Theoretical Foundations of Buffer Stock Saving

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Drawbacks of Numerical Solutions

A Black Box

- Can Construct Solution to Model Without Really Understanding It
- Hard Even To Be Sure Your Numerical Solution Is *Right*
- Little Intuition for How Results Might Change With
  - Calibration
  - Structure
- *Very* Hard To Teach!

I Am A *Big* Fan Of Numerical Methods

- Have Done A Good Deal Of Work With Them Myself
- But As A Result, Have Felt All These Drawbacks Keenly
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The Gap This Paper Fills

Foundations For Microeconomic Household’s Problem With

- Uncertain Labor Income
- No Liquidity Constraints
- CRRA Utility
- (Problem with Liquidity Constraints Is A Limiting Case)
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Key Result

Restrictions on parameter values such that:

- Problem defines a contraction mapping
  - $\Rightarrow \exists$ A unique consumption function $c(m)$
- There is a ‘target’ ratio of assets to permanent income
  - Requires a key ‘impatience’ condition to hold
  - Good news
    - Condition is weaker (easier to satisfy) than previous papers imposed
Restrictions On Parameter Values Such That

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Limit as horizon $T$ goes to infinity of

$$a_t = m_t - c_t$$

$$b_{t+1} = a_t R$$

$$p_{t+1} = p_t \Gamma_{\psi_{t+1}}$$

$$\equiv \Gamma_{t+1}$$

$$m_{t+1} = b_{t+1} + p_{t+1} \xi_{t+1},$$

$$\xi_{t+n} = \begin{cases} 0 & \text{with probability } \varphi > 0 \\ \theta_{t+n}/\varphi & \text{with probability } \varphi \end{cases}$$

$u(\bullet) = \bullet^{1-\rho}/(1-\rho)$; $E_t[\psi_{t+n}] = E_t[\xi_{t+n}] = 1 \forall n > 0$; $\beta < 1, \rho > 1$
Surely This Problem Has Been Solved?

No.

- Can’t Use Stokey et. al. theorems because CRRA utility
- Lit thru Matkowski and Nowak (2011) Can’t Handle Permanent Shocks
- Must Use Boyd’s ‘Weighted’ Contraction Mapping Theorem
- Surprisingly Subtle

Fortunately, the Conclusions Are Simple!
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## Benchmark: Perfect Foresight Model

### Definitions:

- **Absolute Patience Factor**: $\mathcal{P} = (R\beta)^{1/\rho}$
- **Return Patience Factor**: $\mathcal{P}_R = \mathcal{P} / R$
- **Perfect Foresight Growth Patience Factor**: $\mathcal{P}_\Gamma = \mathcal{P} / \Gamma$

### Conditions and Implications:

<table>
<thead>
<tr>
<th>Name</th>
<th>Condition</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>(AIC) Absolute Impatience Condition</td>
<td>$\mathcal{P} &lt; 1$</td>
<td>$c \downarrow$ over time</td>
</tr>
<tr>
<td>(RIC) Return Impatience Condition</td>
<td>$\mathcal{P}_R &lt; 1$</td>
<td>$c/a \downarrow$ over time</td>
</tr>
<tr>
<td>(PFGIC) Growth Impatience Condition</td>
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<td>$c/p \downarrow$ over time</td>
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When Does A Useful Limiting Solution Exist?

Finite Human Wealth (FHWC) condition:

\[ \Gamma < R \quad (3) \]

Return Impatience Condition:

\[ \bar{D}_R < R \quad (4) \]
When Does A Useful Limiting Solution Exist?

Finite Human Wealth (FHWC) condition:

\[ \Gamma < R \]  \hspace{1cm} (3)

Return Impatience Condition:

\[ \mathbf{D}_R < R \]  \hspace{1cm} (4)
What If There Are Liquidity Constraints?

- FHWC is *not* necessary for solution to exist
- Other Key Condition For Useful Solution is ‘Perfect Foresight Finite Value of Autarky Condition (PFFVAC )’:
  \[ \beta \Gamma^{1-\rho} < 1 \]  
  (5)
- Without RIC, Constraints Are Irrelevant
  - Because Wealth Always Wants To Rise, So Constraint Never Binds
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Liquidity Constraints and Uncertainty

- Introduce permanent shocks to income
- Finite Value of Autarky Condition Becomes

\[ \beta \Gamma^{1-\rho} < 1 \]
\[ \beta < \Gamma^{\rho-1} \]
Finite Value of Autarky Condition: Same As In Liq Constr Problem!

\[ \beta \Gamma^{1-\rho} < 1 \]  
\[ \beta < \Gamma^{\rho-1} \]  

\textit{‘Weak Return Impatience Condition’ (WRIC )}

\[ 0 \leq \varphi^{1/\rho} \Phi_R < 1 \]
Requirement For Existence Of A Target

Definitions: ‘Uncertainty-Adjusted’ Growth:

\[ \Gamma = \Gamma \psi \]  \hspace{1cm} (9)

Adjusted Growth Patience Factor:

\[ \Phi_{\Gamma} = \Phi / \Gamma \]  \hspace{1cm} (10)

Growth Impatience Condition:

\[ \Phi_{\Gamma} < 1 \]  \hspace{1cm} (11)

Why? Because it can be shown that

\[ \lim_{m_t \to \infty} E_t \left[ \frac{m_{t+1}}{m_t} \right] = \Phi_{\Gamma} \]  \hspace{1cm} (12)
Five Propositions

1. \( \lim_{m_t \to \infty} \mathbb{E}_t[c_{t+1}/c_t] = \mathbb{P} \)
2. \( \lim_{m_t \to 0} \mathbb{E}_t[c_{t+1}/c_t] = \infty \)
3. \( \exists \) a unique target value of \( m \), called \( \hat{m} \)
4. \( \mathbb{E}_t[c_{t+1}/c_t | m_t = \hat{m}] = \Gamma - \epsilon \)
5. \( \left( \frac{d\mathbb{E}_t[c_{t+1}/c_t]}{dm_t} \right) < 0 \)
The Target Saving Figure

\[ \mathbb{E}_t \left[ \frac{c_{t+1}}{c_t} \right] = (R\beta)^{1/\rho} \]

\[ m_t \]

\[ \tilde{m} \]

\[ \Gamma \]
Bounds On the Consumption Function

\[ \tilde{c}(m) = \kappa m = (1 - \varpi^{1/p} \rho R) m \]

Upper Bound = Min \([\tilde{c}(m), \bar{c}(m)]\)

\[ \bar{c}(m) = (m - 1 + h) \kappa \]

\[ c(m) = (1 - \rho R) \kappa m \]
The Marginal Propensity to Consume

\[ \kappa(m) \equiv c'(m) \]

\[ (1 - \frac{\rho^1}{\rho} \mathbf{p}_R) \equiv \bar{\kappa} \]

\[ \kappa \equiv (1 - \mathbf{p}_R) \]
The Consumption Function and Target Wealth

\[ c(m_t) \]

\[ \mathbb{E}_t[\Delta m_{t+1}] = 0 \]
Szeidl (2012) Proves Existence of an Invariant Distribution of $m, c, a$, etc.
Balanced Growth Equilibrium

Achieved When Cross Section Distribution Reaches Invariance

\[ \frac{Y_{t+1}}{Y_t} = \frac{C_{t+1}}{C_t} = \Gamma \]  

(13)

Fisherman Separation Fails, Even Without Liquidity Constraints!

Insight:

- Precautionary Saving ≈ Liquidity Constraints

If \( \check{c}(m) \) is solution for constrained consumer,

\[ \lim_{\varphi \downarrow 0} c(m; \varphi) = \check{c}(m) \]  

(14)
Achieved When Cross Section Distribution Reaches Invariance

\[ \frac{Y_{t+1}}{Y_t} = \frac{C_{t+1}}{C_t} = \Gamma \]  (13)

Fisherman Separation Fails, Even Without Liquidity Constraints!

Insight:
- Precautionary Saving \( \approx \) Liquidity Constraints
- If \( \dot{c}(m) \) is solution for constrained consumer,

\[ \lim_{\varphi \downarrow 0} c(m; \varphi) = \dot{c}(m) \]  (14)
Achieved When Cross Section Distribution Reaches Invariance

\[ \frac{Y_{t+1}}{Y_t} = \frac{C_{t+1}}{C_t} = \Gamma \]  \hspace{2cm} (13)

Fisherian Separation Fails, Even Without Liquidity Constraints!

Insight:
- Precautionary Saving \( \approx \) Liquidity Constraints
- If \( \hat{c}(m) \) is solution for constrained consumer,

\[ \lim_{\varphi \downarrow 0} c(m; \varphi) = \hat{c}(m) \]  \hspace{2cm} (14)
The MPC Out Of Permanent Shocks

http://econ.jhu.edu/people/ccarroll/papers/MPCPerm.pdf

Lots of Recent Papers Trying to Measure the MPC

Paper Proves:
- MPCP < 1
- But not a lot less:
  - 0.75 to 0.95 (annual rate) for wide range of parameter values
- Defined Conditions Under Which Widely Used Problem Has Solution
  - Finite Value of Autarky Condition Guarantees Contraction (with WRIC)
  - Growth Impatience Condition Prevents $m \to \infty$
- Economy Of Buffer Stock Consumers Exhibits Balanced Growth
  - Even In Absence of General Equilibrium Adj of Interest Rate