Digestible Microfoundations: Buffer Stock Saving in a Krusell–Smith World

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Wealth Heterogeneity and Marginal Propensity to Consume

\[ \frac{\text{Consumption}}{\text{(quarterly) permanent income ratio}} \]

Histogram: empirical (SCF1998) density of \( m_t/(p_tW_t) \) (right scale)

Consumption/(quarterly) permanent income ratio (left scale)

\( m_t/(p_tW_t) \)
Consumption Modeling

Core since Friedman’s (1957) PIH:

- $c$ chosen optimally;
  want to smooth $c$ in light of $y$ fluctuations
- Single most important thing to get right is income dynamics!
- With smooth $c$, income dynamics drive everything!
  - Saving/dissaving: Depends on whether $\mathbb{E}[\Delta y] \uparrow$ or $\mathbb{E}[\Delta y] \downarrow$
  - Wealth distribution depends on integration of saving
- Cardinal sin: Assume crazy income dynamics
  - No end (‘match wealth distribution’) can justify this means
  - Throws out the defining core of the intellectual framework
Matching key micro facts may help understand macro ‘puzzles’ unresolvable in Rep Agent models

Why might heterogeneity matter?

Concavity of the consumption function:
  - Different $m \rightarrow$ HHs behave very differently
  - $m$ affects
    - MPC
    - $L$ supply
    - response to financial change
The Idea—‘Tidewater’ Economics

- Lots of people have cut their teeth on Krusell and Smith (1998) model
- **Our goal:** Bridge KS descr of macro and our descr of micro
To Do List

1. Calibrate realistic income process
2. Match empirical wealth distribution
3. Back out optimal C and MPC out of transitory income
4. Is MPC in line with empirical estimates?

Our Question:
Does a model that matches micro facts about income dynamics and wealth distribution give different (and more plausible) answers than KS to macroeconomic questions (say, about the response of consumption to fiscal ‘stimulus’)?
Friedman (1957): Permanent Income Hypothesis

\[ Y_t = P_t + T_t \]
\[ C_t = P_t \]

Progress since then

- **Micro data:** Friedman description of income shocks works well
- **Math:** Friedman’s words well describe optimal solution to dynamic stochastic optimization problem of impatient consumers with geometric discounting under CRRA utility with uninsurable idiosyncratic risk calibrated using these micro income dynamics (!)
Use the Benchmark KS model with Modifications

Modifications to Krusell and Smith (1998)

1. Serious income process
   ▶ MaCurdy, Card, Abowd; Blundell, Low, Meghir, Pistaferri, . . .
2. Finite lifetimes (i.e., introduce Blanchard (1985) death, D)
3. Heterogeneity in time preference factors
Idiosyncratic (household) income process is logarithmic Friedman:

\[ y_{t+1} = p_{t+1} \xi_{t+1} W \]
\[ p_{t+1} = p_{t} \psi_{t+1} \]

\( p_t = \) permanent income
\( \xi_t = \) transitory income
\( \psi_{t+1} = \) permanent shock
\( W = \) aggregate wage rate
Income Process

Modifications from Carroll (1992):
Trans income $\xi_t$ incorporates unemployment insurance:

$$
\xi_t = \begin{cases} 
\mu & \text{with probability } u \\
(1 - \tau) \tilde{I} \theta_t & \text{with probability } 1 - u
\end{cases}
$$

$\mu$ is UI when unemployed
$\tau$ is the rate of tax collected for the unemployment benefits
Model Without Aggr Uncertainty: Decision Problem

\[ v(m_{t,i}) = \max_{\{c_{t,i}\}} u(c_{t,i}) + \beta \mathbb{D}\mathbb{E}_t \left[ \psi_{t+1,i}^{1-\rho} v(m_{t+1,i}) \right] \]

s.t.

\[
\begin{align*}
    a_{t,i} &= m_{t,i} - c_{t,i} \\
    a_{t,i} &\geq 0 \\
    k_{t+1,i} &= a_{t,i}/(\mathbb{D}\psi_{t+1,i}) \\
    m_{t+1,i} &= (\bar{\gamma} + r) k_{t+1,i} + \xi_{t+1} \\
    r &= \alpha z(K/\bar{L})^{\alpha-1}
\end{align*}
\]

Variables normalized by \( p_t W \)
What Happens After Death?

- You are replaced by a new agent whose permanent income is equal to the population mean.
- Prevents the population distribution of permanent income from spreading out.
What Happens After Death?

- You are replaced by a new agent whose permanent income is equal to the population mean
- Prevents the population distribution of permanent income from spreading out
Ergodic Distribution of Permanent Income

_exists, if death eliminates permanent shocks:

\[ \mathcal{D} \mathbb{E}[\psi^2] < 1. \]

Holds.

Population mean of \( p^2 \):

\[ \mathbb{M}[p^2] = \left( \frac{D}{1 - \mathcal{D} \mathbb{E}[\psi^2]} \right) \]
Parameter Values

- $\beta$, $\rho$, $\alpha$, $\delta$, $\bar{t}$, $\mu$, and $u$ taken from JEDC special volume
- Key new parameter values:

<table>
<thead>
<tr>
<th>Description</th>
<th>Param</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob of Death per Quarter</td>
<td>D</td>
<td>0.005</td>
<td>Life span of 50 years</td>
</tr>
<tr>
<td>Variance of Log $\psi_t$</td>
<td>$\sigma_{\psi}^2$</td>
<td>0.016/4</td>
<td>Carroll (1992); SCF</td>
</tr>
<tr>
<td>Variance of Log $\theta_t$</td>
<td>$\sigma_{\theta}^2$</td>
<td>0.010 $\times$ 4</td>
<td>Carroll (1992)</td>
</tr>
</tbody>
</table>
### Annual Income, Earnings, or Wage Variances

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{\psi}^2$</th>
<th>$\sigma_{\xi}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Our parameters</strong></td>
<td>0.016</td>
<td>0.010</td>
</tr>
<tr>
<td>Carroll (1992)</td>
<td>0.016</td>
<td>0.010</td>
</tr>
<tr>
<td>Storesletten, Telmer, and Yaron (2004)</td>
<td>0.008–0.026</td>
<td>0.316</td>
</tr>
<tr>
<td>Meghir and Pistaferri (2004)*</td>
<td>0.031</td>
<td>0.032</td>
</tr>
<tr>
<td>Low, Meghir, and Pistaferri (2005)</td>
<td>0.011</td>
<td>—</td>
</tr>
<tr>
<td>Blundell, Pistaferri, and Preston (2008)*</td>
<td>0.010–0.030</td>
<td>0.029–0.055</td>
</tr>
<tr>
<td>Implied by KS-JEDC</td>
<td>0.000</td>
<td>0.038</td>
</tr>
<tr>
<td>Implied by Castaneda et al. (2003)</td>
<td>0.029</td>
<td>0.005</td>
</tr>
</tbody>
</table>

*Meghir and Pistaferri (2004) and Blundell, Pistaferri, and Preston (2008) assume that the transitory component is serially correlated (an MA process), and report the variance of a subelement of the transitory component. $\sigma_{\xi}^2$ for these articles are calculated using their MA estimates.*
Typology of Our Models

Three Dimensions

1. Discount Factor $\beta$
   - ‘$\beta$-Point’ model: Single discount factor
   - ‘$\beta$-Dist’ model: Uniformly distributed discount factor

2. Aggregate Shocks
   - (No)
   - Krusell–Smith
   - Friedman/Buffer Stock

3. Empirical Wealth Variable to Match
   - Net Worth
   - Liquid Financial Assets
Dimension 1: Estimation of $\beta$-Point and $\beta$-Dist

$\beta$-Point’ model

- ‘Estimate’ single $\hat{\beta}$ by matching the capital–output ratio

$\beta$-Dist’ model—Heterogenous Impatience

- Assume uniformly distributed $\beta$ across households
- Estimate the band $[\hat{\beta} - \nabla, \hat{\beta} + \nabla]$ by minimizing distance between model ($w$) and data ($\omega$) net worth held by the top 20, 40, 60, 80%

$$\min_{\{\hat{\beta}, \nabla\}} \sum_{i=20, 40, 60, 80} (w_i - \omega_i)^2,$$

s.t. aggregate net worth–output ratio matches the steady-state value from the perfect foresight model
## Results: Wealth Distribution

<table>
<thead>
<tr>
<th>Income Process</th>
<th>Friedman/Buffer Stock</th>
<th>KS-JEDC</th>
<th>KS-Orig*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Point Uniformly</td>
<td>Our solution</td>
<td>Hetero</td>
</tr>
<tr>
<td>Discount Factor\‡</td>
<td>Discount Distributed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta)-Point</td>
<td>(\beta)-Dist</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>10.3</td>
<td>24.9</td>
<td>3.0</td>
</tr>
<tr>
<td>Top 20%</td>
<td>54.9</td>
<td>81.0</td>
<td>40.1</td>
</tr>
<tr>
<td>Top 40%</td>
<td>75.7</td>
<td>93.1</td>
<td>66.0</td>
</tr>
<tr>
<td>Top 60%</td>
<td>88.9</td>
<td>97.4</td>
<td>84.0</td>
</tr>
<tr>
<td>Top 80%</td>
<td>97.0</td>
<td>99.3</td>
<td>95.2</td>
</tr>
</tbody>
</table>

Notes: \‡ : \(\hat{\beta} = 0.9888\). \* : \((\hat{\beta}, \nabla) = (0.9869, 0.0052)\). \*: The results are from Krusell and Smith (1998) who solved the models with aggregate shocks. \*: U.S. data is the SCF reported in Castaneda, Diaz-Gimenez, and Rios-Rull (2003). Bold points are targeted. \(K_t/Y_t=10.3\).
Results: Wealth Distribution

- KS–JEDC →
- β–Dist (solid line)
- β–Point
- ↑ US data (SCF)
- ↑ KS–Orig Hetero

Percentile
Model with KS Aggregate Shocks: Assumptions

- Only two aggregate states (good or bad)
- Aggregate productivity \( z_t = 1 \pm \Delta^z \)
- Unemployment rate \( u \) depends on the state \( (u^g \text{ or } u^b) \)

Parameter values for aggregate shocks from Krusell and Smith (1998)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta^z )</td>
<td>0.01</td>
</tr>
<tr>
<td>( u^g )</td>
<td>0.04</td>
</tr>
<tr>
<td>( u^b )</td>
<td>0.10</td>
</tr>
<tr>
<td>Agg transition probability</td>
<td>0.125</td>
</tr>
</tbody>
</table>
Solution Method

- HH needs to forecast $k_t \equiv K_t/\bar{I}_tL_t$ since it determines future interest rates and wages.

- Two broad approaches
  1. Direct computation of the system’s law of motion
     Advantage: fast, accurate
  2. Simulations (iterate until convergence)
     Advantage: directly generate micro data ⇒ we do this
Marginal Propensity to Consume & Net Worth

Consumption/(quarterly) permanent income ratio for least patient in $\beta$–Dist (left scale)

$\beta$–Point (left scale)

$\uparrow$ for most patient in $\beta$–Dist (left scale)

Histogram: empirical (SCF1998) density of $m_t/(p_t W_t)$ (right scale)
Results: MPC (in Annual Terms)

<table>
<thead>
<tr>
<th>Income Process</th>
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<th>KS-JEDC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>β-Point</td>
<td>β-Dist</td>
</tr>
<tr>
<td>Overall average</td>
<td>0.09</td>
<td>0.19</td>
</tr>
<tr>
<td>By wealth/permanent income ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Top 1%</td>
<td>0.06</td>
<td>0.05</td>
</tr>
<tr>
<td>Top 20%</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>Bottom 1/2</td>
<td>0.12</td>
<td>0.28</td>
</tr>
<tr>
<td>By employment status</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed</td>
<td>0.08</td>
<td>0.16</td>
</tr>
<tr>
<td>Unemployed</td>
<td>0.20</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Notes: Annual MPC is calculated by $1 - (1 - \text{quarterly MPC})^4$. See the paper for a discussion of the extensive literature that generally estimates empirical MPC's in the range of 0.3–0.6.
# Estimates of MPC in the Data: \(\sim 0.2–0.6\)

<table>
<thead>
<tr>
<th>Authors</th>
<th>Nondurables</th>
<th>Durables</th>
<th>Total PCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agarwal, Liu, and Souleles (2007)</td>
<td></td>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td>Coronado, Lupton, and Sheiner (2005)</td>
<td>0.28–0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Johnson, Parker, and Souleles (2006)</td>
<td>0.12–0.30</td>
<td></td>
<td>0.50–0.90</td>
</tr>
<tr>
<td>Johnson, Parker, and Souleles (2009)</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lusardi (1996)‡</td>
<td>0.2–0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parker (1999)</td>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parker, Souleles, Johnson, and McClelland (2011)</td>
<td>0.12–0.30</td>
<td></td>
<td></td>
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<tr>
<td>Sahm, Shapiro, and Slemrod (2009)</td>
<td></td>
<td></td>
<td>0.33</td>
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<td>0.33</td>
</tr>
<tr>
<td>Souleles (1999)</td>
<td>0.09</td>
<td>0.54</td>
<td>0.64</td>
</tr>
<tr>
<td>Souleles (2002)</td>
<td>0.6–0.9</td>
<td></td>
<td></td>
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Notes: ‡: elasticity.
Dimension 2.b: Adding FBS Aggregate Shocks

Friedman/Buffer Stock Shocks

- **Motivation:**
  More plausible and tractable aggregate process, also simpler

- **Eliminates ‘good’ and ‘bad’ aggregate state**

- **Aggregate production function:** $K_t^\alpha (L_t)^{1-\alpha}$
  - $L_t = P_t \Xi_t$
  - $P_t$ is aggregate permanent productivity
  - $P_{t+1} = P_t \Psi_{t+1}$
  - $\Xi_t$ is the aggregate transitory shock.

- **Parameter values estimated from U.S. data:**

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<tr>
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</thead>
<tbody>
<tr>
<td>Variance of Log $\Psi_t$</td>
<td>$\sigma^2_{\Psi}$</td>
<td>0.00004</td>
</tr>
<tr>
<td>Variance of Log $\Xi_t$</td>
<td>$\sigma^2_{\Xi}$</td>
<td>0.00001</td>
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<td>Variance of Log $\Xi_t$</td>
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</tr>
</tbody>
</table>
Results

Our/FBS model

- A few times faster than solving KS model
- The results are similar to those under KS aggregate shocks
- Average MPC
  - Matching net worth: 0.18
  - Matching liquid financial assets: 0.69
Dimension 3: Matching Net Worth vs Liquid Financial Assets

Liquid Assets $\equiv$ transaction accounts, CDs, bonds, stocks, mutual funds
Matching Net Worth vs Liquid Financial Assets

- Buffer stock saving driven by accumulation of liquidity for rainy days
- May make more sense to match liquid assets (Hall (2011), Kaplan and Violante (2011))
- **Average MPC Increases Substantially: 0.19 \uparrow 0.68**

<table>
<thead>
<tr>
<th>(\beta)-Dist</th>
<th>Net Worth</th>
<th>Liq Fin Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall average</td>
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<td>0.68</td>
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</tr>
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<td>Top 20%</td>
<td>0.06</td>
<td>0.28</td>
</tr>
<tr>
<td>Top 40%</td>
<td>0.07</td>
<td>0.39</td>
</tr>
<tr>
<td>Top 60%</td>
<td>0.09</td>
<td>0.50</td>
</tr>
<tr>
<td>Bottom 1/2</td>
<td>0.28</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Notes: Annual MPC is calculated by \(1 - (1 - \text{quarterly MPC})^4\).
Distribution of MPCs

Wealth heterogeneity translates into heterogeneity in MPCs

Annual MPC

Matching liquid financial assets

Matching net worth

KS–JEDC

Percentile

0 25 50 75 100

0

0.25

0.5

0.75

1

0 25 50 75 100

0

0.25

0.5

0.75

1
Conclusions

- Micro-founded income process and heterogeneity in patience help increase wealth inequality.
- The model produces more plausible implications about MPC.
- Version with more plausible aggregate specification is simpler, faster, better in every way!


