HOUSEHOLD RISK MANAGEMENT AND OPTIMAL MORTGAGE CHOICE*

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This paper asks how a household should choose between a fixed-rate (FRM) and an adjustable-rate (ARM) mortgage. In an environment with uncertain inflation a nominal FRM has a risky real capital value, whereas an ARM has a stable real capital value but short-term variability in required real payments. Numerical solution of a life-cycle model with borrowing constraints and income risk shows that an ARM is generally attractive, but less so for a risk-averse household with a large mortgage, risky income, high default cost, or low moving probability. An inflation-indexed FRM can improve substantially on standard nominal mortgages.

I. INTRODUCTION

The portfolio of the typical American household is quite unlike the diversified portfolio of liquid assets discussed in finance textbooks. The major asset in the portfolio is a house, a relatively illiquid asset with an uncertain capital value. The value of the house generally exceeds the net worth of the household, which finances its homeownership through a mortgage contract to create a leveraged position in residential real estate. Other financial assets and liabilities are typically far less important than the house and its associated mortgage contract.

The importance of housing in household wealth is illustrated in Figure I. This figure plots the fraction of household assets in housing and in equities against the wealth percentile of the household. Poor households appear at the left of the figure, and wealthy households at the right. Data come from the 1989 and 1998 Survey of Consumer Finances. The figure shows that middle-class American families (from roughly the fortieth to the eightieth percentile of the wealth distribution) have more than half their assets in the form of housing. Even after the expansion of equity ownership during the 1990s, equities are of negligible importance for these households.[1]

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1. We are grateful to Joe Tracy for providing us with this figure. The methodology used to construct it is explained in Tracy, Schneider, and Chan [1999] and Tracy and Schneider [2001]. Wealth is defined as total assets, without subtracting liabilities and including all assets except human capital and defined-benefit pension plans. Households in the Survey of Consumer Finances are sorted by this

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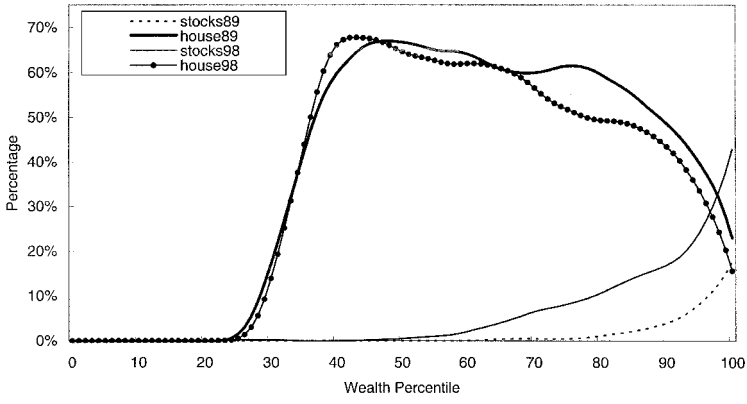


FIGURE I

Fraction of Household Assets in Corporate Equity and Real Estate by Wealth Percentile, 1989 and 1998

The data are from the 1989 and 1998 Survey of Consumer Finances. We would like to thank Joe Tracy for kindly providing us the data for this figure.

Academic economists have explored the effects of illiquid risky housing on saving and portfolio choice (see, for example, Cocco [2001], Davidoff [2002], Flavin and Yamashita [2002], Fratantoni [2001], Goetzmann [1993], Hu [2001], Skinner [1994], and Yao and Zhang [2001]). Some have proposed innovative risk-sharing arrangements in which households share ownership of their home with financial institutions [Caplin, Chan, Freeman, and Tracy 1997] or buy insurance against declines in local house price indexes [Shiller 1998; Shiller and Weiss 1999] in order to reduce their exposure to fluctuations in house prices. Such arrangements have not yet been implemented on any significant scale, perhaps because the occupant of a single-family home has inadequate incentives to maintain the home when he is not the sole owner, or because homeownership protects households from fluctuations in local rents [Sinai and Souleles 2003], or because of barriers to innovation in retail financial markets.

In this paper we consider a household that solely owns a house with an uncertain capital value, financing it with a mort-

measure of wealth; then the median share in real estate and equity is calculated separately for families in each percentile of the wealth distribution. The medians are smoothed across neighboring percentiles in the figure. Equity holdings include direct holdings as well as mutual funds, defined-contribution retirement accounts, trusts, and managed accounts.

gage. We turn attention to the form of the mortgage contract, which can also have large effects on the risks faced by the homeowner. We view the choice of a mortgage contract as a problem in household risk management, and we conduct a normative analysis of this problem. Our goal is to discover the characteristics of a household that should lead it to prefer one form of mortgage over another. We abstract from all other aspects of household portfolio choice by assuming that household savings are invested entirely in riskless assets.

Mortgage contracts are often complex and differ along many dimensions. But conventional mortgages can be broadly classified into two main categories: adjustable rate (ARM) and nominal fixed-rate (FRM) mortgages. In this paper we study the choice between these two types of mortgages, characterizing the advantages and disadvantages of each type for different households. We compare these conventional mortgages with inflation-indexed fixed-rate mortgages of the sort proposed by Fabozzi and Modigliani [1992], Kearl [1979], Statman [1982], and others.

When deciding on the type of mortgage, an extremely important consideration is labor income and the risk associated with it. Labor income or human capital is undoubtedly a crucial asset for the majority of households. If markets are complete such that labor income can be capitalized and its risk insured, then labor income characteristics play no role in the mortgage decision. In practice, however, markets are seriously incomplete because moral hazard issues prevent investors from borrowing against future labor income, and insurance markets for labor income risk are not well developed.

In this paper we solve a dynamic model of the optimal consumption and mortgage choices of a finitely lived investor who is endowed with nontradable human capital that produces a risky stream of labor income. The framework is the buffer-stock savings model of Zeldes [1989], Deaton [1991], and Carroll [1997], calibrated to microeconomic data following Cocco, Gomes, and Maenhout [1999] and Gourinchas and Parker [2002]. The investor initially buys a house with a required minimum downpayment, financing the rest of the purchase with either an ARM or a FRM. Subsequently, the investor can refinance the FRM, if the value of the house less the minimum downpayment exceeds the principal balance of the mortgage,

by paying a fixed cost.[2] We can also allow the investor to take out a second loan, up to the point where total debt equals the value of the house less the required downpayment, and we can allow for a fixed probability each period that the investor will move house. We ask how these options and other parameters of the model affect mortgage choice.

Our results illustrate a basic trade-off between several types of risk. A nominal FRM, without a prepayment option, is an extremely risky contract because its real capital value is highly sensitive to inflation. The presence of a prepayment option protects the homeowner against one side of this risk, because the homeowner can call the mortgage at face value if nominal interest rates fall, taking out a new mortgage contract with a lower nominal rate. However, this option does not come for free; it raises the interest rate on an FRM and leaves the homeowner with a contract that is expensive when inflation is stable, but extremely cheap when inflation increases as occurred during the 1960s and 1970s. This *wealth risk* is an important disadvantage of a nominal FRM.

An ARM, on the other hand, is a safe contract in the sense that its real capital value is almost unaffected by inflation. The risk of an ARM is the *income risk* of short-term variability in the real payments that are required each month. If expected inflation and nominal interest rates increase, nominal mortgage payments increase proportionally even though the price level has not yet changed much; thus, real monthly payments are highly variable. This variability would not matter if the homeowner could borrow against future income, but it does matter if the homeowner faces binding borrowing constraints. Constraints bind in states of the world with low income and low house prices; in these states buffer-stock savings are exhausted, and home equity falls below the minimum required to take out a second loan. The danger of an ARM is that it will require higher interest payments in this situation, forcing a temporary but unpleasant reduction of consumption. We find that households with large houses relative to

2. The fixed cost represents some combination of explicit "points," often charged at the initiation of a mortgage contract, and implicit transactions costs [Stanton 1995]. We do not allow households to choose among mortgages offering a trade-off of points against interest rates [Stanton and Wallace 1998]. Caplin, Freeman, and Tracy [1997] and Chan [2001] emphasize that refinancing can become impossible if house prices fall below mortgage balances so that homeowners have negative home equity.

their income, volatile labor income, or high risk aversion are particularly adversely affected by the income risk of an ARM.

Our model also allows for *real interest rate risk,* the risk that the cost of borrowing will increase during the life of a long-term loan. Merton [1973] pointed out that long-term investors should be just as concerned about shocks to interest rates as about shocks to their wealth; as Campbell and Viceira [2001, 2002] have emphasized, this means that short-term debt is not a safe investment for long-term investors. The same point applies to long-term borrowers. Long-term FRMs protect homeowners against the risk that real interest rates will increase, whereas ARMs do not.

The mobility of a household and its current level of savings also affect the form of the optimal mortgage contract. If a household knows it is highly likely to move in the near future, or if it is currently borrowing-constrained, the most appropriate mortgage is more likely to be the one with the lowest current interest rate. Unconditionally, this is the ARM, since the FRM rate incorporates a positive term premium and the cost of the FRM prepayment option; but if the short-term interest rate is currently high and likely to fall, the FRM might have a lower rate. Thus, our model implies that homeowners should respond to the yield spread between FRM and ARM mortgage rates, which is driven by the yield spread between long-term and short-term bond yields. When this yield spread is unusually high, more homeowners should take out ARMs; when it is unusually low, more homeowners should take out FRMs.

One solution to the risk management problems identified in this paper is an inflation-indexed FRM. This contract removes the wealth risk of the nominal FRM without incurring the income and real interest rate risks of the standard ARM contract. The inflation-indexed FRM should also have a lower mortgage rate than a nominal FRM, since the real term structure is flatter than the nominal term structure and the option to prepay an inflation-indexed mortgage is less valuable. We calibrate our model to U. S. interest data over the period 1962–1999 and find large welfare gains from indexation of FRMs. These results parallel the findings of Campbell and Shiller [1996] and Campbell and Viceira [2001] that inflation-indexed bonds should be attractive to conservative long-term investors.

It is interesting to compare our normative results with historical patterns in mortgage financing, and with the advice that homeowners receive from books on personal finance. The United

States is unusual among industrialized countries in that the predominant mortgage contract is a long-term nominal FRM, usually with a 30-year maturity. The monthly interest rate survey of the Federal Housing Finance Board shows that long-term nominal FRMs accounted for 70 percent of newly issued mortgages on average during the period 1985–2001, while ARMs accounted for 30 percent. Nominal FRMs have a very large secondary market, whose liquidity has been supported by U. S. government policy over many decades, particularly through the government agency GNMA (Government National Mortgage Association or "Ginnie Mae"), and the private but government-sponsored entities FNMA (Federal National Mortgage Association or "Fannie Mae") and FHLMC (Federal Home Loan Mortgage Corporation or "Freddie Mac"). The liquidity of this market likely reduces the rates on nominal FRMs and helps to account for their popularity in the United States.[3]

Figure II plots the evolution of the FRM share over time. The FRM share is strongly negatively correlated with the level of long-term interest rates (the correlation with the ten-year Treasury yield is −0.77 in levels and −0.57 in quarterly changes). Accordingly, the FRM share trended upward during the period 1985–2001 as interest rates trended downward; it averaged around 60 percent in the late 1980s and around 80 percent in the late 1990s. Surprisingly, the FRM share is almost uncorrelated with the yield spread between ten-year and one-year interest rates (the correlation is 0.10 in levels and 0.02 in quarterly changes).[4]

One explanation for the tendency of households to use FRMs when long-term interest rates have recently fallen is that households believe long-term interest rates to be mean-reverting. If declines in long-term interest rates tend to be followed by increases, then it is rational to "lock in" a long-term interest rate

3. Woodward [2001] describes in detail how federal policy has supported the FRM market. Several studies have found important liquidity effects in mortgage markets. Cotterman and Pearce [1996] find a 25–40 basis point spread between private label mortgages and the conforming mortgages that are securitized by FNMA and FHLMC, while Black, Garbade, and Silber [1981] and Rothberg, Nothaft, and Gabriel [1989] find that the initial securitization of mortgages by GNMA lowered mortgage interest rates by 60–80 basis points.

4. During 2002 the FRM share fell even while interest rates declined. This attracted the attention of the business press as a departure from the historical pattern. See, for example, Ruth Simon, "Do You Have the Wrong Mortgage? In Puzzling Move, Homeowners Flock to Riskier Variable Loans Instead of Locking In Low Rates," *Wall Street Journal,* June 18, 2002.

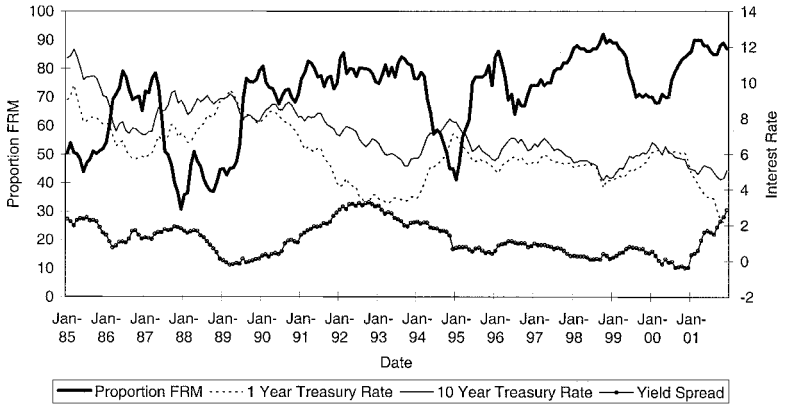


FIGURE II
FRM Share and Treasury Interest Rates

This figure plots the percentage of conventional single-family mortgages originated by major lenders with fixed rates. The data are from the monthly interest rate survey of the Federal Housing Finance Board from January 1985 to December 2001. The figure also plots the one-year Treasury rate, the ten-year Treasury rate, and the yield spread between ten-year and one-year rates.

that is low relative to past history by taking out a FRM. Some personal finance books offer advice of this sort. Irwin [1996], for example, offers the following tip: "When interest rates are low, get a fixed-rate mortgage and lock in the low rate" [p. 143], while Steinmetz [2002] advises "If you think rates are going up, get a fixed-rate mortgage" [p. 84]. The difficulty with this advice, of course, is that movements in long rates are extremely difficult to forecast. The expectations theory of the term structure implies that changes in long-term bond yields should be almost unforecastable; while there is some empirical evidence against this theory (see, for example, Campbell and Shiller [1991] or Campbell, Lo, and MacKinlay [1997, Chapter 10]), it seems overambitious for the average homeowner to try to predict movements in long-term interest rates.

Other recommendations of personal finance books are more consistent with the normative results presented in this paper. Homeowners who expect to move within a few years are often advised to take out ARMs to exploit the low initial interest rate. Tyson and Brown [2000, p. 64], for example, write: "Many homebuyers don't expect to stay in their current homes for a long time. If that's your expectation, consider an ARM. Why? Because an ARM starts at a lower interest rate than does a fixed-rate loan,

you should save interest dollars in the first two years of holding your ARM." ARMs are also recommended for homeowners who are currently borrowing constrained but expect their incomes to grow rapidly: "ARMs are best utilized . . . when your cash flow is currently tight but you expect it to increase as time goes on" [Orman 1999, p. 254]; "Sometimes ARMs have lower initial loan costs. If cash is a big consideration for you, look into them" [Irwin 1996, p. 144].

Personal finance books do not explicitly distinguish different types of risk as we do in this paper. However, some personal finance authors clearly think that income risk and real interest rate risk are important for homeowners, because they describe ARMs as risky assets and FRMs as safe: "An ARM can pay off, but it's a gamble. Sometimes there's a lot to be said for something that's safe and dependable, like a fixed-rate mortgage" [Fisher and Shelly 2002, p. 319].

There is a large academic literature on mortgage choice. Follain [1990] surveys the literature from the 1980s and earlier. Much recent work focuses on FRM prepayment behavior, and its implications for the pricing of mortgage-backed securities (for example, Schwartz and Torous [1989] and Stanton [1995]). One strand of the literature emphasizes that households know more about their moving probabilities than lenders do; this creates an adverse selection problem in prepayment that can be mitigated through the use of fixed charges or "points" at mortgage initiation [Dunn and Spatt 1985; Chari and Jagannathan 1989; Brueckner 1994; LeRoy 1996; Stanton and Wallace 1998].

A few papers discuss the choice between adjustable-rate and fixed-rate mortgages. On the theoretical side, Alm and Follain [1984] emphasize the importance of labor income and borrowing constraints for mortgage choice, but their model is deterministic and thus they cannot address the risk management issues that are the subject of this paper. Stanton and Wallace [1999] discuss the interest-rate risk of ARMs, but without considering the role of risky labor income and borrowing constraints. We are not aware of any previous theoretical work that treats income risk and interest-rate risk within an integrated framework as we do here. On the empirical side, Shilling, Dhillon, and Sirmans [1987] look at micro data on mortgage borrowing and estimate a reduced-form econometric model of mortgage choice. They find that households with a more stable income and households with a higher moving probability are more likely to use ARMs. These findings

are consistent both with our theoretical model and with the typical advice given by books on personal finance.

The organization of the paper is as follows. Subsection II.A lays out the model of household choice, and subsection II.B calibrates its parameters. Section III compares alternative nominal mortgage contracts, while Section IV studies inflation-indexed FRMs. Section V asks whether our results are robust to alternative parameterizations. Section VI concludes.

II. A Life-Cycle Model of Mortgage Choice

II.A. Model Specification

We model the consumption and asset choices of a household, indexed by j, with a time horizon of T periods. We study the decision of how to finance the purchase of a house of a given size $\bar{H}_j$. That is, we assume that buying a house is strictly preferred to renting—perhaps because of tax considerations—so that we do not model the decision to buy versus rent. In addition, we do not study what determines the initial choice of house size, and we assume that the household remains in a house of this size, regardless of the path of household income. Thus, we ignore the possibility that the household can adjust to an income shock by moving to a larger or smaller house.[5]

In each period t, $t = 1, \ldots, T$, the household chooses real consumption of all goods other than housing, C_{jt}. We assume preference separability between housing and consumption. Since the size of the house and the utility derived from it are fixed, we can omit housing from the objective function of the household and write

$$\max E_0 \sum_{t=0}^{T} \beta^t \frac{C_{jt}^{1-\gamma}}{1-\gamma} + \beta^{T+1} \frac{W_{j,T+1}^{1-\gamma}}{1-\gamma}, \tag{1}$$

where β is the time discount factor and γ is the coefficient of relative risk aversion. The household derives utility from terminal real wealth $W_{j,T+1}$, which can be interpreted as the remaining lifetime utility from reaching age $T + 1$ with wealth $W_{j,T+1}$.

FRM and ARM mortgages differ because nominal interest

5. Cocco [2001] studies the choice of house size using a life-cycle model similar to the one in this paper. Sinai and Souleles [2003] study the choice between renting and buying housing.

rates are variable over time. This variability comes from movements in both the expected inflation rate and the ex ante real interest rate. We use the simplest model that captures variability in both these components of the short-term nominal interest rate, and allows for some predictability of interest rate movements. Thus, in our model there will be periods when homeowners can rationally anticipate declining or increasing short-term nominal interest rates, and thus declining or increasing ARM payments.

We write the nominal price level at time t as P_t. We adopt the convention that lowercase letters denote log variables. Thus, $p_t = \log(P_t)$, and the log inflation rate $\pi_t = p_{t+1} - p_t$. To simplify the model, we abstract from one-period uncertainty in realized inflation; thus, expected inflation at time t is the same as inflation realized from t to $t + 1$. While clearly counterfactual, this assumption should have little effect on our comparison of nominal mortgage contracts, since short-term inflation uncertainty is quite modest and affects nominal ARMs and FRMs symmetrically. Later in the paper we consider inflation-indexed mortgages; the absence of one-period inflation uncertainty in our model will lead us to understate the advantages of these mortgages.

We assume that expected inflation follows an AR(1) process. That is,

$$\pi_t = \mu(1 - \phi) + \phi\pi_{t-1} + \epsilon_t, \tag{2}$$

where ϵ_t is a normally distributed white noise shock with mean zero and variance σ_ϵ^2. By contrast, we assume that the ex ante real interest rate is variable but serially uncorrelated. The expected log real return on a one-period bond, $r_{1t} = \log(1 + R_{1t})$, is given by

$$r_{1t} = \bar{r} + \psi_t, \tag{3}$$

where $\bar{r}$ is the mean log real interest rate and ψ_t is a normally distributed white noise shock with mean zero and variance σ_ψ^2.

We make the assumption that real interest rate risk is transitory for tractability. Fama [1975] showed that the assumption of a constant real interest rate was a good approximation for U. S. data in the 1950s and 1960s, but it is well-known that more recent U. S. data display serially correlated movements in real interest rates (see, for example, Garcia and Perron [1996], Gray [1996], or Campbell and Viceira [2001]). However, movements in expected inflation are the most important influence on long-term nominal interest rates [Fama 1990; Mishkin 1990; Campbell and

Ammer 1993], and our AR(1) assumption for expected inflation allows persistent variation in nominal interest rates.

The log nominal yield on a one-period nominal bond, $y_{1t} = \log(1 + Y_{1t})$, is equal to the log real return on a one-period bond plus expected inflation:

$$y_{1t} = r_{1t} + \pi_t. \tag{4}$$

To model long-term nominal interest rates, we assume that the log expectations hypothesis holds. That is, we assume that the log yield on a long-term n-period nominal bond, $y_{nt} = \log(1 + Y_{nt})$, is equal to the expected sum of successive log yields on one-period nominal bonds which are rolled over for n periods plus a constant term premium, ξ:

$$y_{nt} = \left(\frac{1}{n}\right) \sum_{i=0}^{n-1} E_t[y_{1,t+i}] + \xi. \tag{5}$$

This model implies that excess returns on long-term bonds over short-term bonds are unpredictable, even though changes in nominal short rates are partially predictable. Thus, there are no predictably good or bad times to alter the maturity of a bond portfolio, and homeowners cannot reduce their average borrowing costs by trying to time the bond market.

At date 1, household j finances the purchase of a house of size $\bar{H}_j$ with a nominal loan of $(1 - \lambda) P_{j1}^H \bar{H}_j$, where λ is the required downpayment and P_{j1}^H is the date 1 nominal price of the house. The mortgage loan is assumed to have maturity T, so that it is paid off by period $T + 1$.

If the household chooses a nominal FRM, and the date 1 interest rate on a FRM with maturity T is Y_{T1}^F, then in each subsequent period the household must make a real mortgage payment M_{jt}^F of

$$M_{jt}^F = \frac{(1 - \lambda) P_{j1}^H \bar{H}_j}{P_t \sum_{j=1}^{T} (1 + Y_{T1}^F)^{-j}}. \tag{6}$$

Since nominal mortgage payments are fixed at mortgage initiation, real payments are inversely proportional to the price level P_t. This implies that a nominal FRM, without a prepayment option, is a risky contract because its real capital value is highly sensitive to inflation.

We allow for a prepayment option. A household that chooses

an FRM may in later periods refinance at a monetary cost of ρ. Let I^{ρ}_{jt} be an indicator variable which takes the value of one if the household refinances in period t, and zero otherwise. We assume that a refinancing household at date t obtains a new FRM mortgage with the same principal as the remaining principal of the old mortgage, and with maturity $T - t + 1$ such that by the terminal date $T + 1$ the mortgage will have been paid down. We allow refinancing to occur only if the household has positive home equity at time t; that is, if the house price less the minimum downpayment exceeds the principal balance of the mortgage.

We assume that the date t nominal interest rate on a FRM is given by

$$Y^{F}_{T-t+1,t} = Y_{T-t+1,t} + \theta^{F}, \tag{7}$$

where θ^{F} is a constant mortgage premium over the yield on a $(T - t + 1]$-period bond. This premium compensates the mortgage lender for default risk and for the value of the refinancing option.

If the household chooses an ARM, the annual real mortgage payment, M^{A}_{jt}, is given by the following. We write D_{jt} for the nominal principal amount of the original loan outstanding at date t. Then the date t real mortgage payment is given by

$$M^{A}_{jt} = \frac{Y^{A}_{1t} D_{jt} + \Delta D_{j,t+1}}{P_{t}}, \tag{8}$$

where $\Delta D_{j,t+1}$ is the component of the mortgage payment at date t that goes to pay down principal rather than pay interest. We assume that $\Delta D_{j,t+1}$ is equal to the average nominal loan reduction that occurs at date t in a FRM for the same initial loan. While this does not correspond exactly to a conventional ARM, it greatly simplifies the problem since by having loan reductions that depend only on time and the amount borrowed, the proportion of the original loan that has been repaid is not a state variable.

The date t nominal interest rate on an ARM is assumed to be equal to the short rate plus a constant premium:

$$Y^{A}_{1t} = Y_{1t} + \theta^{A}. \tag{9}$$

The ARM mortgage premium θ^{A} compensates the mortgage lender for default risk.

The household is endowed with stochastic gross real labor income in each period, L_{jt}, which cannot be traded or used as

collateral for a loan. As usual, we use a lowercase letter to denote the natural log of the variable, so $l_{jt} \equiv \log(L_{jt})$. Household j's log real labor income is exogenous and is given by

$$l_{jt} = f(t, Z_{jt}) + v_{jt} + \omega_{jt}, \tag{10}$$

where $f(t, Z_{jt})$ is a deterministic function of age t and other individual characteristics Z_{jt}, and v_{jt} and ω_{jt} are stochastic components of income. Thus, log income is the sum of a deterministic component that can be calibrated to capture the hump shape of earnings over the life-cycle, and two random components, one transitory and one persistent. The transitory component is captured by the shock ω_{jt}, an i.i.d. normally distributed random variable with mean zero and variance σ_ω^2. The persistent component is assumed to be entirely permanent; it is captured by the process v_{jt}, which is assumed to follow a random walk:

$$v_{jt} = v_{j,t-1} + \eta_{jt}, \tag{11}$$

where η_{jt} is an i.i.d. normally distributed random variable with mean zero and variance σ_η^2. These assumptions closely follow Cocco, Gomes, and Maenhout [1999] and other papers on the buffer-stock model of savings.

We allow transitory labor income shocks, ω_{jt}, to be correlated with innovations to the stochastic process for expected inflation, ϵ_t, and denote the corresponding coefficient of correlation φ. To the extent that wages are set in real terms, this correlation is likely to be zero. If wages are set in nominal terms, however, the correlation between real labor income and inflation may be negative, and this can affect the form of the optimal mortgage contract.

We model the tax code in the simplest possible way, by considering a linear taxation rule. Gross labor income, L_{jt}, is taxed at the constant tax rate τ. We also allow for mortgage interest deductibility at this rate.

The price of housing fluctuates over time. Let p_{jt}^H denote the date t real log price of house j. Real house price growth is given by

$$\Delta p_{jt}^H = g + \delta_{jt}, \tag{12}$$

a constant g plus an i.i.d. normally distributed shock δ_{jt} with mean zero and variance σ_δ^2. To economize on state variables, we assume that innovations to a household's real house price are perfectly positively correlated with innovations to the permanent component of the household's real labor income so that

$$\delta_{jt} = \alpha \eta_{jt}, \tag{13}$$

where $\alpha > 0$. This assumption implies that states of the world with low house prices are also states with low permanent labor income; in these states an increase in required mortgage payments under an ARM contract can require costly adjustments in consumption. In the next section we use PSID data to judge the plausibility of this assumption.[6]

House prices matter in our model because we impose the realistic constraint, emphasized by Caplin, Freeman, and Tracy [1997] and Chan [2001], that refinancing of a FRM is only possible if the value of the house, less the minimum downpayment, exceeds the principal balance of the mortgage. In addition, we can extend the model to allow households to obtain a second one-period loan to bring total debt up to the value of the house less the minimum downpayment. Recall that D_{jt} is the nominal dollar amount of the original loan outstanding at date t. We allow households at time t to borrow B_{jt} nominal dollars for one period subject to the constraint

$$B_{jt} \leq (1 - \lambda) P_{jt}^{H} \bar{H}_j - D_{jt}. \tag{14}$$

That is, total borrowing cannot exceed the original proportion of house value that could be borrowed at date 1. We assume that the nominal interest rate on the second loan is equal to Y_{1t} plus a constant premium θ^B.

In each period the household decides whether or not to default on the loan. In case of default the bank seizes the house and the household is forced into the rental market for the remainder of its life. We set the rental cost equal to the user cost of housing plus a constant rental premium θ^R. The real rental cost Z_t for a house of size $\bar{H}$ with price P_t^H is given by

$$Z_t = \frac{[Y_{1t} - E_t(\Delta p_{t+1}^H + \pi_{t+1}) + \theta^R] P_t^H \bar{H}}{P_t}, \tag{15}$$

where $Y_{1,t}$ is the one-period nominal interest rate, $E_t(\Delta p_{t+1}^H + \pi_{1,t+1})$ is the expected proportional nominal change in the house price, and $P_t^H \bar{H}$ is the date t value of the house. The rental premium covers the moral hazard problem of renting, that tenants have no incentive to look after a property so that mainte-

6. A large positive correlation between income shocks and house prices is also present in Ortalo-Magné and Rady [2001].

nance becomes more expensive. In addition, and contrary to interest payments on a mortgage loan, the rental cost of housing is not tax-deductible, which increases the after-tax cost of renting.

The date t real profit of lenders of funds, or banks, depends on whether there is default. For an ARM loan to a household with no second loan, it is given by

$$\Pi_{jt} = \frac{(P_{jt}^{H}\bar{H}_j - D_{jt})I_{jt}^{Z} + \theta^{A}D_{jt}(1 - I_{jt}^{Z})}{P_t}, \tag{16}$$

where I_{jt}^{Z} is an indicator variable which takes the value of one if the household defaults in period t and zero otherwise (of course this variable is not defined in case there has been default in a period prior to t). In case of default the bank seizes the house but loses the outstanding mortgage principal. If there is no default, the bank receives the ARM premium on the outstanding loan. For a FRM the household can also refinance the loan, in which case interest payments cease, but the bank receives the outstanding mortgage principal.

We introduce moving in the model in the following simple manner: with probability p the household moves in each period. When this happens, the household sells the house, pays off the remaining mortgage, and evaluates utility of wealth using the terminal utility function. This enables us to study the impact of the likelihood of moving, or of termination of the mortgage contract, on mortgage choice.

In summary, the household's control variables are $(C_{jt},B_{jt},I_{jt}^{0},I_{jt}^{Z})$ at each date t. The problem is somewhat simpler in the case of an ARM, because in this case the refinancing indicator variable I_{jt}^{0} is not a control variable. The vector of state variables can be written as $X_{jt} = (t,y_{1t},W_{jt},P_t,y_{1,t_j'},t_j',v_{jt},S_{jt}^{Z})$ at each date t, where $y_{1,t_j'}$ $(t_j' < t)$ is the level of nominal interest rates when the mortgage was initiated or was last refinanced, t_j' is the period when the mortgage was initiated or was last refinanced, W_{jt} is real liquid wealth or cash-on-hand, P_t is the date t price level, v_{jt} is the household's permanent labor income, and S_{jt}^{Z} is a state variable that takes the value of one if there has been previous default and zero otherwise.

The equation describing the evolution of real cash-on-hand for an ARM when there has not been previous default, and with no second loan, can be written as

(17)

$$W_{j,t+1} = (W_{jt} - C_{jt} - M^A_{jt} + \tau Y^A_{1t} D_{jt}/P_t)(1 + R_{1,t+1}) + (1 - \tau) L_{j,t+1},$$

or when there has been previous default,

(18) $$W_{j,t+1} = (W_{jt} - C_{jt} - Z_{jt})(1 + R_{1,t+1}) + (1 - \tau) L_{j,t+1},$$

and similarly for a FRM.

This problem cannot be solved analytically. Given the finite nature of the problem, a solution exists and can be obtained by backward induction. We discretize the state space and the choice variables using equally spaced grids in the log scale. The density functions for the random variables were approximated using Gaussian quadrature methods to perform numerical integration [Tauchen and Hussey 1991]. The nominal interest rate process was approximated by a two-state transition probability matrix. The grid points for these processes were chosen using Gaussian quadrature. In period $T + 1$ the utility function coincides with the value function. In every period t prior to $T + 1$, and for each admissible combination of the state variables, we compute the value associated with each combination of the choice variables. This value is equal to current utility plus the expected discounted continuation value. To compute this continuation value for points which do not lie on the grid, we use cubic spline interpolation. The combinations of the choice variables ruled out by the constraints of the problem are given a very large (negative) utility such that they are never optimal. We optimize over the different choices using grid search.

II.B. Parameterization

We study the optimal consumption and mortgage choices of investors who buy a house early in life. Adult age in our model starts at age 26, and we let T be equal to 30 years. For computational tractability, we let each period in our model correspond to two years, but we report annualized parameters and data moments for ease of interpretation. In the baseline case we assume an annual discount factor β equal to 0.98 and a coefficient of relative risk aversion γ equal to three. We will study how the degree of risk aversion affects mortgage choice.

Parameter estimates for inflation and interest rates are reported in Table I. Our measure of inflation is the consumer price index. We use annual data from 1962 to 1999, time aggregated to two-year periods, to estimate equation (2). We find average infla-

TABLE I
CALIBRATED AND ESTIMATED PARAMETERS

Description	Parameter	Value
Risk aversion	γ	3
Discount factor	β	0.98
House size ($ thousands)	$\bar{H}$	125, 187.5
Downpayment ratio	λ	0.20
Tax rate	τ	0.20
Mean log inflation	μ	0.046
Standard deviation of log inflation	$\sigma(\pi_{1t})$	0.039
Autoregression parameter	ϕ	0.754
Mean log real yield	$\bar{r}$	0.020
Standard deviation of real log yield	$\sigma(r_{1t})$	0.022
Nominal FRM premium	θ^F	0.018
Term premium	ξ	0.010
Refinancing cost ($ thousands)	ρ	1, ∞
ARM premium	θ^A	0.017
Second loan premium	θ^B	∞
Rental premium	θ^Z	0.030
Mean real house price growth	$\exp(g + \sigma_\delta^2/2)$	0.016
Standard deviation of log real house price growth	σ_δ	0.115
Standard deviation of transitory income shocks	σ_ω	0.141, 0.248
Standard deviation of persistent income shocks	σ_η	0.020
Correlation of transitory income and inflation shocks	φ	0.000

All parameters are in annual terms. The interest rate measure is the one-year Treasury bond rate from 1962 to 1999. The income and house price data are from the PSID from 1970 through 1992. Families that were part of the Survey of Economic Opportunities were dropped from the sample. Labor income in each year is defined as total reported labor income plus unemployment compensation, workers compensation, social security, supplemental social security, other welfare, child support, and total transfers, all this for both head of household and if present his spouse. Labor income and reported house prices were deflated using the Consumer Price Index.

tion of 4.6 percent per year, with a standard deviation of 3.9 percent, and an annual autoregressive coefficient of 0.754. To measure the log real interest rate, we deflate the two-year nominal interest rate using the consumer price index. We measure the variability of the ex ante real interest rate by regressing ex post two-year real returns on lagged two-year real returns and two-year nominal interest rates, and then calculating the variability of the fitted value. We obtain a standard deviation of 2.2 percent per year, as compared with a mean of 2.0 percent. This standard

deviation is surprisingly high, which may be a result of overfitting in our regression; but since our assumption that all real interest rate risk is transitory artificially diminishes the importance of such risk, we use this high standard deviation to partially offset this effect. Our results are not particularly sensitive to changes in the volatility of the real interest rate.

In order to assess how well our model for the term structure matches the data, we have computed the annualized standard deviations of the two-year bond yield, the ten-year bond yield, and the spread between them. The values we obtain are 5.3, 1.9, and 3.5 percent, respectively. The corresponding values in the data are 3.1, 2.9, and 0.7 percent. It appears that our model overstates the volatility of the short rate and understates its persistence, which means that we understate the volatility of the long rate level and overstate the volatility of the long-short yield spread.

In Section V on alternative parameterizations we assess the benefits of mortgage indexation when we calibrate our interest-rate process to a process characteristic of the United States in the 1983–1999 period. As expected, the estimated parameters (reported in Section V) imply considerably lower inflation risk in this period.

Two important parameters of the mortgage contracts are the mortgage premiums, θ^F and θ^A. It is natural to assume that $\theta^F \geq \theta^A$. One can think of θ^A as a pure measure of default risk, while θ^F contains both default risk and the value of the prepayment option.

To estimate the mortgage premiums on the contracts, we use data from the monthly interest rate survey of the Federal Housing Finance Board (FHFB) from January 1986 to December 2001. To estimate the mortgage premium on FRM contracts, θ^F, we compute the difference between interest rates on commitments for fixed-rate mortgages and the yield to maturity on ten-year Treasury bonds. The average annual difference over this period is 1.8 percent.

To estimate the mortgage premium on ARM contracts, θ^A, we compute the difference between the ARM contract rate and the yield on a one-year bond over the same sample period. The average annual difference is equal to 1.7 percent. This number may be biased downward by the fact that ARMs sometimes have low initial "teaser" rates to lure households into the ARM commitment.

The difference between the ARM and FRM premiums is

surprisingly small. This may result in part from measurement error in the survey data or the short sample period of the survey. It may also result from the liquidity of the FRM market which has been supported by U. S. government policy over many decades, particularly through the activities of GNMA and the government's sponsorship of FNMA and FHLMC.

We set the term premium equal to 1.0 percent, the average yield spread between ten-year and one-year Treasury bonds over the period 1986–2001. This term premium increases the average interest cost of FRMs relative to ARMs.

We assume a required downpayment of 20 percent, and we set the rental premium θ^Z to 3.0 percent. In the baseline case we make θ^B infinite and therefore do not allow the homeowner to take out a second loan. We relax this restriction in Section V.

We use house price data from the PSID for the years 1970 through 1992. As with income the self-assessed value of the house was deflated using the Consumer Price Index, with 1992 as the base year, to obtain real house prices. We drop observations for households who reported that they moved in the previous two years since the house price reported does not correspond to the same house. In order to deal with measurement error, we drop the observations in the top and bottom 5 percent of real house price changes.

We estimate the average real growth rate of house prices and the standard deviation of innovations to this growth rate. Over the sample period real house prices grew an average of 1.6 percent per year. Part of this increase is due to improvements in the quality of houses, which cannot be separated from other reasons for house price appreciation using PSID data. The annualized standard deviation of house price changes is 11.5 percent, a value comparable to those reported by Case and Shiller [1989] and Poterba [1991].

We consider two alternative house sizes. In the benchmark case the household purchases a house costing $187,500 using a $150,000 mortgage and paying $37,500 down. (The downpayment is assumed to come from prior savings or transfers from family members, rather than from current income.) In an alternative case, the household purchases a smaller house costing $125,000 using a $100,000 mortgage and a $25,000 downpayment.

To estimate the income process, we follow Cocco, Gomes, and Maenhout [1999]. We use the family questionnaire of the Panel Study on Income Dynamics (PSID) to estimate labor income as a

function of age and other characteristics. In order to obtain a random sample, we drop families that are part of the Survey of Economic Opportunities subsample. Only households with a male head are used, as the age profile of income may differ across male- and female-headed households, and relatively few observations are available for female-headed households. Retirees, nonrespondents, students, and homemakers are also eliminated from the sample.

Like Cocco, Gomes, and Maenhout [1999] and Storesletten, Telmer, and Yaron [2003], we use a broad definition of labor income so as to implicitly allow for insurance mechanisms—other than asset accumulation—that households use to protect themselves against pure labor income risk. Labor income is defined as total reported labor income plus unemployment compensation, workers compensation, social security, supplemental social security, other welfare, child support, and total transfers (mainly help from relatives), all this for both head of household and if present his spouse. Observations which still reported zero for this broad income category were dropped.

Labor income defined this way is deflated using the Consumer Price Index, with 1992 as the base year. The estimation controls for family-specific fixed effects. The function $f(t,Z_{jt})$ is assumed to be additively separable in t and Z_{jt}. The vector Z_{jt} of personal characteristics, other than age and the fixed household effect, includes marital status, household composition, and the education of the head of the household.[7] Figure III shows the fit of a third-order polynomial to the estimated age dummies for singles and married couples with a high school education but no college degree. We use these age profiles for our calibration exercise. Average annual income for married couples is about 40 percent higher than income for singles, starting at around $23,000 and peaking at $32,000. This means that a house of given size is larger relative to income if it is owned by a single person.

The residuals obtained from the fixed-effects regressions of log labor income on $f(t,Z_{jt})$ can be used to estimate σ_η^2 and σ_ω^2. Define l_{jt}^* as

$$l_{jt}^* \equiv l_{jt} - f(t,Z_{jt}). \tag{19}$$

7. Campbell, Cocco, Gomes, and Maenhout [2001] estimate separate age profiles for different educational groups. They also estimate different income processes for households whose heads are employed in different industries, or self-employed. In this version of the paper, we focus on a single representative income process for simplicity.

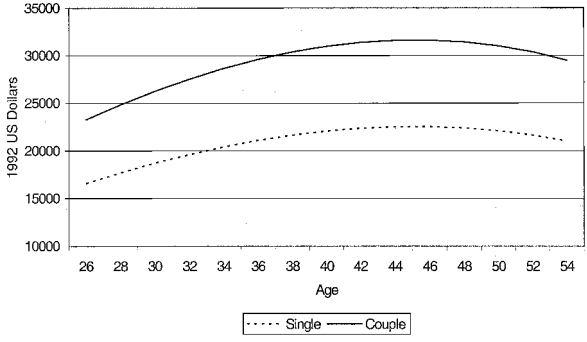


FIGURE III
Labor Income Profile

This figure plots a fitted third-order polynomial to the estimated age dummies for households composed of single individual and for a couple. The data are from the PSID for the years 1970 through 1992.

Equation (10) implies that

$$l^*_{jt} = v_{jt} + \omega_{jt}. \tag{20}$$

Taking first differences,

$$l^*_{jt} - l^*_{j,t-1} = v_{jt} - v_{j,t-1} + \omega_{jt} - \omega_{j,t-1} = \eta_{jt} + \omega_{jt} - \omega_{j,t-1}. \tag{21}$$

We consider several alternative methods for calibrating the standard deviations of the permanent and transitory shocks to income. One approach is to use the standard deviation of income innovations from (21), and the correlation between innovations to income and real house price growth, to obtain estimates for the standard deviations of η_{jt} and ω_{jt}. The estimated correlation is 0.027, with a p-value of 2 percent. Recall that in the model, and for tractability, we have assumed that real house price growth is perfectly positively correlated with innovations to the persistent component of income, and has zero correlation with purely transitory shocks. This assumption, and the standard deviation of $\eta_{jt} + \omega_{jt} - \omega_{j,t-1}$, imply that σ_η and σ_ω are 0.35 percent and 16.3 percent, respectively. This estimate of σ_η, the standard deviation of permanent income shocks, seems too low. The reason is probably that measurement error biases our estimate of the correlation between house price and income growth downward.

An alternative approach is to use household level data on income growth over several periods to estimate σ_η and σ_ω. Following Carroll [1992] and Carroll and Samwick [1997], Cocco,

Gomes, and Maenhout [1999] estimate that σ_η and σ_ω are 10.3 percent and 27.2 percent, respectively. Storesletten, Telmer, and Yaron [2003] have reported similar numbers.[8]

These numbers may be somewhat inflated by measurement error in the PSID. A large standard deviation for permanent income growth is particularly problematic for our model of mortgage choice because we assume a house of a fixed size and ignore the possibility that the household will choose to move to a larger or smaller house. This implies, for example, that our model will tend to overpredict default rates when permanent income is volatile.

To avoid this difficulty, we use a third calibration approach. We assume that all shocks to permanent labor income are aggregate shocks, so that idiosyncratic income risk is purely transitory. This assumption is consistent with the fact that aggregate labor income appears close to a random walk [Fama and Schwert 1977; Jagannathan and Wang 1996]. In this case σ_η can be estimated, as in Cocco, Gomes, and Maenhout [1999], by averaging across all individuals in our sample and taking the standard deviation of the growth rate of average income. Following this procedure, we estimate σ_η equal to 2.0 percent. For our baseline case we set σ_ω equal to 14.1 percent (20 percent over two years), which implies a correlation of house price growth with total income growth of about 0.1. Given the somewhat arbitrary nature of these decisions, we are careful to do sensitivity analysis with respect to the income growth parameters. We consider a higher transitory standard deviation of 24.8 percent (35 percent over two years) in the tables reported below, and in addition we have recomputed some results for a higher permanent standard deviation of 5 percent with results similar to those reported.

In the baseline case we set the correlation between transitory labor income shocks and innovations to expected inflation, φ, equal to zero.

The PSID contains information on total estimated federal income taxes of the household. We use this variable to obtain an estimate of τ. Dividing total federal taxes by our broad measure of labor income and computing the average across households, we obtain an average tax rate of 10.3 percent. This number under-

8. There is a large literature in empirical labor economics that estimates similar parameters, sometimes allowing them to vary over time. See, for example, Abowd and Card [1989], Gottschalk and Moffitt [1994], or MaCurdy [1982].

estimates the effect of taxation because the PSID does not contain information on state taxes and because our model abstracts from the progressivity of the income tax. To roughly compensate for these biases, we set τ equal to 20 percent. All the calibrated parameters are summarized in Table I.

III. Alternative Nominal Mortgages

We now use our model to compare fixed and adjustable rate nominal mortgages. We do so by calculating optimal consumption and refinancing plans, and the associated lifetime expected utilities, under alternative FRM and ARM contracts. We are particularly interested in the effects of house size, income risk, and the level of income on behavior and welfare. Accordingly, we consider two alternative house sizes—$125,000 and $187,500, corresponding to mortgages of $100,000 and $150,000, respectively—two levels of transitory income risk—annual standard deviations of 0.141 and 0.248—and two income levels—calibrated for a couple and a single person.

One way to get a sense for the size of these mortgages in relation to income is to calculate the ratio of total mortgage payments to income, both in the first year of the mortgage and averaged over the life of the mortgage. We have done this for the ARM, averaging across different levels of interest rates. For a couple, the payment on a $100,000 mortgage amounts to 36 percent of income in the first year and 16 percent of income on average over the life of the mortgage, while the payment on a $150,000 mortgage is 53 percent of income initially and 24 percent of income on average. For a single, these mortgages are more burdensome. A $100,000 mortgage costs 50 percent of income initially and 22 percent on average, while a $150,000 mortgage is an extreme case that costs 75 percent of income initially and 33 percent on average.

As a first step toward a welfare analysis, Figure IV plots the distribution of realized lifetime utility, based on simulation of the model across 1,000 households. Each household is assumed to have to finance a $150,000 mortgage on a $187,500 home using either an ARM, or an FRM with a $1,000 refinancing cost. In the top panel of the figure, the household has a couple's income, while in the bottom panel the household has a smaller single person's income. In both cases the lower standard deviation of income growth, 0.141, is assumed.

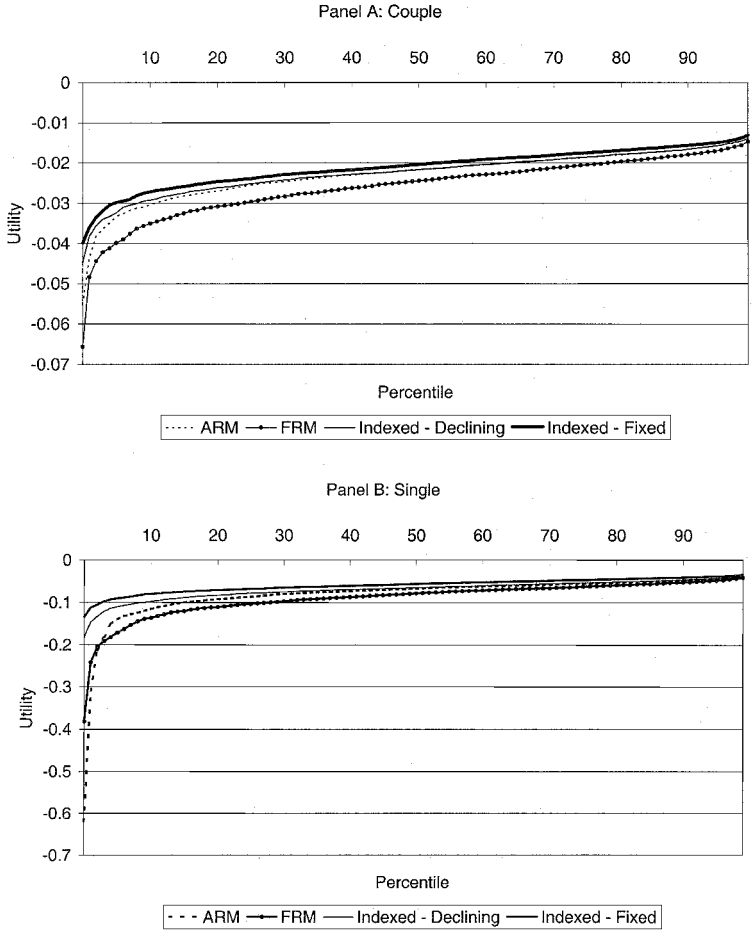


FIGURE IV
Benchmark Utility Distribution

This figure shows various percentiles of the distribution of realized utility when we simulate the model for 1000 households. Panel A shows the results for households composed of a couple, and Panel B for households composed of a single individual. The parameters of the model are given in Table I, with the size of the house that needs to be financed equal to $187,500. The figure illustrates utility for an ARM and a nominal FRM with refinancing cost of $1000, an inflation-indexed FRM whose real payments decline at the average rate of inflation, and an inflation-indexed FRM with fixed real mortgage payments.

Figure IV shows that ARMs have substantial advantages for most households. For couples, an ARM delivers higher utility than a nominal FRM everywhere in the utility distribution. For

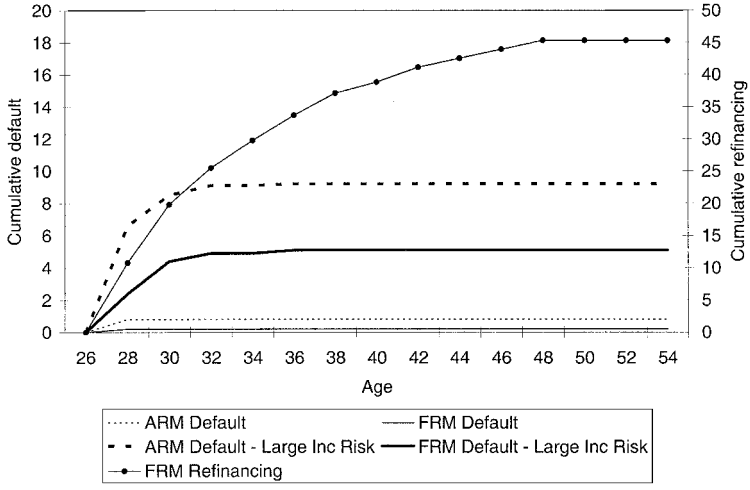


FIGURE V

Cumulative Default and Mortgage Refinancing

This figure shows the cumulative proportion of investors who choose to default under the FRM and ARM contracts for a household composed of a single individual and for two levels of labor income risk. The parameters of the model are given in Table I, with the size of the house that needs to be financed equal to $187,500. The figure also shows the cumulative proportion of households with low income risk that refinance the FRM contract.

singles, with lower income relative to house size, households in the upper part of the utility distribution are better off with an ARM, but a few households at the lower end of the distribution are substantially worse off. These results reflect the chief disadvantage of an ARM, the cash-flow risk that ARM payments will rise suddenly, exhausting buffer-stock savings and forcing an unpleasant cutback in consumption. This risk is important when the mortgage is large relative to income.

The cash-flow risk in ARM payments also implies that the proportion of households who choose to default on each loan tends to be higher under an ARM than under a FRM. Default rates are extremely low for couples, but Figure V plots cumulative default rates for singles with low income risk (dashed lines) and high income risk (solid lines), respectively. It is important to note that these default rates are obtained from simulating the behavior of households who differ in their history of shocks to interest rates, labor income and house prices. Households choose to default when faced with negative labor income shocks, so that buffer-stock savings become low, and with negative house price shocks,

so that home equity becomes negative. In a simulation scenario in which house prices and labor income shocks are mainly positive (negative), default rates are lower (higher). Figure V shows cumulative default over the life-cycle. Since the risk in mortgage payments is higher early in life when buffer-stock savings are smaller, default occurs mainly within the first eight years of the contract.

There are some differences in the circumstances that trigger default under each mortgage contract. While low labor income and house prices trigger default for both types of contract, households with ARMs choose to default when current interest rates and therefore current mortgage payments are high. They do so because default allows them to avoid paying down the principal of their mortgage, and this reduction in payments is particularly valuable when interest rates are high. Households with FRMs, on the other hand, choose to default when current interest rates are low and expected to rise. In these circumstances borrowing constraints are more severe under the FRM contract than in the rental market, because the FRM mortgage payment is based on the long-term interest rate while the rental payment is based on the current short-term interest rate.

Figure V also shows the cumulative refinancing of FRMs by singles with low income risk. The refinancing rate is slightly higher for singles with high income risk, and for couples, because these households accumulate larger savings and thus are more readily able to afford the $1,000 refinancing cost. Over the life of the mortgage, about 45 percent of households refinance their mortgages; almost all of this refinancing activity takes place within the first twenty years of the mortgage, since late refinancing reduces interest payments on a smaller principal balance for fewer years but incurs the same fixed cost as early refinancing. The timing of refinancing is somewhat sensitive to the constraint we have imposed, that homeowners must have positive home equity in order to refinance. If we relax this constraint, we get higher refinancing in the very early years of the mortgage, but the difference diminishes over time and is only about 1 percentage point after twelve years. This reflects the fact that house prices increase on average, while outstanding mortgage principal diminishes, so that very few households are likely to have persistently negative home equity.

Table II reports the average consumption growth rate and the standard deviation of consumption growth for households

TABLE II
CONSUMPTION GROWTH WITH NOMINAL MORTGAGES

	$\overline{\Delta c_t}$			$\sigma(\Delta c_t)$		
Refinancing	ARM	FRM Yes	FRM No	ARM	FRM Yes	FRM No
			Panel A: Couple			
$\bar{H} = 125.0$, $\sigma_\omega = .141$	1.5	1.5	1.5	12.1	13.6	13.7
$\bar{H} = 125.0$, $\sigma_\omega = .248$	2.2	2.2	2.2	17.1	18.6	18.7
$\bar{H} = 187.5$, $\sigma_\omega = .141$	2.0	2.0	1.9	13.9	15.6	15.9
$\bar{H} = 187.5$, $\sigma_\omega = .248$	2.8	2.8	2.8	19.4	20.7	20.9
			Panel B: Single			
$\bar{H} = 125.0$, $\sigma_\omega = .141$	1.9	1.9	1.8	13.5	15.2	15.5
$\bar{H} = 125.0$, $\sigma_\omega = .248$	2.8	2.8	2.8	17.6	18.8	19.0
$\bar{H} = 187.5$, $\sigma_\omega = .141$	3.0	2.9	2.8	17.9	19.5	20.0
$\bar{H} = 187.5$, $\sigma_\omega = .248$	4.1	4.1	4.1	22.4	24.2	24.7

This table shows average annual consumption growth, for goods other than housing, and the standard deviation of annual consumption growth under different mortgage contracts and for different parameter configurations. The data are obtained by simulating the model in Section II. Annual average consumption growth and the standard deviation of annual consumption growth are obtained by dividing the two-year values by two and square root of two, respectively. The FRM contract can allow for refinancing at a $1000 cost, or can prohibit refinancing. Panel A shows the results for households composed of a couple, and Panel B shows the results for households composed of a single individual.

with ARMs, nominal FRMs that allow refinancing, and nominal FRMs without a refinancing option. The top panel of the table is for a couple, while the bottom panel is for a single. Within each panel we consider a small or large house, and low or high income risk. Average consumption growth rates are very similar for all mortgages, since they depend largely on the hump-shaped profile of labor income in the presence of borrowing constraints. In one case with a large house relative to income, the average consumption growth rate is higher for an ARM, reflecting precautionary savings to guard against the cash-flow risk of the ARM. The form of the mortgage has a larger effect on the volatility of consumption growth. Volatility is lowest with an ARM, higher with a refinancing FRM, and highest with a nonrefinanceable FRM. These numbers reflect the dominance of wealth risk over income risk in determining consumption volatility over the life-cycle.

Table III summarizes the welfare implications of these numbers. The table reports the welfare provided by a FRM, with or without a refinancing option, relative to an ARM. We calculate welfare using a standard consumption-equivalent methodology. For each mortgage contract we compute the constant consump-

TABLE III
WELFARE ANALYSIS OF NOMINAL MORTGAGES

Refinancing	FRM: Yes	FRM: No	Refinancing option
		Panel A: Couple	
$\bar{H} = 125.0$, $\sigma_\omega = .141$	−6.34	−6.84	0.50
$\bar{H} = 125.0$, $\sigma_\omega = .248$	−5.72	−6.32	0.59
$\bar{H} = 187.5$, $\sigma_\omega = .141$	−5.96	−6.79	0.83
$\bar{H} = 187.5$, $\sigma_\omega = .248$	−5.40	−6.31	0.91
		Panel B: Single	
$\bar{H} = 125.0$, $\sigma_\omega = .141$	−5.77	−6.51	0.74
$\bar{H} = 125.0$, $\sigma_\omega = .248$	−5.43	−6.20	0.76
$\bar{H} = 187.5$, $\sigma_\omega = .141$	−5.71	−7.29	1.58
$\bar{H} = 187.5$, $\sigma_\omega = .248$	−1.03	−3.16	2.13

This table shows the mean welfare gain delivered by a nominal FRM, with and without refinancing at a \$1000 cost, relative to an ARM. The data are obtained by simulating the model in Section II. Welfare is reported in the form of standard consumption-equivalent variations. We weight the different states by the ergodic or steady-state distribution. For each mortgage contract we compute the constant consumption stream that makes the household as well off in expected utility terms. Utility losses are then obtained by measuring the percentage change in this equivalent consumption stream across mortgage contracts. Panel A shows the results for households composed of a couple, and Panel B shows the results for households composed of a single individual. The last column shows the value of the option to refinance obtained as the welfare difference for the refinancing and no refinancing versions of the FRM.

tion stream that makes the household as well off in expected utility terms, and then measure the percentage change in this equivalent consumption stream across mortgage contracts. In all the cases we consider, the ARM is the best available mortgage contract. The welfare consequences of this can be very large, reflecting the importance of the mortgage decision in the financial life of a household. If we consider a couple with low income risk and a large \$187,500 house as a benchmark case, the couple is 5.96 percent worse off with a nominal FRM that allows cheap refinancing, and 6.79 percent worse off with a FRM that prohibits refinancing.

The welfare advantage of the ARM diminishes when the house is large relative to income and when income is volatile. In the extreme case of a single with high income risk and a large house, the household is only 1.03 percent worse off with a refinanceable FRM than with an ARM. This reflects the fact that the cash-flow risk of an ARM is disproportionately more important when labor income is risky and the house is large relative to income.

By comparing welfare levels across FRMs with alternative refinancing costs, we can obtain the value of the option to refinance. In the benchmark case this option is worth 0.83 percent of consumption; it becomes more valuable when the house is large relative to income, and when income is risky. Note that these numbers assume a fixed FRM rate even while changing the cost of refinancing, and thus they do not impose a zero-profit condition on mortgage lenders.

All the numbers in Table III are averages across states of the world with high and low interest rates. We have also calculated expected utility conditional on an initial interest rate. As one would expect, the ARM is even more advantageous if the interest rate is initially low, since in this case the ARM has a lower cost in the early years when borrowing constraints are most severe. The FRM is more attractive if the interest rate is initially high; in the benchmark case a couple will slightly prefer an ARM even with a high initial interest rate, but a single will strongly prefer a FRM. Thus, our model implies that households' financing decisions should be sensitive to the slope of the term structure. As we discussed in the Introduction, the time-series behavior of U. S. mortgage financing does not match this prediction. However, our ability to explore this issue is limited by the fact that our discretized model allows only two possible levels of interest rates.

Using (16), we have calculated the average annual profit of lenders of funds under each mortgage contract, assuming that the bank borrows funds at the one-period riskless interest rate. In the benchmark case of a couple with low income risk and a large house, the average annual profit is \$745 for the ARM and \$1218 for a refinanceable FRM. The higher average profit on the FRM comes from the term premium that banks earn by borrowing short and lending long; of course, this term premium can be regarded as compensation for the risk that banks take when they mismatch the maturity of their borrowing and lending.

IV. Inflation-Indexed Mortgages

In this section we investigate the welfare properties of inflation-indexed mortgages. In principle, an inflation-indexed FRM can offer the wealth stability of an ARM, together with the income stability of an FRM; it should therefore be a superior vehicle for household risk management.

We consider inflation-indexed FRM contracts in which the

interest rate is fixed in real terms. We study the welfare properties of a standard inflation-indexed FRM contract, with constant real mortgage payments, and also those of an inflation-indexed mortgage whose real payments diminish at the average rate of inflation. We do so because our investor is borrowing constrained; one of the advantages of the standard inflation-indexed FRM contract, relative to the nominal FRM and ARM contracts, is that real payments are lower early in life, when borrowing constraints are more severe. This advantage of the standard inflation-indexed contract is conceptually distinct from the risk-sharing advantage of indexation. Thus, to obtain a pure measure of the risk-sharing advantage of indexation, we consider an inflation-indexed mortgage whose real payments diminish at the average rate of inflation.

If in period 1 household j chooses an inflation-indexed FRM with fixed real payments, and the current real interest rate on an inflation-indexed FRM contract with maturity T is R^I_{T1}, then in each subsequent period the household must make a real mortgage payment, M^I_{jt}, of

$$M^I_{jt} = \frac{(1 - \lambda) P^H_{j1} \overline{H_j}}{\sum_{j=1}^{T} (1 + R^I_{T,1})^{-j}}. \tag{22}$$

Real mortgage payments are fixed at mortgage initiation, and nominal payments increase in proportion to the price level P_t. Thus, unlike a nominal FRM, the real capital value of an inflation-indexed mortgage is not sensitive to inflation.

For the inflation-indexed mortgage contract, we ignore the possibility of refinancing. Given our assumption that real interest rate variation is transitory, the gains from refinancing in our model would be fairly small, and even a small monetary refinancing cost would prevent households from exercising their option. In reality, even with persistent real interest rates, the possibility of refinancing an inflation-indexed contract is likely to be only a minor feature of the contract, given the low volatility of the real interest rate compared with that of nominal yields.

We assume that the date t real interest rate on an inflation-indexed FRM is given by

$$R^I_{T-t+1,t} = R_{T-t+1,t} + \theta^I, \tag{23}$$

where θ^I is a constant mortgage premium over the yield on a $(T - t + 1)$-period real bond, $R_{T-t+1,t}$, which is determined by the

expectations theory of the term structure applied to log real interest rates. We assume that there is no log term premium for long-term real bonds; that is, that the real term structure is flat on average. This is consistent with the observed behavior of real yields on Treasury inflation-protected securities since their issue in 1997 [Roll 2003]. Since we do not allow for the possibility of refinancing the inflation-indexed FRM contract we set θ^I equal to the ARM premium of 1.7 percent. This premium compensates the mortgage lender for the initiation cost of the mortgage and for default risk.

In the inflation-indexed mortgage with real payments which diminish at the average rate of inflation, we have that

$$M_{jt}^D = M_{j,t-1}^D/(1 + \mu), \tag{24}$$

where M_{jt}^D is the date t real mortgage payment and μ is average inflation. The interest rate or internal rate of return for this mortgage contract is assumed to be equal to that for the standard inflation-indexed FRM.

The standard inflation-indexed mortgage with constant real payments eases the household's borrowing constraints. The real payments under this contract are lower than the required real payments on nominal mortgages early in life, when borrowing constraints are more severe. A measure of the degree to which investors are borrowing constrained is consumption growth. Table IV shows that in the benchmark case the average consumption growth rate under the inflation-indexed contract with constant real payments is only 0.8 percent compared with 2.0 percent with an ARM or a nominal FRM. Because the inflation-indexed mortgage allows households to remain in debt later in life, it increases the effect of income risk on consumption; thus, the standard deviation of consumption growth is actually higher with this contract than with an ARM.

The inflation-indexed mortgage with declining real payments has a much smaller effect on average consumption growth. There is some reduction in the average consumption growth rate for households with high income risk and large houses, reflecting the fact that known real mortgage payments require smaller buffer stocks and generate less precautionary saving than the random real payments required by ARMs. The major effect of an inflation-indexed mortgage with declining real payments is to reduce the volatility of consumption growth, since this mortgage eliminates

TABLE IV
CONSUMPTION GROWTH WITH INFLATION-INDEXED MORTGAGES

	$\overline{\Delta c_t}$			$\sigma(\Delta c_t)$		
	ARM	Inflation-indexed FRM		ARM	Inflation-indexed FRM	
		Constant	Declining		Constant	Declining
			Panel A: Couple			
$\bar{H} = 125.0, \sigma_\omega = .141$	1.5	0.8	1.5	12.1	11.7	12.0
$\bar{H} = 125.0, \sigma_\omega = .248$	2.2	1.5	2.1	17.1	17.4	18.2
$\bar{H} = 187.5, \sigma_\omega = .141$	2.0	0.8	1.9	13.9	12.9	14.4
$\bar{H} = 187.5, \sigma_\omega = .248$	2.8	1.6	2.6	19.4	18.5	19.1
			Panel B: Single			
$\bar{H} = 125.0, \sigma_\omega = .141$	1.9	0.8	1.8	13.5	12.5	13.5
$\bar{H} = 125.0, \sigma_\omega = .248$	2.8	1.7	2.7	17.6	15.9	17.6
$\bar{H} = 187.5, \sigma_\omega = .141$	2.9	0.8	2.6	17.9	14.7	17.5
$\bar{H} = 187.5, \sigma_\omega = .248$	4.1	1.9	3.7	22.4	18.4	22.2

This table shows average annual consumption growth, for goods other than housing, and the standard deviation of annual consumption growth under different mortgage contracts and for different parameter configurations. A standard ARM contract is compared with two alternative inflation-indexed FRMs, one with constant real payments and one with real payments that decline at the average rate of inflation. The data are obtained by simulating the model in Sections II and IV. Annual average consumption growth and the standard deviation of annual consumption growth are obtained by dividing the two-year values by two and square root of two, respectively. Panel A shows the results for households composed of a couple, and Panel B shows the results for households composed of a single individual.

both the wealth risk of the nominal FRM and the income risk of the ARM.

The inflation-indexed FRM also reduces default risk. For the parameter values we have considered, households with inflation-indexed mortgages never default. Average annual profits of lenders are \$1081 for the standard inflation-indexed mortgage, and \$971 for the mortgage with declining real payments. These profits are higher than those for the ARM. In equilibrium these lower default rates and higher profits might be translated into lower inflation-indexed mortgage premiums, which would further benefit households.

Figure IV plots the distribution of realized lifetime utility for households with inflation-indexed FRMs with constant or declining real payments. The figure shows that the welfare gains of inflation-indexed mortgages are substantial for both couples and singles. The gains are particularly large for households at the bottom of the welfare distribution, but there are benefits to households across the distribution. The inflation-indexed mortgage

TABLE V
WELFARE ANALYSIS OF INFLATION-INDEXED MORTGAGES

	Nominal FRM	Inflation-indexed FRM	
		Constant	Declining
		Panel A: Couple	
$\bar{H} = 125.0$, $\sigma_\omega = .141$	−6.34	1.40	0.12
$\bar{H} = 125.0$, $\sigma_\omega = .248$	−5.72	2.22	0.24
$\bar{H} = 187.5$, $\sigma_\omega = .141$	−5.96	3.95	0.91
$\bar{H} = 187.5$, $\sigma_\omega = .248$	−5.40	7.40	2.39
		Panel B: Single	
$\bar{H} = 125.0$, $\sigma_\omega = .141$	−5.77	3.49	0.67
$\bar{H} = 125.0$, $\sigma_\omega = .248$	−5.43	6.69	1.69
$\bar{H} = 187.5$, $\sigma_\omega = .141$	−5.71	15.86	7.30
$\bar{H} = 187.5$, $\sigma_\omega = .248$	−1.03	36.42	16.05

This table shows the mean welfare gain delivered by a nominal FRM and by an inflation-indexed FRM, with constant real payments and with real payments which diminish at the average rate of inflation, relative to an ARM. The data are obtained by simulating the model in Sections II and IV. Welfare is reported in the form of standard consumption-equivalent variations. We weight the different states by the ergodic or steady-state distribution. For each mortgage contract we compute the constant consumption stream that makes the household as well off in expected utility terms. Utility losses are then obtained by measuring the percentage change in this equivalent consumption stream across mortgage contracts. Panel A shows the results for households composed of a couple, and Panel B shows the results for households composed of a single individual.

with constant real payments is always superior to the mortgage with declining real payments.

Table V shows the average welfare gains of the inflation-indexed mortgages relative to the ARM in the form of standard consumption-equivalent variations. For ease of reference the earlier comparison of the nominal FRM with the ARM is repeated here. In the benchmark case of a couple with a $187,500 house and low income risk, the inflation-indexed mortgage with constant real payments offers a welfare gain over an ARM equivalent to 3.95 percent of consumption. The welfare gain increases with house size and with income risk, since ARMs are particularly problematic with large houses and risky income. In the extreme case of a single with a large house and high income risk, the welfare gain of inflation indexation exceeds 36 percent of consumption.

Comparing the two inflation-indexed contracts, we see that the average welfare gains of the contract with declining real mortgage payments are considerably smaller than those of the mortgage contract with fixed real payments, but they remain

positive in every case we consider. In the benchmark case Table V shows that households are on average 0.91 percent better off with a declining inflation-indexed FRM than with an ARM. Again the welfare gains increase dramatically with house size and income risk.

These results imply that, with substantial inflation risk of the sort we have estimated for the 1962–1999 period, the risk-sharing advantages of indexation are very large. Households would be able to manage their lifetime risks much more effectively if they had access to inflation-indexed mortgage contracts. Of course, these results depend on the parameters we have estimated. In the next section we assess the benefits of mortgage indexation for alternative parameterizations, including an income process in which nominal wages are temporarily sticky, and an interest-rate process characteristic of the United States in the recent period of declining inflation.

V. Alternative Parameterizations

When nominal wages are inflation-indexed, or equivalently when wages are fixed in real terms, the correlation between real labor income shocks and inflation shocks is zero. We have assumed this in our benchmark parameterization. However, in a world where implicit contracts tend to fix nominal, but not real wages, the correlation between inflation shocks and real labor income shocks is negative. In such a world households may demand nominal mortgage contracts because their wage contracts are nominal. To explore this coordination feature of nominal contracts, we compute the welfare benefits of inflation-indexed mortgages when nominal wages are sticky.

Table VI repeats the welfare comparison of nominal and inflation-indexed FRMs with ARMs for several alternative specifications. We consider the benchmark case of a couple with a large $187,500 house and low income risk. The first row of the table repeats the numbers from Table V for this case. The second row shows the average welfare gains of nominal and inflation-indexed FRMs for a negative correlation coefficient of -1 between temporary real labor income shocks and inflation innovations. In the presence of negative correlation, a nominal ARM becomes much less attractive because positive inflation shocks drive up nominal interest rates and increase mortgage payments at times when real labor income is temporarily low. A nominal FRM becomes

TABLE VI
WELFARE ANALYSIS UNDER ALTERNATIVE PARAMETERIZATIONS

	Nominal FRM	Inflation-indexed FRM	
		Constant	Declining
1. Benchmark	−5.96	3.95	0.91
2. Correlated income and inflation ($\varphi = -1.00$)	−2.87	5.16	1.81
3. 1983–1999 parameters	−6.94	1.01	0.02
4. Lower second loan premium ($\theta^B = 0.01$)	−6.40	1.57	0.97
5. Higher moving probability ($p = 0.10$)	−6.60	8.94	0.86
6. Lower rental premium ($\theta^Z = 0.02$)	−6.08	3.83	0.85
7. Higher rental premium ($\theta^Z = 0.04$)	−5.79	4.14	1.09
8. Higher time discount rate ($\beta = 0.90$)	−6.49	12.46	1.46
9. Higher risk aversion ($\gamma = 5$)	−1.33	8.12	4.58
10. More volatile permanent income ($\sigma_\eta = 0.05$)	−5.11	2.17	0.90
11. Hybrid ARM contract	−6.75	3.09	0.07
12. Correlated income and real rates	−5.90	4.02	0.97
13. Term premium in the real term structure	−5.96	−1.34	−3.72

This table shows the mean welfare gain delivered by a nominal FRM and by an inflation-indexed FRM, with constant real payments and with real payments which diminish at the average rate of inflation, relative to an ARM for different parameterizations. The data are obtained by simulating the model in Sections II and IV. The first row shows the results for our benchmark case, which is a couple needing to finance a house of $187,500 and facing labor income risk $\sigma_\omega = .141$. The second shows the results for a correlation between transitory labor income and inflation shocks equal to minus one. The third row shows the results when the parameters of the inflation and real interest rate processes are calibrated using data for the 1983–1999 period. The annual parameters are $\mu = .034$, $\sigma(\pi_{1t}) = .012$, $\phi = .412$, $\bar{r} = .031$, and $\sigma(r_{1t}) = .016$. In the fourth row we allow homeowners to take out second loans if they have positive home equity, with a second loan premium of 1 percent. The fifth row shows the results for a probability of moving equal to 10 percent. The sixth and seventh rows show the results for a lower and a higher rental premium in case of default. The eighth row shows the results for a higher time discount rate, corresponding to a lower time discount factor of 0.90. The ninth row shows the results for a higher coefficient of relative risk aversion. The tenth row shows the results for a higher standard deviation for permanent income shocks. The eleventh row shows the results for a hybrid ARM characterized by a 2 percent cap per year and a 6 percent lifetime cap. The twelfth row shows the results for a correlation of income growth and real interest rate shocks equal to −0.2. The last row shows the results assuming that the term premium of 1 percent is in the real term structure rather than the nominal term structure.

relatively more attractive, although for the benchmark case reported in Table VI it does not dominate the ARM.

Negative correlation makes an inflation-indexed mortgage less attractive relative to a nominal FRM. The welfare difference between the inflation-indexed FRM with declining real payments and the nominal FRM is 6.87 percent in the benchmark case with a zero correlation, but only 4.68 percent with a correlation of −1. However, the inflation-indexed mortgage becomes more attractive relative to the ARM, which is particularly disfavored by nominal wage stickiness.

While these results are qualitatively unsurprising, it is striking that temporary nominal wage stickiness does not reverse the welfare ordering we found in the previous section, that inflation-

indexed FRMs dominate ARMs, which in turn dominate nominal FRMs. To reverse that ordering, we would need to assume nominal stickiness in the permanent component of labor income, which would imply that inflation shocks permanently reduce real labor income. Such an assumption is much more extreme than the temporary nominal stickiness we consider here.

Clarida, Gali, and Gertler [2000] and Campbell and Viceira [2001] report considerably lower inflation risk during the period since 1983 in which Federal Reserve Chairmen Paul Volcker and Alan Greenspan have brought U. S. inflation under control. We now assess the benefits of mortgage indexation when we calibrate our interest-rate process to a process characteristic of the United States in the 1983–1999 period. For this period we find lower average inflation (3.4 percent as compared with 4.6 percent), less volatile inflation (a standard deviation of 1.2 percent as compared with 3.9 percent), less persistent inflation (an autoregressive parameter of 0.41 as compared with 0.75), a higher average real interest rate (3.1 percent as compared with 2.0 percent), and a less volatile real interest rate (a standard deviation of 1.6 percent as compared with 2.2 percent).

The third row of Table VI changes the interest-rate parameters to those we calibrate for the 1983–1999 period. The first column shows that nominal FRMs are less attractive relative to ARMs than was the case in our benchmark model. Evidently, the stabilization of inflation and interest rates has reduced the income risk of ARMs relatively more than the wealth risk of FRMs. The second and third columns show that inflation-indexed FRMs remain superior mortgage contracts, but the welfare gain is extremely small for the inflation-indexed FRM with declining real payments. It appears that the Volcker-Greenspan monetary policy has reduced the pure risk management advantages of inflation-indexed bonds to a low level.

We now study how allowing homeowners to take out second loans, if they have positive home equity, affects the benefits of mortgage indexation. The fourth row of Table VI shows the welfare gains for a second loan premium, θ^B, of 1 percent in annual terms. For tractability, in this case we eliminate the prepayment option on the nominal FRM. We see that the benefits of constant real payments are smaller when second loans are allowed, since these loans are an alternative way to relax the household's borrowing constraints. However, second loans do not entirely eliminate the income risk of ARMs, because low house prices may

coincide with low income and high inflation, in which case second loans are unavailable precisely when they would be most valuable. Thus, our basic results survive the addition of second loans to our model.

In the fifth row of Table VI we solve our model assuming a moving probability equal to 0.10, meaning that the household moves on average once every ten years. Recall that in all the cases reported in earlier tables, the probability of moving is equal to zero. We find that the welfare gain of an ARM over a nominal FRM is higher when the moving probability is higher. If a homeowner knows he is highly likely to move in the near future, he is more likely to use the kind of mortgage that has the lower current interest rate. On average, this is the ARM or the inflation-indexed FRM since the nominal FRM has a higher yield spread that reflects the slope of the nominal term structure.

Our results are sensitive to the assumption that we make about consumption in the event of a mortgage default. In the model we assume that in case of default the bank seizes the house and the household is forced into the rental market for the remainder of its life. We set the rental premium equal to the user cost of housing plus a constant rental premium, θ^R, which in the benchmark parameterization is equal to 3 percent. The sixth and seventh rows of Table VI consider lower values and a higher value for the rental premium of 2 percent and 4 percent, respectively. We interpret these as roughly capturing the effects of different default costs or exemption levels in the event of personal bankruptcy. Table VI shows that cheaper default makes a nominal FRM less attractive relative to an ARM, because it mitigates the income risk of the ARM. The benefits of FRM indexation are also reduced but remain substantial. Naturally, the cheaper is default the higher is the default rate.

These results suggest that in states or countries where bankruptcy is relatively cheaper one should observe, ceteris paribus, a higher proportion of households choosing ARMs. However, we do not adjust the ARM premium θ^A to compensate lenders for variations in the default rate caused by variations in the rental premium, and thus our results do not capture the full general equilibrium effect of the bankruptcy code.

In the eighth row of Table VI we consider impatient investors with a higher time discount rate and a correspondingly smaller time discount factor of 0.90. Such investors accumulate a smaller buffer stock of liquid financial assets, so they default more often

and are more affected by the wealth risk of nominal FRMs and the income risk of ARMs. They gain more from inflation-indexation, even if real payments decline over time. They particularly benefit from the postponed payments of an inflation-indexed FRM with constant real payments.

In the ninth row of the table, we increase the risk aversion coefficient from 3 to 5. This causes households to become more concerned about the income risk of ARMs. The welfare advantage of ARMs over nominal FRMs diminishes, and the benefits of inflation-indexation increase. Much of the gain from inflation-indexation comes from improved risk management, but these households also have a strong preference for smooth consumption so they benefit from declining real mortgage payments.

The tenth row of Table VI shows the results for a higher standard deviation of permanent income shocks equal to 5 percent. The results are qualitatively similar to those for the benchmark case. The main quantitative difference is a reduced benefit of an inflation-indexed mortgage with constant real payments. This change is explained by the fact that with risky permanent income, the option to default becomes more valuable for the ARM and nominal FRM contracts relative to the inflation-indexed mortgage with constant real payments. Once again, we do not adjust the ARM or FRM premium for variations in default caused by the change in the volatility of income, and thus our results reflect only a partial equilibrium, not a general equilibrium analysis.

In the eleventh row of Table VI, we consider a hybrid ARM in which there is a cap of 2 percent on the annual increase in the interest rate, and a lifetime cap of 6 percent on the cumulative interest rate increase after mortgage initiation. These terms are fairly standard ones for a hybrid ARM. The hybrid ARM is more attractive than either a straight ARM or a nominal FRM, as it mitigates income risk while still limiting wealth risk. The benefits of mortgage indexation are smaller in comparison to a hybrid ARM, and shrink to 7 basis points for an inflation-indexed mortgage with declining real payments.

In practice, ARMs often have more complicated terms including a low initial teaser rate. A teaser rate enables an ARM to capture some of the benefits of an inflation-indexed mortgage with constant real payments, but we do not attempt to capture the full richness of available ARM contracts here.

Our benchmark model assumes that shocks to income growth

are uncorrelated with shocks to real interest rates. If we assume instead that income growth is negatively correlated with real interest rates, this exacerbates the income risk of ARMs, since income will tend to be low precisely when interest rates are high and required ARM mortgage payments are high. The twelfth row of Table VI shows that with a correlation of -0.2 between income and real interest rates, nominal and inflation-indexed FRMs become slightly more attractive relative to ARMs.

The last row of Table VI assumes that the term premium of 1 percent applies to the real term structure as well as the nominal term structure. In this case the spread between long-term nominal and real interest rates is caused only by expected inflation and does not include an inflation risk premium. This increases the cost of an inflation-indexed mortgage relative to a nominal mortgage, and reduces the benefit of indexation. Under this assumption an ARM looks attractive as a way for a homeowner to avoid paying the term premium.

Finally, we consider an alternative specification in which we allow households who choose a nominal ARM to subsequently refinance into a nominal FRM. Recall that our baseline specification compares a nominal ARM to a nominal FRM, without allowing households to switch between the two. In practice, and even though there are transaction costs associated with switching between different types of mortgages, it is possible to do so. The complexity of our model prevents us from considering a period-by-period decision to switch mortgages. However, we can study the welfare effects of allowing a one-time switch from a nominal ARM to a nominal FRM. It may be the case that ARM borrowers find it optimal to choose the ARM when interest rates are low, but plan to switch to a FRM if and when interest rates increase.

The solution to this alternative specification requires that at each date t and for each combination of the state variables, we compare the utility of remaining an ARM borrower to the utility of switching to the FRM contract. More precisely, let $V_t(X_t; FRM)$ denote the lifetime utility of becoming an FRM borrower at date t, when the vector of state variables is given by X_t. Assuming a zero switching cost, the household will at date t switch to the FRM if and only if $V_t(X_t; FRM) > V_t(X_t; ARM)$, where $V_t(X_t; ARM)$ is the lifetime utility of remaining an ARM borrower with the option to switch to the FRM in a subsequent period.

To solve for the optimal mortgage choices under this alternative specification, we set the parameters equal to their bench-

mark values and the switching cost to zero. For these parameters most borrowers prefer an ARM and never switch to a FRM. There are 1.3 percent of households who start off with an ARM and later on switch to a FRM when current interest rates are high. However, not all households find it optimal to switch to the FRM when current rates are high; only those with low current income and financial wealth do so. The intuition for this result is simple: when current interest rates are high the ARM implies a larger current mortgage payment than the FRM. Those consumers who are more borrowing constrained find it optimal to pay the higher average premium on the FRM in exchange for the lower current mortgage payments. This result illustrates once more the importance of borrowing constraints for mortgage choice. As consumers grow older, the labor income profile becomes flatter, and households become less borrowing constrained. For this reason the benefits of switching to the FRM contract are lower. This explains our finding that for the baseline parameters all the switching from the ARM to the FRM takes place before age 38.

We also study the welfare effects of allowing consumers to switch from the ARM to the FRM. We compute the mean welfare gain delivered by the ARM with the option to switch to the FRM, relative to the baseline nominal ARM contract. We find a modest welfare gain of 0.27 percent, reflecting the small number of households that choose to make this switch.

VI. Conclusion

The problem of mortgage choice is both basic and complex. It is basic because almost every middle-class American faces this choice at least once in his or her life. It is complex because it involves many considerations that are at the frontier of finance theory: uncertainty in inflation and interest rates, borrowing constraints, illiquid assets, uninsurable risk in labor income, and the need to plan over a long horizon.

Despite the complexity of the problem, it is important for financial economists to try to offer scientifically grounded advice. If financial economists avoid the topic, homeowners may be guided by unwise commercial or journalistic advice; for example, they may be urged to time the bond market by predicting the direction of long-term interest rates. Mortgage choice should not be left to specialists in real estate, but should be treated as an

aspect of household risk management, a topic that lies at the heart of finance.

In this paper we have shown that the form of the mortgage contract can have large effects on household welfare. We begin by comparing pure forms of the standard nominal ARM and FRM contracts. FRM contracts expose households to wealth risk, while ARM contracts expose them to income risk: the risk that borrowing constraints will bind more severely when high interest rates coincide with low income and house prices. While the exact levels of welfare depend on the particular premiums we have assumed for ARM and FRM mortgages, we can draw general conclusions about the types of households that should be more likely to use ARMs. Households with smaller houses relative to income, more stable income, lower risk aversion, more lenient treatment in bankruptcy, and a higher probability of moving should be the households that find ARMs most attractive.

Interestingly, these results match quite well with empirical evidence reported by Shilling, Dhillon, and Sirmans [1987]. These authors look at micro data on mortgage borrowing and estimate a reduced-form econometric model of mortgage choice. They find that households with coborrowers and married couples (whose household income is presumably more stable) and households with a higher moving probability are more likely to use ARMs.

We have also investigated the welfare properties of innovative inflation-indexed mortgage contracts. An inflation-indexed FRM can offer the wealth stability of an ARM together with the income stability of an FRM, so it is a superior vehicle for household risk management. Using U. S. data from the period 1962–1999, we find very large welfare gains from the availability of an inflation-indexed mortgage contract. Some of these gains arise from the reduced mortgage payments early in the life of the mortgage that are implied by a constant real payment as opposed to a constant nominal payment. Even if we remove this advantage by requiring real payments that decline at the expected rate of inflation, we still find substantial welfare gains from indexation.

This finding raises the question of why inflation-indexed mortgages are not more commonly used. There are several possible answers to this question. First, the yield spreads between inflation-indexed Treasury securities and nominal Treasury securities have been extremely low, generally below 2 percent, since inflation-indexed Treasury securities were introduced in 1997 [Roll 2003]. This suggests that investors have a high degree of

confidence that inflation will remain low in the future, and that the inflation risk premium is low. When we calibrate our interest-rate model to the period since 1983 in which inflation has been relatively well controlled, or when we assume a zero inflation risk premium, we find smaller risk management benefits of indexation. Second, we find that hybrid ARMs with nominal interest rate caps can improve significantly over pure ARMs. These ARMs reduce the benefits of inflation-indexation.

Third, inflation-indexed mortgages can cause the outstanding principal of the mortgage to increase over time in nominal terms (although not in real terms). Financial advisers frequently warn against such "negative amortization," without making any distinction between real and nominal debt. Irwin [1996], for example, writes "Another trap has to do with negative amortization. Some of the adjustable loans keep the monthly payments down by adding the interest to the principal. In other words, you end up owing more than you borrowed!" Even more colorfully, Tyson and Brown [2000] write "Negative amortization has the potential to be a personal financial neutron bomb. It destroys the borrower without harming the property. If you're offered an ARM with negative amortization, emphatically say NO!"[9] Suspicion of negative amortization may have inhibited acceptance of innovative mortgage contracts that might reduce the income risk of standard ARMs.

Although our model captures many of the important factors that should influence mortgage choice, it remains oversimplified in several important respects. First, we have assumed that households remain in a house of fixed size unless they are randomly forced to move, in which case we evaluate their welfare using a terminal utility function. Second, related to this, we assume a relatively low volatility of permanent income growth in order to avoid large mismatches between household income and house size. Third, our model is too stylized to capture teaser rates and other special features of many mortgages that are offered in the marketplace. Fourth, we work in partial equilibrium and do not attempt to use zero-profit conditions for mortgage lenders to solve for equilibrium mortgage premiums. We believe that there is

9. Steinmetz [2002] offers a more nuanced view, stating that "Negative amortization is not inherently bad." However, he, like the other authors quoted here, does not try to distinguish the real value of a debt from its nominal value.

room for further research to model these issues in a more satisfactory manner.

We have calibrated our model to match basic features of the U. S. mortgage market. Other countries have mortgage markets that differ in important respects. For example, long-term nominal fixed-rate mortgages are almost unknown in the United Kingdom and Canada. An interesting area for future research will be to relate these international differences in prevailing mortgage contracts to differences in the risk management problem that households face.

The concept of income risk that we emphasize in this paper has interesting implications for other areas of finance. Corporations, for example, must consider the risk that short-term or floating-rate debt will require high interest payments in circumstances where internal cash flow and collateral are low and external financing is expensive. Here as in the problem of mortgage choice, borrowing constraints both complicate and enrich standard models of risk management.

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