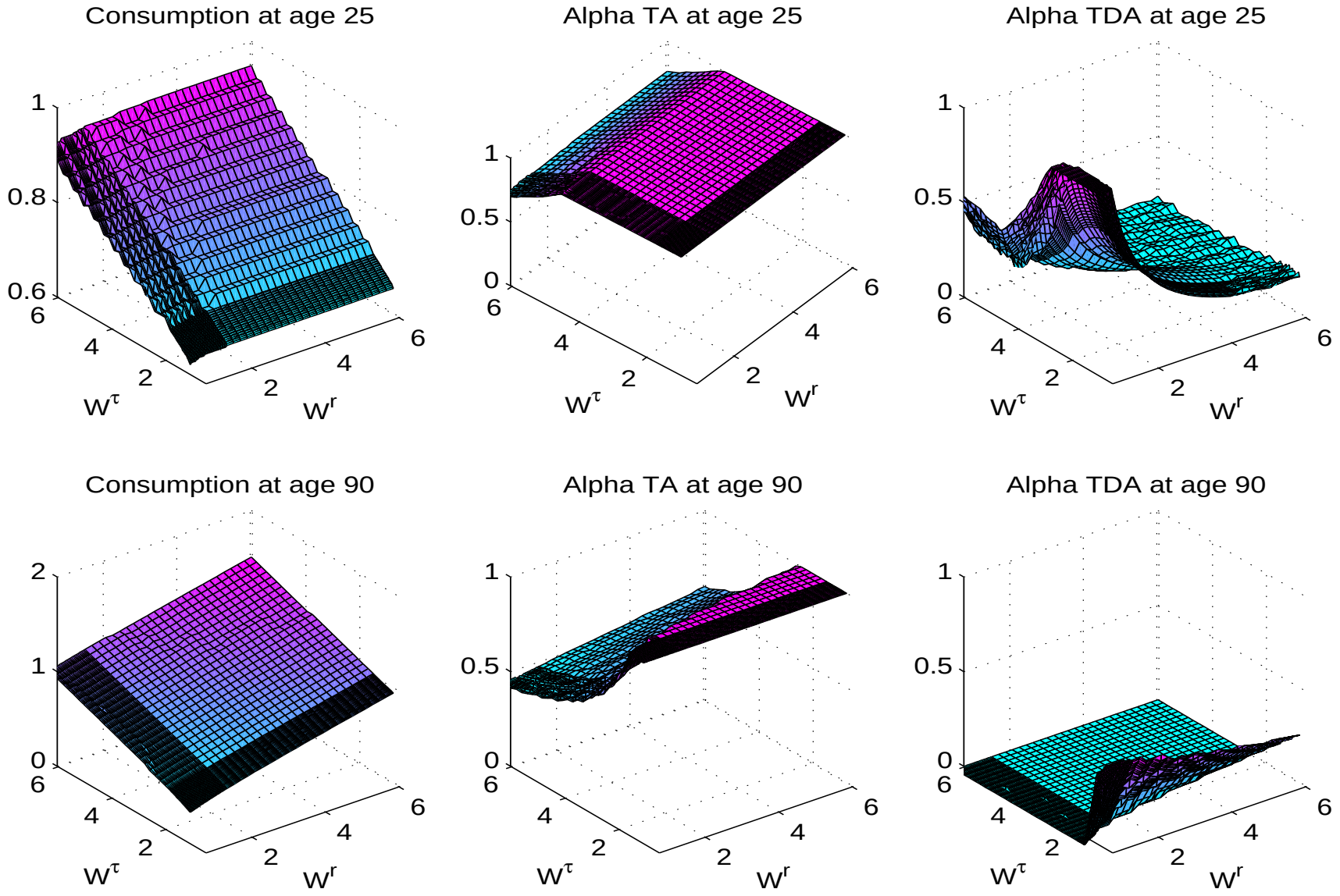


Figure 2: Consumption with and without TDA for direct stockholders.

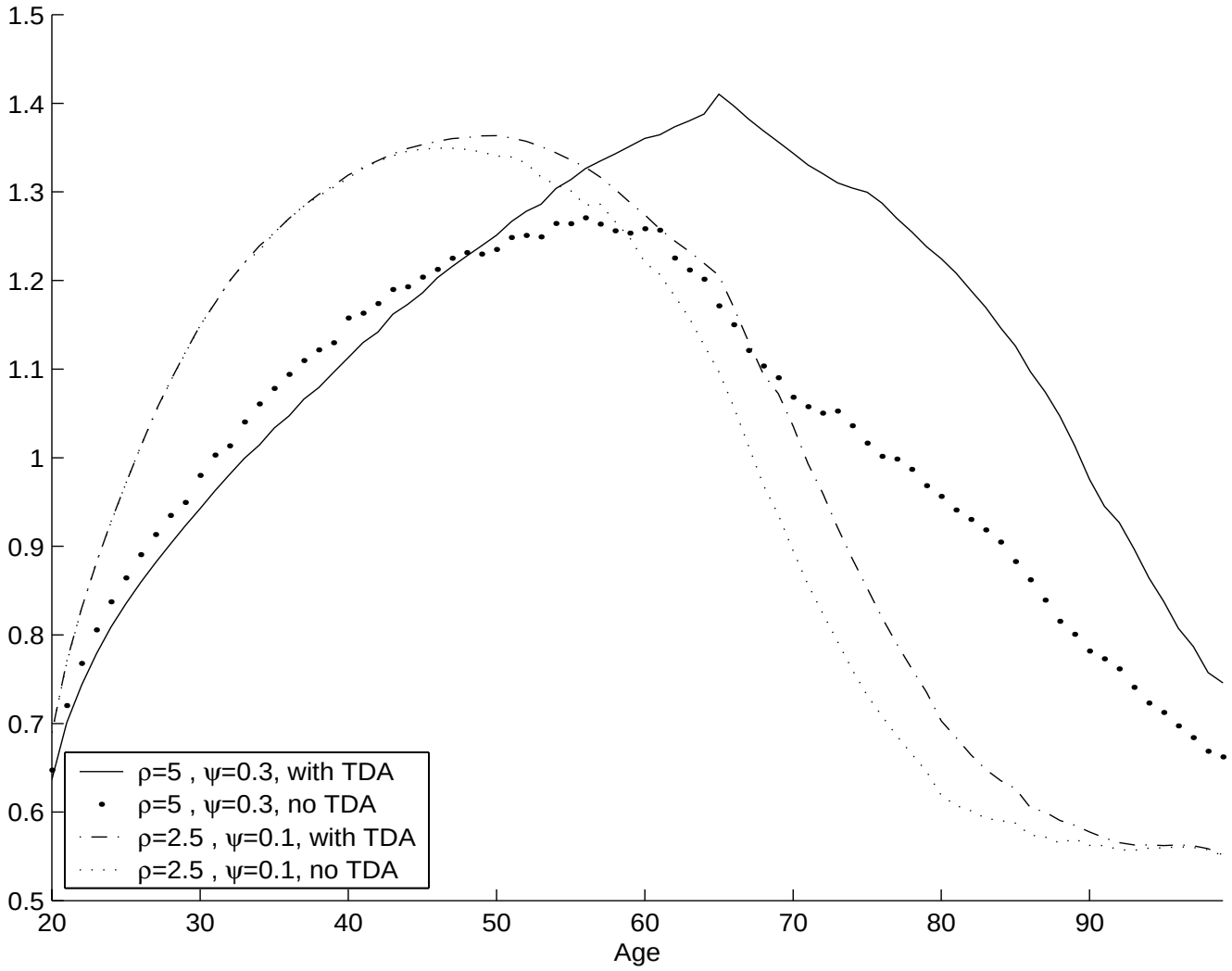


Table 1: Stockholders by TDA ownership and direct/indirect type. Households are classified as follows: (*i*) *direct stockholders* own some equities/equity funds in TA's (also may have some equity in the TDA) – labeled "DS", (*ii*) *indirect stockholders* own equities/equity funds *only* through TDA's – labeled "IS". We use SCF sample weights to compute reported statistics.

SCF year/	*Millions of households*		*Percent of population*		Combined,
TDA status	DS	IS	DS	IS	mil/%
1989					
No TDA	6.7	–	7.3	–	6.7/7.3
Have TDA	12.8	12.7	13.7	13.6	25.5/27.3
Combined	19.5	12.7	21.0	13.6	32.2/34.6
1992					
No TDA	6.8	–	7.2	–	6.8 /7.2
Have TDA	14.3	16.2	14.9	16.9	30.5/31.8
Combined	21.1	16.2	22.1	16.9	37.3/39.0
1995					
No TDA	7.1	–	7.2	–	7.1/7.2
Have TDA	15.8	18.0	16.0	18.2	33.8/34.2
Combined	22.9	18.0	23.2	18.2	40.9/41.4
1998					
No TDA	7.9	–	7.7	–	7.9 /7.7
Have TDA	22.0	20.6	21.5	20.1	42.6/41.6
Combined	29.9	20.6	29.2	20.1	50.5/49.3
2001					
No TDA	7.4	–	7.0	–	7.4/7.0
Have TDA	26.1	22.8	24.5	21.4	48.9/45.9
Combined	33.5	22.8	31.5	21.4	56.3/52.9

Table 2: Components of financial wealth (medians) for groups of households by TDA ownership and stockholding status. Included components of wealth and non-financial income are defined in the text. Households are classified by stock ownership as follows: (*i*) *direct stockholders* own some equities/equity funds in TA's (also may have some equity in the TDA) – labeled "DS", (*ii*) *indirect stockholders* own equities/equity funds *only* through TDA's – labeled "IS". We use SCF sample weights to compute reported statistics.

	Financial Wealth ($)		
SCF year/	Stockholder status		
account	DS	IS	Combined
1989			
TA+TDA	55,550	16,500	31,920
TA	32,290	2,800	13,440
TDA	7,000	9,560	8,000
1992			
TA+TDA	59,930	15,810	36,480
TA	32,650	2,600	12,830
TDA	9,000	10,000	9,300
1995			
TA+TDA	65,500	17,000	35,400
TA	34,050	1,450	10,500
TDA	11,000	12,500	12,000
1998			
TA+TDA	96,300	19,400	52,800
TA	49,350	1,670	16,200
TDA	19,800	14,000	16,200
2001			
TA+TDA	133,950	25,900	66,690
TA	66,000	2,350	18,710
TDA	29,000	18,000	24,000

Table 3: Wealth-income ratios (medians) and portfolio allocation (means and medians) by account, age and stockholding status. Included components of wealth and income are defined in the text. Households are classified by stock ownership as follows: (*i*) *direct stockholders* own some equities/equity funds in TA's (also may have some equity in the TDA) – labeled "DS", (*ii*) *indirect stockholders* own equities/equity funds *only* through TDA's – labeled "IS". We use SCF sample weights to compute reported statistics.

Panel A, Wealth/Income ratios

	DS		IS	
Age	Taxable	Tax-Def.	Taxable	Tax-Def.
< 36	0.79	0.11	0.14	0.12
36 - 50	1.72	0.43	0.66	0.44
51 - 65	3.55	0.84	1.32	0.70
> 65	12.33	0.56	4.64	1.98

Panel B, Portfolio share in stocks (mean/median)

	DS		IS	
Age	Taxable	Tax-Def.	Taxable	Tax-Def.
< 36	0.72/0.79	0.77/1.00	0.0/0.0	0.80/1.00
36 - 50	0.65/0.68	0.78/1.00	0.0/0.0	0.74/0.89
51 - 65	0.61/0.62	0.72/0.94	0.0/0.0	0.76/0.92
> 65	0.58/0.58	0.55/0.50	0.0/0.0	0.76/0.91

Table 4: Distribution of years of participation in employer-provided tax-deferred retirement plans from the 2001 SCF and the corresponding three-point distribution assumed in the model. SCF data includes years of participation in current plans and any previous retirement plans rolled over to the current plans.

Years	% in the SCF	θ	% in the model
1 - 5	48	3	50
6 - 10	23	8	25
> 10	29	15	25

Table 5: Wealth-income ratios (medians) from simulations and SCF data. Preference parameters (risk aversion and EIS) for DS are $\rho = 5.0$ and $\psi = 0.3$, for IS are $\rho = 2.5$ and $\psi = 0.1$. The rest of the parameters is described in the calibration section 3.5.

Panel A, Direct Stockholders

	Model		Data	
Age	Taxable	Tax-Def.	Taxable	Tax-Def.
< 36	0.66	0.24	0.79	0.11
36 - 50	2.69	0.69	1.72	0.43
51 - 65	4.89	0.97	3.55	0.84
> 65	5.37	0.29	12.33	0.56

Panel B, Indirect Stockholders

	Model		Data	
Age	Taxable	Tax-Def.	Taxable	Tax-Def.
< 36	0.22	0.01	0.14	0.12
36 - 50	0.67	0.32	0.66	0.44
51 - 65	0.95	0.88	1.32	0.70
> 65	0.31	0.52	4.64	1.98

Table 6: Average contribution rates (in percentage of income) for direct and indirect stockholders with different time of access to TDA (θ). Preference parameters (risk aversion and EIS) for DS are $\rho = 5.0$ and $\psi = 0.3$, for IS are $\rho = 2.5$ and $\psi = 0.1$. The rest of the parameters is described in the calibration section 3.5.

Panel A, Direct Stockholders			
Age	$\theta = 15$	$\theta = 8$	$\theta = 3$
< 36	2.98	3.25	4.91
36 - 50	5.13	7.84	9.97
51 - 65	8.74	9.33	9.33
Panel B, Indirect Stockholders			
Age	$\theta = 15$	$\theta = 8$	$\theta = 3$
< 36	0.36	0.36	0.43
36 - 50	1.27	1.79	3.37
51 - 65	3.90	6.07	8.59

Table 7: Fraction of portfolio invested in stocks from simulations and SCF data for direct stockholders. Preference parameters (risk aversion and EIS) for DS are $\rho = 5.0$ and $\psi = 0.3$. The rest of the parameters is described in the calibration section 3.5.

	Model (mean)			Data (mean/median)		
Age	Taxable	Tax-Def.	Total	Taxable	Tax-Def.	Total
< 36	0.97	0.91	0.97	0.72/0.79	0.77/1.00	0.79/0.83
36 - 50	0.78	0.51	0.72	0.65/0.68	0.78/1.00	0.70/0.78
51 - 65	0.50	0.49	0.50	0.61/0.62	0.72/0.94	0.66/0.70
> 65	0.66	0.36	0.68	0.58/0.58	0.55/0.50	0.59/0.63

Table 8: Fraction of portfolio invested in stocks in the TDA account from simulations and SCF data for indirect stockholders. Preference parameters (risk aversion and EIS) for IS are $\rho = 2.5$ and $\psi = 0.1$. The rest of the parameters is described in the calibration section 3.5.

Age	Model (mean)	Data (mean/median)
< 36	1.00	0.80/1.00
36 - 50	0.98	0.74/0.89
51 - 65	0.95	0.76/0.92
> 65	0.99	0.76/0.91

Table 9: Minimum wealth-income ratios required to trigger stock market participation in TA for indirect stockholders for various levels of fixed cost F. The thresholds are evaluated at the corresponding levels of retirement wealth from table 3.

Age	$F = 5.0\%$	$F = 2.5\%$	$F = 1.0\%$	Data, 2001 SCF (Txbl. Fin. Assets/Income)
< 35	0.48	0.35	0.18	0.02
35 - 50	0.31	0.19	0.10	0.04
51 - 65	0.26	0.19	0.06	0.08
> 65	0.63	0.39	0.25	0.44

Table 10: Average contribution rates (in percentage of income) for direct stockholders for different preference parameters.

Age	$\rho = 1.5$, $\psi = 0.1$	$\rho = 2.5$, $\psi = 0.1$	$\rho = 5.0$, $\psi = 0.3$	$\rho = 5.0$, $\psi = 0.5$
20 - 29	0.00	0.01	3.66	4.44
30 - 39	0.00	0.86	4.66	5.28
40 - 49	0.00	1.28	6.08	6.19
50 - 59	0.03	2.05	6.86	6.97
60 - 65	4.01	5.40	8.24	7.97

Table 11: Welfare loss from following fixed contribution rule for direct stockholders. The loss is expressed as a percentage of certainty equivalent life-time consumption.

Utility parameters	Fixed contribution rate (%)					Average endogenous contribution (%)
	0.5	1.0	2.0	3.0	5.0	
$\rho = 1.5$, $\psi = 0.1$	-0.47	-0.94	-1.85	-2.75	-4.56	0.45
$\rho = 2.5$, $\psi = 0.1$	-0.43	-0.86	-1.72	-2.66	-4.50	1.53
$\rho = 5.0$, $\psi = 0.3$	-8.09	-4.21	-1.59	-0.23	-0.56	5.64
$\rho = 5.0$, $\psi = 0.5$	-6.99	-5.59	-2.31	-0.46	-0.46	5.97

Table 12: Increase in wealth accumulation from having access to TDA. The table reports average percentage increase in total wealth for every age group and preferences parameter combinations.

Utility parameters	Age group		
	20-35	36-50	51-65
	Panel A, Direct stockholders		
$\rho = 1.5$, $\psi = 0.1$	0.06	0.02	5.47
$\rho = 2.5$, $\psi = 0.1$	0.25	15.45	47.04
$\rho = 5.0$, $\psi = 0.3$	54.75	61.36	77.42
$\rho = 5.0$, $\psi = 0.5$	76.95	80.40	105.78
	Panel B, Indirect stockholders		
$\rho = 1.5$, $\psi = 0.1$	0.00	0.00	5.69
$\rho = 2.5$, $\psi = 0.1$	0.93	14.01	42.32
$\rho = 5.0$, $\psi = 0.3$	43.10	48.98	78.12
$\rho = 5.0$, $\psi = 0.5$	61.62	69.74	102.74

Table 13: Welfare gain from having access to TDA. The gain is expressed as a percentage of certainty equivalent life-time consumption.

Utility parameters	Direct stockholder	Indirect stockholder
$\rho = 1.5$, $\psi = 0.1$	0.04	0.04
$\rho = 2.5$, $\psi = 0.1$	0.08	0.20
$\rho = 5.0$, $\psi = 0.3$	8.26	11.06
$\rho = 5.0$, $\psi = 0.5$	10.00	11.19