Essays on Household Portfolio and Current Account Dynamics

by

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ABSTRACT

This dissertation studies the household behavior of consumption and portfolio choice, and current account dynamics. After a summary in Chapter 1, Chapter 2 develops a model of household portfolio choice between liquid and illiquid assets under income uncertainty. The model demonstrates that households optimally hold the majority of precautionary wealth in the illiquid form, even though they have to pay transaction costs in changing illiquid asset positions upon adverse shocks. I also show that the marginal propensity to consume out of illiquid wealth is much smaller than the MPC out of liquid wealth in the short run.

Chapter 3 puts the implications of the model to the test in the Panel Study of Income Dynamics. I examine the steady-state relationship between income uncertainty and illiquid portfolios. My estimation results confirm that households facing more income uncertainty hold more illiquid portfolios in the steady state. However, consistent with the fact that illiquid wealth is held for the precautionary reasons, I find that households actually reduce their illiquid asset holdings after adverse shocks.

In Chapter 4, which is a joint work with Prof. Christopher Carroll, we propose a new method to estimate the wealth effect on consumption. We argue that the recently popularized cointegration method is misguided since it relies on the unrealistic assumption that consumption, income, and wealth have a unique, linear relationship in the long-run. Alternatively, we argue that a model of habit forming

consumers is a good characterization of consumption dynamics. Using the habit model, we estimate that the immediate MPC out of wealth is 0.9 percent and the long-run MPC is 4 percent.

Finally, Chapter 5 applies the consumption theory to a model of current account dynamics. Simple intertemporal models of the current account commonly fail to match the volatility of actual movements. I extend the simple model to allow for the distinction between durable and non-durable goods and find that the extended model fits the actual data better for a small open economy such as Canada, but not for an large open economy such as the U.S.

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CHAPTER 1

Introduction

This dissertation is a collection of four essays on household consumption behavior. In particular, the first two essays analyze a household portfolio choice decision, presenting a theoretical model and testing the implications of the model in U.S. microeconomic data. The third essay studies the relationship between household wealth and consumption expenditures, proposing a new method to estimate the wealth effect on consumption. The last essay shows that a representative consumer's model can be used to derive the optimal current account movement and tests how well the model fits the actual data of current accounts in the U.S. and Canada.

In the second chapter, I lay out and solve a theoretical model of household portfolio choice under income uncertainty. To prepare for adverse income shocks in the future, households lower current consumption expenditures and hold precautionary wealth. This precautionary wealth may be held in several different forms, given various types of assets available to households. How households optimally allocate their precautionary wealth between different asset types, in particular, between liquid and illiquid assets, is the question to be studied in this chapter.

Most of households hold the bulk of their wealth in the illiquid form such as housing equity and retirement accounts. On average, illiquid assets may generate more returns or services to the owners of the assets than liquid assets. (For example, retirement accounts offer favor-

able tax treatment for the account holders than other liquid accounts.) However, due to the transaction costs associated with every change in illiquid asset positions, when adverse income shocks hit households, illiquid wealth would be less effective or more costly as a buffer against consumption falls than liquid wealth. Therefore, the optimal portfolio choice problem between liquid and illiquid assets for a precautionary household facing income uncertainty involves comparing the costs and benefits of holding one type of assets over the other.

Household portfolio choice problems have been mainly discussed in the finance literature, where the focus is the optimal allocation between risky and risk-free assets, typically an allocation between stocks and bonds. Given the fact that the majority of household wealth is held in the illiquid form, it is also important to understand the factors which determine a portfolio allocation decision between liquid and illiquid assets.

In the consumption/saving literature, there is substantial evidence that precautionary wealth is mostly held in the illiquid form. If one considers only the transaction costs in changing illiquid asset positions and ignores the benefits of holding illiquid assets such as higher returns, this empirical fact seems puzzling. However, there should exist the benefits as well as the costs in holding illiquid assets, as long as both liquid and illiquid assets are held by households. By solving a simple model of household portfolio choice, I demonstrate that only slightly higher returns on illiquid assets than liquid assets can lead a precautionary household to optimally hold a very illiquid portfolio.

I further investigate the relationships between the optimal portfolio and three different types of income uncertainty; Three shocks

considered are unemployment shocks (rarely occurring but disastrous shocks, not affecting permanent income), permanent income shocks (high-frequency and small shocks, affecting permanent income), and temporary income shocks (high-frequency and small shocks, not affecting permanent income). The nature of income uncertainty such as the size and frequency of shocks affects the calculation of the expected costs and benefits of holding illiquid assets over liquid assets, consequently, influences the optimal portfolio allocation between the two asset types. For example, against the unemployment risk, the expected benefits of enjoying slightly higher returns on illiquid assets easily dominate the expected costs of having to pay transaction costs upon receiving shocks, if unemployment shocks hit fairly infrequently. Hence, it makes sense that a household facing only the unemployment risk would want to hold a very illiquid portfolio. This point is confirmed by simulating the model under different assumptions on the nature of income uncertainty. The results show that illiquid assets are preferred as a buffer against unemployment risk and permanent income uncertainty while liquid assets are preferred as a buffer against temporary income uncertainty. Since most of precautionary wealth is held against the first two types of shocks, the optimal portfolio allocation can be quite illiquid.

I examine both the steady states and transitional dynamics of the model of household portfolio choice. In the steady state, a household facing more unemployment risk or permanent income uncertainty tends to hold a more illiquid portfolio, while a household facing more temporary income uncertainty tends to hold a more liquid portfolio. The transitional dynamics analysis concerns a case in which the unemployment risk jumps up unexpectedly and permanently. Eventually

each household would shift its portfolio toward a more illiquid one in the new, riskier environment. However, in the beginning of the transition, households may have more *liquid* portfolios since they increase liquid assets faster than illiquid assets in a rush of building up their precautionary savings.

Using the solutions of the model, I can calculate the marginal propensity to consume (MPC) out of liquid and illiquid wealth, separately. The computed immediate MPC out of liquid wealth is 25 percent which is larger than the immediate MPC out of illiquid wealth by about 60 percent. The larger MPC out of liquid assets than illiquid assets is consistent with intuition.

The third chapter put the model of household portfolio choice developed in the second chapter to the test. Namely, I test the implications of the theoretical model in the U.S. household data drawn from the Panel Survey of Income Dynamics.

First, I examine the steady state implication of the model, that is, whether a household facing more income uncertainty tends to hold not only larger illiquid wealth but also more illiquid portfolio.

Constructing a three-year panel of the household data, I find a significantly positive relationship between income uncertainty and illiquid portfolios as well as illiquid asset holdings. Hence, the findings corroborate what the theoretical model concludes — as a household face larger income uncertainty, she increases precautionary wealth more in the illiquid form than in the liquid form: a household facing the 80th centile level of income uncertainty holds 96 percent more illiquid wealth and 32 percent more illiquid portfolio than a household facing the 20th centile level of income uncertainty. I further try to decompose

income uncertainty into permanent and temporary components at the household level. When these two income uncertainty measures are used as explanatory variables, both permanent and temporary uncertainty indices are found to be positively related to illiquid portfolios; The permanent uncertainty measure has nearly twice as large an effect on illiquid portfolios as the temporary uncertainty measure. Considering that my measure of temporary income uncertainty may also reflect the probability of receiving rarely occurring but disastrous shocks such as unemployment spells, the empirical findings are consistent with what the theoretical model implies about the relationship between permanent income uncertainty, temporary income uncertainty, and illiquid portfolios in the steady state.

As long as illiquid wealth is held for the precautionary reason, households should actually liquidate some of their wealth to keep consumption from falling sharply when adverse shocks hit them. So, finding evidence that households indeed reduce their illiquid wealth temporarily upon adverse shocks supports my argument that precautionary wealth is actually illiquid. To see whether there is such evidence, I use a two-year panel of the same household data and construct the amount of active saving/liquidation of illiquid assets for each household. The estimation result shows that adverse income shocks seem to induce significant amounts of liquidation of illiquid assets within one year.

In sum, the household data from the PSID produce results consistent with the theoretical model's prediction on the relationships between income uncertainty and illiquid asset holdings. Households facing income uncertainty hold the majority of precautionary wealth in the illiquid form in the steady state; They reduce some of illiquid assets in

times of drawing severely adverse income shocks.

In the fourth chapter, which is a joint work with Prof. Christopher Carroll, we empirically investigate the link between household consumption expenditures and wealth. After reviewing the currently popularized method to estimate the wealth effect on consumption, we propose a new, simple method to estimate the wealth effect on consumption in the U.S. macro data.

Recently, parallel to the developments of time-series econometrics techniques, the method to estimate the wealth effect on consumption in a cointegrating regression framework — call it the "VECM" approach — has become widely used. However, this method crucially relies on the unrealistic assumption that consumption, income, and wealth have a unique, linear relationship to which these three variables revert in the long run. Permanent changes in parameter values underlying the economy, such as the steady state interest rate, productivity growth rate, population growth rate, and tax rate, would disrupt the assumed linear relationship between the three variables.

Furthermore, the VECM approach ignores the well-known empirical feature of consumption dynamics: consumption growth does not jump with respect to innovations in income and wealth, rather adjusts gradually to the innovations over time. Our alternative approach to estimate the wealth effect on consumption is based on a theoretical model of habit forming consumers, which can explain such lagged adjustments of consumption growth to past information.

According to the habit forming model, consumption growth is serially correlated. A initial innovation to consumption growth is, thus, carried over into future consumption with its effect decaying gradu-

ally. We estimate this consumption growth serial correlation coefficient caused by the habit forming behavior and use the estimate to obtain the marginal propensity to consume out of a wealth shock. In the estimation process, we also test whether our habit forming model is a reasonable characterization of consumption dynamics.

When the degree of habit formation is strong, the initial response of consumption with respect to shocks is substantially reduced as a habit forming consumer tries to distribute the effect of the shocks over many periods. We estimate that the immediate MPC out of a wealth shock is around 0.9 percent. The long-run MPC out of a wealth shock is found to be more than four times as large as the immediate effect around 4 percent. Our estimate is within the range of the wealth effect values obtained in the existent literature.

The final chapter is an application of the consumption theory to an international model. The current account balance is, by identity, a difference between saving and investment at the national level. To the extent that we can use a representative agent model to explain consumption movements, equivalently saving movements, we can theoretically explain the current account movements. Models of the current account based on the consumption theory is called an intertemporal approach.

When the intertemporal model is brought to the actual current account data in its simplest form, it commonly underpredicts the volatility of current account movements for small open economies. The simple intertemporal model takes into consideration only exogenous shocks to output. The failure to match the actual volatility of current accounts may suggest that the simple model misses some other important shocks

which also drive the current account movements. Bergin and Sheffrin (1999) extend the simple model to allow for shocks to the real exchange rates, introducing the distinction between tradable and non-tradable goods. They argue that the extension can improve the model's ability to fit the actual current account data for several small open economies.

I propose another extension to allow for shocks to the relative prices between durable and non-durable consumption goods. Since expenditures on durable goods fluctuate more than those on nondurable goods, I expect that the model with the distinction between durable and non-durable goods can generate more dynamics driving the current account fluctuations. I test my extended intertemporal model of the current account in the U.S. and Canadian data. As expected, the introduction of durable goods imparts more volatility to the theoretical current account movements. For Canada, the extended model fits the actual current account movements significantly better than the simple model. However, for the U.S., the extended model does no better job in matching the data than the simple model and generates even excess volatility in the current accounts. Actually, the simple model already slightly overpredicts the actual volatility of the current accounts. This result is consistent with a fact that the model is suitable for a small open economy such as Canada, but not for a large open economy such as the U.S.

CHAPTER 2

Household Portfolio Choice with Illiquid Assets

2.1 Introduction

Household portfolio choice behavior has been extensively studied in the finance literature. Recently an increasing number of studies point out that income uncertainty plays an important role in determining the optimal portfolio allocation. However, analysis has been predominantly on the effect of income uncertainty on the optimal portfolio allocation between risky and risk-free assets, such as the allocation between stocks and bonds. In reality, for a typical household most of stocks and bonds are held in quasi-illiquid retirement accounts. In contrast to the rich literature in finance on portfolio allocation between risky and risk-free assets, there is no thorough theoretical work on household portfolio choice behavior with liquid and illiquid assets. This paper provides a theoretical model to analyze the effect of income uncertainty on the optimal portfolio allocation between liquid and illiquid assets.

Households hold the majority of their wealth in illiquid forms. The largest component of wealth is typically housing equity. Even those who do not have their own housing are likely to hold their retirement savings in quasi-illiquid forms, such as Individual Retirement Accounts (IRAs) or 401(k) plans. Illiquid assets are by definition more costly to convert into consumption than liquid assets since they require transaction costs (eg. time spent in finding a buyer, or monetary costs of processing a sale). Therefore, households are reluctant to change

their illiquid asset positions once they invest into these assets. At the same time, households also hold liquid assets in order to smooth consumption against small income fluctuations. However, when they are hit by a series of adverse shocks and liquid assets holdings are too low to absorb the shocks, they can liquidate all or a part of their illiquid assets to support consumption.

In the consumption/saving literature, empirical work has been done on measuring the precautionary effect of income uncertainty on household wealth. Estimation results on the size of precautionary wealth vary across studies and seem to be sensitive to the definition of wealth used for analysis. When narrowly defined wealth consisting of only liquid assets is used in estimation, the estimated effect of uncertainty on wealth is found to be small or insignificant. On the other hand, when broadly defined wealth including not only liquid assets but also illiquid assets is used, the precautionary effect is estimated to be larger and more significant. (Carroll and Samwick (1995, 1998) and Carroll, Dynan, and Krane (2003))

The finding that precautionary wealth is held in the illiquid form rather than in the liquid form might seem puzzling at first, particularly since liquid assets are easier and less costly to convert into consumption hence seem more effective as a buffer against income shocks. However, my theoretical model demonstrates that it is not necessarily a puzzle when illiquid assets generate more returns/benefits than liquid assets as long as they are held, and when the occurrence of seriously adverse income shocks is a fairly rare event. A good example of illiquid assets is Individual Retirement Accounts (IRAs) and 401(k) plans. The important features of these illiquid assets include tax-deductable con-

tributions, tax-free accrual of interest (benefits), and penalty on early withdrawal (costs).[1] I show that households might prefer to hold a very illiquid portfolio when the benefits of enjoying higher returns dominate the expected costs of being forced to liquidate, that is, when there is a small probability of drawing a large adverse income shock.

The key determinants of the optimal portfolio choice are factors that affect the cost-benefit calculation of holding two types of assets. The characteristics of income uncertainty such as the size of shocks, frequency, and persistence are all important and relevant factors for determining the optimal portfolio allocation. My model includes three types of income shocks: large but rare transitory adverse income shocks, relatively small temporary income shocks, and permanent income shocks. By simulating the model, I show how the optimal portfolio choice responds to each type of income uncertainty.

The consumption function obtained by solving the model can be used to compare the marginal propensity to consume out of liquid resources and out of illiquid resources. On average the MPC out of liquid resources is found 60 percent larger than the MPC out of illiquid resources. This finding implies that the wealth effect caused by an illiquid assets value appreciation could be low compared to the income effect on consumption.

[1] IRAs were first introduced in 1974 in the U.S. only for individuals without access to a pension. In 1981 eligibility was expanded to include nearly all individuals, and total IRA contributions increased greatly until eligibility for IRAs was again restricted by the Tax Reform Act of 1986. In 1986 the contributions to IRAs (about $ 38 billion) accounted for 30 percent of the aggregate personal saving. 401(k) plans were first authorized in 1974 in the U.S. Since the Department of Treasury clarified the rules for the use in 1981, 401(k) plans have grown popular, partly replacing IRAs after 1986. By 1996 the contributions were $ 104 billion. There exist similar saving incentive plans in other countries, such as Registered Retirement Saving Plan in Canada and Personal Equity Plans in U.K.

The portfolio choice decision with illiquid assets is partially irreversible since households have to incur transaction costs whenever they change illiquid asset positions. The theory on investment under uncertainty predicts that investment is postponed when future profits become more uncertain.[2] This point also apples to the portfolio allocation decision in investing into illiquid assets. I conduct an experiment in which there is a one-time rise in income uncertainty and derive the transitional dynamics towards the new steady state consistent with the higher income risk. I find that in the short run a portfolio allocation becomes more liquid since accumulation of illiquid assets takes longer than that of liquid assets, though in the long run the optimal portfolio with the higher risk is more illiquid.

The rest of the paper is organized as follows. The next section reviews the literature related to this paper. Section 2.3 describes the baseline model of portfolio choice between liquid and illiquid assets. The characteristics of the solution of the model are discussed in Section 2.4. The model is simulated and the steady state results are presented in Section 2.5. I compare the marginal propensities to consume out of two types of assets in Section 2.6. The dynamic adjustment of portfolio allocation with respect to a rise in income risk is examined in Section 2.7. Finally, Section 2.8 summarizes and discusses future research ideas.

2.2 Literature Review

Many studies have tried to measure the precautionary effect of income uncertainty on household consumption/saving behavior. The estimation results, however, are mixed: Some find that a significant

[2]see Dixit and Pindyck (1994)

increase in saving is associated with higher income risk (Carroll and Samwick (1997,1998), Engen and Gruber (1997), and Lusardi (1997,1998)) while others argue that higher uncertainty doesn't induce significant precautionary saving (Dynan (1993) and Guiso, Jappelli, and Terlizzese (1992)). The mixed findings are due to the differences in methods to construct an uncertainty measure, estimation techniques, data sets used, and definitions of wealth. On the last point, several studies suggest that a bulk of precautionary wealth might exist in the illiquid form rather than in the liquid form. For example, Carroll, Dynan, and Krane (2003) estimate the effects of job loss risk on household wealth and find that a strong precautionary response shows up in total wealth including both liquid and illiquid assets but that no significant response is observed in a liquid part of wealth.[3]

There is other evidence that illiquid assets might be held as a precautionary buffer.[4] Hurst and Stafford (2002) argue that housing equity, a major component of household illiquid wealth, is effectively used as a buffer in income downturns even though accessing home equity is quite costly. They present a theoretical model with housing equity and show that households short of liquidity are likely to tap into home equity when they are hit by adverse income shocks. The implication of their model is further tested with the 1996 PSID data set; they confirm that households actually liquidate housing assets, paying expensive transaction costs, when they need to smooth consumption against in-

[3]Similarly, Carroll and Samwick (1997,1998) find that the precautionary effect on household wealth is significant only when a comprehensive definition of wealth including illiquid assets is used.

[4]Illiquid assets might be held for other reasons than a precautionary motive. For example, Laibson (1997) argues that illiquid assets might be held by precautious households as a commitment device to force a future self to accumulate the desirable amount of saving.

come shocks. Although their model has both illiquid and liquid assets, the amount of illiquid assets (home equities) is treated as exogenous, hence their model cannot be used to analyze the endogenous portfolio choice between illiquid and liquid assets. Skinner (1996) also finds a similar result that households consider housing wealth as consumable resources to rely on in case they draw adverse income shocks.

In the finance literature, a growing number of studies focus on a portfolio choice problem under uncertain labor income. (Cocco (2000), Heaton and Lucas (1997), Guiso et al. (1996), and Vissing-Jorgensen(2000)) They use an extended version of the standard stochastic household consumption/saving model which allows for multiple asset types. However, they are interested in portfolio allocation between risky assets (stocks) and risk-free assets (bonds) and analyze whether introduction of income uncertainty can explain the well-documented under-holdings of risky assets in household financial portfolios. They argue that income uncertainty plays an important role in reducing the optimal share of risky assets in financial portfolios. [5] This paper takes up another portfolio choice problem, the allocation between liquid and illiquid assets, and shows that income uncertainty is also important in determining the optimal portfolio in this dimension.

2.3 A Model of Household Portfolio Choice

In this section, I present a simple model of the household who maximizes expected lifetime utility, given a stochastic labor income process, allocating her savings between two types of assets: liquid as-

[5]The level of income uncertainty is commonly referred as "background risks" in the literature.

sets and illiquid assets. While illiquid assets generate higher returns than liquid assets, they require transaction costs whenever the household wants to change positions of illiquid assets holdings. When the household chooses the optimal portfolio, she tries to balance the expected costs against benefits of holding the two types of assets under income uncertainty. This cost-benefit calculation in choosing the optimal portfolio depends on not only parameters characterizing the asset types, such as the wedge between two asset returns and the amount of transaction costs, but also the household's preference parameters and the characteristics of income uncertainty that the household assumes. For example, if income shocks are of high frequency, causing the household to liquidate the accumulated illiquid assets quite often, the expected costs of holding illiquid assets increase and the optimal portfolio becomes less illiquid. On the contrary, if income shocks are of low frequency, leading to only a small probability of liquidation, the expected costs of holding illiquid assets decreases and the optimal portfolio would be more illiquid. Even though the household is exposed to a very large adverse income shock such as a spell of unemployment, if the occurrence of such a disastrous event is fairly rare, the expected benefits of holding illiquid assets might outweigh the expected costs of being forced to liquidate with a very small probability. To see how the characteristics of income uncertainty affect the optimal portfolio allocation, I consider three types of income shocks: large adverse shocks which occur with a very small probability, relatively small temporary shocks which arise with high frequency, and permanent shocks which also arise with high frequency.

Consider the household at the beginning of period t who holds

an initial portfolio of liquid assets (L_t) and illiquid assets (K_t). In each period, she chooses how much to consume (C_t) and what fraction of illiquid assets to liquidate or what fraction of liquid resources to invest into illiquid assets. Let ΔK_t denote the amount of accumulation in illiquid assets in period t. When ΔK_t is negative, the household is liquidating and drawing down her illiquid assets holdings. When ΔK_t is positive, she is investing into illiquid assets. Because of the liquidity constraint, the household would need to liquidate some of the accumulated illiquid wealth in order to support consumption when an adverse income shock hits and the initial amount of liquid assets holdings is too low to absorb the shock. In both accumulating and divesting illiquid assets, the household is required to pay transaction costs ($I(\Delta K_t)$) out of liquidity. The transaction costs function $I(\cdot)$ takes zero only when $\Delta K_t = 0$ and takes a positive value otherwise. Two assets are different not only in terms of transaction costs but also in terms of rates of return. Liquid assets earn a net interest rate of r^L per period while illiquid assets earn a net interest rate of r^K per period. I assume that $r^K > r^L$ so that holding illiquid assets is more profitable than holding liquid assets if the household can avoid liquidation of the accumulated illiquid wealth.[6]

In each period, the household receives labor income Y_t which is decomposed into two multiplicative components: a permanent component of income P_t and a temporary income shock Ψ_t.

$$Y_t = P_t \cdot \Psi_t \tag{2.1}$$

[6]In a general equilibrium setting, the higher returns for illiquid assets could be endogenously generated as the equilibrium outcome in which the household holds both assets since the household needs to be compensated for the transaction costs.

To introduce a large but rare adverse shock, I assume that the temporary shock to income Ψ_t takes the value of zero with a very small probability of p.[7] With a probability of $(1-p)$, Ψ_t takes a strictly positive value and is a lognormally distributed white noise with the standard deviation of σ_Ψ and the mean of one. (i.e. $E[\Psi_t] = 1$)

$$\Psi_t = \begin{cases} 0 & \text{with a probability of p} \\ \sim lognormal(\mu_\Psi, \sigma_\Psi) & \text{with a probability of 1-p} \end{cases} \tag{2.2}$$

On the other hand, the permanent component of income P_t grows at the deterministic gross rate of G and is subject to a permanent shock to income Π_t.

$$P_t = G \cdot P_{t-1} \cdot \Pi_t \tag{2.3}$$

where the permanent shock to income Π_t is a lognormally distributed white noise with the mean value of one and the standard deviation of σ_Π. [8] Hence, in total there are three parameters which characterize the distribution of income shocks: p controlling the small probability of large adverse shocks, σ_Ψ controlling the uncertainty level of relatively small and frequent temporary shocks, and σ_Π controlling the uncertainty level of permanent shocks. There is no other source of uncertainty such as life expectancy or health risks in my model. The rates of return for both liquid and illiquid assets are assumed to be constant. I briefly mention the possibility of extending the base model to incorporate other uncertainty in the last section.

[7] When the household is hit by a zero Ψ_t event, her current income Y_t is zero regardless of the values of the permanent income P_t.

[8] Due to the existence of the temporary shock, the growth rate of labor income exhibits a negative serial correlation, which is a requirement for saving to be procyclical. (see Deaton (1991))

The household's optimization problem at the beginning of period t is shown below.

$$\begin{aligned} V(L_t, K_t, P_t) &= \max_{\{C_s, \Delta K_s\}_{s=t,\cdots,\infty}} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right] \\ &= \max_{\{C_t, \Delta K_t\}} u(C_t) + \beta E_t \left[V(L_{t+1}, K_{t+1}, P_{t+1}) \right] \end{aligned} \quad (2.4)$$

subject to

$$L_{t+1} = (1 + r^L)(L_t - \Delta K_t - I(\Delta K_t) - C_t) + Y_{t+1} \quad (2.5)$$

$$K_{t+1} = (1 + r^K)(K_t + \Delta K_t) \quad (2.6)$$

$$0 < C_t <= L_t - \Delta K_t - I(\Delta K_t) \quad (2.7)$$

$$-K_t <= \Delta K_t <= L_t \quad (2.8)$$

The model is an extension of the standard buffer-stock saving model developed by Deaton (1991) and Carroll (1992) to incorporate multiple types of assets as saving instruments. All variables are in real terms. V is the value function; β is the discount factor taking a value between 0 and 1; E_t is the rational expectation operator conditioned on all information available at the beginning of period t. The total liquid resources at the beginning of period t (L_t), which are the sum of current liquid assets holdings carried from the last period and current labor income, is conventionally called "liquidity-on-hand".

Equation (2.5) and (2.6) describe the evolutions for each type of assets holdings. The amount in the first bracket in Equation (2.5), $L_t - \Delta K_t - I(\Delta K_t) - C_t$, is liquid saving, which is the amount of liquid resources left after obtaining extra liquidity by tapping into illiquid wealth or taking out extra liquidity by investing into illiquid assets

$(-\Delta K_t)$, paying the required transaction costs $(-I(\Delta K_t))$, and spending on consumption $(-C_t)$. The corresponding amount in Equation (2.6), $K_t + \Delta K_t$, is illiquid saving. Liquidation ($\Delta K_t < 0$) reduces illiquid saving and investment ($\Delta K_t > 0$) adds to illiquid saving.

Liquidity constraints are represented by two equations (2.7) and (2.8). The former stipulates that liquid saving $(L_t - \Delta K_t - I(\Delta K_t) - C_t)$ must be positive, prohibiting the household from borrowing liquidity at the net interest rate of r^L. The second constraint limits the amount of liquidation or investment: the first inequality sets the maximum amount of liquidation to the current size of illiquid assets holdings while the second inequality sets the maximum amount of investment to the current size of liquid assets holdings. In short the household cannot invest nor liquidate more than what she currently owns.[9]

I assume that the time-separable utility function is of the constant relative risk aversion (CRRA) form.

$$u(C_t) = \frac{C_t^{1-\rho}}{1-\rho} \tag{2.9}$$

where ρ is the coefficient of relative risk aversion. The CRRA preference exhibits prudence [10] hence induces precautionary behavior in the

[9] In other words there is no short-selling of either asset. The existence of liquidity constraints is essential in the model of portfolio choice between liquid and illiquid assets. Without the constraints, the household would always prefer illiquid assets because she could avoid paying transaction costs by borrowing liquidity whenever needed. Only when the household is liquidity-constrained, does the portfolio choice decision become non-trivial with a serious tension between two types of assets.

[10] Technically, prudence is equivalent to a positive third derivative of a utility function, which guarantees that the marginal utility function is convex. The degree of precautionary saving is controlled by this convexity of the marginal utility function which in turn depends on the value of ρ for the CRRA utility. Furthermore, the liquidity constraint reinforces the degree of precautionary saving. (see Deaton (1991))

presence of uncertainty.[11] The choice of a CRRA form has another advantage of reducing the number of state variables in the original model from three to two by allowing me to rewrite the model in terms of ratios to permanent income P_t. For example, the normalized consumption at period t is $c_t = C_t/P_t$. The normalized version of the model becomes

$$\begin{aligned} v(l_t, k_t) &= \max_{\{c_s, \Delta k_s\}_{s=t,\cdots,\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \\ &= \max_{\{c_t, \Delta k_t\}} u(c_t) + \beta E_t \left[(G\Pi_{t+1})^{1-\rho} v(l_{t+1}, k_{t+1}) \right] \end{aligned} \tag{2.10}$$

subject to

$$l_{t+1} = \frac{(1+r^L)}{G\Pi_{t+1}}(l_t - \Delta k_t - I(\Delta k_t) - c_t) + \Psi_{t+1} \tag{2.11}$$

$$k_{t+1} = \frac{(1+r^K)}{G\Pi_{t+1}}(k_t + \Delta k_t) \tag{2.12}$$

$$0 < c_t <= l_t - \Delta k_t - I(\Delta k_t) \tag{2.13}$$

$$-k_t <= \Delta k_t <= l_t \tag{2.14}$$

where the lower case variables are the normalized values of the corresponding upper case variables.

[11] Gale and Scholz (1994) presents the model of IRAs and other saving for a household which lives for three periods. They use the quadratic utility function, hence their model cannot address the question how the precautionary motive affects IRA choices.

2.4 Numerical Solution for the Optimal Portfolio Allocation

2.4.1 Parameter Values

Baseline parameter values used for the model are shown in Table 2.1.[12] The parameters of particular interest are the ones controlling the stochastic labor income process: σ_Ψ (the standard deviation of temporary income shock), σ_Π (the standard deviation of permanent income shock), and p (the probability of the zero income event). An increase in any of these three parameters means more uncertainty for future income and should affect precautionary saving behavior. I solve the model under different sets of these three uncertainty parameter values and compare the results.

Preference parameters such as ρ and β, the constant real interest rate for liquid assets r^L, and the constant growth rate of permanent income G are set to the values within the range commonly assumed in the literature.

I specify the form of the transaction costs function $I(\Delta K_t)$ as follows,

$$I(\Delta K_t) = \begin{cases} \chi \cdot |\Delta K_t| + F & if\ \Delta K_t < 0\,(liquidating) \\ 0 & if\ \Delta K_t = 0 \\ F & if\ \Delta K_t < 0\,(investing) \end{cases} \tag{2.15}$$

Transaction costs consist of two parts: a fixed component (F), which

[12] To make sure that the problem has a converged solution, a condition has to be imposed on the parameter values. Namely, the parameter values need to satisfy the following inequality; $r^L - \delta < \rho(g - 0.5(1+\rho)\sigma_\Pi^2)$. δ is the discount rate.($\beta = 1/(1+\delta)$) This condition basically means that the household is impatient enough not to accumulate saving forever.

is paid whenever an amount of illiquid assets holdings is changed, and a variable component, which is paid only upon liquidation and proportional to the size of adjustment. [13] The fixed component F captures the costs of preparing documents, the costs of opening an account in the case of IRAs, and the psychological decision-making costs while the variable component captures the difficulty in selling illiquid assets in the secondary market or penalty on early withdrawal in the case of IRAs. For the fixed cost (F), I assume a small value of 0.001 since it mostly reflects the phycological decision costs. I choose 10 percent for the proportional rate in liquidation costs (χ) since this number matches the early withdrawal penalty for IRAs.

The constant real interest rate for illiquid assets r^K is set to 4 percent (twice as large as the return for liquid assets), which is lower than the average rate of returns on stocks and bonds typically held in IRAs. However, I think that it is reasonable to assume this lower value since the return for illiquid assets is risk-free in my model, as opposed to the high volatility of stock returns in general.

2.4.2 Numerical Solution

The model is solved by numerical dynamic programming for two main policy functions: The consumption function $c(l_t, k_t)$ and the liquidation/investment function $\Delta k(l_t, k_t)$. Each period of the maximization problem is divided into two sub-stages. In the first stage, the household chooses whether she wants to get more liquidity by taping into some of the accumulated illiquid wealth or to shift extra liquidity into

[13] Cocco (2000) models the transaction costs of selling a house, which he assumes are 8 percent of the unit being exchanged. However, he doesn't include fixed costs.

the illiquid form by investing. (i.e. the household chooses $\Delta k(l_t, k_t)$.) Then, at the end of the first stage the adjusted amount of "liquidity-on-hand" ($l_t - \Delta k(l_t, k_t) - I(\Delta k(l_t, k_t)) = x_t$) realizes. In the second stage, the problem reduces to a simple consumption/saving choice problem, given the adjusted "liquidity-on-hand" x_t. The household chooses how much to consume out of the liquidity without violating the liquidity constraint. (i.e. the household chooses $c(l_t, k_t)$ subject to $c_t \leq x_t$.) Further technical details of the solution method are contained in Appendix.

Two policy functions, $c(l_t, k_t)$ and $\Delta k(l_t, k_t)$, represent the optimal responses of the household in period t given the beginning-of-period portfolio position, l_t and k_t.

The consumption function has a general shape shown in Figure 2.1. It is concave and increasing in both liquid (l_t) and illiquid resources (k_t). The slope of the consumption function corresponds to the marginal propensity to consume (MPC) out of liquid resources in the direction of l [14] and the MPC out of illiquid resources in the direction of k. Figure 2.3 shows the consumption function along l_t holding k_t at 0 and the same consumption function along k_t holding l_t at 0. Both functions are below the 45 degree line, which implies that the household always makes a positive amount of saving in either liquid or illiquid assets.[15] When I compare the two consumption curves in Figure 2.3, the MPC out of liquid resources seems slightly larger than the MPC out of illiquid resources. However, this figure doesn't carry much information

[14] The MPC out of liquid resources is actually equivalent to how much consumption responds to a temporary income shock Ψ_t since $l_t = ((1 + r^L)/(G\Pi_t))s_{t-1} + \Psi_t$ where $s_{t-1} = l_{t-1} - \Delta k_{t-1} - I(\Delta k_{t-1}) - c_{t-1}$. (i.e. $\partial c/\partial l_t = \partial c/\partial \Psi_t$)

[15] More precisely, the consumption function at $l_t = 0$, $c(0, k_t)$, should be below the $(1-\chi)k_t - F$ line if there is a strictly positive amount of liquid or illiquid saving.

about the actual MPCs observed for households since these curves are conditioned on either $l_t = 0$ or $k_t = 0$ and the household might hold non-zero amounts of both assets for most of time. The household would hold a varying combination of l_t, k_t given a stochastic income process and the actual MPCs would vary with the portfolio position. I show another two-dimensional consumption functions (Figure 2.4) which are conditioned on the average realization of l_t or k_t[16], not on "$l_t = 0$ or $k_t = 0$". In this figure, we can see a much clearer difference in the MPCs out of two types of assets. The consumption curve along illiquid resources (the dashed curve) is everywhere much flatter than the consumption curve along liquid resources (the solid curve).

The shape of the solved liquidation function $\Delta k(l_t, k_t)$ (Figure 2.2) embodies the tradeoff which the household has to face in choosing the optimal portfolio; Although the household is attracted by the higher rate of return for illiquid assets, she is deterred from holding an overly illiquid portfolio in the anticipation of being forced to liquidate and incur the transaction costs when hit by adverse income shocks. This tradeoff implies that there exists the target portfolio. The household invests into illiquid assets when her portfolio is too liquid, and divests extra illiquid assets when her portfolio is too illiquid, which explains why the liquidation/investment function is on the whole decreasing in l_t and increasing in k_t.[17]

The existence of the target level in household saving behavior was first pointed out by Carroll (1997) for a precautionary wealth to

[16] These two dimensional consumption functions are obtained by cutting the three dimensional consumption function through the dot in two ways.

[17] As illiquid asset holdings increase, the household is more likely to divest illiquid assets. (i.e. $\partial \Delta k / \partial k < 0$). As liquid asset holdings increase, the household is more likely to invest into illiquid assets. (i.e. $\partial \Delta k / \partial l > 0$.)

permanent income ratio.[18] The mechanism of generating the target level for a wealth to income ratio is a tension between impatience and prudence, two commonly assumed characteristics of households' preference. While impatience makes households prefer current consumption over saving (i.e. future consumption), prudence makes her give up some of current consumption and save against future adverse income shocks. This tradeoff leads households to try to maintain the target wealth to income ratio, which is not too much from the point of impatience but not too little from the point of prudence. While the same logic applies to my model in determining the target level for a total wealth to income ratio, my model has one more tension, namely, the tradeoff between the costs and the benefits of holding a more illiquid portfolio which determines the target level for a household portfolio.[19] In sum, my model features two targets corresponding to two kinds of tradeoffs. Interestingly, both targets are highly sensitive to the nature of income uncertainty. The target wealth to income ratio should be unambiguously positively related to any increase in income uncertainty due to the precautionary motive of saving. Whether the target portfolio is negatively or positively related to an increase in income uncertainty is not clear, and the answer is likely to depend on the certain characteristics of income uncertainty. Therefore, I need to simulate the model to

[18] Formally, the definition of the target level of a wealth to income ratio denoted by w^* is given as,

$$E_t[w_{t+1}] = w^* \quad if \quad w_t = w^* \tag{2.16}$$

Furthermore, this target level is stable in the sense that if $w_t > w^*$, $E_t[w_{t+1}] < w_t$ and vice versa. (Carroll (1997))

[19] In the same way as the target wealth-to-income ratio is defined in the previous footnote, the target levels for the liquid wealth-to-income ratio l^* and the illiquid wealth-to-income ratio k^* can be defined and their stabilities can be checked. Then, the target portfolio is simply $l^*/(l^* + k^*)$.

see the effects of income uncertainty on the optimal household portfolio allocation.

The shape of the liquidation/investment function has another feature; it has a visible flat area in which the household finds it optimal not to change at all illiquid assets holding positions. (i.e. $\Delta k_t = 0$) This flat area is created by the existence of the fixed costs and the kinked asymmetric form assumed for the transaction costs. (Equation (2.15)). Hence, adjustment of illiquid asset holding positions is sluggish, and an actual portfolio might deviate from the target level for quite a long time.

Figure 2.5 shows the time series paths of main variables in the model for one typical household who experienced zero income events in two consecutive periods. The temporary shock to income (Ψ_t), consumption (c_t), liquid saving (s_t), illiquid saving (m_t), and liquidation/investment (Δk_t), all as ratios to permanent income, are drawn with levels adjusted so that the lines do not overlap.

The two consecutive zero income events occur in the middle of the entire period shown. When zero income shock hits the household, she liquidates most of illiquid assets. Even though she nearly depletes her wealth, consumption has to fall dramatically during these two periods. However, once the disastrous periods are over, she accumulates her wealth back quickly.

Consumption has the most smooth path among all with its standard deviation being less than a half of the standard deviation of the temporary shock process excluding two zero income periods. To smooth the consumption path, both liquid and illiquid saving are fluctuating. On average, the household makes a liquid saving of 0.06 and

an illiquid saving of 0.13 relative to permanent income.[20] The total consumable resources relative to income in period t can be defined as $l_t + \max\{k_t - I(\Delta k_t), 0\} = w_t$ and averages about 1.41, which is about 44 percent larger than the average consumption to income ratio of 0.98. Liquidation or investment is not frequently made; for 41 percent of the entire periods shown, the household is not actively adjusting her illiquid assets positions. (For 36 percent $\Delta k_t < 0$, and for 23 percent $\Delta k_t > 0$.) Furthermore, changes in illiquid assets holdings (Δk_t) are positively correlated with consumption. In the good states, consumption increases, and investment is likely to be made. In the adverse states, consumption drops, and liquidation is likely to be observed. [21]

2.5 The Effect of Income Uncertainty on the Optimal Portfolio Allocation

Both liquid and illiquid assets can be used as a precautionary buffer; hence, the household facing a riskier income stream is likely to hold more of both assets. However, which asset type is more preferred as a precautionary measure against income uncertainty is not a simple question. Since the household tries to balance the costs against the benefits of holding two types of assets, and the characteristics of income uncertainty affect this cost-benefit calculation, the relationship between income uncertainty and the optimal portfolio has to be examined by simulating the model under different assumptions on income

[20] As a result, the household holds the average liquidity on hand l_t of 1.05, which by definition includes a mean one temporary income draw Ψ_t, and the average illiquid assets holdings k_t of 0.41.

[21] Liquidation is made to support consumption in the adverse states but generally not enough to totally avoid a consumption fall.

uncertainty and comparing the results. In my model, three parameters control the characteristics of income uncertainty: The standard deviation of temporary income shocks σ_Ψ, the standard deviation of permanent income shocks σ_{Pi}, and the probability of a zero income event p. I simulate the model under different configurations of these three parameter values for 10,000 households and compare the means of simulated portfolio allocations across the configurations. In simulation, all households start with zero liquid and illiquid assets holdings and in each period receive income shocks drawn randomly and independently across households from the assumed distribution. Although they are initially identical and follow the same policy rules, their assets holding positions diverge as they draw different realizations of income shocks over time. As a result, the distribution of assets holdings among 10,000 households evolves period by period, converging towards the steady state distribution. The summary results from this steady state distribution are presented in Table 2.2.

Three main points can be observed in the simulation results. First, the majority of precautionary response with respect to the probability of large but rare negative income shocks is recorded in a change in illiquid assets holdings. An Increase in the probability of the zero income event from the baseline value of 0.5 percent to 1 percent induces 32 percent more illiquid assets holdings as opposed to 7 percent more liquid assets holdings on average. As a result, total consumable resources, a combined measure for total precautionary wealth (defined as $l_t + \max\{k_t - I(\Delta k_t), 0\}$), increases by 9 percent, and the average portfolio shifts towards a more illiquid one. The result of a decrease in the probability of the zero income event from the baseline value of 0.5

percent to 0.1 percent confirms this observation. Only illiquid assets holdings negatively respond to the reduction of uncertainty, dropping nearly to a half. Liquid assets holdings even increase slightly, though total consumable resources decrease by 16.4 percent. Intuitively, illiquid assets serve a preferred buffer against disastrous but rare income shocks since the expected benefits of enjoying higher returns dominate the expected costs of having to incur transaction costs with a very small probability.

Second, liquid assets holdings respond mainly as temporary shocks to income become more volatile. When the standard deviation of temporary shocks to income increases from the baseline value of 0.1 to 0.2, liquid assets holdings more than double. On the contrary, illiquid assets holdings increase only by 12 percent. The results in the case of reduction of the standard deviation of temporary shocks reinforce this observation. With the standard deviation of temporary shocks half as small as the baseline value, the household reduces liquid assets holdings nearly by half but increases illiquid assets holdings a little. (Total consumable resources decrease still with less uncertainty.) As a result, the optimal portfolio becomes more liquid as temporary shocks to income become more volatile. This result implies that temporary income shocks can be smoothed out less costly by accumulation of liquid assets rather than illiquid assets. Intuitively, in order to absorb temporary shocks of small magnitude, the household needs to hold a small amount of liquid assets and adjust them frequently.

Third, illiquid wealth is preferred over liquid wealth as a buffer against permanent income uncertainty. A rise in the standard deviation of permanent shocks from the baseline value of 0.1 to 0.15 induces

a huge increase in illiquid assets holdings by 187 percent in contrast with an increase in liquid assets holdings by 65 percent. The results of a decrease in the standard deviation of permanent shocks reassure this point. When permanent shocks are less volatile than in the baseline case, the household reduces illiquid assets holdings by 32 percent but slightly increases liquid assets holdings. Total consumable resources increase monotonically with uncertainty, and the optimal portfolio becomes more illiquid as permanent income gets riskier.

Previous studies show that permanent income uncertainty has a much larger impact on total precautionary wealth than temporary income uncertainty.[22] Adding to that fact, my paper finds that the impact on precautionary wealth induced by permanent income uncertainty shows up mainly in illiquid assets holdings not in liquid assets holdings. Against permanent income shocks, the effect of which lasts long, the household needs to hold precautionary wealth for a long term. Illiquid assets are preferred as a buffer against such a long-term risk since they accumulate faster with higher returns than liquid assets.

The marginal propensity of consumption (MPC) out of liquid and illiquid resources evaluated at the average portfolio allocation are shown in the sixth and eighth column of Table 2.2. The MPCs out of liquid resources (varying from .17 to .33) are always found higher than the MPCs out of illiquid resources (varying from .09 to .27). In addition, the MPCs out of liquid resources become smaller as the household faces more uncertainty. However, the MPCs out of illiquid resources don't

[22]This fact can be observed in my results too. An increase in total consumable resource induced by 5 percent rise in the standard deviation of permanent shocks is about 17 (!) times as large as an increase induced by 5 percent rise in the standard deviation of temporary shock.

show such a clear pattern. The observation that the MPCs out of liquid resources are larger than the MPCs out of illiquid resources was also inferred from the shape of the consumption function shown in Figure 2.4.

The last column of Table 2.2 reports what fraction of households are actually facing the binding liquidity constraint. (i.e. $s_t = l_t - \Delta k_t - I(\Delta k_t) - c_t = 0$) In the steady state, 3 percent (for $\sigma_\Pi = 0.05$) to 23 percent (for $p = 0$) of households actually face the binding liquidity constraint, making no liquid saving. Almost all of those with zero liquid saving are liquidating some amount of illiquid wealth to support consumption.

The distributions of simulated illiquid resources, liquid resources, and total consumable resources in the steady state are shown in Figure 2.6 for the baseline set of parameter values.[23] The steady state distribution of liquid savings has concentration on zero and very small values. This is because households hold very little liquid savings in general and 15 percent of them face the binding constraint (i.e. $s_t = 0$). Note that the steady state distribution of liquid assets holdings (k_t) is also skewed to the right. Unlike the model without frictions, my model features expensive transaction costs associated with changes in illiquid assets positions, which create a flat area in the shape of the optimal liquidation/investmnet function. (see Figure 2.1) This flat area could be substantial. Indeed, for the baseline parameter values, about 52 percent of households fall in this flat area, choosing not to liquidate nor invest. The black dot on the surface of the liquidation/investmnet

[23]The distributions for the simulated variables are computed with the kernel density procedure in Stata. I assume that after 150 periods the distributions converge to the steady states.

policy function corresponds to the average values of two state variables and falls on the flat area too. Furthermore, if households are out of the flat area, they are more likely to be liquidating than investing.[24] The fact that a considerable proportion of households are not adjusting their illiquid assets holdings frequently suggests that illiquid assets holdings could, on average, deviate from the target level for a while.[25] And the asymmetric nature of liquidation/investment choices leads to the skewness of the steady state distribution of k_t.

2.6 The Marginal Propensity to Consume out of Liquid and Illiquid Resources

In the previous sections, I show that the MPCs out of liquid resources are on average larger than the MPCs out of illiquid resources. The analysis on the MPC so far was only concerning an impact (one-time) response to a shock to each type of wealth. In this section, I will show the impulse response of consumption over time with respect to a shock to liquid wealth and illiquid wealth. When I trace the consumption response to each shock, I assume all the stochastic variables except the initial shock take their mean values during the period considered. All the analysis in this section is based on the solution under the baseline parameter setting. However, the similar result can be obtained with different parameter choices.

The first picture of Figure 2.7 shows simple responses of consump-

[24] About 34 percent of households are liquidating ($\Delta k_t < 0$)), and about 14 percent are investing ($\Delta k_t > 0$) under the baseline parameter setting. The same observation can be made under all other parameter values.

[25] In all configurations considered, the target level of illiquid asset holdings computed in the way represented by Equation (2.16) is smaller than the average level of the simulated illiquid asset holdings shown in Table 2.2. (not shown)

tion after shocks to wealth. The dotted line is the consumption response with respect to one standard deviation shock to liquid wealth; the solid line is the consumption response to a same size shock to illiquid wealth. The same consumption function and liquidation/investment function are used to derive the impulse response functions. The household starts from the average consumption level in period zero; the one-time shock occurs in period one, and the two lines diverge from period one. The consumption paths are for the normalized ones.[26]

To see the difference in the dynamic MPCs out of each type of wealth more clearly, I draw the deviation of each impulse response function from the consumption path which would have realized without a shock. (the second picture of Figure 2.7) The consumption response to a shock to liquid wealth (the dotted line) is larger than the response to a shock to illiquid wealth (the solid line) for the first four periods. However, in the long run the effect of an illiquid wealth shock on consumption overtakes and ends up slightly higher than the effect of a liquid wealth shock.

The difference in the consumption response to a shock to different types of wealth clarifies the point that the MPC out of liquid wealth is larger, especially in the short term, than the MPC out of illiquid wealth. This finding implies that appreciation of illiquid assets values, such as housing prices or stock prices if stocks are mostly held in the quasi-liquid retirement accounts, doesn't have as large an impact on consumption as a rise in income does especially in the short term.

[26]Non-normalized consumption paths would share the positive trend since permanent income grows at 2 percent in the model, and in the steady state consumption also grows at the same rate.

2.7 The Transitional Dynamics of Portfolio Allocation

In simulation, I verify that in the steady state the household facing a more risky income process holds more precautionary wealth in both the liquid and illiquid form than the household facing a less risky income process. However, the steady state analysis cannot address the question as to what happens to consumption and portfolio choice behavior during transition after there is a fundamental change in income uncertainty. For example, when households perceive that a job loss risk will be higher in the future, how will they respond to the larger risk and adjust their portfolios over time? It is clear that, eventually in the new steady state with the higher unemployment risk, households would accumulate more precautionary wealth in both forms and end up with more illiquid portfolios. While the long-run response is known, short-run adjustment during the transition from the old to the new steady state is not obvious, and a simulation experiment is warranted. Households might hold more liquid portfolios in the beginning of the transition since building up illiquid assets is more costly than accumulating liquid assets, and households fear a higher possibility of being forced to liquidate the just accumulated illiquid wealth.

The theory of investment predicts a negative relationship between uncertainty and illiquid assets investment. Carroll and Dunn (1997) argue that housing investment is delayed when a household becomes more pessimistic about future income prospect. Robst, and et al. (1999) also shows a result indicating that income uncertainty reduces the likelihood of owning homes. These studies imply that the household might hold

a more liquid portfolio temporarily upon an increase in income risk.

To see the transitional dynamics, I run a simulation experiment in which the household experiences a one-time permanent rise in zero income probability p from the baseline value of 0.5 percent to 1 percent. Again the experiment is simulated for 10,000 households, and the results are summarized in terms of the means of simulated variables. Initially, all the variables have the steady state distributions under the baseline parameter values ($p = 0.005$ and $\sigma_\Psi = \sigma_{Pi} = 0.1$). Upon the increase in job loss risk, all the households change their behavior and start following the new policy functions corresponding to the new parameter values ($p = 0.01$ and $\sigma_\Psi = \sigma_{Pi} = 0.1$). As the households adjust their portfolios over time, the distribution of asset holding positions evolves gradually towards the new steady state distribution consistent with the higher job loss risk.

Figure 2.8 shows the dynamic adjustment paths for liquid assets saving, illiquid assets saving, and illiquid asset shares after the zero income probability rises from 0.5 percent to 1 percent.

The dynamic paths are drawn separately by the initial amount of total consumable resources. The first period corresponds to the steady state before the increase in job loss risk. The change in income uncertainty occurs between period one and period two and has an impact on consumption and portfolio choice behavior from period two. As is clear in Figure 2.8, there is overshooting in liquid assets saving especially for the households starting with low resources. In contrast, illiquid assets saving increases gradually towards the new steady state level. Given the dynamic paths for liquid and illiquid assets saving, illiquid assets shares in portfolios fall during the initial phase of the adjustment, gradually

climb up, and in the new steady state end up higher than the shares before p changes. The transition process takes quite a long time. (See the third panel of Figure 2.8)

This experiment shows that when the household perceives a rise in job loss risk, it accumulates liquidity first then illiquid assets later. The process of adjusting the amount of precautionary wealth upwards is slow since accumulation of wealth is costly, requiring the household to give up current consumption and to pay transaction costs. In the long run, the household should see a relatively large increase in illiquid assets holdings and a moderate increase in liquid assets holdings. However, in the beginning of adjustment, the household is reluctant to hold a more illiquid portfolio since she fears a larger possibility of having to liquidate and pay transaction costs when hit by the zero income shocks. Therefore, the household starts building up illiquid assets only after she accumulates enough liquid wealth as a buffer and feels comfortable to do so.

In sum, the short run precautionary response to a increase in income risk is very different from the long run precautionary response. Accumulation of illiquid assets takes longer than accumulation of liquid assets. Therefore, when an income process becomes riskier, the household might temporarily hold a more liquid portfolio, even though the long run optimal portfolio is more illiquid.

2.8 Conclusion

I present a theoretical model of household portfolio choice between liquid and illiquid assets and show that illiquid assets can be optimally

held as a precautionary buffer against income uncertainty, even though they are more costly than liquid assets to convert into consumption. In particular, illiquid assets are preferred when shocks to income are persistent or when households face large adverse shocks with a very small probability as in a case of unemployment shocks. My model provides a theoretical support for the empirical finding that a bulk of precautionary wealth might be held in the illiquid form rather than in the liquid form.

I also show that the MPC out of liquid resources is larger than the MPC out of illiquid resources especially in the short run. This result implies that stock price appreciation causes a smaller boost in consumption if stocks are mostly held by households in the quasi-illiquid retirement accounts.

In the steady state, households facing higher unemployment risk should hold more illiquid portfolios than those facing lower unemployment risk. However, when all the households experience a one-time rise in unemployment risk, they temporarily hold more liquid portfolios during the transition to the new steady state since they make liquid saving faster than an illiquid saving. Such a short run precautionary response of household portfolio might have important implications for short run price movements of liquid and illiquid assets. For example, the overshooting of liquid assets demand upon a rise in aggregate income risk might cause a corresponding overshooting of liquid assets prices relative to illiquid assets prices.

There are limitations to the analysis of this paper. Some assets might be held for reasons different from a precautionary concern and interest earnings. For example, housing assets might be held for a

bequest motive; moreover, housing assets produce not only a stream of returns but also a flow of consumption service. Nor does my model deal with risk characteristics of assets.[27] It is desirable to extend the model to allow for risky assets towards a more comprehensive account of household portfolio choice behavior.

[27]If illiquid assets are riskier than liquid assets, the paper's result would be weakened. However, as long as the risk is not too large, enough high returns for illiquid assets guarantee the main point of the paper.

Table 2.1: Baseline Parameter Values

preference parameters	
β	0.95
ρ	2
asset characteristics parameters	
r^L	0.02
r^K	0.04
χ	0.1
F	0.001
income process parameters	
G	1.02
σ_Ψ	0.1
σ_Π	0.1
p	0.005

Table 2.2: Steady State Results for Alternate Uncertainty Parameter Values

parameter value	averages				MPC out of		% of liquidity constrained
	liquid	illiquid	illiquidity ratio	total resources	liquid	illiquid	
$\sigma_\Psi = .05$	.038	.476	91.8%	1.48	.29	.18	17.6%
$\sigma_\Psi = .10$	.075	.463	86.3%	1.50	.29	.20	15.0%
$\sigma_\Psi = .15$	.114	.477	82.1%	1.56	.27	.20	14.4
$\sigma_\Psi = .20$	.152	.516	79.2%	1.63	.26	.20	11.6%
$\sigma_\Pi = .05$	.083	.317	81.2%	1.37	.34	.28	3.3%
$\sigma_\Pi = .10$	.075	.463	86.3%	1.50	.29	.20	15.0%
$\sigma_\Pi = .15$	.125	1.327	92.8%	2.37	.18	.10	17.2%
$p = .001$	.077	.232	76.9%	1.29	.32	.24	9.3%
$p = .005$	.075	.463	86.3%	1.50	.29	.20	15.0%
$p = .01$	.081	.609	88.0%	1.65	.26	.19	6.7%

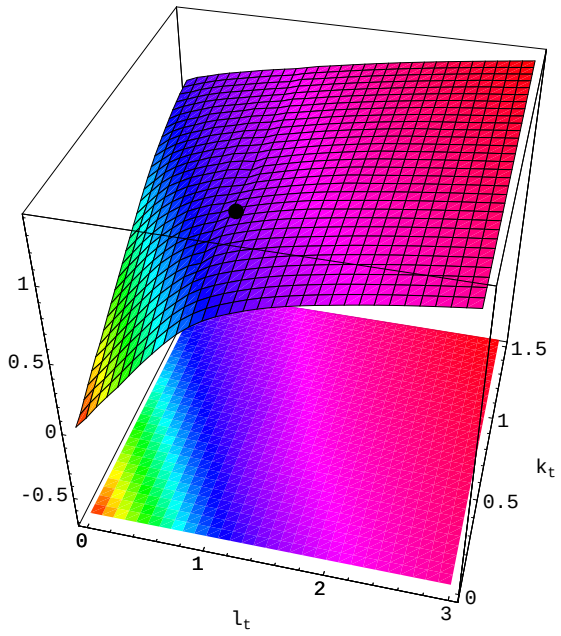


Figure 2.1: Consumption Function $c(l_t, k_t)$

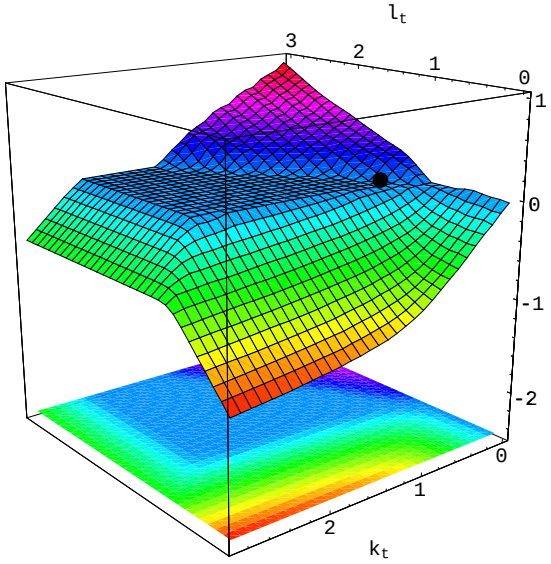


Figure 2.2: Liquidation/Investment Function $\Delta k(l_t, k_t)$

Note: A dot on the surface corresponds to the average value for portfolio allocation.

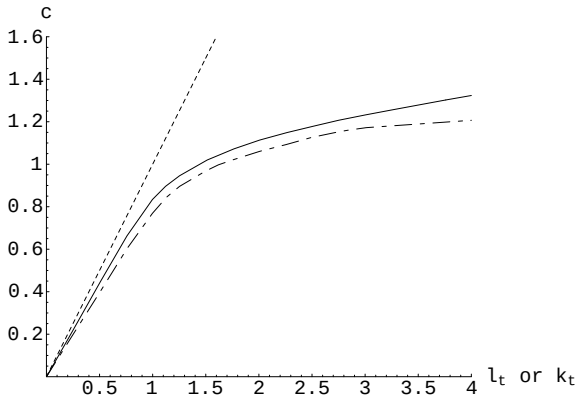


Figure 2.3: Consumption Functions at $l_t = 0$ or $k_t = 0$

Note: The dotted line is the 45 degree line; the solid line is the consumption function evaluated at $k_t = 0$, and the dashed line is the consumption function evaluated at $l_t = 0$.

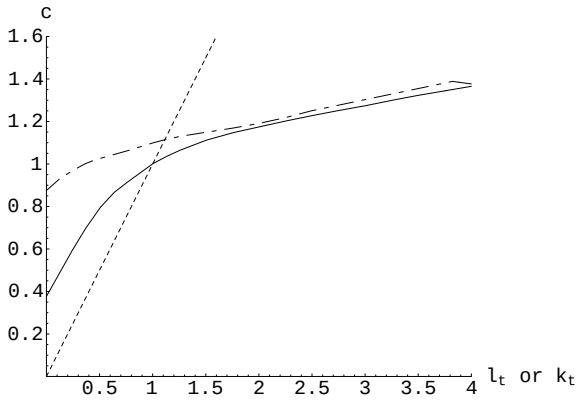


Figure 2.4: Consumption Functions at the Average l_t or k_t

Note: The dotted line is the 45 degree line; the solid line is the consumption function evaluated at the average level of k_t, and the dashed line is the consumption function evaluated at the average level of l_t.

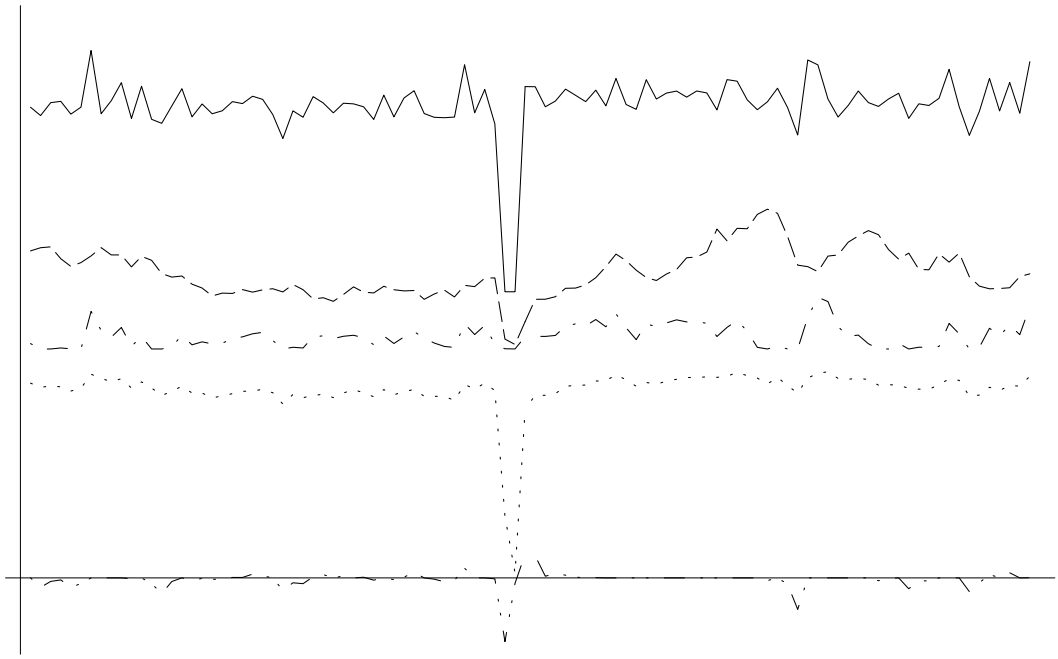


Figure 2.5: Simulations of Main Variables

Note: From the top to the bottom, temporary shocks (Ψ_t), consumption (c_t), liquid savings (s_t), illiquid savings (m_t), and liquidation/investment (Δk_t) in order.

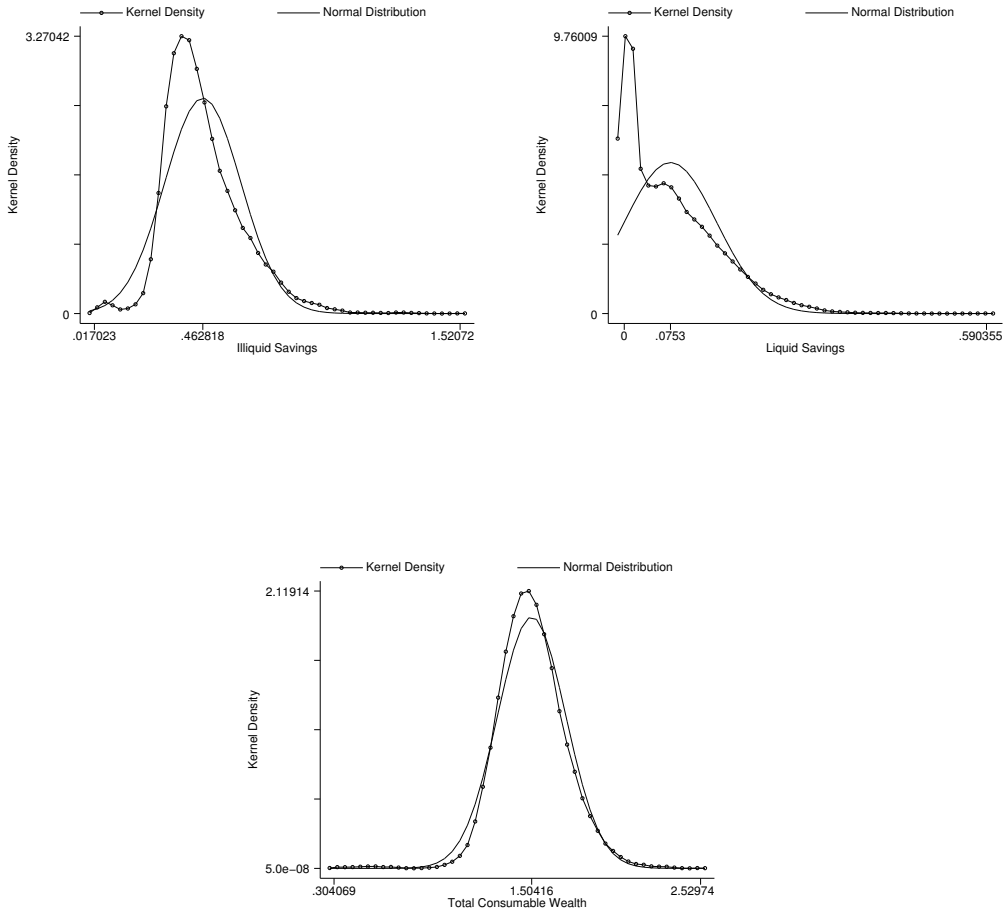


Figure 2.6: The Steady State Distributions

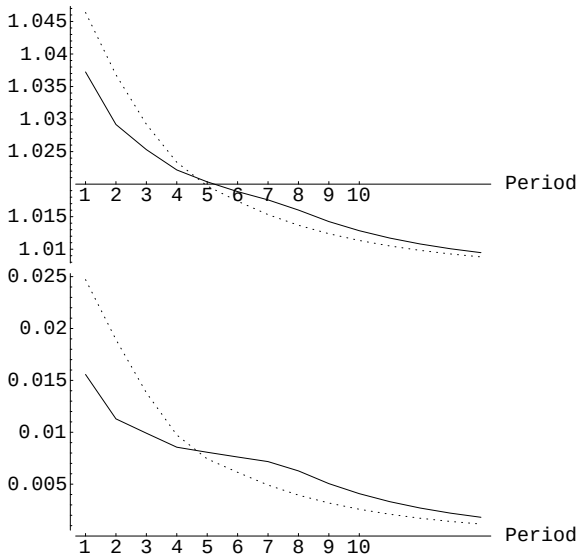
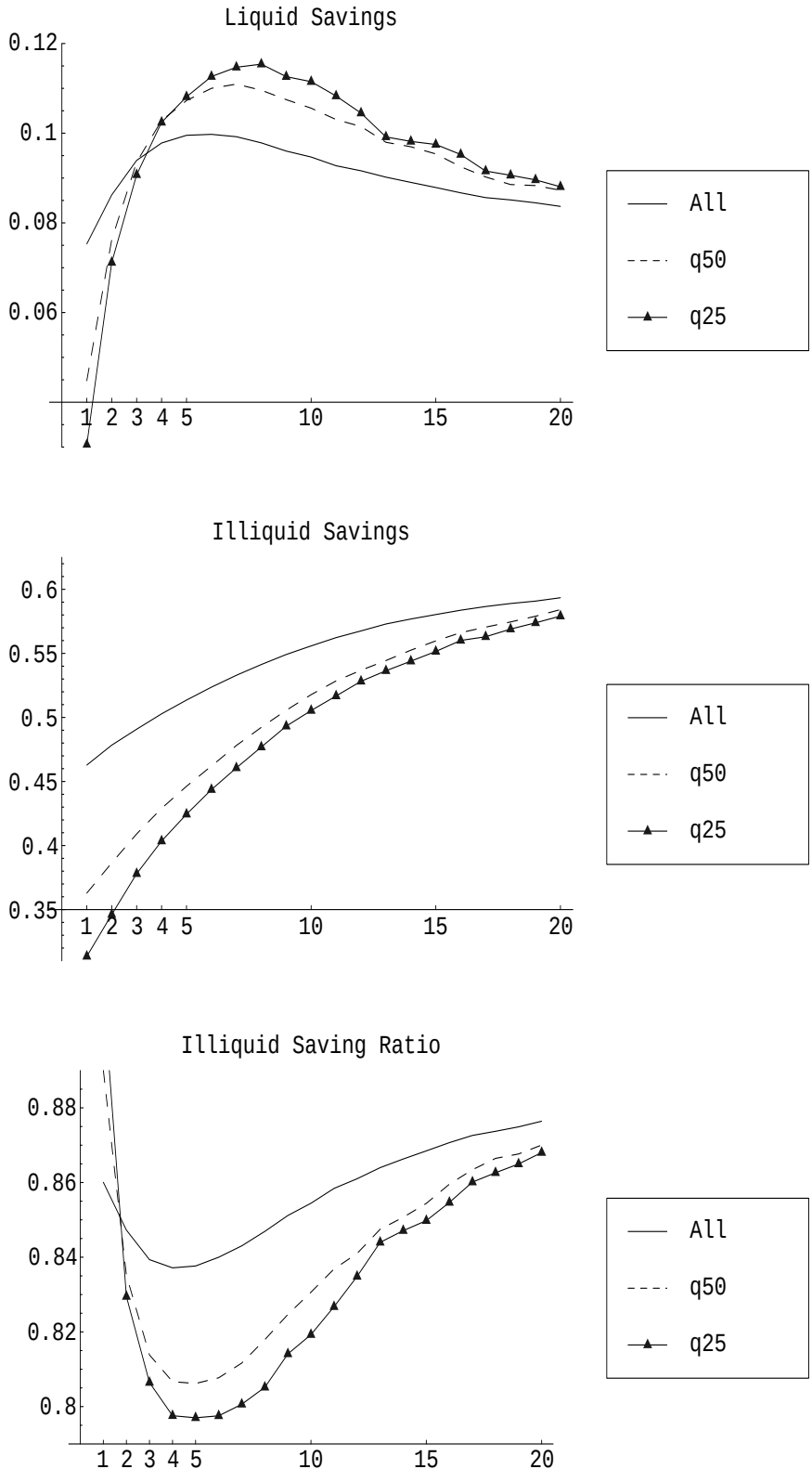


Figure 2.7: The Impulse Response to a Shock to Wealth

Note: The dotted line is the impulse response to one standard deviation shock to liquid wealth, and the solid line is the impulse response to a same size shock to illiquid wealth.

Figure 2.8: The Dynamic Paths of Asset Holdings



CHAPTER 3

Empirical Facts on Household Portfolios

3.1 Introduction

Recently many papers have been written in search for the determinants of household portfolios among different financial assets. Theoretical models of a utility-maximizing household can explain that the optimal portfolio allocation varies across households due to different preferences, levels of wealth holdings, planning horizons, and characteristics of background risks. In particular, income uncertainty has been emphasized as one of the crucial factors in understanding the vast heterogeneity in portfolio allocations between several assets with various risk characteristics. Since consumption/saving decisions and portfolio allocation decisions of savings are simultaneously made, it is natural to argue that one of the major determinants for consumption/saving decisions for a precautious household, namely income uncertainty also influences portfolio allocation decisions. This chapter uses the Panel Study of Income Dynamics (PSID) to provide a new insight into household portfolio choice behavior between liquid and illiquid assets and focuses on the relationship between household portfolio allocations and income uncertainty. I look at the broad range of wealth including housing equity, transportation assets, business assets in my empirical work, not just financial wealth.

Empirical tests performed in this chapter is backed by the theoretical work in the previous chapter. Using numerical techniques to

solve for the optimal solution of a household portfolio choice problem with liquid and illiquid assets, I find that most of precautionary savings is held in the illiquid form rather than in the liquid form as a buffer against income shocks — especially against permanent shocks and rarely occurring, but disastrous unemployment shocks. The simulation results show that illiquid assets can be optimally held as precautionary wealth even though households have to incur liquidation costs every time they need to convert illiquid assets into consumption. If large negative shocks hit households fairly infrequently, they can enjoy slightly higher returns from illiquid assets most of the time, hence holding illiquid precautionary wealth makes economic sense. Consistent with the theoretical results, I find direct evidence that household illiquid asset ratios are positively related to income uncertainty.

My empirical exercise consists of two tests. One is to test the systematic relationship between illiquid portfolio allocations and income uncertainty, using a three-year panel of the household wealth data collected from the PSID. In the first step, I measure income uncertainty at the household level by using the annual income series from 1993 to 2001. Then, in the second step, this measure of uncertainty is used as one of regressors to explain household illiquid portfolio allocations and wealth holdings. The regression is expected to capture the steady state implication from the theoretical model, namely, that households with higher income uncertainty tend to hold more illiquid portfolios. The panel data structure allows us to control for unobserved household specific heterogeneity, which has been a persistent problem plaguing wealth regressions with cross sectional data sets. With cares to measurement errors and potential endogeneity problems, the data support

the model's steady state prediction. Households facing the 80th percentile level of total income uncertainty would hold 96 percent more illiquid wealth and 32 percent more illiquid portfolios than household facing the 20th percentile level of total income uncertainty. If assets are held in part for the precautionary reasons, they should be actually consumed when households are hit by large negative income shocks. Therefore, I perform the second test to see if there exists evidence of liquidating illiquid assets in times of adverse income shocks. Constructing "active" changes in illiquid asset holdings between 1999 and 2001, I find that households indeed liquidate part of illiquid assets immediately after they experience adverse income shocks.

This chapter is organized as follows. In Section 3.2, I briefly review related literature and highlight what is different in my approach. In Section 3.3, I summarize the theoretical results. In Section 3.4, I describe the data set and explain how I construct variables used in the following empirical tests. Econometric results on household portfolio choice in the steady state are given in Section 3.5. Section 3.6 presents another empirical result on asset liquidation behavior. Finally, Section 3.7 provides concluding remarks.

3.2 Empirical Evidence

Empirical relationships between household wealth and income uncertainty have been investigated by many. Finding a positive effect of income uncertainty on wealth is interpreted as the existence of a precautionary saving motive. (Carroll and Samwick (1997), Carroll, Dynan, and Krane (1999), Engen and Gruber (2001), Kazarosian (1997),

Lusardi (1997)) The specification of empirical tests is normally a linear regression of household wealth holdings on some measure of income uncertainty. Since each study uses different data sets, time periods, empirical methods, strategies of measuring household specific income uncertainty, and definitions of wealth, each yields different results on the significance of the precautionary motive. In particular, whether the wealth definition includes only liquid wealth or includes both liquid and illiquid wealth seems to affect the conclusions of empirical work significantly in a sense that the size of precautionary wealth is estimated quite differently. (Carroll, Dynan, and Krane (1999)) Hurst and Srafford (2002) examine the 1996 PSID which contains supplementary questions on financial situations of households and show that housing equity is effectively used as a buffer against adverse income shocks. Their result implies the existence of illiquid precautionary wealth such as home equity. Skinner (1996) also argues that households tend to consider their housing wealth as consumable resources to draw on in case they experience adverse income shocks.

The relationships between household portfolio choice and income uncertainty have been increasingly studied in finance literature. Empirical work has pointed out the importance of income risks on household portfolio choice behavior. (Cocco (2000), Heaton and Lucas (1997), Guiso, Jappelli, and Terlizzese (1996), and Vissing-Jorgensen(2000)) The primary interest in finance is to see how portfolio allocations between risky assets (mainly stocks) and risk-free assets (mainly bonds) are affected by income uncertainty. Using the 1983 Survey of Consumer Finance, Haliassos and Bertaut (1997) find that people working in riskier occupation categories have lower shares of risky assets in their

financial portfolios. Using a sample of stock holders from the PSID, Vissing-Jorgensen (2000) investigates the role of nonfinancial income uncertainty in explaining cross-sectional differences in equity shares and shows that larger income uncertainty has a negative effect on risky asset shares. Gakidis (1998) finds that the probability of experiencing zero income events has a large negative effect on equity holding shares in the 1984 PSID wealth supplement. Souleles (2001) studies the role of a hedging motive in household portfolio choice behavior. He finds that households with more volatile consumption, larger covariances between consumption and asset returns, or more pessimistic feelings about the future hold as well as buy fewer securities. Guiso, Jappelli, and Terlizzese (1996) construct a subjective measure of uncertainty from the 1989 Bank of Italy Survey of Household Income and Wealth and show that risky asset shares are negatively related to the subjective income uncertainty. In sum, these studies commonly point out that risky asset portfolios in financial wealth decrease with the degree of background risks such as income uncertainty, which is consistent with the prediction derived as the optimal behavior from theoretical models of a risk-averse household.

In this chapter, I focus on the relationship between "liquid or illiquid" portfolio choice and income uncertainty, which has not been examined in detail. It is partly because illiquid assets have quite heterogeneous characteristics and are hard to analyze as one category. While I am aware of the risk of lumping heterogeneous asset types into one variable, I believe that most of illiquid asset types are an important part of precautionary wealth and that the work on simple "liquid or illiquid" asset portfolio allocations is worthwhile towards understand-

ing more complicated household portfolio problems. The use of panel data allows me to control for unobserved household heterogeneity in asset holdings and portfolio choice. Another advantage of using a panel data set in finding the steady state relationship is that I can control for transitory "out of the steady state" movements.

In addition to the steady state analysis of the relationship between portfolio allocation and income uncertainty, I conduct a complementary test to see whether changes in illiquid asset holdings occur immediately after actual income shocks hit households. Using the Consumer Expenditure Survey, Souleles (2001) estimates an (S,s)-type model of active transactions of equity holdings over one year. He adopts the (S,s)-type estimation technique since transaction costs and sluggish adjustments are considered to be typical features in determining whether households buy or sell their equity holdings. Since the transaction costs of changing positions of illiquid asset holdings are expected to be severe, I also adopt (S,s)-type models to estimate the regressions of changes in illiquid asset holdings.

3.3 Theoretical Model

I now present a simple model of household portfolio choice with illiquid and liquid assets and summarize the predictions obtained from the theory on which the following empirical work is based. I consider an infinitely living household who has a precautionary motive, makes consumption/saving choice in each period, and allocates savings between liquid and illiquid assets. In each period, the household receives exogenous income which is subject to temporary and permanent shocks

and gets zero (unemployment event) with a small probability. Under uncertainty about the future income, the precautionary household has to make the optimal decision on the size and the composition of its savings. By holding illiquid assets rather than liquid assets, the household can enjoy slightly higher returns, however, she is required to pay transaction costs whenever she changes positions of illiquid asset holdings. Therefore, the household weighs the expected benefit of holding illiquid assets rather than liquid assets against the expected cost. This cost-benefit calculation is affected by the nature of income uncertainty faced by the household, since income risk would determine how frequently the household has to adjust precautionary savings to smooth consumption. In the steady state analysis of the model, I demonstrate that a household with a higher permanent income risk or unemployment risk tends to hold not only more precautionary wealth but also a more illiquid portfolio; Against permanent income shocks, a household needs to adjust consumption rather than savings, incurring less transaction costs. Against unemployment shocks, a household would have to use savings at the times of shocks, however, the shocks rarely occur. Hence, it makes sense to put precautionary savings in the illiquid form against these two types of income shocks.

The model of household portfolio choice is specified as follows.

$$\max_{\{C_s, \Delta K_s\}_{s=t,\cdots,\infty}} E_t \left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \right] \tag{3.1}$$

subject to

$$L_{t+1} = R^L(L_t - \Delta K_t - I(\Delta K_t) - C_t) + Y_{t+1} \tag{3.2}$$

$$K_{t+1} = R^K(K_t + \Delta K_t) \tag{3.3}$$

$$0 < C_t <= L_t - \Delta K_t - I(\Delta K_t) \tag{3.4}$$

$$-K_t <= \Delta K_t <= L_t \tag{3.5}$$

where the current liquid asset position is L_t[28], the current illiquid asset position is K_t. Each asset earns the constant gross interest rate of R^L and R^K respectively. $R^K > R^L$ is assumed as the benefit of holding illiquid assets over liquid assets. Current income is Y_t which follows an exogenous stochastic process described in detail later in this section. In each period, a household decides how much of illiquid assets she wants to liquidate and convert into liquid resources or how much of liquid assets she wants to invest into illiquid assets. The amount of the change in illiquid asset positions is denoted by ΔK_t. The transaction costs associated with ΔK_t is specified as an $I(\cdot)$ function.

$$I(\Delta K_t) = \begin{cases} \chi \cdot |\Delta K_t| + F & \text{if } \Delta K_t < 0 \text{ (liquidating)} \\ 0 & \text{if } \Delta K_t = 0 \\ F & \text{if } \Delta K_t > 0 \text{ (investing)} \end{cases} \tag{3.6}$$

The household also faces a strict version of the liquidity constraint (Equation (3.4)) and cannot invest or liquidate more than she has (Equation (3.5)). The time preference factor is β. The utility function is of the CRRA form $u(C_t) = C^{1-\rho}/(1-\rho)$, where ρ is the coefficient of relative risk aversion.

The structure of the exogenous income process follows Carroll

[28]Total liquid resource at the beginning of period t (L_t), which is the sum of the liquid asset holdings carried from the last period and current labor income draw Y_t, is conventionally called "liquidity-on-hand".

(1992). Labor Income Y_t consists of permanent income P_t and a multiplicative temporary shock Ψ_t. The temporary shock may take a zero value with a small probability of p, otherwise is distributed lognormally with a standard deviation of σ_Ψ. Permanent income P_t is equal to the last period's permanent income multiplied by the deterministic gross growth rate G and a permanent income shock Π_t. The permanent income shock is also distributed lognormally with a standard deviation of σ_Π.

$$Y_t = P_t \cdot \Psi_t \tag{3.7}$$

$$P_t = G \cdot P_{t-1} \cdot \Pi_t \tag{3.8}$$

$$\Psi_t = \begin{cases} 0 & \text{with a probability of } p \\ \sim lognormal(\mu_\Psi, \sigma_\Psi) & \text{with a probability of } 1-p \end{cases} \tag{3.9}$$

The degree of uncertainty is controlled by three parameters concerning the income process: σ_Ψ (a standard deviation of temporary income shocks), σ_Π (a standard deviation of permanent income shocks), and p (a probability of the zero income event). An increase in any of theses three parameters leads to more uncertainty in the future income and induces more precautionary behavior. Solving the model under different assumptions on the income uncertainty parameters, I show that at the steady state an increase in the standard deviation of permanent shock σ_Π or the probability of unemployment shock p makes the optimal portfolio more illiquid. On the other hand, an increase in the standard deviation of temporary shock σ_Ψ makes the optimal portfolio less illiquid. Next, I would like to test these implications derived from the theoretical model, using a panel of the household wealth data from

the PSID.

3.4 Data

The sample of households in the following analysis is drawn from the Panel Study of Income Dynamics (PSID) from 1993 to 2001. Data on wealth and portfolio come from the supplementary files collected in the years of 1993, 1999, and 2000. The PSID is a longitudinal survey of a representative sample of U.S. individuals and families in which they reside. Data were collected for the first time in 1968 and annually afterwards until 1997 when the data collection frequency switched to every other year. Starting with an initial sample of approximately 4,800 households in 1968, the PSID has traced individuals from these initial households since then. The PSID provides a wide variety of information at the family and individual levels such as economic and demographic conditions (e.g., income, employment, family composition). Some information other than core topics (e.g., health, wealth, retirement planes, flows of time and money help among families and their friends, and so on) is gathered intermittently as supplements.

In my selected sample from 1993 to 2001, I have seven years (waves) of core topic information for each household.[29] I use income information from all seven waves to construct a measure of income uncertainty for each household. The details of construction are explained below. Income data collected in a given wave refer to the previous calender year, therefore the income variables are dated actually from 1992 to 2000. I use the family files rather than the individual files since wealth information is available only at the family level. I define non-

[29]Namely, 1993, 1994, 1995, 1998, 1997, 1999, 2001

financial family income as all taxable income plus transfer income of all family members minus income from dividends, interest, trust funds, and royalties.

Disaggregate wealth information is recorded as a supplement only in the years of 1994, 1999 and 2001 and not available in the other years. In each wealth supplement file, the categories of wealth include housing equity, business assets, checking/savings accounts, stocks and bonds, transportation assets, real estates other than residential home, and others. Of these asset types, included into the liquid asset category are cash (checking/savings accounts, money market funds, CDs, savings bonds, and Treasury bills), stocks (shares of stocks in publicly held corporations, mutual funds or investment trusts, including stocks in IRAs), and other investments (trusts or estates, bond funds, life insurance policies, special collections), while included into the illiquid asset category are home equity, transportation assets, business or farm assets, and real estates other that main home. The main dependent variable used in the analysis, an illiquid portfolio share, is constructed as a ratio of all illiquid assets divided by total wealth.[30] All income and wealth variables are converted to the real terms by deflating with the consumer price index for all urban consumers.[31]

After stacking three cross-sectional data for which wealth information is available into one panel data set, I make sample selection of households; A household is dropped if an income variable is missing in any of seven years, if family structure is not stable from 1993 to 2001[32],

[30]Total wealth includes debts such as credit cards, student loans, medical or legal bills, and personal loans.

[31]The CPI is taken from the BLS website. The series is annual for all items and the base period is 1982-84.

[32]I define a break in family structure as changes in the head of household or the

if in any year family income gets below 20 percent of the seven-year average level[33], finally if a household is in the SEO sample.[34] After the sample selection, I have 1,457 households in each year. The summary statistics for selected variables in the sample of households are given in Table 3.1.

The second empirical test requires changes in illiquid asset holdings between two waves as a dependent variable. I decide to use the most recent two waves, the 1999 and 2000 PSID panel, to construct the dependent variable. This is the shortest period of time in which wealth information is consecutively available. The longer a time span is for measured changes in asset holdings, the less clear it is whether the changes in asset holdings are forced by adverse income shocks or by other factors. Upon adverse shocks, households might actually liquidate some of illiquid assets to keep consumption from falling sharply. However, after a while they will try to gradually accumulate back asset positions towards the optimal level. When the time span between two wealth positions to measure the change is too large, I would fail to capture much of immediate responses after the adverse shocks. A simple difference of illiquid asset holdings in two years includes passive changes caused by capital appreciation/depreciation as well as active changes in asset holdings, hence is not suitable for an (S-s)-type analysis. I therefore construct active changes in illiquid asset holdings from 1999 to 2001 by carefully adding net active changes in housing equity,

marital status of the head.

[33]In this way, I can exclude households experiencing large negative shocks. Otherwise, the measure of income uncertainty would be dominated by the large negative shock.

[34]The Survey of Economic Opportunity (SEO) sample are low-income families included in the original PSID.

business asset holdings, and real estate holdings.[35]

3.5 Empirical Results: The Steady State

In this section, I investigate the empirical relationship between household portfolio choice behavior and income uncertainty. The theoretical model concludes that in the steady state households facing more risks in permanent income or unemployment shocks tend to hold more illiquid portfolios, while households facing more risks in temporary income fluctuations tend to hold more liquid portfolios. The category of illiquid assets should include types of assets which require significant transaction costs in selling and buying. I include housing equity[36], net values of cars and other vehicles, net values of other real estates, and net values of business assets into the illiquid category. The rest of asset types which consist of checking/savings account balances, stock and bond holdings, and other miscellaneous investments are defined as liquid in this analysis. In my selected sample of the PSID households, the majority (55%) of illiquid assets is held in housing equity[37], the second largest component (32%) is transportation assets, other real estates and business assets occupy very small portions (7% and 6%) of total illiquid wealth. Illiquid portfolio shares are defined as illiquid wealth divided by total wealth[38] and are my main dependent variable. The average of

[35]Unfortunately, I could not construct net active changes in transportation assets over these two years due to a lack of information.

[36]Housing equity is defined as the value of house minus total outstanding mortgage balances.

[37]This calculation uses only households with positive illiquid asset wealth but includes negative housing equity due to the mortgage value exceeding the housing value. (4108 data points pooling three years)

[38]Total wealth includes debts on credit cards, but excludes mortgages. To get illiquid portfolio shares, only data with positive total wealth are considered. (4222 data points pooling three years)

illiquid portfolio shares is about 70 percent, the median is 77 percent and the standard deviation is about 30 percent. The large standard deviation suggests large heterogeneity in household portfolio allocation behavior.

3.5.1 Framework for Estimating Income Uncertainty

One of the most important but hard-to-measure explanatory variables is a degree of income uncertainty. I follow a framework proposed by Carroll and Samwick (1997) and adopted by other researchers (Vissing-Jørgensen (2002)) to measure total income uncertainty for each household by utilizing the panel structure of the data. Furthermore, I try to decompose total income uncertainty into temporary and permanent components for each household. Unfortunately, however, a probability of being unemployed cannot be measured with reasonable accuracy at the household level since only few households record such an event over eight years in sample.[39] The selected data set from the PSID contains information on household income annually from 1992 to 1996 plus 1998 and 2000, giving seven data points for each household. The family income is converted to the real terms, using the consumer price indices of corresponding years. To understand how to measure household specific uncertainty from these income data, it is useful to review the income process assumed in the theoretical model. Equation (3.7) and (3.8) are rewritten in terms of logarithms below. The prob-

[39] Actually, I eliminated households whose total income ever drops below the 20 % level of the average household income over eight years so that income shocks wouldn't include big adverse income shocks such as unemployment spells. To measure the probability of unemployment shocks, it is needed to combine information from other household data sets which have larger samples on labor market conditions such as the Current Population Survey.

ability of getting a zero income event p is set to zero since I do not measure this probability.

$$
\begin{aligned}
y_{it} &= p_{it} + \psi_{it} && (3.10)\\
p_{it} &= g_{it} + p_{it-1} + \pi_{it} && (3.11)\\
\psi_{it} &\sim N(-2/\sigma_i^{\psi}, \sigma_i^{\psi}) && (3.12)\\
\pi_{it} &\sim N(-2/\sigma_i^{\pi}, \sigma_i^{\pi}) && (3.13)
\end{aligned}
$$

where lowercase variables are the logarithms of corresponding uppercase variables. For example, y_{it} is the logarithm of nonfinancial income of household i in year t. The logarithm of permanent income p_{it} is a random walk with a drift term of g_{it}. The logarithms of both temporary shock ψ_{it} and permanent shock π_{it} are i.i.d. random variables with normal distributions.[40]

The d-year income growth rate is then defined as

$$
\begin{aligned}
y_{it} - y_{it-d} &= p_{it} + \psi_{it} - p_{it-d} - \psi_{it-d} && (3.14)\\
&= \sum_{s=1}^{d} g_{it-s+1} + \sum_{s=1}^{d} \pi_{it-s+1} + \psi_{it} - \psi_{it-d} && (3.15)
\end{aligned}
$$

Assuming that two shocks ψ_{it} and π_{it} are not correlated with each other at all lags and leads and setting d = 2, the expected two-year growth rate of income and its variance have the following forms.

$$
\begin{aligned}
E[y_{it} - y_{it-2}] &= g_{it} + g_{it-1} = G_{2it} && (3.16)\\
V[y_{it} - y_{it-2}] &= 2\sigma_i^{\pi 2} + 2\sigma^{\psi 2} && (3.17)
\end{aligned}
$$

[40]Note that the means of two normal distributions are adjusted so that the means of the levels (not logs) of the variables are one.

I further assume that the two-year expected growth rate G_{2it}, which is a sum of two drift terms g_{it} and g_{it-1}, is perfectly predictable as a linear function of a set of variables predetermined at time $t-2$. (i.e., $G_{2it} = X_{it-2} \cdot \delta$.) For X_{it-2}, I use age of household head, age of household head squared, a dummy for head being female, one digit occupation and industry dummies, a dummy for being self-employed, family size, number of children under 18, a dummy for being married, a dummy for being white, three educational dummies (16+ years, 13-15 years and 9-12 years of education), and some of the regressors interacted with age of head.

$$G_{2it} = X_{it-2}\delta \tag{3.18}$$

I work with two-year growth rates rather than annual growth rates, since income data are available only two years apart in the most recent PSID files. First, I regress G_{2it} on X_{it-2} and obtain the coefficient estimates $\hat{\delta}$. Then, I construct the predicted two-year income growth rate as $\hat{G}_{2it} = X_{it-2}\hat{\delta}$. Finally, I can estimate the variance of two-year income growth rates (Equation (3.17)) as the second moment of the residuals $\hat{\nu}_{it-2}$ obtained from the regression (Equation (3.18)).

$$\begin{aligned} \hat{V}[y_{it} - y_{it-2}] &= \frac{1}{T}\sum_t (y_{it} - y_{it-2} - \hat{G}_{2it})^2 && (3.19) \\ &= \frac{1}{T}\sum_t (\hat{\nu}_{it-2})^2 && (3.20) \\ &= \Sigma_{2i} && (3.21) \end{aligned}$$

I have the estimated variance of two-year income growth rates denoted by Σ_{2i}, which is what I call total income uncertainty.[41] By construction,

[41] In the following regressions, I use Σ_{2i} divided by 2 so that the measure becomes

the total income uncertainty for each household is based on only five two-year growth rates (seven income data points). Hence, I should expect Σ_{2i} to be a very noisy estimate for true income uncertainty. Parameters for the income process such as σ_Π and σ_Ψ might change over time and it would be ideal to estimate a time-varying uncertainty index for each household. Given only short time-series data available, it is practically impossible to compute accurate uncertainty measures for different time spans at the household level.[42] The constructed total income uncertainty has an interquartile range extending from 0.01 to 0.10 with an average of 0.09.

Further, I try to decompose total income uncertainty into temporary and permanent components, following the method used by Carroll and Samwick (1997). The variance of income growth rates can be generalized for a longer time span (d years) as follows:

$$V[y_{it} - y_{it-d}] = d\sigma_i^{\pi 2} + 2\sigma_i^{\psi 2} \tag{3.22}$$

This equation can be seen as an estimating regression with coefficients being $\sigma_i^{\pi 2}$ and $\sigma_i^{\psi 2}$. Therefore, I estimate the following regression for each household i,

$$\hat{V}[y_{it} - y_{it-d}] = \theta_{1i}d + \theta_{2i}2 + \eta_i \tag{3.23}$$

$$\Sigma_{di} = \theta_{1i}d + \theta_{2i}2 + \eta_i \tag{3.24}$$

an annual value.

[42]It might be feasible to obtain reasonably accurate uncertainty measures for different periods at the group level if not at the household level.

where

$$\Sigma_{di} = \frac{1}{T}\sum_{t}(y_{it} - y_{it-d} - \hat{G}_{dit})^2 \tag{3.25}$$

The expected d-year income growth rate ($\hat{G}_{dit} = \hat{E}[y_{it} - y_{it-d}]$) is estimated in the same way as I estimate the expected two-year growth rates. (see Equation (3.18)) The coefficient estimates for θ_{1i} and θ_{2i} in the above regression (3.24) are unbiased estimates for $\sigma_i^{\pi 2}$ and $\sigma_i^{\psi 2}$ respectively. When I use a value of d from 1 to 8, the sample size for each household regression is 21, which may still be too small to obtain the accurate measures of permanent and temporary income uncertainty.

3.5.2 Estimation Results

With the estimated measures of income uncertainty, I regress household portfolio shares on uncertainty and other explanatory variables. Since I have a three-year panel, the random-effect panel estimation method is used. One of the important issues to be handled is the measurement error in the uncertainty index. As I construct uncertainty indices from only seven income observations for each household, my uncertainty measures are likely to be very noisy variables and cause a severe downward bias in coefficient estimation without appropriate correction. I use instrumental variables to handle the measurement error problem in the following regressions.

In addition to the uncertainty index, a set of regressors include demographic variables, educational background, and a measure of permanent income. Two exogenous shocks — a dummy indicating whether a head's health deteriorated or not and an estimated income shock[43]

[43]Income shock is defined as current family income minus permanent income. Permanent income is constructed in two ways; (1) the average of family income

— are included to control for variations out of the steady state. The estimating equations look like

$$\log W_{it}^{I} = \alpha_1 \sigma_i + \alpha_2 \log P_{it} + X_{it}\beta + \epsilon_{it} \quad (3.26)$$

$$S_{it}^{I} = \alpha_1 \sigma_i + \alpha_2 \log P_{it} + X_{it}\beta + \epsilon_{it} \quad (3.27)$$

$$\epsilon_{it} = u_i + \eta_{it} \quad (3.28)$$

where W_{it}^{I} is illiquid asset holdings for household i at year t ($t = 1994, 1999, 2001$), S_{it}^{I} is an illiquid asset portfolio share, P_{it} is an estimated permanent income level, σ_i is a measured uncertainty level for household i, and X_{it} includes all the other regressors to control for factors affecting household portfolio choice behavior. Finding a positive coefficient estimate on the uncertainty index in Equation (3.26) suggests that illiquid assets are actually held for the precautionary motive. Furthermore, obtaining a positive coefficient estimate on the uncertainty index in Equation (3.27) suggests that precautionary wealth is held more in the illiquid form than in the liquid form.

To correct for measurement errors in σ_i and P_{it}, I need to find appropriate instruments which are correlated with σ_i and P_{it} but not correlated with the error term ϵ_{it}. For identification, the set of instruments should include at least some variables which are not in X_{it}. I decide to use occupation and industry codes, and a dummy indicating whether a household head is self-employed or not along with all

over 8 years, or (2) the average of family income which is strapped of time effects plus the predicted time effect for each year. Whichever permanent income is used, the difference between current income and permanent income would not contain the household fixed effect. Otherwise, constructed income shock would be correlated with the household random effect, causing bias in estimation.

the variables in X_{it} as instruments in the panel regressions. [44] The simple OLS and random effect panel estimation results with and without instruments for Equations (3.26) and (3.27) are presented in Table 3.2. The total income uncertainty measure σ_i affects both illiquid asset holdings and portfolio shares positively and significantly, which is consistent with what the theoretical model of precautionary households predicts in the steady state. Furthermore, when I use separately measured permanent and temporary income indices in place of the total uncertainty index (the last column of Table 3.2), both measures have positive and significant effects on illiquid asset holdings, with a larger coefficient estimate found for permanent than temporary uncertainty. The downward-bias caused by measurement errors in the uncertainty index turns out serious as the coefficient estimate on the uncertainty measure jumps up from 0.48 to 7.87 when instruments are used. Within the sample, the 20th and the 80th percentiles of σ_i are 0.9 percent and 7.6 percent respectively. An increase in the total uncertainty index of this magnitude would increase log illiquid wealth by 96 percent. As predicted, the (adverse) exogenous shocks (d(health) and log(shock)) have negative effects on illiquid asset holdings, implying that illiquid wealth is actually liquidated upon drawing negative shocks. This point is further investigated in the next section.

The results for illiquid portfolio shares are presented in Table 3.3. The total income uncertainty index enters positively in all regressions. As before, correcting measurement errors in uncertainty increases a co-

[44] The first stage regression shows that a dummy indicating whether a household head is self-employed or not supplies the most important identification information. Actually, for the sample without self-employed households (about 12 percent of the sample is dropped), the measure of uncertainty becomes insignificant.

efficient estimate to the level six times as large as without correction. An increase in total uncertainty from the 20th percentile to the 80th percentile would make a portfolio share 32 percent larger.[45] When I use the separate permanent and temporary uncertainty indices, both have positive effects on illiquid portfolio shares. Though the effect of permanent income uncertainty is significant only at the 19 percent level, the estimate is about twice as large as the effect of temporary income uncertainty. According to the theory, temporary income uncertainty should have a negative effect on illiquid asset portfolios while permanent income uncertainty should have a positive effect. Therefore, the positive estimate on temporary uncertainty does not seem to corroborate the theoretical implication. This conflicting result could be due to the difficulty in separating low-frequency temporary income shocks such as unemployment risk from high-frequency temporary shocks. In the empirical analysis, I did not try to decompose temporary income shocks into two types, while I do in the theoretical analysis. (I have a probability of getting unemployed p and a standard deviation of temporary income shocks σ_Ψ.) A rarely occurring large negative shock such as an unemployment shock is likely to be positively related to the optimal illiquid portfolio allocation, while the standard deviation of temporary income shocks, apart from such an unemployment shock, should be negatively related to the optimal illiquid portfolio. If the estimated standard deviation of temporary income shocks is strongly correlated with the unemployment risk, there is an omitted variable bias and it might affect the estimation.

The use of instrument variables can correct for the bias caused

[45]The rate of increase is calculated at the 20th percentile level of portfolio shares.

by correlations between explanatory variables and an error term, which might be either due to measurement errors in explanatory variables or endogeneity of explanatory variables. Appropriate instruments should be correlated with explanatory variables in question but should not be correlated with an error term. There are concerns as to whether the instrument set which I use for uncertainty indices are actually appropriate in that sense. In the panel estimation, the error term ϵ_{it} includes the household random effect u_i which might capture some unobserved household characteristics such as household preferences and risk tolerance. These unobserved characteristics might be correlated with not only illiquid asset holdings (or portfolio shares) but also the instruments (namely, occupation and industry codes). For example, consider a degree of risk tolerance; A more risk-averse individual would tend to self-select himself into less risky occupation or industry and consequently have a lower standard deviation of labor income, at the same time he would prefer to hold more precautionary wealth. Then, there could be correlations between instrument variables such as occupation or industry codes and the unobserved household random effect term capturing partly the degree of risk tolerance.

One way to handle such endogeneity bias caused by correlations between some of regressors and the random effect errors in panel data is to use the Hauseman-Taylor estimation method. (Hauseman and Taylor (1981))[46] They introduce the estimator which allows for correlations between some of explanatory variables and the random effect

[46]Another way to handle in this specific case is to include a measure of risk tolerance in the set of regressors. There is such a measure available in the 1996 PSID file. However, the measure is not perfect and there would be other unobserved household characteristics reflected in the random effect term.

errors, without losing the ability to estimate the coefficients of time-invariant variables unlike the fixed effect estimators.[47] I re-estimate the panel regressions using the Hauseman-Taylor method and present the results in Table 3.4. The coefficient of uncertainty is estimated smaller, though still positive, than in the IV estimation (Table 3.2). The smaller estimate of the effect of uncertainty is likely to be due to the measurement error bias. In all, the possibility of correlations between instruments and the family random effect does not overturn the major results.

3.6 Empirical Results: Out of the Steady State

In the previous section, direct evidence is found that a household facing larger income uncertainty allocates more precautionary savings into the illiquid form of assets. This finding applies to the steady state implication derived from the theoretical model. The theory also suggests that when a household is actually hit by large adverse income shocks, she would liquidate a part of illiquid assets to support consumption expenditures. This out-of-the-steady-state implication can be tested by looking at the relationship between changes in illiquid asset holdings and actual income shocks. The finding that households which experience negative income shocks tend to reduce illiquid asset holdings will confirm that illiquid assets are indeed held for the precautionary reasons.

For regressions in this section, I use the two most recent PSID

[47] The exogeneity assumption required for consistency of their estimator is actually $plim(\frac{1}{N}\sum_{i=1}^{N}\bar{X}_{i1}u_i) = 0$, where $\bar{X}_{i1}$ is the household mean of time-variant explanatory variables. The cost of using this estimator is that I cannot now correct for correlation between regressors and the time-variant error term η_{it}

waves, 2001 and 1999. A sample of households are reselected by the same rules as I construct the three-year panel. The sample size is 1,367 households.

A change in illiquid asset holdings includes a capital gain/loss as well as an active change. For my purpose, only active changes in illiquid asset holdings should be used as the dependent variable. I manually construct the active change in illiquid asset holdings between 1999 and 2001 as the sum of (1) a change in housing equity through mortgage financial activity, home sale and purchase, (2) a change in real estate holdings through sale and purchase, and (3) a change of business asset holdings through sale and purchase. One advantage of using active changes in asset holdings is that, like first differences, they would purge out any fixed household effects on the level of asset holdings.

The existence of transaction costs would make active changes in illiquid asset holdings sluggish, implying S-s type behavior; A household would change illiquid asset holding positions only when the actual amount of holdings diverges far enough from the desired amount. In this way, the household would avoid frequent payments of transaction costs. As a result, active changes in illiquid asset holdings tend to take a zero value in many cases, making the OLS method inappropriate. In my sample, 36 percent of households do not adjust the amount of its illiquid asset holdings over two years. 44 percent of households adjust upwards, while 20 percent of households adjust downwards. To handle the frequent occurrence of zero observations and the gap between the desired and the actually observed value, I choose to use the latent variable approach. Let ΔW_i^{I*} denote the unobservable desired amount of a change in illiquid asset holdings. Then, I consider the following

regression model.

$$\Delta W_i^{I*} = \gamma X_i + \varepsilon_i \tag{3.29}$$

$$\varepsilon_i \sim Normal(0, \sigma_\varepsilon) \tag{3.30}$$

where X_i represents a vector of variables which have effects on the desired change in illiquid asset holdings for a household i. The exogenous income shock is included among X_i and is of particular interest in this study. A household who is hit by a large adverse income shock should be more likely to reduce her illiquid asset position in order to keep consumption from falling sharply.

First, I use the ordered probit estimation method based on the latent variable model (Equation (3.29)). The dependent variable is discrete, taking 1 for households with positive adjustments of illiquid assets, taking -1 for households with negative adjustments, and taking 0 for households with no adjustment.

$$D_i = \begin{cases} 1 & \text{if } \Delta W_i^{I*} > k_u \\ 0 & \text{if } k_u \geq \Delta W_i^{I*} \geq -k_l \\ -1 & \text{if } -k_l \geq \Delta W_i^{I*} \end{cases} \tag{3.31}$$

According to this specification, an actual asset adjustment is not made if the desired amount of adjustment W_i^{I*} is small enough to be in the range between two unobserved thresholds, k_l and k_u.

The ordered probit model is consistent, however, it does not utilize the information on how much households adjust illiquid asset holding positions. Therefore, I also consider the Rosset model, which is proposed to estimate models of friction. (Maddala (1983)) Like the

ordered probit model, the Rosset model assumes that the dependent variable takes a non-zero value only when the unobserved latent variable is out of the range between the upper and lower thresholds. The idea is illustrated by Figure 3.1.

The Rosset model is expressed as:

$$\Delta W_i^I = \begin{cases} \Delta W_i^{I*} - k_u & \text{if } \Delta W_i^{I*} > k_u \\ 0 & \text{if } k_u \geq \Delta W_i^{I*} \geq -k_l \\ \Delta W_i^{I*} + k_l & \text{if } -k_l \geq \Delta W_i^{I*} \end{cases} \tag{3.32}$$

where both k_u and k_l are positive.[48]

Among variables included in X_i other than the exogenous income shock, I use changes in family structure (a change in family size and a change in the number of kids), changes in economic conditions (a dummy for getting retired and a change in permanent income), and the state variable predetermined in the year 1999 (asset balances, employment status, health condition, age of household head). In particular, the initial illiquid asset portfolio is expected to capture the portfolio rebalancing motive which is common in (S,s)-type behavior; If the initial portfolio is too illiquid, the household tends to decrease illiquid asset holdings. If the initial portfolio is too liquid, the household tends to increase illiquid asset holdings. Hence, the initial portfolio balance should negatively enter the regression. The exogenous income shock is constructed as the permanent income computed for 2000[49] minus the actual real income in 2000. Therefore, having a positive value of the

[48] I can allow for shifts in the slope coefficients.

[49] The permanent income is essentially the average of real income streams from 1992 to 2000.

real income shock means that the household draws an adverse income shock in 2000. The coefficient of the adverse income shock is expected to be negative.

The results for the ordered probit model and the Rosset model, along with the OLS estimation are shown in Table 3.5. The estimation problems caused by the non-linearity of the underlining model are clear; In the OLS estimation, few variables are significant. On the other hand, the Rosset model can generate significant coefficients and the coefficient estimates are larger than the corresponding OLS estimates. The adverse shock enters negatively in all cases as expected. According to the estimate in the Rosset Model, upon an adverse income shock the household reduces illiquid asset holdings by 13 percent of the size of the income shock.

The portfolio rebalancing motive is captured by the coefficient of illiquid portfolios in 1999. Households starting with larger illiquid portfolios in 1999 tend to liquidate some illiquid assets over two years, while households starting with smaller illiquid portfolios tend to increase illiquid asset holdings.

The ordered probit model and the Rosset model also estimate the upper and lower thresholds which are shown in the last rows of Table 3.5. In both models, the lower threshold is highly significant, while the upper threshold is insignificant, implying that liquidation requires more transaction costs than investment in illiquid assets.

3.7 Conclusion

This chapter investigated empirical relationships between income uncertainty and portfolio choice behavior of U.S. households in the recent PSID panel. Guided by the theoretical results in the previous chapter, I focused on the role of income uncertainty and the precautionary motive to explain household portfolio choice behavior between illiquid and liquid assets. The theory tells us that most of precautionary wealth is held in the illiquid form rather than in the liquid form against rarely occurring large adverse shocks such as unemployment shocks, and permanent shocks. By constructing the degree of income uncertainty at the household level, I found positive effects of income uncertainty on not only total illiquid asset holdings but also proportions of household wealth invested in illiquid assets. A household facing the 80th percentile level of total income uncertainty would hold 96 percent more illiquid wealth and 32 percent more illiquid portfolio than a household facing the 20th percentile level of total income uncertainty.

To confirm that illiquid wealth is indeed held for the precautionary reasons, I tested whether illiquid assets are actually consumed immediately after households are hit by adverse income shocks. Using active changes in illiquid asset holdings between 1999 and 2001, my estimation results showed that households who experienced adverse income shocks in 2000 reduced illiquid wealth by 13 percent of the size of the income shock. I also found evidence of significant transaction costs in reducing illiquid wealth.

Overall, the empirical results corroborate the theoretical implications except the effect of temporary uncertainty on illiquid portfolio

allocations. The theory suggests that households facing more volatile temporary income tend to hold more liquid portfolios. However, I found the opposite in the regressions with the constructed household specific standard deviation of temporary income shocks. I argued that the difficulty in measuring unemployment risk separately from relatively small fluctuations of temporary income is responsible for the failure of finding the negative effect of temporary income uncertainty on illiquid portfolio allocations. I did not measure unemployment risk for each household simply due to the lack of enough information in the PSID data set. If one wants to investigate the relationship between income uncertainty and unemployment risk, It is feasible to construct a measure of unemployment risk by group of households who have similar characteristics from a larger data set and combine the measured unemployment risk at the group level with household wealth data. Since the size of precautionary wealth could be highly sensitive to even a small unemployment risk, such analysis will be desired in future research.

Table 3.1: Summary Statistics of Variables in the PSID Sample

Variable	Mean	Std.Dev.	1994 Mean	1999 Mean	2001 Mean
age of head	49.5	13.9			
sex of head	.177	.382			
education of head 1	.442	.497			
education of head 2	.212	.408			
education of head 3	.304	.460			
family size	2.89	1.385	2.98	2.88	2.81
number of kids	.887	1.127	1.00	.876	.780
married	.763	.425			
white	.913	.281			
self-employed	.123	.329	.124	.126	.118
total income (nominal)	65431	77081	52950	71617	78492
illiquid wealth	141761	660356	95678	147412	182637
liquid wealth	104056	762153	57728	111201	143176
illiquid portfolio	.692	.295	.703	.684	.687
uncertainty index	.084	.128			
permanent	.019	.113			
temporary	.068	.241			
sample size	4371	4371	1457	1457	1457

Note: Education level 1 is for completed grades between 9 and 12. Education level 2 is for grades between 13 and 15. Education level 3 is for all the higher grades.

Table 3.2: Results for $\log(W^I)$

variable	OLS	RE	RE-IV	RE-IV
uncertainty	0.49**	0.55**	7.91***	
	(2.16)	(1.99)	(5.37)	
permanent				21.74**
				(2.19)
temporary				8.44***
				(5.00)
year 1999	-0.07	−0.16***	−0.14*	-0.08
	(-1.04)	(-2.34)	(-1.41)	(-0.58)
year 2001	-0.01	-0.14	-0.10	-0.03
	(-0.21)	(-1.99)	(-0.88)	(-0.16)
d(health)	−0.25***	−0.12**	−0.11*	−0.12*
	(-3.19)	(-1.74)	(-1.45)	(-1.47)
log(shock)	−0.36***	−0.56***	−0.64***	−0.58***
	(-3.89)	(-10.55)	(-7.20)	(-4.85)
d(edu1)	0.59***	0.70***	0.62***	0.70**
	(3.72)	(3.95)	(2.81)	(2.12)
d(edu2)	0.60***	0.81***	0.75***	0.84**
	(3.63)	(4.26)	(2.73)	(2.07)
d(edu3)	0.42***	0.71***	0.76**	0.96**
	(2.55)	(3.71)	(2.26)	(1.91)
female head	0.11	0.07	0.17	0.10
	(0.78)	(0.39)	(0.84)	(0.39)
age	0.09***	0.14***	0.14***	0.15***
	(5.88)	(8.27)	(7.51)	(6.97)
age^2	−0.00***	−0.001***	−0.001***	−0.001***
	(-2.77)	(-5.98)	(-5.83)	(-5.33)
family size	−0.15***	-0.05	-0.06	0.04
	(-2.89)	(-1.01)	(-1.16)	(0.55)
♯ of kids	0.13***	0.01	0.03	0.04
	(2.52)	(0.19)	(0.40)	(0.55)
d(married)	0.85***	1.01***	1.21***	1.51***
	(5.69)	(6.02)	(4.67)	(3.87)
race	0.74***	0.86***	0.91***	0.91***
	(7.37)	(6.95)	(6.03)	(5.16)
log(permanent income)	1.26***	0.95***	0.94***	0.70*
	(22.87)	(14.86)	(3.15)	(1.43)

Notes:t statistics are in parentheses. {*,**,***}=Statistical significance at {10,5,1} percent.

Table 3.3: Results for S^I

variable	OLS	RE	RE-IV	RE-IV
uncertainty	0.20^{***}	0.20^{***}	1.18^{***}	
	(5.97)	(4.26)	(5.43)	
permanent				2.14^{*}
				(1.51)
temporary				1.08^{***}
				(5.19)
year 1999	-0.01	-0.01^{*}	-0.01	-0.01
	(-1.21)	(-1.39)	(-1.10)	(-1.18)
year 2001	0.00	0.00	0.00	0.00
	(-0.11)	(0.00)	(0.45)	(0.31)
d(health)	-0.01	0.00	0.00	-0.00
	(-0.40)	(0.22)	(0.23)	(-0.10)
log(shock)	0.00	0.00	-0.02	-0.02
	(0.23)	(0.27)	(-1.25)	(-1.22)
d(edu1)	-0.05^{**}	-0.05^{*}	-0.07^{**}	-0.08^{*}
	(-2.15)	(-1.56)	(-1.81)	(-1.60)
d(edu2)	-0.08^{***}	-0.08^{***}	-0.10^{**}	-0.12^{**}
	(-3.23)	(-2.37)	(-2.22)	(-2.04)
d(edu3)	-0.16^{***}	-0.16^{***}	-0.17^{***}	-0.18^{***}
	(-6.82)	(-4.94)	(-3.09)	(-2.47)
female head	-0.01	-0.00	0.01	0.01
	(-0.32)	(-0.16)	(0.41)	(0.38)
age	0.01^{***}	0.01^{***}	0.01^{***}	0.01^{***}
	(5.00)	(4.01)	(2.90)	(2.35)
age^2	-0.00^{***}	-0.00^{***}	-0.00^{***}	-0.00^{***}
	(-6.00)	(-5.06)	(-3.72)	(-2.65)
family size	0.03^{***}	0.02^{***}	0.02^{**}	0.02^{**}
	(3.72)	(2.49)	(1.86)	(1.69)
♯ of kids	-0.01^{*}	-0.01	-0.01	-0.01
	(-1.63)	(-0.98)	(-0.66)	(-0.53)
d(married)	0.09^{***}	0.11^{***}	0.13^{***}	0.15^{***}
	(3.86)	(3.85)	(2.92)	(2.37)
race	-0.03^{**}	-0.03^{*}	-0.04^{*}	-0.04^{**}
	(-1.64)	(-1.47)	(-1.48)	(-1.71)
log(permanent income)	-0.10^{***}	-0.10^{***}	-0.09^{*}	-0.10^{*}
	(-11.30)	(-8.14)	(-1.48)	(-1.30)

Table 3.4: Results with the Hausman-Taylor Method

	$\log(W^I)$	S^I
uncertainty	1.00^{*}	0.29^{**}
	(1.29)	(1.83)
year 1999	0.89^{**}	-0.07
	(2.31)	(-0.88)
year 2001	0.93^{***}	-0.05
	(2.57)	(-0.74)
d(health)	-0.01	0.02
	(-0.19)	(1.20)
log(shock)	-0.35^{***}	0.01
	(-6.43)	(0.82)
d(time)	-0.01^{***}	-0.00
	(-2.51)	(-0.14)
d(employment)	-0.03	-0.03^{**}
	(-0.29)	(-1.79)
g(permanent income)	8.60^{**}	-0.50
	(2.05)	(-0.62)
d(edu1)	0.69^{***}	-0.08^{***}
	(4.09)	(-2.33)
d(edu2)	0.47^{***}	-0.14^{***}
	(2.39)	(-3.42)
d(edu3)	0.02	-0.25^{***}
	(0.10)	(-5.21)

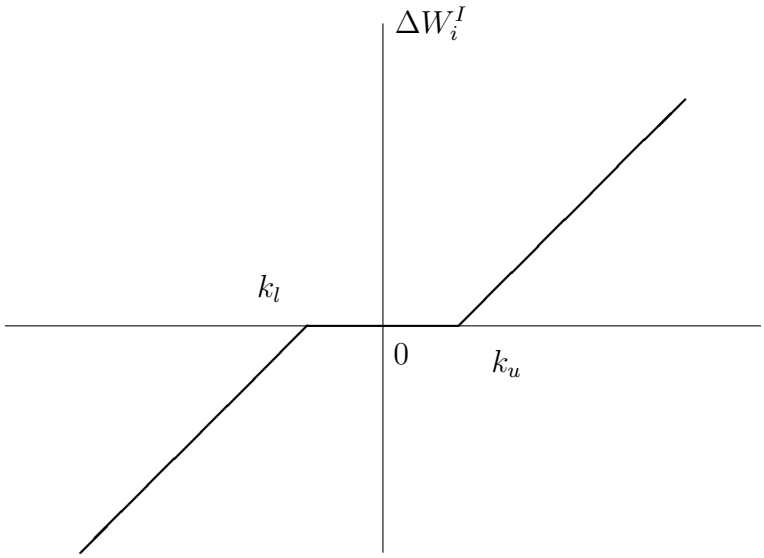
female head	-0.25^{*}	0.02
	(-1.41)	(0.47)
age	0.10^{***}	0.00
	(2.83)	(0.32)
age^2	-0.00^{**}	-0.00
	(-2.18)	(-1.00)
family size	0.06	0.01
	(0.89)	(0.98)
♯ of kids	-0.31^{**}	0.01
	(-2.17)	(0.38)
d(married)	-0.02	0.04
	(-0.10)	(1.04)
race	0.62^{***}	-0.06^{**}
	(4.46)	(-2.03)
log(permanent income)	0.28^{***}	0.00
	(7.60)	(0.05)
d(home)	1.90^{***}	
	(24.16)	

Notes: t statistics are in parentheses. {*,**,***}=Statistical significance at {10,5,1} percent. Occupation and industry code dummies and a dummy for being self-employed are included in the regressors but not shown. I allow for occupation and industry code dummies, a dummy for being self-employed, permanent income and uncertainty index to be correlated with the family random effect.

Table 3.5: Ordered Probit and Rosset Model

	OLS	Ordered Probit	Rosset
adverse shock	-58.45	-0.208^{**}	-127.48^{*}
	(-0.95)	(-1.763)	(-1.54)
$\triangle$ permanent income	67.15	0.552	315.68
	(0.18)	(0.978)	(0.64)
$\triangle$ family size	-61.21	-0.120^{*}	-115.94^{**}
	(-1.26)	(-1.633)	(-1.75)
$\triangle$ ♯ of kids	116.7^{**}	0.103^{**}	163.87^{**}
	(2.03)	(1.179)	(2.11)
d(retired)	-31.59	-0.201	-129.95
	(-0.29)	(-1.220)	(-0.87)
d(retired99)	-9.92	-0.197^{**}	-109.82
	(-0.13)	(-1.708)	(-1.02)
d(health99)	-2.66	-0.107	-53.84
	(-0.05)	(-1.250)	(-0.68)
d(unemployed99)	77.90	-0.448^{*}	-109.72
	(0.40)	(-1.510)	(-0.41)
d(debt99)	-2.96	-0.182^{***}	-79.48^{*}
	(-0.07)	(-2.764)	(-1.37)
portfolio99	-28.78	-0.371^{***}	-216.27^{**}
	(-0.40)	(-3.416)	(-2.20)
mortgage99	-0.15	0.002^{***}	0.39
	(-0.55)	(4.118)	(1.06)
age99	-1.93	-0.004	-3.89^{*}
	(-0.87)	(-1.258)	(-1.28)
d(white)	35.01	0.129	98.58
	(0.48)	(1.204)	(1.00)
d(female head)	-13.59	-0.105	-63.66
	(-0.24)	(-1.278)	(-0.83)
first cut off		-1.289^{***}	
		(-6.100)	
second cut off		-0.259	
		(-1.237)	
k_l			1165.96^{***}
			(6.00)
k_u			149.92
			(0.79)
σ			997.90^{***}
			(40.39)

Figure 3.1: Rosset Model



CHAPTER 4

Estimating a Wealth Effect on Consumption

4.1 Introduction

The amount of asset holdings is one of essential factors which determine household consumption expenditures. The theoretical link between household consumption and wealth is clearly specified in life cycle models developed during the 1950s in a number of important contributions such as Modigliani and Brumberg (1954) and Modilgliani and Ando (1963). They model a consumer's decision as a problem in which the consumer intertemporally allocates the total amount of resources available over the lifetime. Since a consumer with a concave utility function wants to smooth consumption expenditures over time, she does not spend all of an increase in wealth immediately. Solving the standard life-cycle model, the marginal propensity to consume out of an increase in wealth can be derived as a function of preference parameters, interest rate, and the remaining length of lifetime.[50] Since the life cycle model provided solid microfoundations to the study of consumption which was previously dominated by the Keynesian models[51], the wealth effect on consumption has been built into almost all practical models of consumption.

Recent developments in time series econometrics — in particu-

[50]For example, a typical consumption function has a form: $C_t = (1 - R^{-1}(R\beta)^{1/\rho})(W_t + H_t)$ where W_t is wealth holdings, H_t is present discount value of labor income, for an infinitely living consumer with a CRRA utility function and without uncertainty.

[51]The Keysian models assume that consumption is a simple function of current disposable income with no wealth effect.

lar, substantial progress in cointegration analysis — have led to a new method to estimate the wealth effect on consumption.

The new approach, or "Vector Error Correction Model (VECM)" approach, relies on the existence of a stable long-run relationship between consumption, income, and wealth. We show a typical estimate in this framework and explain how to interpret the results. One of the implications derived from the estimation results is that consumption responds immediately and fully to permanent innovations in income and wealth rather than adjusting sluggishly to the shocks.

A separate, and older, literature developed after Hall (1978) pointed out the random-walk property implied by theoretical models of consumption. According to the pure permanent income hypothesis, variables lagged $t-1$ or earlier should not help predict consumption growth in period t. However, in practice, most of forecast models of consumption use lagged macro variables including lagged wealth growth in regressions since consumption spending seems to adjust sluggishly in the data. (Brayton and Tinsley (1996)) They justify their estimating specification by arguing that consumers may not always behave exactly as predicted by the simplest version of the PIH theory and may adjust their spending with a delay since it takes time for them to make a decision on purchase. (Davis and Palumbo (2001))

It is fair to say that the VECM approach of estimating the wealth effect has recently become very popular among economists. However, this approach does not have a theoretically coherent background in the same sense that typical macro forecast models ignore what a microeconomic consumption theory tells about consumption dynamics by assuming the ad-hoc adjustment lags in spending. This paper ar-

gues that the new method is misguided in estimating the MPC out of wealth shock and propose an alternative approach which is theoretically founded and consistent with important empirical features of consumption such as the excess sensitivity of consumption to past information and the excess smoothness of consumption movements.

Our approach proposes estimating the wealth effect based on the Euler equation of a habit formation model. We demonstrate that the habit model can explain the forecastability of consumption by lagged wealth growth. Furthermore, we present a simple method to estimate the MPC out of wealth shock in our framework and generate a wealth effect estimate which is comparable to the estimates found in the existent literature. The immediate MPC out of a wealth shock is estimated around 0.9 percent while the long-run MPC out of a wealth shock is found to be around 4 percent. Habits reduce the immediate MPC significantly and make adjustments of consumption with respect to an initial shock gradual and smooth.

The rest of the paper proceeds as follows. Section 4.2 lays out the VECM approach and estimates a system of three variables: consumption, income and wealth. Then, we point out several problems in this approach, motivating our preferred method. Section 4.3 describes our approach, the Euler equation approach with a habit-forming consumer, and estimates both the immediate and long-run MPC out of wealth shocks. Finally, Section 4.4 concludes.

4.2 The Vector Error Correction Model Approach

In this section, we start by deriving the fundamental formula on which the VECM approach is based, then describe the estimation method used for statistical analysis. This approach builds up on the notion that a general household budget constraint *with a couple of strict assumptions* implies that log consumption, log labor income, and log asset wealth are cointegrated. Hence, first, we show how a simple wealth accumulation equation can be manipulated to arrive at the cointegrating relationship among consumption, income, and wealth in logs. After deriving the relationship, we explain why the assumptions on which it is based are problematic.

Let us start from the following general budget constraint:

$$W_{t+1} = R_{t+1}(W_t + Y_t - C_t) \tag{4.1}$$

where W_t is the amount of asset holdings at the beginning of period t, Y_t is labor income, C_t is consumption, and R_{t+1} is the gross rate of return on savings between time t and $t+1$. By iterating the above budget constraint forward, we can obtain the intertemporal budget constraint:

$$W_t + \sum_{i=0}^{\infty} \frac{1}{R_{t+1}R_{t+2}...R_{t+i}} Y_{t+i} = \sum_{i=0}^{\infty} \frac{1}{R_{t+1}R_{t+2}...R_{t+i}} C_{t+i} \tag{4.2}$$

Let us call the second term on the left hand side Γ_t and the term on the right hand side Φ_t. Notice that Γ_t evolves as follows.

$$\Gamma_{t+1} = R_{r+1}(\Gamma_t - Y_t) \tag{4.3}$$

$$\frac{\Gamma_{t+1}}{\Gamma_t} = R_{t+1}\left(1 - \frac{Y_t}{\Gamma_t}\right) \tag{4.4}$$

Taking the logarithm of both sides,

$$\begin{aligned}
\log \Gamma_{t+1} - \log \Gamma_t &\approx r_{t+1} + \log(1 - \exp(\log Y_t - \log \Gamma_t)) && (4.5)\\
&\approx r_{t+1} + \left(1 - \frac{1}{1 - Y/\Gamma}\right)(\log Y_t - \log \Gamma_t - \log Y + \log \Gamma) && (4.6)\\
&= r_{t+1} + \left(1 - \frac{1}{\gamma}\right)(\log Y_t - \log \Gamma_t) + k && (4.7)
\end{aligned}$$

where we use Taylor expansion around $\log Y - \log \Gamma$ from the first to the second row; $1 - Y/\Gamma = \gamma$; k absorbs all the unimportant constant terms. The left hand side can be expanded as

$$\log \Gamma_{t+1} - \log \Gamma_t = \Delta \log Y_{t+1} + \log Y_t - \log \Gamma_t - (\log Y_{t+1} - \log \Gamma_{t+1}) \quad (4.8)$$

Substituting this expression in Equation (4.7), we can reach

$$\frac{1}{\gamma}(\log Y_t - \log \Gamma_t) = (\log Y_{t+1} - \log \Gamma_{t+1}) - \Delta \log Y_{t+1} + r_{t+1} + k \quad (4.9)$$

Iterating this forward, we obtain

$$\log Y_t - \log \Gamma_t = \sum_{i=1}^{\infty} \gamma^i (r_{t+1} - \Delta \log Y_{t+i}) + k_y \quad (4.10)$$

Proceeding in the same way for Ψ_t[52],

$$\log C_t - \log \Psi_t = \sum_{i=1}^{\infty} \gamma^i (r_{t+1} - \Delta \log C_{t+i}) + k_c \quad (4.11)$$

Going back to the original intertemporal budget constraint (4.2) and

[52] We assume $\gamma = 1 - Y/\Gamma = 1 - C/\Psi$

applying log-linearization to it,

$$W_t + \Gamma_t = \Psi_t \tag{4.12}$$

$$\log \Psi_t - \frac{\Gamma}{\Psi} \log \Gamma_t - \frac{W}{\Psi} \log W_t = k \tag{4.13}$$

$$\log C_t - \frac{\Gamma}{\Psi} \log Y_t - \left(1 - \frac{\Gamma}{\Psi}\right) \log W_t = \tag{4.14}$$

$$\sum_{i=1}^{\infty} \gamma^i \left(\frac{\Gamma}{\Psi} \Delta \log Y_{t+i} - \Delta \log C_{t+i} + \left(1 - \frac{\Gamma}{\Psi}\right) r_{t+i} \right) \tag{4.15}$$

We substitute Equation (4.10) and (4.11) for $\log \Gamma_t$ and $\log \Psi_t$ in the last row. Then, Equation (4.15) is the one used as a rationale for the VECM approach. If $\Delta \log Y_t$, $\Delta \log C_t$, and r_t are all stationary variables, the right hand side is stationary, and so is the left hand side. In that case, three variables on the left hand side, $\log C_t$, $\log Y_t$, and $\log W_t$, should be cointegrated with a single cointegrating vector and each variable may be expressed as a function of two common trends and one temporary shock. Given the above result, the VECM framework focuses on the system of these three variables, trying to estimate the cointegrating relationship and the short-run adjustment process when a deviation from the long-run trend occurs. With a couple of more assumptions on the structure of the error process, one can also identify permanent and temporary components of shocks to the three-variable system.

A theoretical problem of the cointegrating approach can be pointed out in the above derivation. In obtaining Equation (4.15), we had to assume that the ratio of market wealth to total wealth Γ/Ψ, or equivalently the ratio of market wealth to human wealth is stationary. Neither partial nor general equilibrium theory provides a plausible rational for

this stationarity assumption.

In any sensible macroeconomic models, the long-run relationship between (the logs of) consumption, income and wealth is bound to be determined by the steady state interest rate, productivity growth rate, population growth rate, and tax rate and so on. If any of these parameters change permanently over the time frame examined, and that is highly likely to be the case, considering the fact that there is much empirical evidence of structural breaks in the US economy (Stock and Watson (1996)), any efforts to find one unique cointegrating relationship between three variables would be futile. Even without any important changes in fundamental parameter values of the model, it is doubtful that transitional dynamics towards the steady state is fast enough to identify the steady state relationship with the amount of data available.

4.2.1 Data and Preliminary Analysis

The appendix contains a detailed description of the data used in this paper. The net value of wealth, W_t, is a real, per capita household net worth, which includes financial wealth as well as tangible wealth (housing wealth and consumer durables). We use a real, per capita consumption expenditure on nondurables and services as a measure of C_t. Labor income, Y_t, is wage and salary disbursement plus transfer income after subtracting off personal contributions for social insurance. To deflate nominal variables, we use the implicit price deflator for personal consumption. Our data span from the second quarter of 1952 to the third quarter of 2003.

Figure 4.1 shows the time-series movements of log real wealth, log

real stock wealth, and log real non-stock wealth. As one can see, stock wealth is the main source of total fluctuations in asset values, though it is smaller than non-stock wealth as a fraction of total wealth.

All the variables used in this paper are tested for the existence of a unit root and the results are presented in Table 4.1. ADF test statistics indicate that all of the variables except consumer sentiment index and unemployment rate (in levels) reject a null of no unit root at the 1 percent significant level hence should be first-differenced.

4.2.2 Estimation in the VECM Approach

Commonly, econometric analysis in the VECM approach is done in two steps. The first step is to estimate the long-run cointegrating relationship among consumption, labor income, and wealth (in logs). The second step is to estimate the short-run adjustment process towards the long-run common trend, assuming that the cointegrating relationship estimated in the first step is true. Rigorously speaking, there could be one or two cointegrating vectors among three variables, hence, one needs to first test the number of cointegrating vectors before proceeding further. Lettau and Ludvigson (2004) argue that there exists only one cointegrating vector, which we also assume in this section for the sake of argument.[53] However, the very existence of unique cointegrating relationship among three variables is yet questionable as we explain later in this section.

In the first step, the long-run cointegrating relationship among $\log C_t$, $\log Y_t$, and $\log W_t$ is estimated by the dynamic least squares

[53] They use Johansen's procedures to estimate the number of cointegrating relationships.

procedure (Stock and Watson (1993)), where the coefficient on consumption is normalized to one.

$$\log C_t = \mu + \beta_y \log Y_t + \beta_w \log W_{t-1} + \epsilon_t \tag{4.16}$$

If three variables are cointegrated, the dynamic least squares estimates for β_y and β_w are superconsistent and shown in Table 4.2. Moreover, the cointegraing residual, $\hat{ec}_t = \log C_t - \hat{\beta}_y \log Y_t - \hat{\beta}_w \log W_{t-1}$, should be an I(0) variable and capture temporary deviations from the long-run trend. The cointegrating vector is denoted as $\beta = (1, -\beta_y, -\beta_w)'$. According to the estimate, $\hat{\beta} = (1, -0.608, -0.404)'$, consumption would increase by 6.6 cents in the long run with respect to a permanent dollar increase in wealth and by 61 cents with resect to a dollar increase in income.[54]

A cointegrated set of variables $X_t = (\log C_t, \log Y_t, \log W_{t-1})'$ have a Vector Error Correction Model (VECM) specification:

$$\begin{aligned} \Delta X_t &= \nu + \alpha\beta' X_{t-1} + \Pi(L)\Delta X_{t-1} + e_t \\ &= \nu + \alpha \cdot ec_{t-1} + \Pi(L)\Delta X_{t-1} + e_t \end{aligned} \tag{4.17}$$

where ΔX_t is a vector of log first differences, $(\Delta \log C_t, \Delta \log Y_t, \Delta \log W_t)'$. $\Pi(L)$ is a finite order distributed lag operator. The vector, $\alpha = (\alpha_c, \alpha_y, \alpha_w)'$, captures adjustment of each variable to a one-period lagged deviation from the long-run relationship. The VECM is estimated after substituting the cointegrating residual $\hat{ec}_{t-1}$ obtained from the first step in Equation (4.17). Table 4.3 presents the results with a first-order

[54]These numbers are converted into levels from logs by using the consumption-wealth and consumption-income ratios in the most recent quarter (second quarter 2001).

specification of $\Pi(L)$. By comparing the magnitude of three $\alpha's$, it is known which variable does most of adjustment when a deviation from the long-run trend occurs. The larger the α is, the more of adjustment the variable does to restore the long-run relationship. According to the estimates, only α_w is statistically significantly different from zero, meaning that only wealth adjusts to eliminate a temporary deviation from the long-run trend. The similar results are found by Lettau and Ludvigson (2004).

Given the estimated VECM parameters, Lettau and Ludvigson (2004), Fernandez-Corugedo, Price and Blake (2003) and others further try to decompose asset value fluctuations into permanent and temporary components with the methodology which exploits the cointegrating relationship in the system. (King, Plosser, Stock and Watson (1991), Gonzaro and Granger (1995) and Gonzalo and Ng (2001)) Since the cointegrating relationship imposes certain restrictions on the long-run multipliers of shocks to the system, it is possible to identify two permanent shocks and one temporary shock out of three estimated residuals from the VECM.[55] Applying the permanent/transitory decomposition, Lettau and Ludvigson (2004) find that the majority of asset fluctuations is of temporary nature and have little effect on consumption spending. Consumption would certainly respond to *permanent* movements in income and wealth, as warranted by the cointegrating relationship, but almost none to temporary movements in income and wealth. Since most of asset value movements are estimated as

[55] A permanent shock η_{Pt} should satisfy $\lim_{h\to\infty} \partial E_t[x_{t+h}]/\partial \eta_{Pt} \neq 0$. A temporary shock η_{Tt} should satisfy $\lim_{h\to\infty} \partial E_t[x_{t+h}]/\partial \eta_{Tt} = 0$. Gonzaro and Granger (1995) suggest a matrix $G = \begin{bmatrix} \alpha'_\perp \\ \beta' \end{bmatrix}$ to transform residuals into permanent and temporary shocks, where $\alpha'_\perp \gamma = 0$.

temporary, asset value fluctuations are largely unrelated to consumption movements. Such a conclusion crucially hinges on the estimated values of adjustment coefficients, α. A large magnitude of α means, in the permanent/temporary decomposition framework, that the variable is driven more by temporary shocks than by permanent shocks. On the other hand, a small magnitude of α indicates that the variable consists more of permanent components, or common trends, than of temporary component. Similar to their results, our estimate of α_c is not significantly different from zero, suggesting that consumption would not do much adjustment when a deviation from the long-run trend occurs and that consumption movements are mainly due to permanent innovations in income and wealth.[56] In sum, based on the zero adjustment of consumption to temporary deviations from the common trend, one might argue that consumption does not adjust sluggishly, rather responds immediately to permanent innovations in income and wealth. In contrast to $\hat{\alpha}_c$ and $\hat{\alpha}_y$, the adjustment coefficient of wealth $\hat{\alpha}_w$ is significantly positive, by which one might interprets that most of temporary deviations from the common trend are equilibrated by subsequent movements in asset values.

4.2.3 Problems with the VECM Approach

There are a couple of serious problems in the VECM approach. In this section, we point out these problems and motivate our preferred approach to estimate the wealth effect on consumption, which will be discussed in detail in the next section.

[56] Lettau and Ludvigson (2004) calculate that over 95 percent of the variance for the forecast error of consumption is explained by two permanent innovations at all horizons.

First of all, there is a question on the very existence of unique, stable cointegrating relationship among consumption, income, and wealth. As we already mentioned in the theory section, theoretical grounds for the existence of long-run stable relationship between three variables is questionable in either partial or general equilibrium models. If there is no such relationship, the entire argument in the VECM approach would be problematic, since this approach highly relies on the estimation of a stable cointegrating vector.

According to the formula (Equation (4.15)), the cointegrating vector, $(1, -\beta_y, -\beta_w)$, should correspond to $(1, \Gamma/\Psi, 1-\Gamma/\Psi)$, where Γ/Ψ is the ratio of the present discounted value of human wealth to total wealth (total wealth = human wealth plus market wealth). There, we had to assume that the human to physical wealth ratio is stationary in order to obtain the stable cointegrating vector among the logs of consumption, income, and wealth. However, stationarity of the wealth ratio is a rather strong assumption and a recent literature has found substantial evidence of instability in such cointegrating vectors. (Brennan and Xia (2002), Hahn and Lee (2001))

Secondly, the standard VECM approach looks at only three variables and ignores other variables which can predict consumption growth. We can extract the consumption dynamics equation from the VECM system.

$$\Delta \log C_t = \nu_c + \alpha_c \hat{ec}_{t-1} + \Gamma_c(L)[\Delta \log C_{t-1}\ \Delta \log Y_{t-1}\ \Delta \log W_{t-2}]' + e_{ct} \tag{4.18}$$

The consumption equation includes one error correction term and lagged growth rates of consumption, income, and wealth. As shown in the

bottom row of Table 4.3, these regressors explain only 13 percent of variation in consumption growth. In consumption literature, it is well known that variables other than lagged wealth and income growth can also explain consumption growth. (Carroll, Fuhrer, and Wilcox (1994) and Bacchetta and Gerlach (1997)) If a set of variables constituting the system are actually larger than just three variables included, an omitted variable bias would make any interpretation of estimation results hard to believe. Furthermore, if any of variables in the system contains measurement error, one would get downward-biased parameter estimates and the residuals would not translate themselves into identifiable white noise shocks. As we will discuss below, when aggregate consumption expenditures contain measurement error, the residuals from consumption growth equation would follow a serially correlated process mixing up white noise shocks and measurement errors.

Finally, this approach ignores the optimal consumption dynamics which should be derived from the consumer theory. The consumer model solves for the optimal path of consumption as a result of intertemporal optimization, or an Euler equation. In the framework of the pure PIH theory, the optimal consumption path follows a random walk with a drift term. ($\Delta \log C_t = \nu_c + \Delta \log C_{t-1} + e_{ct}$) Though more elaborate consumer models would lead to consumption dynamics different from a simple random walk, it is not clear which consumer model would justify the consumption growth equation in the VECM without restrictions (Equation (4.18)).

4.3 The Euler Equation Approach

In this section, we introduce an alternative approach to estimate the wealth effect on consumption. Our approach is based on the Euler equation of a habit-forming consumer.

The simplest habit formation model assumes that a consumer maximizes an infinite stream of discounted utility, subject to the budget constraint (4.1).

$$\max_{\{C_s\}} E_t \sum_{s=t}^{\infty} \delta^{s-t} U(C_s - \rho C_{s-1}) \tag{4.19}$$

where δ is the time discount factor, C_{t-1} is the habit stock to which the consumer compares the amount of current consumption, and ρ captures the degree of habit persistence. For a given amount of current consumption, a larger habit stock lowers utility. Assuming a CRRA outer utility function for $U(\cdot)$, the Euler equation for the habit-forming consumer is approximated as,

$$\Delta \log C_t = \mu + \rho \Delta \log C_{t-1} + \epsilon_t \tag{4.20}$$

where ϵ_t is a white noise process capturing the innovations to income and wealth. (Dynan (2000)) The Euler equation tells us that the growth rate of consumption is serially correlated through habit forming behavior captured by ρ. Simple ordinary least squares estimation of the above Euler equation might not yield a consistent value for ρ for two reasons. First, time aggregation in quarterly data would cause ϵ_t to be a MA(1) process with a positive coefficient rather than a white noise process. (Working(1960)) Then, $\log C_{t-1}$ would be correlated with the

error term and an estimation of ρ would be upward-biased. Second, aggregate consumption expenditure data might suffer from a large measurement error, as a substantial proportion of quarterly consumption data is either imputed or interpolated. (See Wilcox(1992) and Sommer (2003))[57] Also, the actual consumption data is subject to transitory fluctuations such as weather-related fluctuations that are not incorporated into the theoretical specification. The existence of a transitory measurement error or a transitory noise in the growth rate of consumption would bias an estimate of ρ downwards. Let us specify these two sources of biases in the following manner.

$$\Delta \log C_t^* = \mu + \rho \Delta \log C_{t-1}^* + \upsilon_t + \theta \upsilon_{t-1} \tag{4.21}$$

$$\log C_t = \log C_t^* + \tau_t \tag{4.22}$$

where $\log C_t^*$ is a "true" log-level consumption, θ captures the MA(1) coefficient caused by time-aggregation, and a measurement error in consumption expenditure is specified as a white noise τ_t.[58] Combining two equations above, the Euler equation with habit formation in terms of measured consumption expenditures becomes,

$$\begin{aligned} \Delta \log C_t &= \mu + \rho \Delta \log C_{t-1} + \upsilon_t + \theta \upsilon_{t-1} \\ &\quad \tau_t + (1-\rho)\tau_{t-1} - \rho \tau_{t-2} \qquad (4.23) \\ &= \mu + \rho \Delta \log C_{t-1} + \xi_t \qquad (4.24) \end{aligned}$$

[57] In retail sales data which account for approximately one half of aggregate consumption, imputation is used to fill missing values for non-respondents, amounting to as high as 25 percent of the total retail sales. (Bureau of Economic Analysis, 2000) The major component of service consumption, housing, is not directly measured but imputed.

[58] Sommer (2003) assumes that the measurement error process follows an MA(1) rather than a white noise.

The error term ξ_t in the Euler Equation has a MA(2) structure which mixes innovations to income and wealth v_t and consumption measurement errors τ_t. To obtain a consistent estimate of ρ, we need to utilize instruments for the lagged consumption growth.

4.3.1 Wealth Effect in the Euler Equation Approach

Before we proceed to estimating the Euler equation (4.23), we would like to show how a concept of the wealth effect fits in our approach.

There is substantial evidence that consumption growth is forecastable against the random-walk property of consumption in the pure permanent income hypothesis. (Campbell and Mankiw (1989), and Carroll, Fuhrer, and Wilcox (1994)) Lagged wealth growth is among many variables which can forecast consumption growth. Then, significant coefficient estimates on lagged and/or contemporaneous wealth growth terms in regressions of consumption growth have been interpreted as a measure of the wealth effect.

In past empirical work, wealth effect estimation in the Euler equation framework has been mostly done with cross-section or panel data sets. Case, Quigley and Shiller (2001) estimate a stock wealth effect and a housing wealth effect on consumption separately by regressing consumption growth on changes in stock and housing wealth and changes in income in a panel of countries as well as states of the US. They find a statistically significant and rather large effect of housing wealth on consumption, but a relatively small or insignificant effect of financial wealth on consumption. Using household panel data from the Consumer Expenditure Survey, Dynan and Maki (2001) find a direct impact

of contemporaneous and lagged changes in stock prices on the spending of households that own stock market wealth. Their estimate of the stock wealth effect is 5-15 cents with respect to a one dollar increase in stock wealth, with the effect occurring gradually over a couple of years.

To see if there are any significant lagged wealth effects on consumption growth in our aggregate quarterly data, the following regression is run,

$$\Delta \log C_t = \mu + A(L)\Delta \log W_{t-1} + \epsilon_t \tag{4.25}$$

where A(L) is a lag operator. We do not include the contemporaneous wealth growth term in the regression, since the term would be seriously correlated with the error ϵ_t. As Table 4.4 displays, lagged wealth effects are significantly positive up to four quarters back and we soundly reject a null that there is no lagged wealth effect on consumption growth with a p-value of 0.000. This result is another evidence against the pure random walk property of consumption. We noted earlier that time-aggregation in quarterly data is likely to cause an innovation to consumption growth ϵ_t to be a MA(1) process rather than a white noise. Hence, Column 2 of Table 4.4 shows the estimation result when lagged wealth growth at $t-1$ is instrumented, and we again reject the null of no wealth effect at a less than 1 percent significance level. The finding that up to four quarters past information about wealth value movements is significant in consumption growth regressions simply confirms a well-known puzzle that consumption exhibits "excess smoothness": Consumption seems to respond with a delay with respect to wealth shocks, while, according to the standard PIH model, consumption should jump immediately to current "news" about lifetime resources.

There are more variables other than lagged wealth growth which have been found to predict consumption growth well. Campbell and Mankiw (1989) argue that any variables which can predict current income growth also predict current consumption growth since a fraction of consumers spend all of their disposable income at every period, hence for these "rule-of-thumb" consumers current consumption is equal to current income. Campbell and Mankiw's interpretation still leaves some empirical features of consumption dynamics unanswered. For example, Carroll, Fuhrer and Wilcox (1994) show that lagged consumer sentiment indices have a forecasting power for current consumption growth above and beyond its ability to predict current income growth. Furthermore, micro data sets show no evidence that many households behave as a "rule-of-thumb" consumer and set $C = Y$ making no saving. Some other papers point out that not only the predicted current income growth but also its lagged terms also forecast consumption growth. (Demery and Duck (2000) and Reis (2003)) The excess sensitivity of consumption to past information is again suggestive of slow adjustment of consumption to shocks. Using the VAR estimation, Furher (2003) demonstrates that consumption expenditure exhibits a significant delay and hump-shaped response to shocks. He further argues that the observed short-run consumption dynamics to shocks can be explained well by including habit formation in the consumer utility function.

Sommer (2003) also argues that a simple habit formation model can explain the correlation of consumption growth with predicted current income growth and lagged consumer sentiment indices. However, he does not consider another correlation of consumption growth with

lagged wealth growth. Instead, our focus is to reinterpret the correlation between consumption and wealth growth in a habit formation model. As the Euler equation (4.20) clearly shows, habits give an incentive to smooth growth rate of consumption. Consequently, consumption growth responds to innovations in income and wealth only gradually, therefore, lagged variables that contain some information on innovations to income and wealth can forecast current and future consumption growth unlike the case of the pure PIH. Moreover, the habit formation model tells us that after controlling the lagged consumption term, lagged information should not help forecast current consumption growth any more. Hence, if the simple habit formation model is true, the lagged wealth effects previously found in the consumption growth regression (4.25) should all be gone when we also include the lagged consumption growth term.

$$\Delta \log C_t = \mu + \rho \Delta \log C_{t-1} + A(L) \Delta \log W_{t-1} + \xi_t \tag{4.26}$$

We therefore want to test whether the coefficients in A(L) are all equal to zeros. As we mentioned in the previous section, the error term ξ_t follows an MA(2) process reflecting both time aggregation and measurement errors in consumption expenditures. Therefore, we will test a null that the simple habit forming model is true by using the two stage least squares method. To ensure that our instruments are not correlated with ξ_t (i.e. the instrument set are valid), variables in the instrument set are all lagged for at least three quarters. A set of instruments include variables which are commonly used in the literature. Namely we choose to include consumer sentiment index, changes in fed-

eral funds rate, and lagged growth rates of consumption, income, and wealth.

The regression results are reported in Column 3 and 4 of Table 4.4. Since wealth growth rates at $t-1$ and $t-2$ could be correlated with the MA(2) error term ξ_t, they are also instrumented in Column 3.[59] The estimated autoregressive coefficient ρ is statistically significant around 0.78. The test statistics are shown in the row labelled "P-Value" and fail to reject the null that all the lagged wealth effects are jointly zeros at a larger than 10 percent significant level. This finding supports an interpretation that the simple habit formation model is a good characterization of the actual consumption dynamics. In the other words, the lagged wealth effects found in the regression (4.25) are due to serial correlation in consumption growth caused by habit forming behavior. Column 4 drops the instrumented lagged wealth growth terms and keep only the wealth effects orthogonal to ξ_t. Again, the p value is large enough not to reject the null of no lagged wealth effects on consumption beyond habit persistence.

Column 5 and 6 restrict all lagged wealth effects to zeros as they are found jointly insignificant in the former columns. The degree of habit persistence ρ is estimated as high as 0.756 when instrumented in Column 6. Comparing with an ρ estimate of 0.410 without instrumenting in Column 5, it seems that the measurement error (downward) bias is dominantly large relative to the time aggregation (upward) bias.

Sommer (2003) finds that once the habit term $\Delta \log C_{t-1}$ is included, the excess sensitivity of consumption to lagged consumer senti-

[59]We add changes in stock price index to the instrument set to predict wealth growth rates separately from consumption growth.

ment indices disappears although lagged sentiment indices have a substantial predicting power for consumption growth in themselves. Our findings that lagged wealth effects are also eliminated after including the habit term strengthen the case for habit formation models of consumption.

4.3.2 The Marginal Propensity to Consume out of Wealth Shock

We saw that all the lagged wealth effects are gone once serial correlation in consumption growth is properly considered as the Euler equation of a habit-forming consumer suggests. Then, how can we know about the size of a wealth effect on consumption expenditures in our framework? The fact that lagged wealth growth rates have a predicting power for consumption growth as in Equation (4.25) means that there exists a wealth shock affecting consumption growth in the first place. Such an initial wealth shock would continue to have its effect on future consumption growth through habit forming preference. Since we already have an estimate of the habit persistence parameter ρ in hand, we know how much of the initial shock will be carried over into future consumption growth. Hence, the answer is that we can back out the immediate marginal propensity to consume out of wealth shock given the estimated ρ.

Go back to the Euler equation of a habit-forming consumer.

$$\Delta \log C_t = \mu + \rho \Delta \log C_{t-1} + \epsilon_t \tag{4.27}$$

Any initial shocks to consumption should show up in the error term ϵ_t

and be carried over into future consumption through ρ.

$$\begin{aligned}
\Delta \log C_t - \rho \Delta \log C_{t-1} &= \mu + \epsilon_t && (4.28)\\
\Delta \log C_t - \rho^2 \Delta \log C_{t-2} &= \mu(1+\rho) + \epsilon_t + \rho\epsilon_{t-1} && (4.29)\\
&\vdots \\
\Delta \log C_t - \rho^k \Delta \log C_{t-k} &= \tilde{\mu} + \epsilon_t + \rho\epsilon_{t-1} + \cdots + \rho^{k-1}\epsilon_{t-k+1} && (4.30)
\end{aligned}$$

Past shocks ϵ_{t-s} have decreasing effects on the "adjusted" current consumption growth, $\Delta \log C_t - \rho^k \Delta \log C_{t-k}$, with a decaying rate of ρ.

Let us define the immediate effect on log consumption out of log wealth shock as $\alpha = \partial\epsilon_t / \partial \Delta \log W_t$. Log wealth shock at time $t-s$ should constitute part of ϵ_{t-s}. Therefore, the effect of log wealth shock at time $t-1$ on the adjusted consumption growth at time t is $\rho\alpha$. In the same way, the effect of log wealth shock at time $t-s$ on the adjusted consumption growth at time t is $\rho^s\alpha$ for any integer s. Using this logic, we can collect the accumulated effects of lagged log wealth shocks on the adjusted consumption growth as follows.

$$\begin{aligned}
& \Delta \log C_t - \rho^k \Delta \log C_{t-k} \\
= \; & \tilde{\mu} + \rho^2\alpha\Delta \log W_{t-2} + \cdots + \rho^{k-1}\alpha\Delta \log W_{t-k+1} + \tilde{\eta}_t \\
= \; & \tilde{\mu} + \alpha(\rho^2 \Delta \log W_{t-2} + \cdots + \rho^{k-1}\Delta \log W_{t-k+1}) + \tilde{\eta}_t \quad (4.31)
\end{aligned}$$

Consider a set of equations from $k=3$ to $k=5$.

$$\begin{aligned}
\Delta \log C_t - \rho^3 \Delta \log C_{t-3} &= \mu_3 + \alpha(\rho^2 \Delta \log W_{t-2}) + \tilde{\eta}_{3t} \\
\Delta \log C_t - \rho^4 \Delta \log C_{t-4} &= \mu_4 + \alpha(\rho^2 \Delta \log W_{t-2} + \rho^3 \Delta \log W_{t-3}) + \tilde{\eta}_{4t}
\end{aligned}$$

$$\Delta \log C_t - \rho^5 \Delta \log C_{t-5} = \mu_5 + \alpha(\rho^2 \Delta \log W_{t-2} + \rho^3 \Delta \log W_{t-3} + \rho^4 \Delta \log W_{t-4}) + \tilde{\eta}_{5t} \quad (4.32)$$

Each of the above four equations can be estimated separately, after substituting the estimated ρ. If the simple habit formation model is true and the immediate effect on log consumption out of log wealth shock α is stable, all the equations share the same coefficient on the term involving lagged wealth shocks. Hence, we can also test if all the equations share the common coefficient α, and if that is true, we can estimate the set of equations simultaneously by imposing the common effect on log consumption out of log wealth shock. The results are shown in Table 4.5. In all regressions, we also include income shock terms in addition to wealth shock terms since these two shocks are major ones to drive consumption dynamics and likely to be correlated with each other. We confirm that the immediate wealth effects on log consumption are stable across the regressions and estimate that the α is about 6 percent and the immediate effect on log consumption out of log income shock is around 12 percent. The estimated α is not itself an MPC, since the model is written in terms of growth rates and α measures the relationship between the growth rate of consumption and the growth rate of wealth. To obtain an estimate of the MPC from α, we multiplied it by the ratio of consumption to wealth in the most recent data. (Or the ratio of consumption to income in the case of the MPC out of income shock.) The estimated immediate MPC out of wealth shock as such is about 1 percent. However, there is a simple way to obtain an MPC directly from regressions: We can use the ratio of changes in wealth to the initial level of consumption in place of the

growth rates. Namely, we use

$$dC_t = (C_t - C_{t-1})/C_{t-6} \tag{4.33}$$

$$dW_t = (W_t - W_{t-1})/C_{t-6} \tag{4.34}$$

in place of $\Delta \log C_t$ and $\Delta \log W_t$. We use 6 periods lagged consumption as the initial consumption since C_{t-6} is the earliest lagged variable appearing in a set of equations (4.32). The results are in Columns 2 and 2-R of Table 4.5. It turns out that the directly estimated MPCs are very close to the calculated MPCs from the growth rate regressions. The MPC out of wealth shock is about 0.9 percent and the MPC out of income shock is about 12 percent.

So far we assumed that wealth shock (or income shock) is simply equal to a growth rate or a change in wealth (or income). However, the simple wealth (or income) growth rate would contain both a predictable component and an unpredictable component. Ideally, only the unpredictable part of wealth (or income) movements should be used in estimating the MPCs out of wealth (or income) shocks.

Identifying predictable and unpredictable movements of wealth (or income) requires having a model of what income or wealth is expected to be. A model for expected income or wealth growth may be given as a function of a set of variables known at $t-1$, $\{E_{t-1}[\Delta \log Z_t] = f_Z(X_{t-1}) \ ; \ Z = Y, W\}$. Then, unpredictable shocks to income and wealth may be defined as the one-step ahead forecast errors in income and wealth. (i.e. $e_{Zt} = \Delta \log Z_t - f_Z(X_{t-1}) \ ; \ Z = Y, W$) We also show the results in which we use the estimated shocks in place of the simple growth rates in Columns 3 and 3-R of Table 4.5. The estimation result

for the MPC out of unexpected wealth shock is not considerably different from the one with simple growth rates, as wealth growth rates are hard to predict. On the other hand, the MPC out of unexpected income shocks are estimated larger than the one with simple growth rates.

For habit forming consumers, the short-run MPCs are smaller than the long-run MPCs, since it takes several periods to adjust fully to a single shock. The long-run MPCs out of shocks are defined as $MPC_Z^L = MPC_Z^S/(1-\rho)$ for $Z = Y, W$, where MPC_Z^S is a short-run MPC. As summarized in the last rows of Table 4.5, the long-run MPCs are estimated to be 4 percent for wealth shock and 48 percent for income shock. Carroll (2000) demonstrates that habits dramatically reduce the immediate MPCs out of permanent shocks to income. Indeed, with the strong habit persistence parameter estimated in this paper, the short-run MPC out of wealth shock is less than one-quarter of the long-run MPC, which is within the range of common estimates in the wealth effect literature.

4.4 Conclusion

In this paper, we explored two approaches to estimate the wealth effect on consumption. We argue that an increasingly used VECM approach does not give us reliable estimates of the MPCs out of wealth shock. Instead, we propose an alternative approach to estimate the short-run and long-run MPCs out of wealth shock, which accommodates empirical features of consumption dynamics such as the excess sensitivity to past information and the excess smoothness of consump-

tion movements. We use the Euler equation of a habit-forming consumer to characterize consumption dynamics.

We show that in a habit model consumption growth is serially correlated and that wealth shock to consumption has a long-lasting effect on consumption growth. We demonstrate how to obtain the short-run and long-run MPCs out of wealth shock in two stages. First, we estimate the consumption serial correlation coefficient, which tells us how much of an initial shock is carried over into future consumption growth. In the second step, given the serial correlation parameter, we can back out the MPC out of wealth shock.

Habits can reduce the immediate MPC out of shocks substantially. Our estimate of the immediate MPC out of wealth shock is around 0.9 percent, while the long-run MPC out of wealth shock is more than four times as large as the immediate effect, amounting to around 4 percent. Our estimate is within the range of typical estimates of the MPC out of wealth.

Table 4.1: Unit Root Test

	level	first difference
$\log C_t$	-0.109	-6.090
$\log Y_t$	-1.066	-7.109
$\log W_t$	-0.532	-6.140
r_t	-2.437	-5.000
sp_t	-0.233	-3.855
u_t	-3.014	-6.273
sentiment_t	-2.951	-5.937

Note: ADF statistics are shown. Four lags are included in regressions. Critical values are -3.487 and -2.885 for 1 percent and 5 percent levels. r_t is federal funds rate; sp_t is S&P's stock market index; u_t is unemployment rate.

Table 4.2: Dynamic Least Squares Regression

$$\log C_t = \mu + \beta_y \log Y_t + \beta_w \log W_{t-1} + \epsilon_t$$

μ	β_y	β_w
-0.805	0.608	0.404
(-4.697)	(6.254)	(4.328)

Note: Sample period is 1953q1-2003q1. t statistics are corrected in the way described in Hayashi (2000) and shown in parenthesis. We use four lags and leads of first differences of $\Delta \log Y_t$ and $\Delta \log W_t$ in the regression.

Table 4.3: Estimation of the VECM

	$\Delta \log C_t$	$\Delta \log Y_t$	$\Delta \log W_{t-1}$
$\hat{ec}_{t-1}$	-0.006	-0.015	0.200
	(-0.427)	(-0.619)	(3.323)
$\Delta \log C_{t-1}$	0.256	0.484	0.985
	(3.073)	(3.477)	(2.833)
$\Delta \log Y_{t-1}$	0.098	0.067	0.208
	(1.920)	(0.790)	(0.977)
$\Delta \log W_{t-2}$	-0.002	0.012	0.004
	(-0.123)	(0.405)	(0.050)
R^2	0.126	.109	.100

Note: Sample period is 1953q2-2003q2. t statistics are in parenthesis. Each equation is estimated with ordinary least squares method. $\hat{ec}_{t-1}$ is an estimated deviation from common trend shared by three variables. Wealth is a stock value measured at the end of period, while other variables are flow values over a quarter. Hence, wealth is lagged one period more than others.

Table 4.4: Consumption Euler Equation

$\Delta \log C_t$	1-OLS	2-IV	3-IV	4-IV	5-OLS	6-IV
$\Delta \log C_{t-1}$			0.782	0.694	0.410	0.756
			(5.674)	(5.816)	(6.350)	(6.675)
$\Delta \log W_{t-1}$	0.066	0.106	-0.003			
	(3.885)	(2.146)	(-0.056)			
$\Delta \log W_{t-2}$	0.014	0.011	-0.055			
	(1.130)	(0.884)	(-1.313)			
$\Delta \log W_{t-3}$	0.037	0.039	0.027	0.025		
	(3.104)	(3.033)	(1.696)	(1.629)		
$\Delta \log W_{t-4}$	0.033	0.030	0.005	0.010		
	(2.859)	(2.389)	(0.366)	(0.714)		
$\bar{R}^2$	.149	.228	.531	.464	.170	.533
OID test	NA	.465	.698	.565	NA	.533
P Value	0.000	0.000	0.215	0.120	NA	NA

First Stage Statistics

	$\Delta \log W_{t-1}$	$\Delta \log C_{t-1}$	$\Delta \log C_{t-1}$		$\Delta \log C_{t-1}$
Partial R^2	.147	.213	.276		.305
F-stats(p value)	1.56(.069)	5.58(.000)	6.07(.000)		5.73(.000)
		$\Delta \log W_{t-1}$			
Partial R^2		.107			
F-stats(p value)		1.34(.183)			
		$\Delta \log W_{t-2}$			
Partial R^2		.121			
F-stats(p-value)		1.69(.054)			

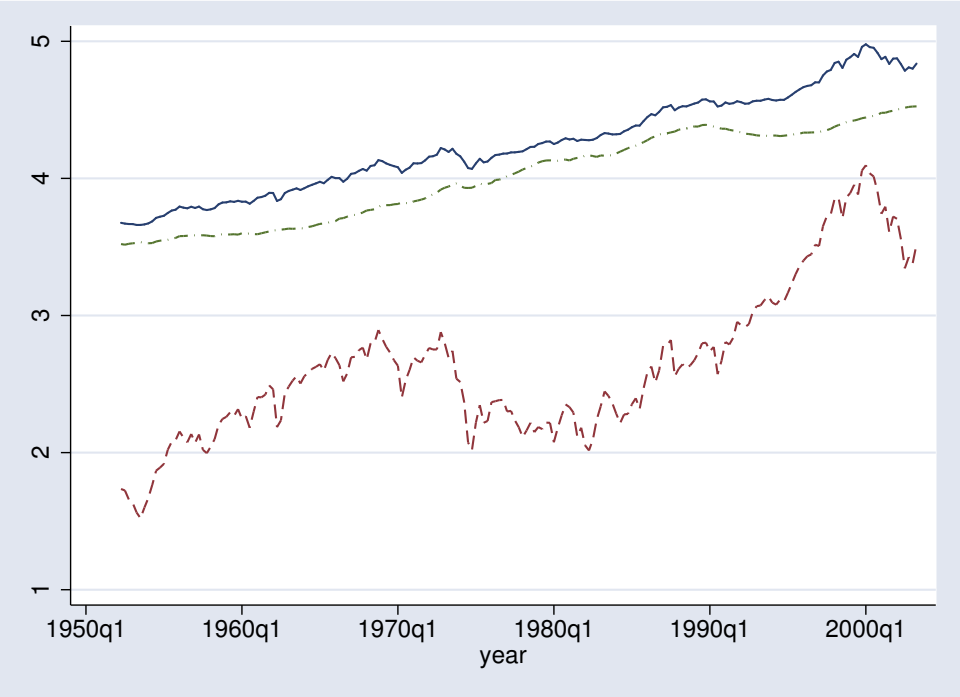
Note: Sample period is from 1962q2 to 2003q3. t statistics are in parenthesis for the second stage regressions. p values are for a null that all lagged wealth effects are zeros. Instruments include consumption growth, income growth wealth growth, changes in federal funds rate, and consumer sentiment index (unemployment expectation). All instruments are lagged for an appropriate number of periods.

Table 4.5: MPCs out of Wealth and Income Shocks

	1	1-R	2	2-R	3	3-R
Eq 1						
Wealth shock	.063	.057	.009	.009	.050	.049
	(4.04)	(5.32)	(3.90)	(5.16)	(3.32)	(4.74)
Income shock	.071	.120	.072	.116	.220	.233
	(1.73)	(4.82)	(7.84)	(5.10)	(5.23)	(8.03)
Eq 2						
Wealth shock	.056		.009		.048	
	(4.65)		(4.51)		(4.16)	
Income shock	.103		.101		0.234	
	(3.51)		(3.71)		(7.19)	
Eq 3						
Wealth shock	.052		.008		.049	
	(4.60)		(4.46)		(4.61)	
Income shock	.118		.114		.233	
	(4.70)		(4.94)		(7.88)	
P Value						
Wealth	.62		.67		.99	
& Income	.58		.61		.99	
SR-MPC						
Wealth		.0092		.0086		.0079
Income		.1200		.1162		.2327
LR-MPC						
Wealth		.0378		.0353		.0325
Income		.4924		.4770		.9550

Note: Column 1 uses a simple growth rate as a shock; Column 2 uses a normalized change defined as Equation (4.33) and (4.34); Column 3 uses an estimated shock. p values are for a null that all the coefficients on aggregated wealth shocks across equations are the same and that all the coefficients on aggregated income shocks are also the same.

Figure 4.1: Log Per Capita Real Wealth



— total wealth, – · – · – non stock wealth, – – – stock wealth

CHAPTER 5

Current Account Dynamics with Durable Goods

5.1 Introduction

This paper extends the simple intertemporal model of the current account to incorporate both durable and nondurable goods, and examines how this modification changes the implications of the model. The intertemporal approach of the current account has been a standard way of modelling the current account dynamics. Although the intertemporal model can be extended in many ways, only the simplest model has empirically been tested on the actual current account data in the past, except several recent works. (Bergin and Sheffrin (1999)) Since the simplest model seems unable to account for the observed high volatility of current account movements, it is interesting to know what extension to the model can improve on the model's performance, especially in predicting the volatility of the current account. Intertemporal models of the current account are based on the permanent income consumption theory. In a small open economy, which can borrow and lend at the world interest rate, the current account dynamics can be explained by what determines saving for a representative consumer.[60] Hence, the same method employed to test the implications of the permanent income hypothesis can be used to test the intertemporal models of the current account.

Introducing the distinction between durables and nondurables

[60]By accounting identity, Saving - Investment = CA. Aside from investment dynamics, saving movements are directly related to current account movements.

into the simple model is worthwhile for the following reasons. In the previous attempts to test the intertemporal model, the simple model with only one nondurable good tends to underpredict the actual volatility of the current account. This fact leads one to think that the simple model might lack in some important channels which have a significant explanatory power for the current account movements. The extension of the model to incorporate an additional channel, in this paper, the distinction between durables and nondurables, is one direction worth looking at. Secondly, the U.S. data suggest that consumer durables constitute a significant part of the trade deficits, reflecting the increase in expenditures on consumer durables. (Sadka and Yi (1996))[61] Hence, the explicit treatment of durables separately from nondurables is desirable.

When durables are included into the model, the movement of aggregate consumption expenditures is expected to become more volatile than when there is only one nondurable good in the model. This is simply because durable consumption expenditures are far more volatile than nondurable expenditures. While, for nondurables, the optimal consumption path is equivalent to the optimal expenditure path by definition, it is not the case for durables. Individuals try to smooth durable good consumption, not durable good expenditures. As a result, the optimal path of durable good expenditures can be quite lumpy. This high volatility of durable good expenditures can naturally feed into the high volatility of the current account. This is the straightforward reason why the presence of durables is expected to increase the volatil-

[61] They argue that the decline in the average consumer durable trade balance from 1973-80 to 1981-88 accounts for almost 60 percent of the decline in the average overall trade balance.

ity of the current account predicted by the theory, with other things being equal. I find that the extended model with the distinction between durables and nondurables, indeed, raises the magnitude of the theoretical volatility of the current account.

The organization of this chapter is as follows. The next section reviews the past literature on the present value test of the current account. Section 3 outlines my theoretical model with durables and nondurables and derives the testable implication. Section 4 explains the econometric framework. Section 5 discusses the data and estimation results from the present value tests based on my model. Section 6 concludes.

5.2 Review of the Previous Literature

Empirical tests of the intertemporal models of the current account take advantage of the methodological developments of the tests on the permanent income hypothesis of consumption. The model is a version of the rational expectation/permanent income consumption models applied to a small open economy in which investment is treated as exogenous. Derived from the forward-looking agent's optimal consumption rule, the current account movement reflects the agent's consumption smoothing motive; Temporarily high output causes a current account surplus, since the agent wants to spread this temporary gain over time by saving some of the current output. A temporary fall in output causes a current account deficit, since the agent borrows from the rest of the world to cushion the drop of consumption.

One useful feature of the model is that the current account can

be expressed as the expected decline of the present discounted value of net output of the country, which I will derive in the next section. Given the expected future path of net output, I can calculate the theoretical path of the current account (or the fundamental values of the current account as Ghosh (1995) defines.) Then, I can compare the theoretical series with the actual series of the current account, testing how closely these two series move over time. One can also device a set of tests on the implications of the theory, which have been originally used in the consumption literature.

5.2.1 Tests with the Simple Model

Sheffrin and Woo (1990) first test the model's implication that the current account should be equal to the discounted present value of expected decline of the country's net output, using the econometric approach developed by Campbell and Shiller (1987) to estimate the present discounted value models. They analyze a small open economy producing a single nondurable good and facing the constant world interest rate. The path of net output is subject to uncertainty and treated as exogenous in the model. In other words, the model uses the partial equilibrium approach. The stochastic process of net output is estimated with the unrestricted bivariate VAR of changes in net output and levels in the current account. Their use of a quadratic utility function allows them to get a closed form solution for the current account from the representative agent's optimization problem. Then, using the estimates of the VAR, they can construct the period-by-period exact prediction of the current account. Under their assumptions on the model, the current

account is linear in the expected future net output:

$$CA_t = -\sum_{i=1}^{\infty} R^{-i} E_t \Delta NO_{t+i} \tag{5.1}$$

where $R = 1 + r$ and $\beta(1 + r) = 1$ are assumed. This expression gives rise to a set of interesting testable implications:

- The most stringent test can be constructed by combining the estimated time-series properties of net output with the formula (Equation (5.1)). This is to test a null that the theoretical current account is equal to the actual current account with the Wald statistics.

- The less stringent and more general test on Equation (5.1) is to check the implied restriction,

$$E_t \left[CA_t - \Delta NO_t - R \cdot CA_{t-1} \right] = 0 \tag{5.2}$$

  which is just the orthogonality condition.

- The least formal test can be done by visually comparing the actual current account with the theoretical series computed from Equation (5.1).

They examine the current account data for four countries: Belgium, Canada, Denmark, and the U.K. Their main finding is that the model performs fairly well for Belgium and Denmark, but does poorly for Canada and the U.K.

Otto (1992) summarizes the testable implications derived from the same simple present value model of the current account. Three

of them are already listed above. The other is to test whether the current account Granger-causes changes in net output, furthermore, whether the sum of the coefficients on the lags of the current account is negative. These statements should be true under Equation (5.1), as long as consumers have richer information about the evolution of net output than the lags of changes in net output contain.

He uses quarterly data for the U.S. and Canada to test the present value relationship. His empirical results show that the U.S. data can provide at least informal evidence in favor of the intertemporal model (i.e. there appears to be a close relationship between the actual and the theoretical series for the U.S.) But the Canadian data cannot provide any support for the model. He finds that the theoretical current account is too smooth, failing to match the volatility of the actual current account movements. The difference between two countries is puzzling, considering that the model is more suitable for Canada (a small open economy) than for the US (a large open economy). He attributes this rather contradictory findings to shocks specific to Canada such as temporary changes in the resource prices and terms of trade.

Ghosh (1995) uses the same basic intertemporal model to argue that the current account forecasts computed by the relationship (5.1) is the benchmark against which the actual series is to be compared. He reasons that the deviation of the two series is due to a lack of adequate capital mobility across countries. The key statistics used in his paper is the variance ratio of the actual current account to the theoretical current account. If this ratio is found larger than one, it means that the current account has not varied enough to smooth consumption in the face of shocks to net output.

He tests the simple model on quarterly data for five major industrial countries: the U.S., Japan, Germany, the U.K., and Canada. He finds that, for all of the countries except the U.S., the actual current account fluctuates more than theoretically justified, given the estimated shocks to net output.

Test results from the simple intertemporal model in the previous literature are summarized as follows: the model has worked fairly well in tracking the directions and the major turning points of the actual current account movements, however, it fails to explain the magnitude of its volatility. In other words, the simple model tends to underpredict the volatility of the current account movements.

5.2.2 Tests with the Extended Models

Bergin and Sheffrin (1999) extend the simple model to allow for variable interest rates and exchange rates by introducing the distinction between tradables and nontradables. This modification enables the current account to respond not only to net output shocks but also to exchange rate and interest rate shocks, which are not captured in the simple model. In their augmented model, where an agent chooses both optimal tradable and nontradable consumption with a CRRA utility function, an expression for the current account is approximated as:

$$ca_t^* = -\sum_{i=1}^{\infty} \beta^i E\left[\Delta no_{t+i} - \sigma \cdot r_{t+i}^*\right] \tag{5.3}$$

where σ is the elasticity of intertemporal substitution, β is the discount rate, r^* is the effective real interest rate index (a function of interest rate, exchange rate, and preference parameters). In contrast with the

expression in the simple model (5.1), their expression is written in logarithm due to a log-linear approximation.[62]

As in the simple model, given the series of expected changes in future net output and the real interest rate index, the period-by-period forecasts of the current account can be computed by the above relationship (5.3). Since they have one additional variable in the current account expression, the stochastic processes of three variables, $ca^*, \Delta no$, and r^* are estimated with the unrestricted VAR method. The test procedure is the same as before.

They use quarterly data for three small open economies, Canada, Australia, and the U.K. to test the performance of their augmented model. They find that including the variable interest rate index improves the fit of the model to the data for all three countries. This result is obtained mainly because the explicit treatment of relative price shocks can provide a way to increase the volatility of the current account explained by the model, and partly because they choose a value for the elasticity of substitution so that they can fit the model's restriction best.

5.3 The Model with Durables and Nondurables

I try another extension of the simple model of the current account. My extension is to introduce a distinction between durables and nondurables and allow for a variable user cost index of durables. Expenditures on durable goods are known to be far more volatile than those on nondurables. This fact itself offers enough reason to believe that

[62] no_{t+i} in Equation (5.3) is $\log(NO_{t+i})$. But ca_t^* is not just $\log(CA_t)$. The transformed form will become clear in the next section.

the inclusion of durables into the model would increase the predicted volatility of the current account.

When goods are durable, the relevant price is a user cost, not a purchase price. The user cost is written as a function of the current and next period purchase prices of durables, depreciation rate, and interest rate. Therefore, my extended model allows the current account to respond to shocks to the user cost as well as net output. To make a clear comparison with Bergin and Sheffrin's model, I keep the interest rate constant. In the following, I outline the augmented model with durables and nondurables and derive a testable expression for the current account similar to Equation (5.1) and (5.3).

The model is based on Obstfeld and Rogoff (1996). Consider a small open economy consuming durable and nondurable goods, both of which are tradable. The country is assumed to be able to borrow and lend against the foreign countries at the constant interest rate r. The representative agent in the country solves the following intertemporal maximization problem by choosing the optimal paths for durables, nondurables, and debt.

$$\max_{\{C_s, D_s, B_s\}_{s=t}^{\infty}} E_t \left[\sum_{i=0}^{\infty} \beta^i U(C_{t+i}, D_{t+i}) \right]$$

subject to

$$B_s - B_{s-1} = rB_{s-1} + Y_s - C_s - Z_s - I_s - G_s$$

$$U(C_s, D_s) = \gamma \log(C_s) + (1-\gamma) \log(D_s)$$

$$Z_s = P_s(D_s - (1-\delta)D_{s-1})$$

$$0 \;<\; \gamma, \delta \;<\; 1$$

where consumption of nondurables is denoted by C_s, consumption of durables which is equal to a stock amount of durables is D_s, and I assume that the service flow from durables is proportional to the stock and normalized accordingly. Y_s denotes a value of output including both types of goods, I_s is an amount of investment, and G_s is government expenditures. All are measured in terms of nondurable goods. The relative price of durables in terms of nondurables is P_s, and Z_s stands for durable consumption expenditures. An amount of external assets is B_s. The left hand side of the budget constraint $(B_s - B_{s-1})$ corresponds to the current account in period s.

From the first order conditions with respect to nondurables, durables and external asset holdings, I can obtain the following set of equations. Assuming $\beta(1+r) = 1$,

$$C_t^{-1} = E_t \left[C_{t+1}^{-1} \right] \tag{5.4}$$

$$\frac{(1-\gamma)}{D_t} = E_t \left[\frac{\Upsilon_{t+1} \cdot \gamma}{C_{t+1}} \right] \tag{5.5}$$

where Υ_t is the user cost of durables and defined as $\Upsilon_t = P_{t-1} - \left(\frac{1-\delta}{1-r}\right) P_t$.[63] Purchase of the durable good involves an investment-like decision; As is the case for any kind of investment decisions, the effective user cost of durable goods includes a speculative motive. When the relative price of durables is expected to increase in the next period, the current user cost is low, making a current durable good purchase more

[63]Time notation is a little confusing. Obstfeld and Rogoff (1996) denote the user cost in time t as a price change from time t to time $t+1$. I choose the time subscript t+1 for the same price change, because this term is known only in time $t+1$.

attractive.[64]

Equation (5.4) is the familiar Euler equation for nondurables. Because of the simple log specification for a utility function, the marginal utility of nondurables is separable from durables and follows a martingale process. If I log-linearize these two equations, (5.4) and (5.5), they are rewritten as,

$$E_t \left[\Delta c_{t+1}\right] = 0 \tag{5.6}$$

$$d_t = E_t \left[c_{t+1}\right] - E_t \left[\tau_{t+1}\right] \tag{5.7}$$

where I use lower case letters for the natural logarithm of corresponding variables.

As is clear from the last equation (5.7), the optimal path of durable good consumption should depend on the expected path of the user cost. Hence, this model introduces another variable which is not considered in the simple model, namely, the user cost of durables. While for nondurables the usual consumption smoothing behavior still applies, for durables the consumer might want to change her consumption profile upon expected changes in the user cost and not to choose a smooth path. Combining Equations (5.6) and (5.7), I get

$$\Delta d_t = -E_t \left[\Delta \tau_{t+1}\right] \tag{5.8}$$

When an increase in the user cost is expected in the next period, the current durable good consumption is induced to rise due to the substitution effect.

[64] Υ_t can be called the implicit rental price of the durable goods. I assume the existence of the resale market for durable goods, no transaction cost, and no adjustment cost.

The intertemporal budget constraint is obtained by applying forward substitution on the budget constraint, taking the expected values of both sides conditioned on time t information, and imposing the transversality condition,

$$\sum_{i=0}^{\infty} E_t \left[R^i \left(C_{t+i} + Z_{t+i} \right) \right] = R \cdot B_{t-1} + \sum_{i=0}^{\infty} E_t \left[R^i NO_{t+i} \right] \tag{5.9}$$

where NO_t is net output, and defined as $NO_t = Y_t - I_t - G_t$. I log-linearize the above intertemporal budget constraint around the steady state values, following the method in Campbell and Mankiw (1989) and Huang and Lin (1993):

$$-\sum_{i=0}^{\infty} \beta^i E_t \left[\Delta no_{t+i} - \Delta \log(C_{t+i} + Z_{t+i}) \right] = no_t - \log(C_t + Z_t) = ca_t^* \tag{5.10}$$

where again I use lower case letters for the logs of upper case letter counterparts. The detailed derivation is contained in the Appendix. Note that I ignore unimportant constant terms from the log-linearization, focusing only on the dynamic relationship. The right hand side is a transformed form of the current account, labelled as ca_t^*.[65]

Now, I substitute the two Euler conditions (5.6) and (5.8) into the log-linearized intertemporal budget constraint, obtaining the following final form:

$$ca_t^* = -\sum_{i=1}^{\infty} E_t \left[\Delta no_{t+i} - \frac{\overline{Z}}{\overline{C} + \overline{Z}} \Delta \tau_{t+i+1}^* \right] \tag{5.11}$$

where $\Delta \tau_{t+i+1}^* = \Delta p_{t+i} - \frac{1}{\delta} \Delta \tau_{t+i+1} + \frac{1-\delta}{\delta} \Delta \tau_{t+i}$. This Equation (5.11)

[65] Bergin and Sheffrin (1999) also use the same transformation for the current account. How to get this form is explained in the Appendix too.

is my expression for the current account and constitutes the basis of the present value test. The above relationship states that when the user cost index τ_t^* is expected to rise, or the net output is expected to decrease, the current account should move towards surplus. The former effect, absent in the simple model, exists because of the distinction between durables and nondurables introduced into my extended model. Intuitively, when consumers expect a future increase in the user cost, they start accumulating foreign assets by running a current account surplus to finance the future expected increase in the user cost.

5.4 Econometric Method to Test the Model

Following the method employed by all the previous studies, I estimate an unrestricted VAR with three variables which appear in the system: changes in net output, the current accounts, and changes in the user cost index. The VAR estimate is then used to forecast a series of changes in net output and changes in the user cost index. Once I am equipped with the two forecast series, the period-by-period forecast of the current account consistent with my extended model can be computed by the relationship (5.11). Finally, the restrictions imposed by the model can be tested in several ways.

First, I estimate an unrestricted VAR of the following form:

$$\begin{bmatrix} \Delta no_t \\ ca_t^* \\ \Delta\tau_t^* \end{bmatrix} = A \cdot \begin{bmatrix} \Delta no_{t-1} \\ ca_{t-1}^* \\ \Delta\tau_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{3t} \end{bmatrix} \tag{5.12}$$

where A is the 3-by-3 matrix of coefficients to be estimated. This is eas-

ily generalized for higher orders of VARs, and in the actual estimation, I choose the optimal order of VARs according to the information criteria. The i period ahead forecasts of the left hand side variables, conditional on time t information, are calculated by using the estimated A matrix, $\hat{A}$, as $E_t[X_{t+i}] = \hat{A}^i X_t$, where $X_t = \begin{bmatrix} \Delta no_t & ca_t^* & \Delta\tau_t^* \end{bmatrix}'$. Then, the expression for the current account implied by the model (5.11) becomes,

$$\begin{aligned} \widehat{ca^*}_t &= -\left(g_1 \cdot I - \frac{\bar{Z}}{\bar{C}+\bar{Z}} \cdot g_2 \cdot \hat{A}\right) \sum_{i=1}^{\infty} \beta^i \hat{A}^i X_t \\ &= -\left(g_1 \cdot I - \frac{\bar{Z}}{\bar{C}+\bar{Z}} \cdot g_2 \cdot \hat{A}\right) \beta\hat{A}\left[I - \beta\hat{A}\right]^{-1} X_t = \widehat{K} \cdot X_t \end{aligned} \tag{5.13}$$

where $g_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $g_2 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$, and I is a 3-by-3 identity matrix. If my model with the VAR estimation can predict the true current account path, $\widehat{ca}_t^* = ca_t^*$ should hold. This restriction is equivalent to $\widehat{K} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$, and the corresponding Wald statistics can be computed by the delta method.

$$W = \left(\widehat{K} - \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\right) \left[\frac{\partial \widehat{K}}{\partial \alpha}\right] V \left[\frac{\partial \widehat{K}}{\partial \alpha}\right]' \left(\widehat{K} - \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\right)'$$

where $\frac{\partial \widehat{K}}{\partial \alpha}$ is a derivative matrix of the $\widehat{K}$ vector with respect to the underlying parameters of the VAR, and V is the variance-covariance matrix of the underlying parameters. Under a null hypothesis that the restriction is true, the W should have a Chi-square distribution with three degrees of freedom.

5.5 Data and Empirical Results

The present value relationship derived above is tested using quarterly data for the U.S. and Canada. The data covers a period from the fourth quarter of 1947 to the fourth quarter of 1999 for the U.S. and from the fourth quarter of 1961 to the second quarter of 1997 for Canada. All data for the U.S. are taken from the National Income and Product Accounts. All data except prices of durables and nondurables for Canada are taken from the International Financial Statistics. A series of the relative price of durables for Canada is constructed from the consumer price of durables and the consumer price of all items obtained from the OECD Main Economic Indicators. The description of all the series used is contained in the Appendix.

Net output no_t is constructed by subtracting investment and government expenditures from GDP, deflating with the GDP deflator, converting to a per-capita value, and taking the logs. The current account $\widehat{ca}_t^*$ is constructed by subtracting the log of the deflated per-capita consumption expenditures from the constructed net output. To construct the user cost index, I need a series of relative prices of durables against nondurables. The relative price is constructed as a ratio between the durable good deflator and the nondurable good deflator for the U.S., though I include service goods into the nondurable category in the model.[66] The user cost index τ_t^* is then constructed by the formula given in Equation (5.8). Following the previous literature, I only analyze the dynamic relationship implied by the intertemporal model. Thus, all the series appearing in the current account expression are

[66]Therefore, the price of service goods is not reflected in the relative price of durable goods for the U.S.

demeaned.

In order to implement the present value test, it is also necessary to specify two parameter values used in the model: the constant world interest rate r, and the depreciation rate of consumer durables δ. Following Otto (1992), I use an annual interest rate of 4.04 percent, which implies that β is equal to about 0.96. Regarding the depreciation rate, I choose an annual rate of 22.5 percent, which is taken from Sadaka and Yi (1996).

5.5.1 Unit Root Test

The first task is to check a stationary assumption on the three variables in the VAR estimation. For that purpose, I perform the Augmented Dickey Fuller test on the three variables for various numbers of lags. The results are reported in Table 5.1. For all the variables in both countries, the ADF test statistics reject the presence of a unit root at the 1 percent significant level for all numbers of lags considered, except for the current account in Canada with 5 lags. (For this case, the rejection is at the 5 percent level.)

5.5.2 Present Value Model Test

The results from the VAR analysis are reported in Table 5.2 for both countries. According to the information criteria, I choose 2 lags for the U.S. and 3 lags for Canada. The present value relationship implies that the current account Granger-causes changes in net output. Moreover, the sum of the coefficients on the lags of the current account is expected to be negative, since a current account surplus can be interpreted as a signal for a future decline in net output.

The null hypothesis that the current account does not Granger-cause changes in net output is rejected at the 5 percent significant level and the sum of the coefficients on the lagged current account has a negative sign for the U.S. For Canada, the sign of the sum of the coefficients on the lagged current account is estimated negative, but the current account is not found to Granger-cause changes in net output with a p-value of 28.3 percent.

The formal test of the present value model is based on the estimated $\widehat{K}$ vector defined by Equation (5.13). The theory suggests that the $\widehat{K}$ vector should be equal to $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$ for the U.S. and $\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ for Canada. The estimated $\widehat{K}$ vector for each country is reported in Table 5.3. The second element of $\widehat{K}$ vector is estimated at 1.741 for the U.S., which is not significantly different from 1 with a p-value of 23.5 percent, and at 0.767 for Canada, which is not significantly different from 1 either with a p-value of 42.0 percent. Therefore, looking at only the second element of the $\widehat{K}$ vector, the estimate result is in favor of the model, although the Wald statistics for the restrictions on the $\widehat{K}$ vector on the whole reject the model for both countries.

The volatility of the current account explained by the model is reported as a ratio to the volatility of the actual data. The volatility ratio exceeds one for the U.S., while the extended model underpredicts the volatility for Canada. (also shown in Table 5.3) The standard deviation of the current account derived from our extended model is 1.90 times as high as the actual one for the U.S., and 0.82 times as high as actually recorded for Canada. Figure 5.1 and 5.3 depict the current

account predicted from the extended model and the actual series. The fit seems to be better for the U.S. than for Canada, as reflected on the high correlation coefficient between the theoretical current account and the actual one; The correlation coefficient is 0.957 for the U.S., much higher than 0.704 for Canada.

How is my extended model compared with the simple model in terms of the ability to explain the actual movement of the current account? To answer this question, I do the same analysis with the simple model and the results are shown in the same format. On the whole, the simple model works fairly well for the U.S. data, which was also the conclusion obtained by Otto (1992) and Gosh (1995). The simple model passes the formal test based on the Wald statistics with a p-value of 43.8 percent. The correlation between the model's predicted path and the actual current account is quite high, estimated to be 0.997. The good fit of the model is reassured by looking at Figure 5.2. The simple model predicts the current account to be 1.73 times as volatile as that of the actual series, overpredicting rather than underpredicting the actual volatility. The case of the U.S. data is rather an exception in the previous studies of the present value model test, since the simple model is typically rejected by the formal test and underpredicts the actual volatility of the current account in most of cases. Comparing the simple model with the extended model, first, there does not seem a huge difference between them. Introducing the distinction between durables and nondurables works in the expected way, increasing the volatility of the theoretical current account, but not much. The correlation is a little lower in the extended model than in the simple model. Comparing Figure 5.2 with Figure 5.1, it can be reconfirmed that the extended

model amplifies the volatility of the current account only by a little.

For Canada, for which the model is more suitable than the U.S., there is a clearer difference between the extended model and the simple model. The simple model performs fairly well in predicting the directions, but fails to capture the magnitude of fluctuations of the actual current account, which is a typical result in the past studies. The simple model's forecast is only about half as volatile as actually observed, as is graphically observed in Figure 5.3. Therefore, the extended model with durables has more ability to explain the volatility of the actual current account than the simple model, successfully increasing the model's predicted current account fluctuations from 51.1 percent to 82.4 percent (measured as a ratio to the volatility of the actual series). In exchange of the better fit for the volatility, the extended model has to sacrifice the correlation between the theoretical forecast and the actual current account, reducing the correlation coefficient from 0.968 to 0.704. This is due to the fact that the model's forecast spikes too much at turning points. (see Figure 5.3)

5.6 Conclusion

I have examined that introducing a distinction between durables and nondurables changes the model's prediction little for the U.S. but substantially for Canada. The extended model's implication is that when a consumer responds optimally to expected changes in the user costs of durables, consumption expenditures would exhibit a larger fluctuation than predicted by the simple model with only one nondurable good. Hence, it is speculated that such a high volatility of durable

consumption expenditures translates into a high volatility of the current account. The speculation turned out to be true by comparing the estimation results of the simple model with the extended model for two countries, the U.S. and Canada. The extended model can generate larger volatility for the current account than the simple model. While a difference between two models is almost insignificant for the U.S., it is substantial for Canada. For Canadian data, I find that the extended model outperforms the simple model in terms of explaining the volatility of the actual current account, although it still under-predicts the actual volatility.

Durable good purchase has the characteristics similar to investment. The user cost is a relevant variable and the speculative motive is involved. The simple intertemporal model does not offer an explicit treatment of such investment dynamics, which are more volatile and harder to predict than consumption. For theoretical models to be able to explain the large fluctuations of the current account, taking appropriate account of investment dynamics seems promising.

Table 5.1: Unit Root Test

Number of lags		1	3	5
U.S.	Δno^*	-10.182^{**}	-8.913^{**}	-7.374^{**}
	ca^*	-4.597^{**}	-4.635^{**}	-3.686^{**}
	$\Delta\tau^*$	-20.85^{**}	-13.75^{**}	-12.29^{**}
Canada	Δno^*	-9.938^{**}	-5.510^{**}	-4.165^{**}
	ca^*	-2.603^{**}	-2.738^{**}	-2.255^{*}
	$\Delta\tau^*$	-16.15^{**}	-15.30^{**}	-8.275^{**}

Note: These are test statistics to be compared with the appropriate Dickey-Fuller critical values. I do not include a time trend and constant in the regression, because three variables are already demeaned. ** indicates that unit root is rejected at the 1 percent level. * indicates rejection at the 5 percent level.

Table 5.2: VAR Estimation Result

	U.S.						Canada					
	Δno^*		ca^*		$\Delta\tau^*$							
Δno^*_{t-1}	-0.071	(-0.689)	-0.013	(-0.450)	0.692	(2.135)	-0.299	(-2.661)	-0.231	(-2.423)	1.141	(1.043)
Δno^*_{t-2}	0.121	(1.712)	0.042	(2.145)	-0.196	(-0.878)	-0.037	(-0.325)	-0.022	(-0.226)	1.533	(1.380)
Δno^*_{t-3}							-0.014	(-0.166)	-0.109	(-1.501)	-0.871	(-1.046)
ca^*_{t-1}	-0.059	(-0.159)	0.849	(8.292)	-1.874	(-1.595)	0.045	(0.336)	1.060	(9.270)	-2.626	(-1.996)
ca^*_{t-2}	-0.358	(-0.996)	-0.070	(-0.707)	1.948	(1.716)	-0.133	(-0.670)	-0.049	(-0.306)	2.225	(1.201)
ca^*_{t-3}							0.002	(0.014)	-0.119	(-1.022)	0.440	(0.327)
$\Delta\tau^*_{t-1}$	-0.060	(-3.045)	-0.018	(-3.387)	-0.873	(-13.95)	-0.022	(-3.307)	0.006	(1.063)	-1.046	(-16.25)
$\Delta\tau^*_{t-2}$	-0.070	(-3.488)	-0.018	(-3.315)	-0.452	(-7.156)	-0.028	(-3.634)	0.004	(0.654)	-0.935	(-12.46)
$\Delta\tau^*_{t-3}$							-0.020	(-3.229)	0.006	(1.164)	-0.644	(-10.61)
Granger	7.030	(0.030)					3.809	(0.283)				

Note: t-statistics are in parenthesis for coefficient estimates. In the last row, p-values for the Granger causality test is shown in parenthesis.

Table 5.3: Present Value Test for the Extended Model

	U.S.		Canada	
K vector				
Δno_t^*	-0.049	(0.106)	0.067	(0.320)
ca_t^*	1.741	(1.024)	0.767	(1.154)
$\Delta\tau_t^*$	0.067	(0.023)	0.057	(0.013)
Δno_{t-1}^*	-0.043	(0.089)	-0.175	(0.196)
ca_{t-1}^*	0.143	(0.325)	-0.115	(0.207)
$\Delta\tau_{t-1}^*$	0.058	(0.022)	0.078	(0.014)
Δno_{t-2}^*			0.033	(0.185)
ca_{t-2}^*			-0.084	(0.134)
$\Delta\tau_{t-2}^*$			0.067	(0.010)
$k_2 = 1$	0.724	(0.235)	-0.202	(0.420)
Wald statistics	29.215	(0.000)	102.82	(0.000)
correlation	0.957		0.704	
$\frac{\sigma(\widehat{ca}^*)}{\sigma(ca^*)}$	1.896		0.824	

Note: standard errors are in parenthesis for the K vectors. p-values are shown in parenthesis for the test statistics.

Table 5.4: Present Value Test for the Simple Model

	U.S.		Canada	
K vector				
Δno_t^*	0.035	(0.082)	0.263	(0.098)
ca_t^*	1.470	(0.512)	0.384	(0.236)
Δno_{t-1}^*	-0.032	(0.049)	0.117	(0.077)
ca_{t-1}^*	0.304	(0.240)	0.064	(0.078)
Δno_{t-2}^*			0.035	(0.051)
ca_{t-2}^*			0.033	(0.078)
$k_2 = 1$	0.920	(0.179)	-2.603	(0.005)
Wald statistics	3.773	(0.438)	11.001	(0.088)
correlation	0.997		0.968	
$\frac{\sigma(\widehat{ca}^*)}{\sigma(ca^*)}$	1.730		0.511	

Figure 5.1: Extended Model (U.S.)

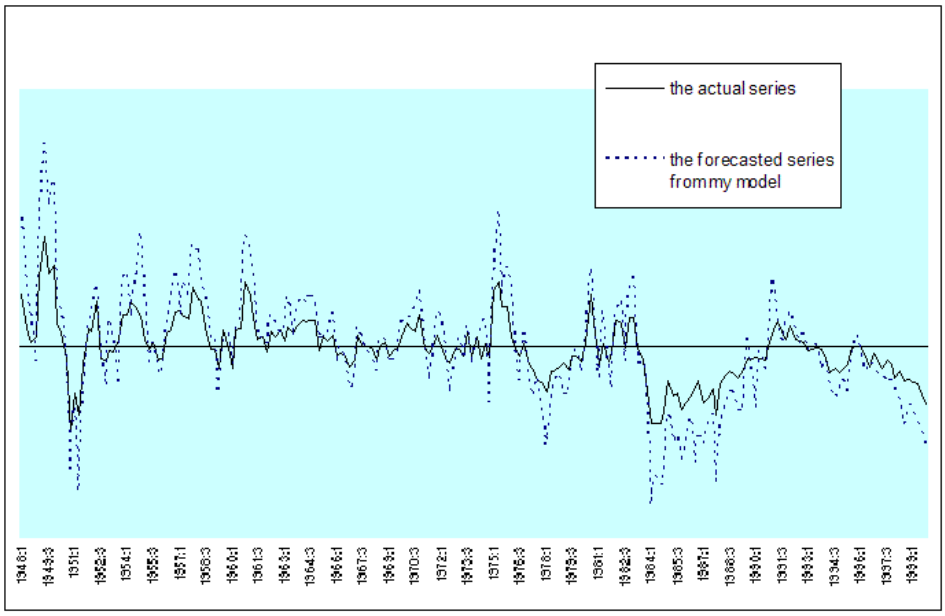


Figure 5.2: Simple Model (U.S.)

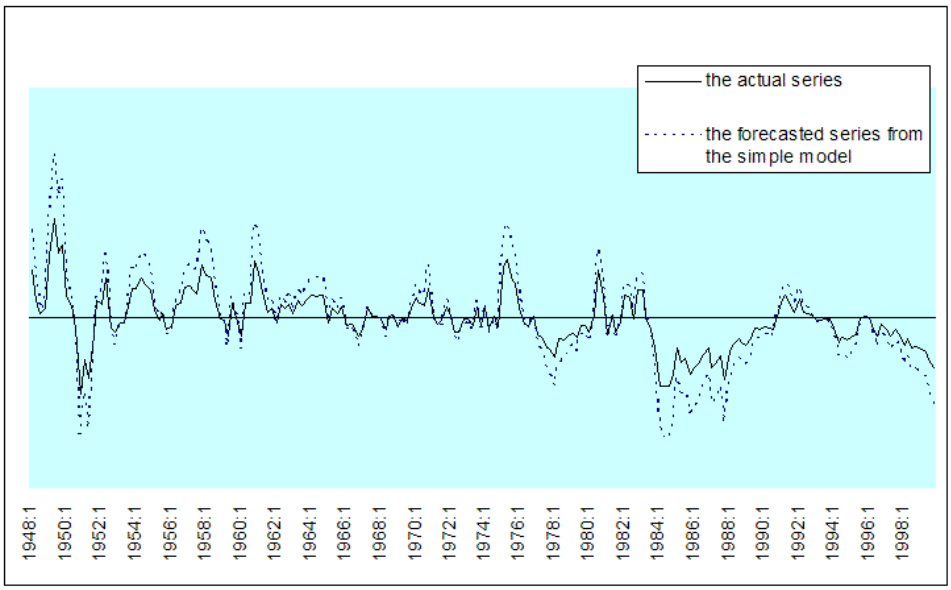


Figure 5.3: Extended Model (Canada)

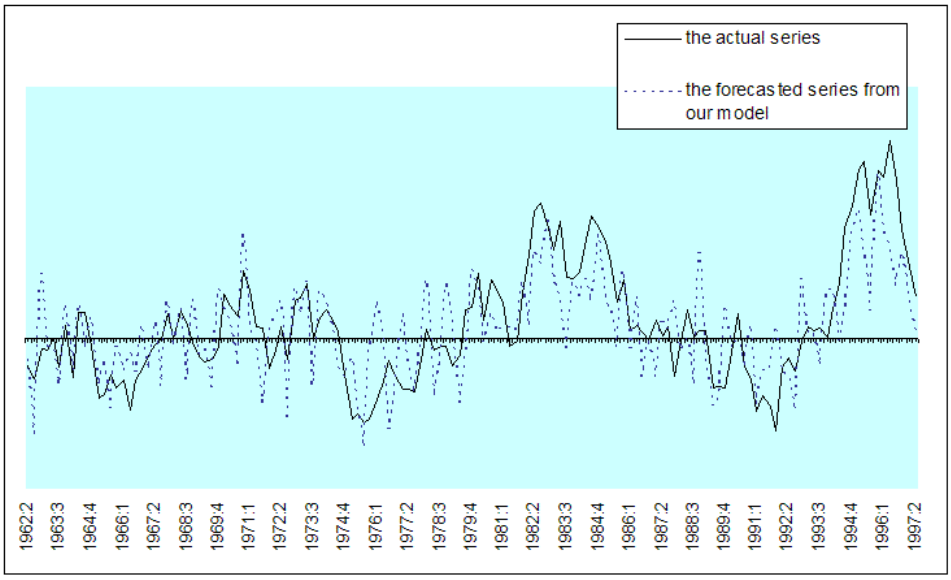
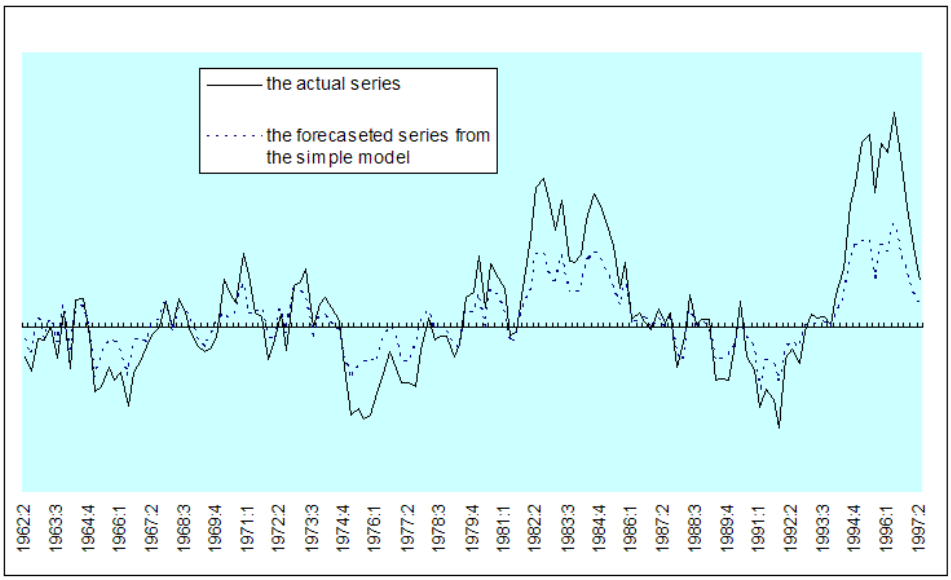


Figure 5.4: Simple Model (Canada)



APPENDIX A

Technical Detail for Chapter 2

A normalized version of the model to be solved by numerical dynamic programming is copied here from Section 2.3 of Chapter 2.

$$\begin{aligned} v(l_t, k_t) &= \max_{\{c_s, \Delta k_s\}_{s=t,\cdots,\infty}} E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \\ &= \max_{\{c_t, \Delta k_t\}} u(c_t) + \beta E_t \left[(G\Pi_{t+1})^{1-\rho} v(l_{t+1}, k_{t+1}) \right] \end{aligned} \quad (A.1)$$

such that

$$l_{t+1} = \frac{(1+r^L)}{G\Pi_{t+1}}(l_t - \Delta k_t - I(\Delta k_t) - c_t) + \Psi_{t+1} \quad (A.2)$$

$$k_{t+1} = \frac{(1+r^K)}{G\Pi_{t+1}}(k_t + \Delta k_t) \quad (A.3)$$

$$0 < c_t <= l_t - \Delta k_t - I(\Delta k_t) \quad (A.4)$$

$$-k_t <= \Delta k_t <= l_t \quad (A.5)$$

In solving, the problem is divided into two sub-stages so that each sub-stage has only one control variable. The first stage determines the optimal amount of liquidation or investment (Δk_t), and the second stage determines the optimal amount of consumption (c_t). These two stages are solved backward. Hence, I start with the second stage problem. In the second stage, the optimal amount of liquidation or investment has been already made, then, given the adjusted amount of liquidity-on-hand $x_t(= l_t - \Delta k_t - I(\Delta k_t))$ and the illiquid saving $m_t(= k_t + \Delta k_t)$, the household chooses consumption. The second stage gives the opti-

mal consumption as a function of x_t and m_t. In the first stage, the optimal choice of liquidation is made, using the obtained consumption function.

Formally, the second stage problem, in the dynamic programming presentation, is written as,

$$\begin{aligned} v^*(x_t, m_t) &= \max_{\{c_t\}} u(c_t) + \beta E_t \left[(G\Pi_{t+1})^{1-\rho} v(l_{t+1}, k_{t+1}) \right] \\ &= \max_{\{c_t\}} u(c_t) + \beta\, \Omega(s_t, m_t) \end{aligned} \tag{A.6}$$

where

$$l_{t+1} = \frac{(1+r^L)}{G\Pi_{t+1}}(x_t - c_t) + m_t + \Psi_{t+1} \tag{A.7}$$

$$k_{t+1} = \frac{1}{G\Pi_{t+1}} m_t \tag{A.8}$$

$$0 < c_t <= x_t \tag{A.9}$$

$$s_t = x_t - c_t \tag{A.10}$$

A new value function $v^*(x_t, m_t)$ is defined from the original $v(l_t, k_t)$. Two new state variables, x_t and m_t, are used for this problem since Δk_t is assumed to be given in this stage. The new state variables are related to the original state variables through Equation (A.7) and (A.8). Furthermore, a new function $\Omega(s_t, m_t)$ is defined to replace the expected term, $E_t[(G\Pi_{t+1})^{1-\rho} v(\frac{(1+r^L)}{G\Pi_{t+1}} s_t + \Psi_{t+1}, \frac{(1+r^K)}{G\Pi_{t+1}} m_t)]$. Then, the second stage problem looks very similar to the standard consumption problem with the liquidity constraint (Equation (A.9)), except the m_t term. The optimal solution for the second stage problem, $c^*(x_t, m_t)$, is

obtained by iterating on the first order condition of the problem,

$$u'(c^*(x_t, m_t)) = \max\{u'(x_t), \beta\,\Omega^s(x_t - c^*(x_t, m_t), m_t)\} \qquad \text{(A.11)}$$

where Ω^s is the derivative of the Ω function with respect to the first argument s_t.

The first stage problem is to choose the optimal Δk_t, given the second stage results, and can be written as,

$$v(l_t, k_t) \;=\; \max_{\{\Delta k_t\}} v^*(l_t - \Delta k_t - I(\Delta k_t), k_t + \Delta k_t) \qquad \text{(A.12)}$$

subject to

$$-k_t \;<=\; \Delta k_t <= l_t \qquad \text{(A.13)}$$

Unlike the second stage, this problem cannot be solved by using the first order condition due to the discontinuous shape assumed for the transaction costs function $I(\Delta k_t)$. (see Equation (2.15)) Instead, I iterate on the value function to solve for the optimal $\Delta k(l_t, k_t)$. Finally, the optimal consumption function in terms of the original state variables is obtained as $c(l_t, k_t) = c^*(l_t - \Delta k(l_t, k_t) - I(\Delta k_t, l_t), k_t + \Delta k(l_t, k_t))$.

APPENDIX B

Data Description for Chapter 4

Consumption (C_t)	Real per capita consumption is measured as expenditures on nondurables and services, divided by price index and population. The data are quarterly, seasonally adjusted at annual rate. The source is National Income and Product Accounts from Bureau of Economic Analysis.
Income (Y_t)	Real per capita labor income is measured as wage and salary disbursement + transfer payments to persons - personal contributions for social insurance, divided by price index and population. The data are quarterly, seasonally adjusted at annual rate. The source is National Income and Product Accounts from the Bureau of Economic Analysis.
Wealth (W_t)	Real per capita wealth is measures as net worth held by households and NPOs, divided by price index and population. The data are quarterly, amount outstanding at the end of period. The source is Flow of Funds Accounts from the Board of Governors of the Federal Reserve System.

Price Index	Price Index used to deflate nominal variables is the implicit price deflator for personal consumption expenditures. (1996 = 100) The source is National Income and Product Accounts from the Bureau of Economic Analysis.
Population	The number of population is in millions. The source is National Income and Product Accounts from the Bureau of Economic Analysis.
Federal funds rate	The average over monthly series. The source is the Board of Governors of the Federal Reserve System.
Consumer Sentiment Index	The source is the Surveys of Consumers from the Survey Research Center at the University of Michigan.
Unemployment Rate	The average over monthly series. The source is from the Bureau of Labor Statistics.
Stock Price Index	Standard & Poor's stock price index is taken from Robert Shiller's web site.

APPENDIX C

Data Description for Chapter 5

USA

All the data used for the U.S. are taken from the DRI database, originally from National Income and Product Accounts published by the Bureau of Economic Analysis.

Gross Domestic Product (GDP)	-	nominal value
Gross Private Capital Formation (GPI)	-	nominal value
Increase in Inventory (GV)	-	nominal value
Government Expenditure (GGE)	-	nominal value
GDP deflator (GDPD)	-	100 in 1987
Population (GPOP)		
Expenditures on nondurable good (GCN)	-	nominal value
Expenditures on durable good (GCD)	-	nominal value
Expenditures on service good (GCS)	-	nominal value
Price of nondurable good (GDFCN)	-	chain-type price index
Price of durable good (GDFDFC)	-	chain-type price index

— no_t appearing in Equation (5.8) is constructed as $\log\left(\frac{GDP_t - GPI_t - GV_t - GGE_t}{GDPD_t \cdot GPOP_t}\right)$.

— The transformed current account series, ca_t^*, is given as

$no_t - \log\left(\frac{GCN_t + GCD_t + GCS_t}{GPOP_t \cdot GDPD_t}\right)$.

— The ratio $\frac{\overline{Z}}{\overline{C}+\overline{Z}}$ in Equation (5.8) is given as the average of $\frac{GCN_t}{GCN_t + GCD_t + GCS_t}$, which is about 0.13. The relative price of durables against nondurables, P_t, is calculated as a ratio, $\frac{GDFDFC_t}{GDFCN_t}$.

CANADA

For Canada, the data are also taken from the DRI database. Most of them are originally from the IFS published by the IMF. Some of them are originally from the OECD main indicators.

Gross Domestic Product (L99BC@C156) - nominal value

Gross Private Capital Formation (L93EC@C156) - nominal value

Increase in Inventory (L93FC@C156) - nominal value

Government Expenditure (L91FC@C156) - nominal value

Real GDP (L99FR@C156) - at constant 1986 prices

GDP deflator - nominal GDP/real GDP

Population (L99Z@C156)

Aggregate Consumption Expenditure (L99FC@C156) - nominal value

Consumer price index on all items (CPALTT01|XOB@C156)

Consumer price index on durable goods (CPGDDU01|XOB@C156)

— no_t is constructed in the same way as for the U.S.

— The transformed current account series, ca_t^*, is given by $no_t - \log\left(\frac{L99FC@C156_t}{L99Z@C156_t \cdot GDPdeflator_t}\right)$.

— For the ratio, $\frac{\overline{Z}}{\overline{C}+\overline{Z}}$, we use the same number as calculated for the U.S. data.

— The relative price of durables against nondurables, P_t, is calculated as a ratio, $= \frac{CPGDDU01|OB@C156_t}{CPALTT01|XOB@C156_t}$.

(Note: Series names in the DRI database are given in parenthesis.)

APPENDIX D

Mathematical Detail for Chapter 5

This section explains step-by-step derivation of Equation (5.10), the log-linearized current account expression. The technique is the same one used by Cambell and Mankiw (1989) and Huang and Lin (1993).

First, I show how to log-linearize the present discounted value expression $\sum_{i=0}^{\infty} R^i NO_{t+i}$. Let us denote this expression as Γ_t. Then, the following holds.

$$\Gamma_{t+1} = R^{-1}(\Gamma_t - NO_t)$$

Dividing both sides by Γ_t and taking the logarithm,

$$\begin{aligned}
\log \frac{\Gamma_{t+1}}{\Gamma_t} &= -\log R + \log(1 - \frac{NO_t}{\Gamma_t}) \\
\log \Gamma_{t+1} - \log \Gamma_t &= -\log R + \log(1 - \exp(\log NO_t - \log \Gamma_t)) \\
\psi_{t+1} - \psi_t &= -\log R + \log(1 - \exp(no_t - \psi_t))
\end{aligned}$$

where $\psi_t = \log \Gamma_t$ and $no_t = \log NO_t$. Applying Taylor approximation to the last term of the right hand side around the steady state level, $\overline{no} - \overline{\psi}$,

$$\begin{aligned}
\psi_{t+1} - \psi_t &\approx -\log R + \log \rho + \frac{\rho - 1}{\rho}(no_t - \psi_t - \overline{no} + \overline{\psi}) \quad \text{(D.1)} \\
&= k + \frac{\rho - 1}{\rho}(no_t - \psi_t)
\end{aligned}$$

where $\rho = 1 - \exp(\overline{no} - \overline{\psi}) = 1 - \frac{\overline{NO}}{\overline{\Gamma}} = R$, and $k = -\log R + \log \rho -$

$\frac{\rho-1}{\rho}(\overline{no} - \overline{\psi})$.

By identity, $\psi_{t+1} - \psi_t = \Delta no_{t+1} + (no_t - \psi_t) - (no_{t+1} - \psi_{t+1})$. Combine this with Equation (D.1),

$$\begin{aligned} k + \frac{\rho-1}{\rho}(no_t - \psi_t) &= \Delta no_{t+1} + (no_t - \psi_t) - (no_{t+1} - \psi_{t+1}) \\ no_t - \psi_t &= \rho(no_{t+1} - \psi_{t+1} - \Delta no_{t+1} + k) = -\sum_{i=1}^{\infty} \rho^i (\Delta no_{t+i} - k) \end{aligned}$$

$$\psi_t = no_t + \sum_{i=1}^{\infty} \rho^i (\Delta no_{t+i} - k) \tag{D.2}$$

From the second to the last equation, I use the assumption that $\lim_{i\to\infty} no_{t+i} - \psi_{t+i} = 0$.

In the same way, $\sum_{i=0}^{\infty} R^i (C_{t+i} + Z_{t+i})$, denoted by Φ_t, can be approximated as

$$\phi_t = \log(C_t + Z_t) + \sum_{i=1}^{\infty} \rho^i (\Delta \log(C_{t+i} + Z_{t+i}) - k) \tag{D.3}$$

where $\phi_t = \log \Phi_t$, and I used the fact that $\frac{\overline{C+Z}}{\overline{\Phi}} = \frac{\overline{NO}}{\overline{\Gamma}} = 1 - R$.

Next, I log-linearize the intertemporal budget constraint (Equation (5.9)), rewritten as $\Phi_t = RB_{t-1} + \Gamma_t$, around the steady state values of Φ, RB, and Γ.

$$\begin{aligned} \overline{\Phi}(\phi_t - \overline{\phi}) &= R\overline{B}(\log B_{t-1} - \log \overline{B}) + \overline{\Gamma}(\psi_t - \overline{\psi}) \\ (R\overline{B} + \overline{\Gamma})\phi_t - \overline{\Gamma}\psi_t &= R\overline{B} \log B_{t-1} - R\overline{B} \log \overline{B} - \overline{\Gamma\psi} + \overline{\Phi\phi} \\ \psi_t - \phi_t &= -\frac{R\overline{B}}{\overline{\Gamma}}(\log B_{t-1} - \phi_t) + \frac{R\overline{B} \log \overline{B} + \overline{\Gamma\psi} - \overline{\Phi\phi}}{\overline{\Gamma}} \end{aligned}$$

$$\psi_t = \frac{1}{\Omega}\phi_t + (1 - \frac{1}{\Omega}) \log B_{t-1} + k' \tag{D.4}$$

where $\Omega = 1 - \frac{R\overline{B}}{\overline{\Phi}}$, and k' is a collection of unimportant constant terms.

Substitute (D.2) and (D.3) into (D.4),

$$\begin{aligned}
& no_t + \sum_{i=1}^{\infty} \rho^i (\Delta no_{t+i} - k) \\
&= \frac{1}{\Omega} \left\{ \log(C_t + Z_t) + \sum_{i=1}^{\infty} \rho^i (\Delta \log(C_{t+i} + Z_{t+i}) - k) \right\} + (1 - \frac{1}{\Omega}) \log B_{t-1} + k' \\
& - \sum_{i=1}^{\infty} \rho^i (\Delta no_{t+i} - k) + \frac{1}{\Omega} \sum_{i=1}^{\infty} \rho^i (\Delta \log(C_{t+i} + Z_{t+i}) - k) - k' \\
&= no_t - \frac{1}{\Omega} \log(C_t + Z_t) - (1 - \frac{1}{\Omega}) \log B_{t-1}
\end{aligned}$$

Ignoring the constant terms, using the fact that $\rho = 1 - \frac{\overline{NO}}{\overline{\Gamma}} = 1 - \frac{\overline{C+Z}}{\overline{\Phi}} = R = \beta$ (the last equality is due to the assumption made in the main text), and finally assuming $\overline{B} = 0$ (then, $\Omega = 1$),

$$- \sum_{i=1}^{\infty} \beta^i \left[\Delta no_{t+i} + \Delta \log(C_{t+i} + Z_{t+i}) \right] = no_t - \log(C_t + Z_t)$$

Taking the expectation of both sides conditional on time t information gives me Equation (5.10).

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