Digestible Microfoundations:
Buffer Stock Saving in a Krusell–Smith World

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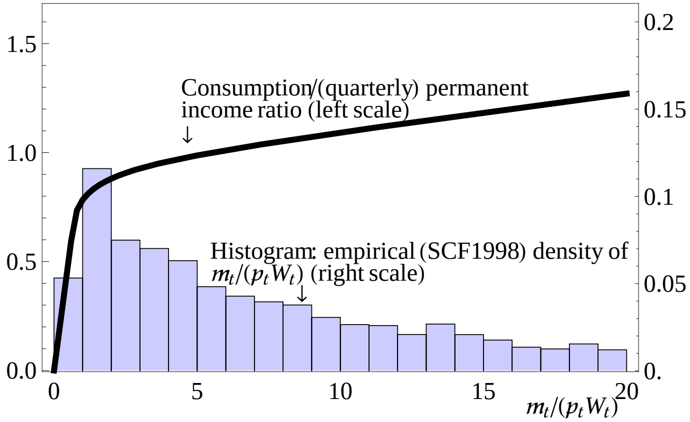
Wealth Heterogeneity and Marginal Propensity to Consume



Consumption Modeling

Core since Friedman's (1957) PIH:

- c chosen optimally;
  want to smooth c *in light of* y fluctuations
- Single most important thing to get right is income dynamics!
- With smooth c, income dynamics drive everything!
  - Saving/dissaving: Depends on whether $\mathbb{E}[\Delta y] \uparrow$ or $\mathbb{E}[\Delta y] \downarrow$
  - Wealth distribution depends on integration of saving
- Cardinal sin: Assume crazy income dynamics
  - No end ('match wealth distribution') can justify this means
  - Throws out the defining core of the intellectual framework

Heterogeneity Matters

- Matching key micro facts may help understand macro 'puzzles' unresolvable in Rep Agent models
- Why might heterogeneity matter?
- Concavity of the consumption function:
  - Different $m \rightarrow$ HHs behave very differently
  - m affects
    - MPC
    - L supply
    - *response to financial change*

The Idea—'Tidewater' Economics

- Lots of people have cut their teeth on Krusell and Smith (1998) model
- **Our goal:** Bridge KS descr of macro and our descr of micro

To Do List

1. Calibrate realistic income process
2. Match empirical wealth distribution
3. Back out optimal C and MPC out of transitory income
4. Is MPC in line with empirical estimates?

Our Question:

Does a model that matches micro facts about income dynamics and wealth distribution give different (and more plausible) answers than KS to macroeconomic questions (say, about the response of consumption to fiscal 'stimulus')?

Friedman (1957): Permanent Income Hypothesis

$$
\begin{aligned}
Y_t &= P_t + T_t \\
C_t &= P_t
\end{aligned}
$$

Progress since then

- Micro data: Friedman description of income shocks works well
- Math: Friedman's words well describe optimal solution to dynamic stochastic optimization problem of impatient consumers with geometric discounting under CRRA utility with uninsurable idiosyncratic risk calibrated using these micro income dynamics (!)

Use the Benchmark KS model with Modifications

Modifications to Krusell and Smith (1998)

1. Serious income process
   - MaCurdy, Card, Abowd; Blundell, Low, Meghir, Pistaferri, ...
2. Finite lifetimes (i.e., introduce Blanchard (1985) death, D)
3. Heterogeneity in time preference factors

Income Process

Idiosyncratic (household) income process is logarithmic Friedman:

$$\begin{aligned} \boldsymbol{y}_{t+1} &= p_{t+1}\xi_{t+1}\mathrm{W} \\ p_{t+1} &= p_t\psi_{t+1} \end{aligned}$$

p_t = permanent income
ξ_t = transitory income
ψ_{t+1} = permanent shock
W = aggregate wage rate

Income Process

Modifications from Carroll (1992):
Trans income ξ_t incorporates unemployment insurance:

$$\begin{aligned} \xi_t &= \mu \text{ with probability } u \\ &= (1-\tau)\bar{l}\theta_t \text{ with probability } 1-u \end{aligned}$$

μ is UI when unemployed
τ is the rate of tax collected for the unemployment benefits

Model Without Aggr Uncertainty: Decision Problem

$$
\begin{aligned}
\mathrm{v}(m_{t,i}) &= \max_{\{c_{t,i}\}} \ \mathrm{u}(c_{t,i}) + \beta \cancel{D} \mathbb{E}_t \left[\psi_{t+1,i}^{1-\rho} \mathrm{v}(m_{t+1,i}) \right] \\
&\text{s.t.} \\
a_{t,i} &= m_{t,i} - c_{t,i} \\
a_{t,i} &\geq 0 \\
k_{t+1,i} &= a_{t,i} / (\cancel{D} \psi_{t+1,i}) \\
m_{t+1,i} &= (\daleth + r) k_{t+1,i} + \xi_{t+1} \\
r &= \alpha z (\boldsymbol{K} / \bar{l} \boldsymbol{L})^{\alpha - 1}
\end{aligned}
$$

Variables normalized by $p_t \mathrm{W}$

What Happens After Death?

- You are replaced by a new agent whose permanent income is equal to the population mean
- Prevents the population distribution of permanent income from spreading out

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Ergodic Distribution of Permanent Income

Exists, if death eliminates permanent shocks:

$$\cancel{D}\mathbb{E}[\psi^2] < 1.$$

Holds.

Population mean of p^2:

$$\mathbb{M}[p^2] \quad = \quad \left(\frac{\mathsf{D}}{1 - \cancel{D}\mathbb{E}[\psi^2]}\right)$$

Parameter Values

- β, ρ, α, δ, $\bar{l}$, μ , and u taken from JEDC special volume
- Key new parameter values:

Description	Param	Value	Source
Prob of Death per Quarter	D	0.005	Life span of 50 years
Variance of Log ψ_t	σ_ψ^2	0.016/4	Carroll (1992); SCF
Variance of Log θ_t	σ_θ^2	0.010×4	Carroll (1992)

Annual Income, Earnings, or Wage Variances

	σ_{ψ}^2	σ_{ξ}^2
Our parameters	0.016	0.010
Carroll (1992)	0.016	0.010
Storesletten, Telmer, and Yaron (2004)	0.008–0.026	0.316
Meghir and Pistaferri (2004)*	0.031	0.032
Low, Meghir, and Pistaferri (2005)	0.011	—
Blundell, Pistaferri, and Preston (2008)*	0.010–0.030	0.029–0.055
Implied by KS-JEDC	0.000	0.038
Implied by Castaneda et al. (2003)	0.029	0.005

* Meghir and Pistaferri (2004) and Blundell, Pistaferri, and Preston (2008) assume that the transitory component is serially correlated (an MA process), and report the variance of a subelement of the transitory component. σ_{ξ}^2 for these articles are calculated using their MA estimates.

Typology of Our Models

Three Dimensions

1. Discount Factor β
   - 'β-Point' model: Single discount factor
   - 'β-Dist' model: Uniformly distributed discount factor
2. Aggregate Shocks
   - (No)
   - Krusell–Smith
   - Friedman/Buffer Stock
3. Empirical Wealth Variable to Match
   - Net Worth
   - Liquid Financial Assets

Dimension 1: Estimation of β-Point and β-Dist

'β-Point' model

- 'Estimate' single $\grave{\beta}$ by matching the capital–output ratio

'β-Dist' model—Heterogenous Impatience

- Assume uniformly distributed β across households
- Estimate the band $[\grave{\beta} - \nabla, \grave{\beta} + \nabla]$ by minimizing distance between model (w) and data (ω) net worth held by the top 20, 40, 60, 80%

$$\min_{\{\grave{\beta},\nabla\}} \sum_{i=20,40,60,80} (w_i - \omega_i)^2,$$

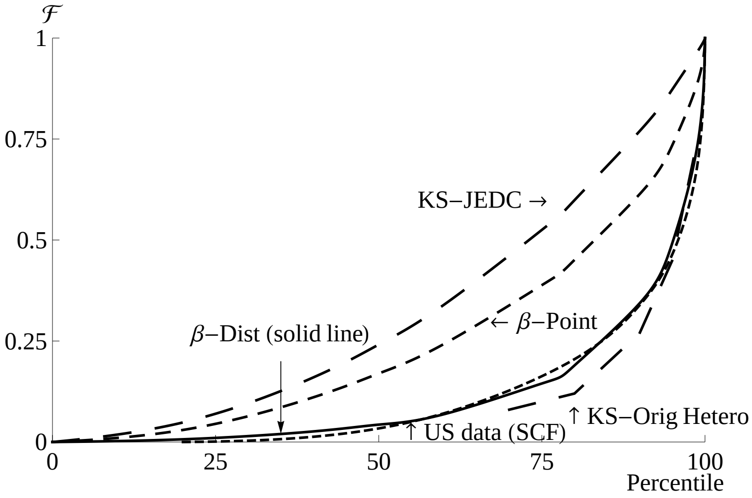
s.t. aggregate net worth–output ratio matches the steady-state value from the perfect foresight model

Results: Wealth Distribution

	Income Process					
	Friedman/Buffer Stock		KS-JEDC	KS-Orig$^{\diamond}$		
	Point Discount Factor‡	Uniformly Distributed Discount Factors*	Our solution		Hetero	
	β-Point	β-Dist				U.S. Data*
Top 1%	10.3	24.9	3.0	3.0	24.0	29.6
Top 20%	54.9	81.0	40.1	35.0	88.0	79.5
Top 40%	75.7	93.1	66.0			92.9
Top 60%	88.9	97.4	84.0			98.7
Top 80%	97.0	99.3	95.2			100.4

Notes: ‡ : $\grave{\beta} = 0.9888$. * : $(\grave{\beta}, \nabla) = (0.9869, 0.0052)$. $^{\diamond}$: The results are from Krusell and Smith (1998) who solved the models with aggregate shocks. * : U.S. data is the SCF reported in Castaneda, Diaz-Gimenez, and Rios-Rull (2003). Bold points are targeted. $\boldsymbol{K}_t/\boldsymbol{Y}_t$=10.3.

Results: Wealth Distribution



Dimension 2.a: Adding KS Aggregate Shocks

Model with KS Aggregate Shocks: Assumptions

- Only two aggregate states (good or bad)
- Aggregate productivity $z_t = 1 \pm \triangle^z$
- Unemployment rate u depends on the state (u^g or u^b)

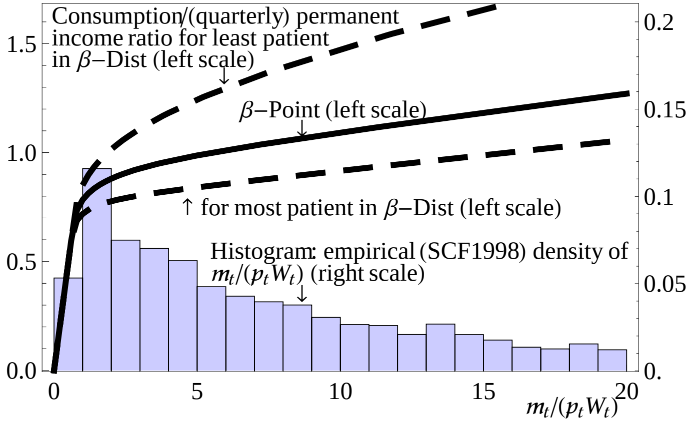
Parameter values for aggregate shocks from
Krusell and Smith (1998)

Parameter	Value
$\triangle^z$	0.01
u^g	0.04
u^b	0.10
Agg transition probability	0.125

Solution Method

- HH needs to forecast $\boldsymbol{k}_t \equiv \boldsymbol{K}_t / \bar{l}_t \boldsymbol{L}_t$ since it determines future interest rates and wages.
- Two broad approaches
  1. Direct computation of the system's law of motion
     Advantage: fast, accurate
  2. Simulations (iterate until convergence)
     Advantage: directly generate micro data $\Rightarrow$ we do this

Marginal Propensity to Consume & Net Worth



Results: MPC (in Annual Terms)

	Income Process		
	Friedman/Buffer Stock		KS-JEDC
	β-Point	β-Dist	Our solution
Overall average	0.09	0.19	0.05
By wealth/permanent income ratio			
Top 1%	0.06	0.05	0.04
Top 20%	0.06	0.06	0.04
Top 40%	0.06	0.07	0.04
Top 60%	0.07	0.09	0.04
Bottom 1/2	0.12	0.28	0.05
By employment status			
Employed	0.08	0.16	0.05
Unemployed	0.20	0.44	0.06

Notes: Annual MPC is calculated by $1 - (1-\text{quarterly MPC})^4$. See the paper for a discussion of the extensive literature that generally estimates empirical MPC's in the range of 0.3–0.6.

Estimates of MPC in the Data: $\sim$0.2–0.6

	Consumption Measure		
Authors	Nondurables	Durables	Total PCE
Agarwal, Liu, and Souleles (2007)			0.4
Coronado, Lupton, and Sheiner (2005)			0.28–0.36
Johnson, Parker, and Souleles (2006)	0.12–0.30		0.50–0.90
Johnson, Parker, and Souleles (2009)	0.25		
Lusardi (1996)‡	0.2–0.5		
Parker (1999)	0.2		
Parker, Souleles, Johnson, and McClelland (2011)	0.12–0.30		
Sahm, Shapiro, and Slemrod (2009)			0.33
Shapiro and Slemrod (2009)			0.33
Souleles (1999)	0.09	0.54	0.64
Souleles (2002)	0.6–0.9		

Notes: ‡: elasticity.

Dimension 2.b: Adding FBS Aggregate Shocks

Friedman/Buffer Stock Shocks

- Motivation:
  More plausible and tractable aggregate process, also simpler
- Eliminates 'good' and 'bad' aggregate state
- Aggregate production function: $\boldsymbol{K}_t^{\alpha}(\boldsymbol{L}_t)^{1-\alpha}$
  - $\boldsymbol{L}_t = P_t \Xi_t$
  - P_t is aggregate permanent productivity
  - $P_{t+1} = P_t \Psi_{t+1}$
  - Ξ_t is the aggregate transitory shock.
- Parameter values estimated from U.S. data:

Description	Parameter	Value
Variance of Log Ψ_t	σ_{Ψ}^2	0.00004
Variance of Log Ξ_t	σ_{Ξ}^2	0.00001

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- Parameter values estimated from U.S. data:

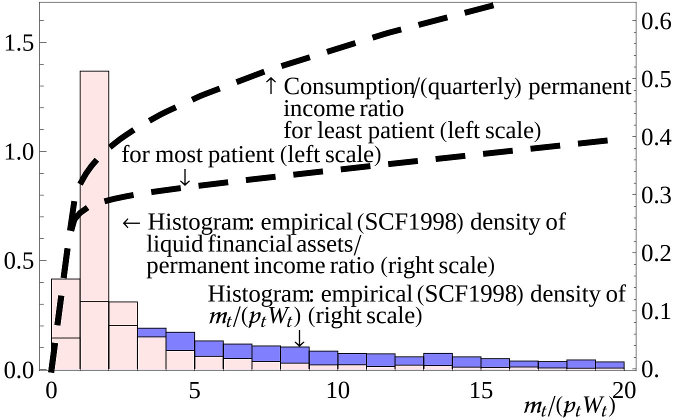
Description	Parameter	Value
Variance of Log Ψ_t	σ^2_{Ψ}	0.00004
Variance of Log Ξ_t	σ^2_{Ξ}	0.00001

Results

Our/FBS model

- A few times faster than solving KS model
- The results are similar to those under KS aggregate shocks
- Average MPC
  - Matching net worth: 0.18
  - Matching liquid financial assets: 0.69

Dimension 3: Matching Net Worth vs Liquid Financial Assets



Liquid Assets ≡ transaction accounts, CDs, bonds, stocks, mutual funds

Matching Net Worth vs Liquid Financial Assets

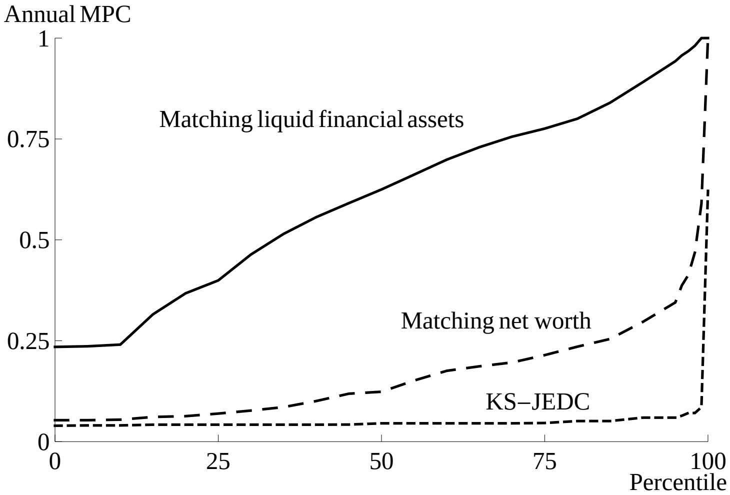
- Buffer stock saving driven by accumulation of liquidity for rainy days
- May make more sense to match liquid assets (Hall (2011), Kaplan and Violante (2011))
- **Average MPC Increases Substantially: 0.19 $\uparrow$ 0.68**

	β-Dist	
	Net Worth	Liq Fin Assets
Overall average	0.19	0.68
By wealth/permanent income ratio		
Top 1%	0.05	0.23
Top 20%	0.06	0.28
Top 40%	0.07	0.39
Top 60%	0.09	0.50
Bottom 1/2	0.28	0.83

Notes: Annual MPC is calculated by $1-(1-\text{quarterly MPC})^4$.

Distribution of MPCs

Wealth heterogeneity translates into heterogeneity in MPCs



Conclusions

- Micro-founded income process and heterogeneity in patience help increase wealth inequality.
- The model produces more plausible implications about MPC.
- Version with more plausible aggregate specification is simpler, faster, better in every way!

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