Preliminary
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# Comparison Utility in a Growth Model

Christopher D. Carroll
ccarroll@jhu.edu
Department of Economics
The Johns Hopkins University
Baltimore, MD 21218-2685

Jody Overland
University of Colorado at Denver
jodyo@econ.pstc.brown.edu

David N. Weil
Brown University
david_weil@brown.edu
Department of Economics
Box B, Brown University
Providence, Rhode Island, 02912

June 2, 1997

JEL D91 E21 O40
Keywords: habit formation, relative consumption, endogenous growth
We are grateful to Xavier Sala-i-Martin, seminar participants at Yale, Wharton, and the University of Chicago, and especially to Harl Ryder for extensive comments and advice.

**Abstract**

This paper compares the dynamics of two general equilibrium models of endogenous growth in which agents have “comparison utility.” In the “inward-looking” economy, individuals care about how their consumption in the current period compares to their own consumption in the past (one way to describe this is “habit-formation” in consumption). In the “outward-looking” economy, individuals care about how their own level of consumption compares with others' consumption. While steady state growth rates are identical in the two economies, transition paths differ. For example, consider the effect of negative shock to capital. In an endogenous growth model with standard preferences, there will be no effect on the saving rate or the growth rate of output. In both of the models that we consider, however, saving and growth will temporarily fall in response to the shock. The initial decline in saving and growth will be larger in the inward-looking case. However, since agents in the outward-looking case do not take into account the externality effect of their consumption, higher growth in this case will lead to lower utility than in the inward-looking case.

# 1 Introduction

At the core of most modern growth theories are agents solving intertemporal optimization problems. Decisions about consumption determine savings, which in turn becomes capital for future production. Much of the theoretical literature on growth has been devoted to exploring the implications of alternative assumptions about the aggregate production function. Far less attention has been payed to the utility function which agents are assumed to be maximizing; generally, it has been taken to be time separable, and usually of the Constant Relative Risk Aversion form.

In this paper we examine the implications for growth of an alternative form of preferences which has recently been proposed as an explanation for a variety of puzzles in both the asset-pricing and consumption literatures. Our consumers' utility depends not only on the level of their consumption, but also on how their consumption *compares* to some standard, which we will designate the "reference stock." We call this general class of utility functions "comparison utility." In this paper we consider two specific versions of the comparison utility model. In the first, which we call the "inward-looking" version, the consumer's reference stock is his own past consumption. In this version of the model, consumers experiencing faster consumption growth will be happier, *ceteris paribus*.[1] In the second formulation, which we call the "outward-looking" version, individuals compare their consumption with the consumption of others. As Duesenberry (1949) puts it "it is not only true that 'what you don't know can't hurt you,' but that what you do know does hurt you." (p. 27).

The idea that individual utility is determined by comparing current consumption to some standard has a long pedigree in economics. In classifying goods for purposes of taxation,

[1] Inward-looking utility more frequently goes by the name "habit formation." We use the former designation to maintain parallelism. Outward-looking utility is sometimes called "interdependent preferences," "external habit formation," "Keeping up with the Joneses," or "the Relative Income Hypothesis." The general class of utility that we call "comparison utility" is also called "endogenous utility." See Pollak (1978) and Kapteyn, Wansbeek and Buyze (1980) for surverys of previous literature in this area.

Smith (1776) defined "necessaries" as

> not only the commodities which are indispensably necessary for the support of life, but whatever the custom of the country renders it indecent for creditable people, even of the lowest order, to be without. A linen shirt, for example, is, strictly speaking not a necessary of life. The Greeks and Romans lived, I suppose, very comfortably, though they had no linen. But in the present times,... a creditable day-labourer would be ashamed to appear in public without a linen shirt.[2]

Veblen (1899) advanced a similar view of outward-looking utility:

> The accepted standard of expenditure in the community or in the class to which a person belongs largely determines what his standard of living will be. It does this directly by commending itself to his common sense as right and good, through his habitually contemplating it and assimilating the scheme of life in which it belongs; but it does so also indirectly through popular insistence on conformity to the accepted scale of expenditure as a matter of propriety, under pain of disesteem and ostracism.[3]

The outward-looking approach was formalized by Duesenberry who proposed that utility is dependent on the ratio of an individual's own consumption to the average consumption of those around him. But Duesenberry also explored what we call the inward-looking view. He argued that consumption data are well explained by the "fundamental psychological postulate" that "it is harder for a family to reduce its expenditures from a high level than for a family to refrain from making high expenditures in the first place." (p. 84-85). Attempts to define "poverty" frequently use a fully or partially relative standard – see Sen (1983). In sociology, the model of "relative deprivation" of Runciman (1966) closely corresponds to what we call the outward-looking model.

In recent years, comparison utility has also been invoked to explain phenomena in asset markets. Campbell and Cochrane (1995) try to explain aggregate stock prices using a model in which individuals compare their own consumption to a habit stock based on past aggregate consumption. Constantinides (1990) considers a similar model in which individuals care about

[2]Modern Library Edition, p. 821-22.
[3]p. 111. See also Pigou (1903).

how their current consumption compares to their consumption in the recent past. Abel (1990) presents a model which individuals make both types of comparisons.

Several recent empirical papers in the consumption literature have argued that some form of comparison utility may play an important role in determining consumption. Deaton and Paxson (1992), Dynan (1993), and Carroll and Weil (1994) argue that habit formation may be necessary to explain various time-series features of consumption data. Kapteyn, van de Geer, and van de Stadt (1985) estimate a model in which both one's own past consumption and the consumption of others influences utilty. They find that the weight on the former is roughly twice the weight on the latter. Further, they cannot reject the proposition that utility is entirely relative, that is, the absolute level of consumption plays no role in determining utility. Finally, Robert Frank (1985) presents a comprehensive array of qualitative evidence suggesting that utility is strongly affected by relative status, and that income or consumption are important indicators of status.

The presence of a reference stock in the utility function changes the individual's optimization problem and thus the dynamics of an economy populated by such individuals. Our goal in this paper is to explore these dynamics in the inward- and outward-looking cases, to see how they compare with both each other and with the standard time-separable benchmark. The utility function that we use, introduced by Abel (1990), nests the two polar cases where the agent cares only about the level of consumption (the reference stock is irrelevant), and where the agent cares only about how consumption compares to the reference stock (the level of consumption is irrelevant).

In the inward-looking case, each household's reference stock is simply an exponential moving average of its own past consumption. In making their consumption decisions, households take into account the effects of their current consumption on their future reference stock. In the outward-looking case, the household's reference stock is determined by the past consumption of others. Each household takes the future path of the reference stock as given, but because an individual does not take into account the effect of his own current consumption on the future reference stock of others, there is a consumption externality which leads to utility lower than that achieved in the inward-looking case. We also explore the limiting case, which is a model in which an individual's utility is determined by comparing his consumption to the current consumption of others.

Since our concern is with the nature of the utility function and how this affects economy-wide dynamics, we keep the production side of the model as simple as possible by using the endogenous growth framework of Rebelo (1991). The Rebelo framework greatly simplifies the analysis in comparison with models with a neoclassical production function, e.g. Ryder and Heal (1973).

The rest of the paper is organized as follows. In Section 2, we set up the household's optimization problem, which is solved in Section 3. We show that allowing for either kind of comparison utility changes the qualitative properties of the economy. For example, in response to a negative shock to the capital stock, a model with a CRRA utility function would predict no change in the saving rate, while both our inward-looking and outward-looking comparison utility models predict a transitory decline in saving. Section 4 compares the out-of-steady-state dynamics of the two economies. We show that in the outward-looking economy, the reactions of the saving and growth rates to a shock to capital are larger than in the inward-looking economy. Section 5 concludes.

## 2 The Model

We begin by setting up the problems solved in different versions of the model. All of the economies that we consider are populated by identical, infinitely-lived dynasties. Households' utility depends on the comparison of their consumption at each point in time to a reference stock. Below we discuss how the reference stock is determined in different versions of the problem.

The instantaneous utility function for household $i$ is

$$U(c_i, z_i) = \frac{\left(\frac{c_i}{z_i^{\gamma}}\right)^{1-\sigma}}{1-\sigma} \tag{1}$$

where $z$ is reference stock. The parameter $\gamma$ indexes the importance of comparison utility. If $\gamma = 0$, then only the absolute level of consumption is important, while if $\gamma = 1$, then consumption relative to the reference stock is all that matters. For values of $\gamma$ in between zero and one, both are important. For example, if $\gamma = .5$, then a person with consumption of 2 and reference stock of 1 would have the same utility as a person with both consumption and reference stock equal to 4. We assume $0 \leq \gamma < 1$. Finally, we assume that $\sigma > \frac{1}{1-\gamma}$. This condition guarantees that the utility function is strictly concave in both arguments.

The production function is

$$y = Ak. \tag{2}$$

Capital depreciates at rate $\delta \geq 0$. Because there are constant returns to capital, there are no interactions between households through the production side of the economy. Household $i$'s stock of capital, $k_i$, evolves according to

$$\dot{k_i} = (A - \delta)k_i - c_i. \tag{3}$$

Households maximize the discounted, infinite stream of utility:

$$\max \int_0^{\infty} U(c_i, z_i)e^{-\theta t}dt. \tag{4}$$

The versions of the model that we consider differ only in the manner in which the reference stock, $z_i$, evolves over time. In the outward-looking case, household reference stocks are based on the average level of consumption in the economy. Because agents are small relative to the economy, none can influence aggregate consumption appreciably. Thus agents take the path of the reference stock to be exogenous. In the inward-looking case, households base their reference stocks on their own past consumption, and take into account the effect of their current actions on their future reference stock when making their consumption decisions. The difference between the two cases can be thought of as resulting from an externality: In the outward-looking model, the consumption of each individual household has a negative externality on the utility of all other households. It is this neglected consumption externality that is the underlying source of the differences between the solutions to the models.[4]

In the outward-looking case, the evolution of the reference stock for household $i$ is given by

$$\dot{z}_i = \rho(c - z_i). \tag{5}$$

where $c$ is the average level of consumption for all households.

In the inward-looking case, there are no interactions between households. The equation governing the evolution of reference stock is

$$\dot{z}_i = \rho(c_i - z_i). \tag{6}$$

The only difference between equations (5) and (6) is the subscript $i$ on $c$ in equation (6)

In both versions of the model, the reference stock is a weighted average of past consumption – the only difference between the models is *whose* past consumption (the household's or the average in the economy) determines the reference stock. In both versions of the model,

[4]An alternative interpretation of the inward-looking case that we describe here is that it is an economy in which each household cares about a reference stock based on the past consumption of other housholds in the economy, but in which a social planner enforces optimal consumption on each household taking into account its externality effect. Taking note of this interpretation makes it clear that, *ceteris paribus,* utility will be higher in an economy composed of outward-looking people who behaved like inward looking people than in the outward-looking economy.

the parameter $\rho > 0$ determines the relative weight of consumption at different times. The larger is $\rho$, the more important is consumption in the recent past. If $\rho = .1$, for example, then consumption over the last ten years receives 63% of the weight in determining the reference stock. If $\rho = .3$, then consumption over the last ten years receives 95% of the total weight.

For the outward-looking case, the current value Hamiltonian is:

$$H = U(c_i, z_i) + \pi((A - \delta)k_i - c_i). \tag{7}$$

There is one state variable that is affected by the household's choices, $k_i$, and an associated co-state variable, $\pi$. The evolution of $z_i$ is taken as exogenous by the household, as given by equation (5) above. An equilibrium path will be one in which the household's expectation of average consumption, $\{c\}$, (which determines the household's expected path of its reference stock, $z_i$,) is equal to the households own planned consumption path, $\{c_i\}$.

In the inward-looking case, the current value Hamiltonian is:

$$H = U(c_i, z_i) + \psi[(A - \delta)k_i - c_i] + \lambda\rho(c_i - z_i). \tag{8}$$

In contrast to the outward-looking problem, both state variables, $k_i$ and $z_i$, are affected by the household's decisions, and so there are two associated co-state variables, $\psi$ and $\lambda$.

# 3 Solution to the Household's Problem

## 3.1 Evolution of Consumption and the Reference Stock with Outward-Looking Utility

The necessary conditions for an optimum are

$$\frac{\partial H}{\partial c_i} = U_c - \pi = 0 \tag{9}$$

$$\dot{\pi} = -\frac{\partial H}{\partial k_i} + \theta\pi = (\theta + \delta - A)\pi, \tag{10}$$

The transversality condition, which in the limit can be written as

$$\frac{\dot{k}_i}{k_i} + \frac{\dot{\pi}}{\pi} - \theta < 0,$$

is satisfied so long as $c_i/k_i > 0$.

The reference stock affects household decisions via its effect on the marginal utility of consumption. Differentiating (9) with respect to time gives

$$\dot{U}_c = \dot{\pi} \tag{11}$$

combining this with (10),

$$\frac{\dot{U}_c}{U_c} = \frac{\dot{\pi}}{\pi} = \theta + \delta - A. \tag{12}$$

Since

$$\dot{U}_c = U_{cc}\dot{c}_i + U_{cz}\dot{z}_i, \tag{13}$$

(11) can be rewritten

$$\frac{U_{cc}\dot{c}_i + U_{cz}\dot{z}_i}{U_c} = (\theta + \delta - A). \tag{14}$$

Substituting

$$\frac{U_{cc}}{U_c} = -\frac{\sigma}{c_i} \quad \text{and} \quad \frac{U_{cz}}{U_c} = \frac{\gamma(\sigma - 1)}{z_i} \tag{15}$$

into (14) gives

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\sigma}\left(A - \delta - \theta + \gamma(\sigma - 1)\frac{\dot{z}_i}{z_i}\right). \tag{16}$$

Equation (16) characterizes the growth rate of consumption along the optimal path in terms of the parameters and the growth rate of the reference stock, which is exogenous from the household's point of view.[5] Of course, since all households are identical, the economy-wide average levels of $c$ and $z$ are equal to the values for any single household. Thus we can drop subscripts and take equation (16) to describe the growth rate of average consumption.

[5] In the cases where the reference stock is constant or where $\gamma = 0$, that is, where the reference stock is irrelevant to utility, this condition reduces to that in Rebelo (1991).

Combining (16) with the equation of motion for the reference stock gives the relation between consumption growth and $c/z$ along the optimal path:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}\left(A - \delta - \theta + \rho\gamma(\sigma - 1)\left(\frac{c}{z} - 1\right)\right). \tag{17}$$

These two variables, $c/z$ and $\dot{c}/c$, are convenient for analyzing the dynamics of the model, since both ratios are constant in steady state. To find their values in steady state, we differentiate $c/z$ with respect to time and substitute in (5),

$$\dot{\left[\frac{c}{z}\right]} = \frac{c}{z}\left(\frac{\dot{c}}{c} - \rho(\frac{c}{z} - 1)\right). \tag{18}$$

Setting $(\dot{c/z}) = 0$ in equation (18), and combining with equation (17), we can solve for the steady state growth rate of consumption (which is also the steady state growth rate of income, capital, and reference stock) and the steady state ratio of consumption to reference stock:

$$\left(\frac{\dot{c}}{c}\right) = \frac{A - \theta - \delta}{\gamma(1-\sigma) + \sigma} \qquad \left(\frac{c}{z}\right) = 1 + \frac{1}{\rho}\left(\frac{A - \theta - \delta}{\gamma(1-\sigma) + \sigma}\right) = 1 + \frac{1}{\rho}\left(\frac{\dot{c}}{c}\right). \tag{19}$$

Equation (19) shows the effect of the parameters on the steady state growth rate of consumption, which is also the steady state growth rate of capital, output, and the reference stock.[6] Recall that, the higher is $\gamma$, the more individuals care about how consumption *compares* to the reference stock, and the less they care about the absolute level of consumption. Higher values of $\gamma$ will lead to a higher growth rate of consumption in the steady state. One way to interpret this effect is as an increase in the long-horizon intertemporal elasticity of substitution

[6]Equation (19) makes clear that the degree of comparison utility interacts with risk aversion. In steady state, $\gamma$ and $\sigma$ appear only as part of the expression $\gamma(1-\sigma)+\sigma$. Thus, for given values of $\dot{c}/c, \theta, \delta$, and $A$, a higher value of $\gamma$ will imply a higher value of $\sigma$. Estimates of $\sigma$ based on the growth rate of consumption which ignore comparison utility may understate the true value of $\sigma$. This is the effect that allows Constantinides (1990) to use habit formation to explain the equity premium puzzle.

in consumption. Consumers are more willing to subsitute intertemporally because the gain or loss in utility associated with a given increase or decrease in consumption will be diminished by the adjustment of the reference stock.[7] One surprising point is that $\rho$, the speed with which the reference stock adjusts to current consumption, does not affect the steady state growth or saving rates (though $\rho$ does affect transitional dynamics). The explanation for this phenomenon is that there are offsetting effects of $\rho$ on growth. Consider what would happen if the value of $\rho$ were reduced. First, for a given level of consumption growth, the steady-state level of $c/z$ would be higher. This would make individuals want to "consume" less growth. However, a decline in $\rho$ would also lower the "price" of $c/z$ in terms of consumption growth – that is, a given increment to consumption growth would raise $c/z$ by more. This effect would tend to make consumption growth rise.

Equation (17), which is graphed in Figure 1, gives the relation between the consumption/reference stock ratio and the rate of consumption growth outside of steady state. For values of $c/z$ that are below the steady state, $\dot{c}/c$ will be higher than would be consistent with constant $c/z$, but will be lower than the steady state level of consumption growth. $c/z$ will be rising in this case.

Note that the capital stock does not appear in the equations describing the dynamics of consumption and reference stock. The intuition for this is simple: in an endogenous growth model with constant returns to capital, the marginal product of capital (and thus the interest rate) is invariant to the level of capital. Thus, given initial levels of $c$, $\dot{c}$, and $z$, we can trace out the dynamic evolution of consumption and reference stock without paying attention to the level of capital. Of course, the initial level of capital will be necessary to determine the initial

[7] Galí (1994) makes a similar point in the context of asset pricing. He shows that equilibrium asset prices and returns in the presence of a consumption externality of the type we consider in our outward-looking economy (in which an increase in average consumption raises the marginal utility of a household's consumption) will be identical to those in an externality-free economy with a lower degree of risk aversion. Lower risk aversion corresponds to higher intertemporal elasticity of substitution.

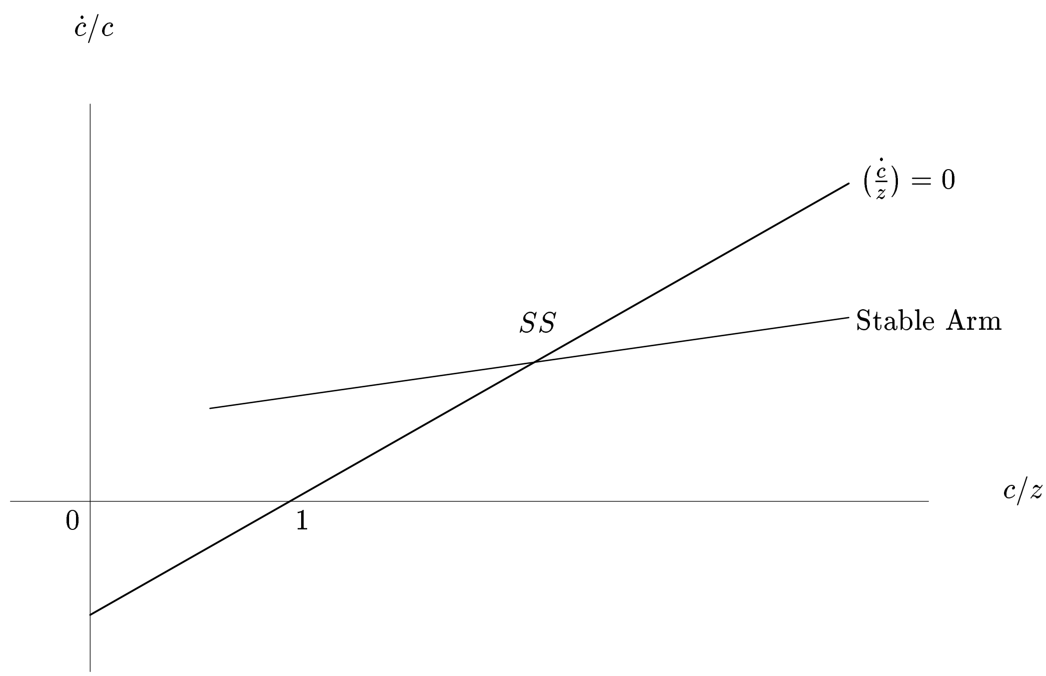


Figure 1: Phase Diagram in $(c/z, \dot{c}/c)$ Space

values of $c$ and $\dot{c}$ that are on the optimal path. In section (3.3 ) we incorporate capital, and show how initial values of $c$ and $\dot{c}$ are chosen.

### 3.2 Evolution of Consumption and the Reference Stock with Inward-Looking Utility

As noted in the introduction, the case where the reference stock corresponds to an average of the agent's own past levels of consumption can be interpreted as a model of habit formation, and the reference stock can be intuitively designated the "habit stock." Since all households are identical, and, further, households with inward-looking utility have no effect on each other, we can drop the household subscript in this version of the problem.

The current value Hamiltonian is

$$H = U(c, z) + \psi[(A - \delta)k - c] + \lambda\rho(c - z). \tag{20}$$

The necessary and sufficient conditions for optimization are the first order conditions

$$\frac{\partial H}{\partial c} = U_c - \psi + \rho\lambda = 0, \tag{21}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial z} + \theta\lambda = (\rho + \theta)\lambda - U_z, \text{and} \tag{22}$$

$$\dot{\psi} = -\frac{\partial H}{\partial k} + \theta\psi = (\theta + \delta - A)\psi \tag{23}$$

along with the transversality conditions:

$$\psi(t) \geq 0, \tag{24}$$

$$\lim_{t \to \infty} e^{-\theta t}\psi(t)k(t) = 0 \tag{25}$$

$$\lambda(t) \leq 0, \text{and} \tag{26}$$

$$\lim_{t\to\infty} e^{-\theta t}\lambda(t)z(t) = 0, \tag{27}$$

and the equations of motion for $z$ and $k$, (6) and (3).

We begin by solving out the costate variables from the first order condtions. Readers not interested in the details of this derivation can skip ahead to equation (35). Differentiating (21) with respect to time gives

$$\dot{\psi} = \dot{c}U_{cc} + \dot{z}U_{cz} + \rho\dot{\lambda} = \dot{U}_c + \rho\dot{\lambda}. \tag{28}$$

Combining this with (22),

$$\dot{\psi} = \dot{U}_c + \rho[(\rho+\theta)\lambda - U_z]. \tag{29}$$

Combining (29) and (21) gives

$$\dot{\psi} = \dot{U}_c + (\rho+\theta)(\psi - U_c) - \rho U_z. \tag{30}$$

Combining (30) and (23),

$$\psi(\delta - A - \rho) = \dot{U}_c - (\rho+\theta)U_c - \rho U_z. \tag{31}$$

Differentiating (31) with respect to time, dividing by (31), and combining with equation (23) gives

$$\begin{aligned}
&\left[\tfrac{\dot{c}}{c}\right]^2 \sigma(\sigma+1) + 2\left[\tfrac{\dot{c}}{c}\right]\left[\tfrac{\dot{z}}{z}\right]\sigma\gamma(1-\sigma) - \left[\tfrac{\ddot{c}}{c}\right]\sigma + \gamma(1-\sigma)(\gamma(1-\sigma)+1)\left[\tfrac{\dot{z}}{z}\right]^2 \\
&+\gamma(\sigma-1)\left[\tfrac{\ddot{z}}{z}\right] - \rho\gamma(\gamma(1-\sigma)+1)\left[\tfrac{c}{z}\right]\left[\tfrac{\dot{z}}{z}\right] - \rho\gamma(\sigma-1)\left[\tfrac{\dot{c}}{z}\right] \\
&= (2\theta+\rho+\delta-A)(\gamma(\sigma-1)\left[\tfrac{\dot{z}}{z}\right] - \sigma\left[\tfrac{\dot{c}}{c}\right]) - (\theta+\delta-A)((\rho+\theta) - \rho\gamma\left[\tfrac{c}{z}\right]).
\end{aligned} \tag{32}$$

Substituting

$$\frac{\dot{z}}{z} = \rho(\frac{c}{z} - 1) \tag{33}$$

and

$$\frac{\ddot{z}}{z} = \rho(\frac{\dot{c}}{z} - \rho(\frac{c}{z} - 1)) \tag{34}$$

into (32) and simplifying,

$$\begin{aligned}
&\rho(\tfrac{c}{z}-1)\left(\rho\gamma(\gamma(1-\sigma)+1)(\sigma(1-\tfrac{c}{z})-1)+\gamma(1-\sigma)(2\theta+2\rho+\delta-A)\right)\\
&+\tfrac{\dot{c}}{c}\left[\sigma(\sigma+1)\tfrac{\dot{c}}{c}+2\sigma\gamma(1-\sigma)\rho(\tfrac{c}{z}-1)+\sigma(2\theta+\rho+\delta-A)\right]\\
&-\tfrac{c}{z}\rho\gamma(\theta+\delta-A)-\sigma\tfrac{\ddot{c}}{c}+(\rho+\theta)(\theta+\delta-A)=0.
\end{aligned}\tag{35}$$

This equation gives the relationship between consumption, the reference stock, and the first and second time derivatives of consumption along an optimal path. Note that it differs from the usual first order condition that emerges from a Ramsey model, where only the first time derivative of consumption is given by the first order condition. This result arises because the consumer takes into account how the growth rate of consumption affects the ratio of consumption to reference stock, so the temporal *evolution* of the growth rate must satisfy an optimality condition, just as in the Ramsey model the temporal evolution of the level of consumption (the first deriviative of consumption with respect to time) must satisfy an optimality condition. Setting either $\gamma=0$, so that the reference stock is irrelevant to utility, or $\rho=0$, so that the reference stock is unchanging, causes (35) to collapse to the first-order condition for consumption growth from the model of Rebelo (1991).[8]

As in the outward-looking problem, we can once again examine the dynamics of $c$ and $z$ without reference to the level of capital. That is, given initial values of $c$, $\dot{c}$, and $z$, we

[8]Setting $\gamma$ to zero, equation (35)reduces to

$$\left[\frac{\dot{c}}{c}\right]^2\sigma(\sigma+1)+\left[\frac{\dot{c}}{c}\right]\sigma(2\theta+\rho+\delta-A)-\frac{\ddot{c}}{c}\sigma+(\rho+\theta)(\theta+\delta-A)$$

Further substituting $\frac{\ddot{c}}{c}=\left(\frac{\dot{c}}{c}\right)^2$, which holds true when the growth rate of consumption is constant, yields a quadratic equation in $\dot{c}/c$. One of the roots of this equation is

$$\frac{\dot{c}}{c}=\frac{A-\delta-\theta}{\sigma},$$

which is the growth rate of output in the endogenous growth model of Rebelo (1991). The other root of the quadratic equation corresponds to the second steady state examined below. In the Appendix we show that this second steady state violates the transversality conditions for an optimum.

can trace out the trace out the evolution of consumption and reference stock without knowing how capital evolves. Of course the initial choices of $c$, $\dot{c}$, and $z$, will depend on the level of $k$ through the intertemporal budget constraint. We examine the determination of these intial values below. The dynamics of consumption relative to the reference stock are the same as in the outward-looking problem:

$$\dot{\left[\frac{c}{z}\right]} = \frac{c}{z}\left(\frac{\dot{c}}{c} - \rho(\frac{c}{z} - 1)\right). \tag{36}$$

Differentiating $\dot{c}/c$ with respect to time and substituting in (35),

$$\begin{aligned} \dot{\left[\frac{\dot{c}}{c}\right]} &= \sigma\left(\frac{\dot{c}}{c}\right)^2 + \frac{\dot{c}}{c}(2\theta + \rho + \delta - A - 2\gamma\rho(1-\sigma)) - \rho^2\gamma(\gamma(1-\sigma)+1)\left(\frac{c}{z}\right)^2 \\ &+ 2\gamma\rho(1-\sigma)\frac{\dot{c}}{c}\frac{c}{z} + \left(\frac{\rho\gamma}{\sigma}\right)\frac{c}{z}(\rho\gamma(1-\sigma)(2\sigma-1) + \theta + \rho + -\sigma(2\theta + \delta - A)) \\ &+ \frac{1}{\sigma}((\rho+\theta)(\theta+\delta-A) + \rho\gamma(1-\sigma)(\rho(\gamma(1-\sigma)+1) - (2\theta + 2\rho + \delta - A))) \end{aligned} \tag{37}$$

Figure 2 shows the phase diagram produced by equations (18) and (37) in $(c/z, \dot{c}/c)$ space. The locus of points where $\dot{\left[\frac{c}{z}\right]}$ is zero is a straight line with slope $\rho$. The intuition is simple: the larger is $\rho$ (that is, the more quickly reference stock catches up with consumption), the faster consumption must grow to keep $c/z$ constant. Equation (37) is nonlinear, and the locus of points for which $\dot{\left[\frac{\dot{c}}{c}\right]}$ is zero is a pair of symmetric hyperbolas, each of which crosses the $\dot{\left[\frac{c}{z}\right]} = 0$ locus once. Thus the dynamical system has two steady states.

The phase diagram shows that the first steady state is knife- edge stable. The second steady state, by contrast, is a vortex, the basin of attraction for which is all of the points that lie below the stable arm associated with the first steady state. In the Appendix, we show that the second steady state violates one of the transversality conditions, and hence paths that lead to it cannot be optimal.[9] Clearly, paths of $c/z$ and $\dot{c}/c$ that lie above the stable arm would eventually lead to growth rates of consumption higher than the maximum possible growth rate

[9] We also show in the Appendix the constraints under which the first steady state does not violate the transversality conditions. These turn out to be very loose.

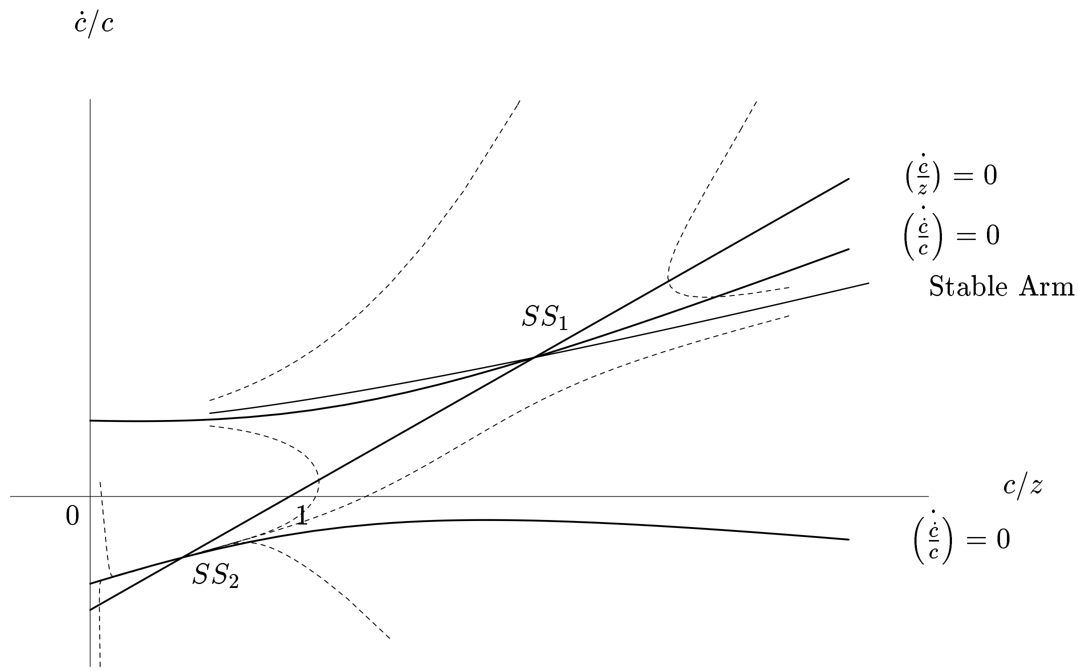


Figure 2: Phase Diagram in $(c/z, \dot{c}/c)$ Space

of output, and thus eventually to a negative capital stock. We thus focus our attention on the first steady state and the stable arm leading to it.

The values of $c/z$ and $\dot{c}/c$ in the steady state can be determined by setting the two dynamic equations, (18) and (37), equal to zero. The unique optimal steady state values are the same in the inward-looking as in the outward-looking problem. (The transition to the steady state, however, is not the same in the two problems. We discuss the transition below.)

Figure 3 shows how stable arms, viewed in $(\frac{c}{z}, \frac{\dot{c}}{c})$ space, compare in the two cases. We show in the Appendix that the stable arm for the outward looking case always lies above that for the inward-looking case for values of $c/z$ lower than the steady state. The difference between these two stable arms will, in turn, determine the different behavior of the two economies outside of steady state.

### 3.3 Dynamics of Consumption, the Reference Stock, and Capital

Clearly, as $c$ and $z$ evolve, $k$ does also. We look at capital's dynamics in terms of $k/z$, which also turns out to have a constant value in the steady state. Differentiating $k/z$ with respect to time gives

$$\left[\dot{\frac{k}{z}}\right] = \left(\frac{\dot{k}}{z}\right) - \frac{k}{z}\frac{\dot{z}}{z} = \frac{k}{z}\left(\frac{\dot{k}}{k} - \frac{\dot{z}}{z}\right). \tag{38}$$

Substituting in (6) and (3),

$$\left[\dot{\frac{k}{z}}\right] = \frac{k}{z}\left(A - \delta - \rho(\frac{c}{z} - 1)\right) - \frac{c}{z}. \tag{39}$$

Setting (39), to zero and combining with the steady state value of $c/z$ derived above (which is the same in the inward- and outward-looking cases) gives the steady state value for $k/z$.

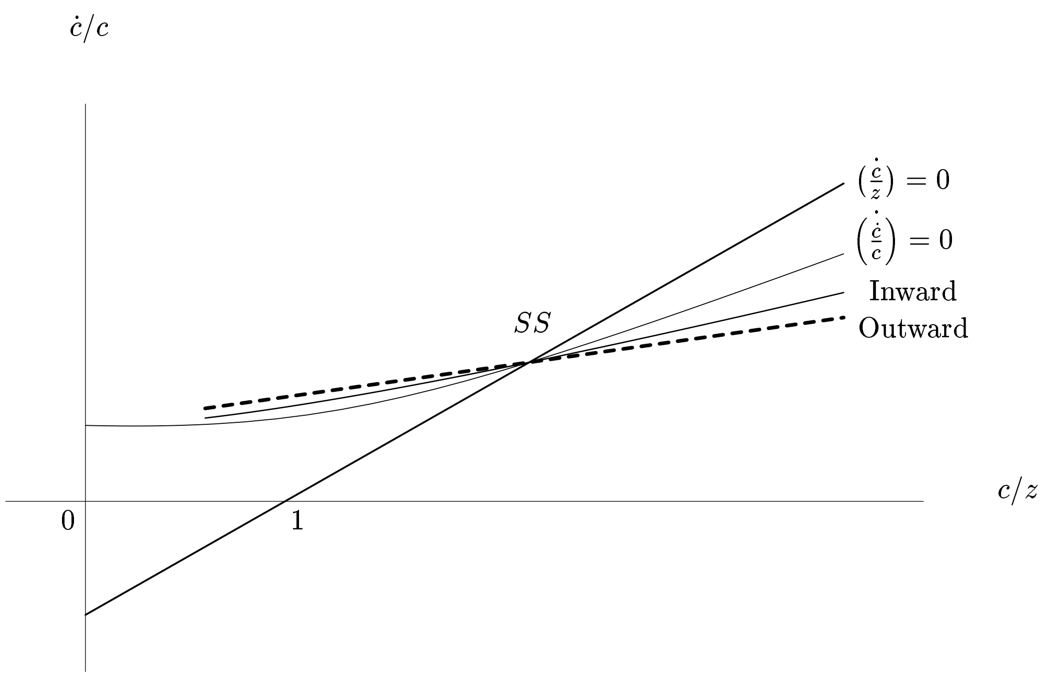


Figure 3: Phase Diagram in $(c/z, \dot{c}/c)$ Space

$$\frac{k}{z} = \frac{1}{\rho}\left[\frac{\rho(\gamma(1-\sigma)+\sigma)+(A-\delta-\theta)}{(A-\delta)(1-\sigma)(\gamma-1)+\theta}\right] \tag{40}$$

Figure 4 shows the locus of points for which $k/z$ is constant, and the dynamics of $\left(\frac{k}{z}\right)$ in $(c/z,\ k/z)$ space.[10] The $\dot{\left(\frac{k}{z}\right)} = 0$ locus never goes beyond $1 + (A - \delta)/\rho$ because for this value of $\frac{c}{z}$, $z$ will grow at rate $A - \delta$, which is the maximum possible growth rate for capital (that is, the rate at which capital would grow if there were no consumption). Thus, for higher values of $c/z$, the reference stock would necessarily be growing faster than capital, and the $k/z$ ratio could not be constant.

Figure 5 shows the phase diagram in three dimensions. The gray lines correspond to the economy under outward-looking utility, while the black lines correspond to the economy under inward-looking utility. The stable arms in $(\frac{c}{z}, \frac{\dot{c}}{c})$ space, which were presented in Figure (3), are shown in the horizontal plane. The back plane of the figure shows the projection of the stable arms onto the $(c/z, k/z)$ plane that was used in Figure 3.

There are two state variables in the maximization problem: the level of the capital stock and the level of the reference stock. However, the assumption of homotheticity in the utility function, in combination with the Rebelo production function, implies that the problem is homogeneous of degree zero in the level of capital and the reference stock; in other words, viewed in $(c/z, \dot{c}/c, k/z)$ space, there is a single state variable, $k/z$. Thus an initial condition for the system can be specified in terms of the initial value of $k/z$ and an analytically uninteresting scaling term.

In $(c/z, \dot{c}/c, k/z)$ space an initial condition of $k/z = \omega$ corresponds to a plane at height $\omega$. Figure 5 depicts portions of two such planes, one with $\omega$ below the steady-state value of $k/z$ and one with $\omega$ above the steady-state value. The intersections of these planes with the stable arm determine the initial values of $\dot{c}/c$ and $c/z$ that correspond to the given initial value of $k/z$.

[10] The dynamics of $c/z$ depend on the value of $\dot{c}/c$ and thus cannot be depicted in this diagram.

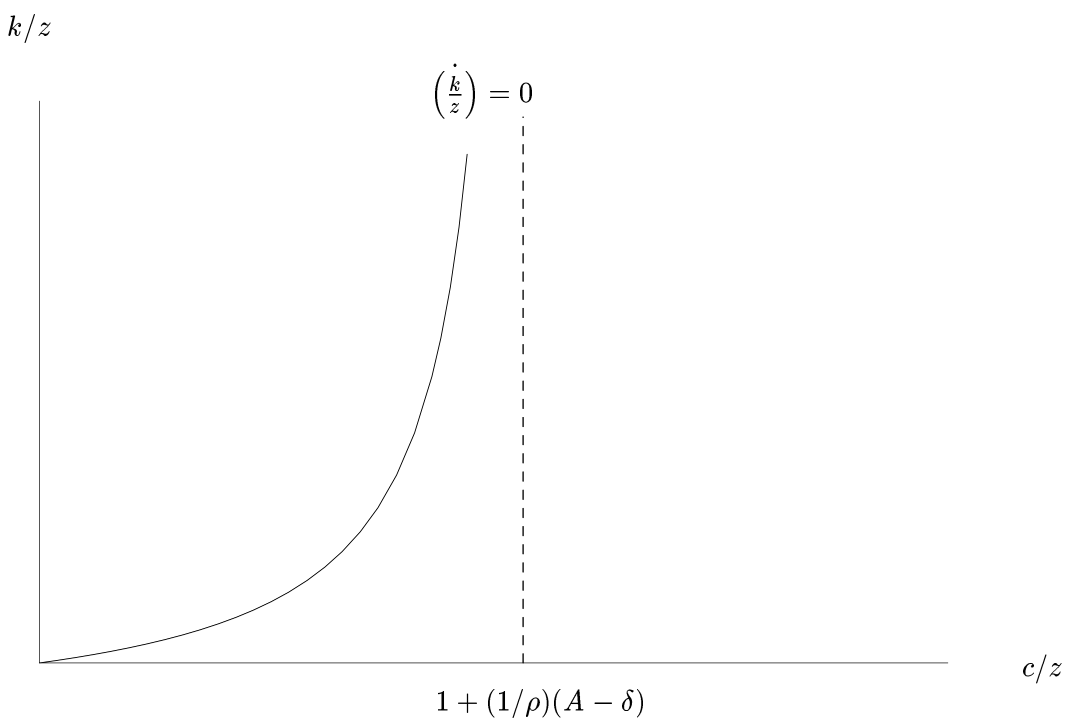


Figure 4: Phase Diagram in $(c/z, k/z)$ Space

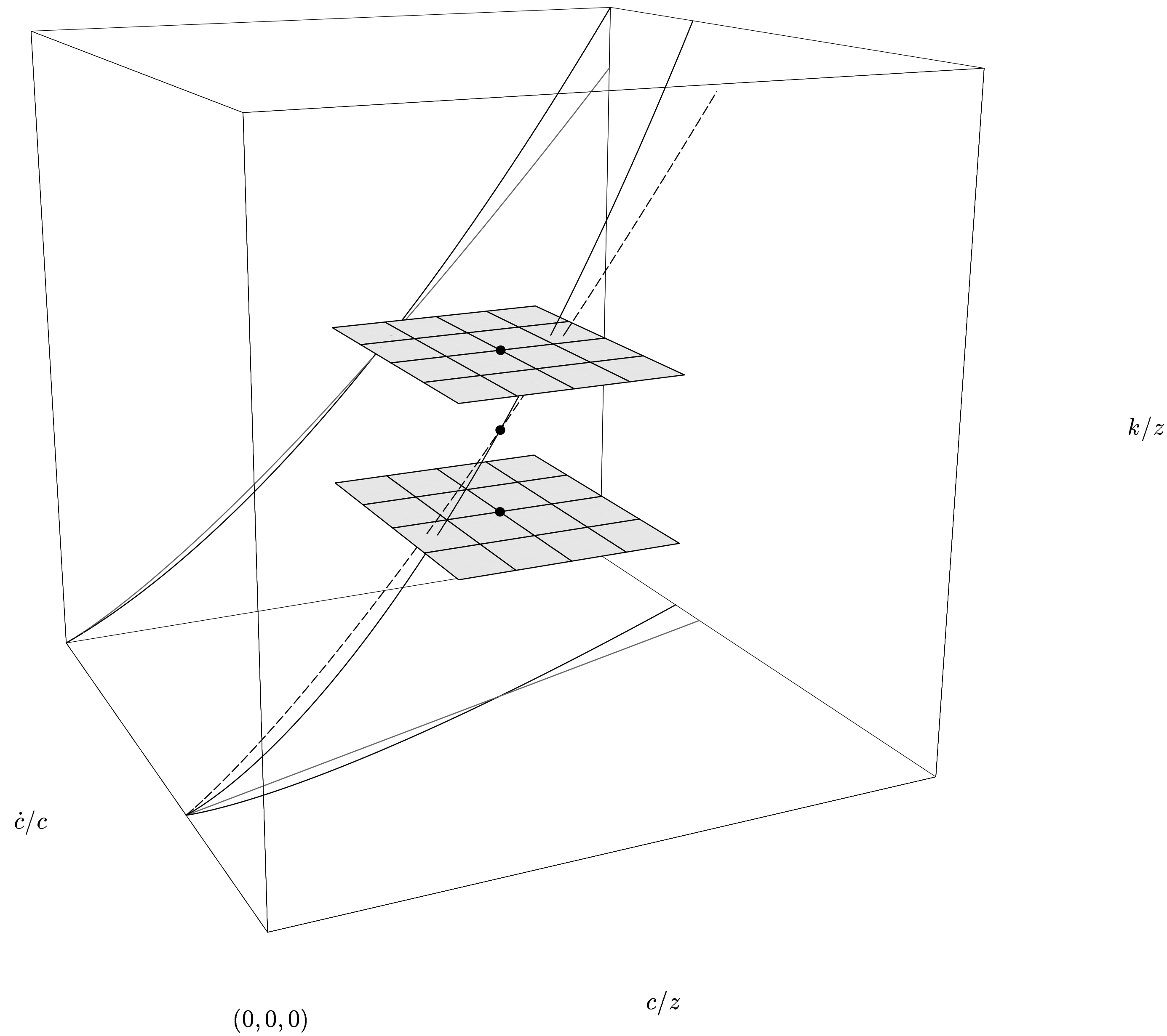


Figure 5: Stable Arms in $(c/z, \dot{c}/c, k/z)$ Space

From the figure it is apparent that a low initial value of $k/z$ will imply a lower-than-steady-state initial value of $c/z$ and a lower-than-steady-state initial value of consumption growth, while the opposite will apply for a high initial value of $k/z$. The intuition for this is simple: with comparatively low capital, the consumer chooses to take some of the pain in the form of a low level of consumption ($c/z$), and to take some in the form of a low growth rate of consumption. Maintaining the low level of consumption moves the $k/z$ ratio up both by fostering capital accumulation and by slowing the growth of the reference stock. As $k/z$ approaches the steady-state value, consumption needs to be depressed less, so $c/z$ grows toward its steady-state value. Examining the figure, it is clear that if the initial value of $k/z$ is smaller than the steady-state value (i.e. if the capital stock is small relative to the reference stock) the intial value of $c/z$ will be lower and the initial value of $\dot{c}/c$ higher in the outward-looking economy than in the inward-looking economy. Similarly, if the initial value of $k/z$ is above its steady state value, then the outward-looking economy will have a higher initial value of $c/z$ and a lower initial value of $\dot{c}/c$ than the inward-looking economy. We discuss the logic for this result in section 4.

### 3.4 Policy Functions

Another way of presenting the solutions to the models is to trace out the relationship between the state variable $k/z$ and the optimal values of the control variable $c/z$. Similarly, we can trace the relationship between $k/z$ and any transformation of the control variable along the optimal path. Several of these policy functions are depicted in figures 6- 7.

The dots represent equally spaced points in time as the system evolves toward the steady-state; the darker dots correspond to the inward-looking version of the problem and the lighter dots to the outward-looking version. To understand the figure, choose a dot and imagine that it represents the position of the economy at time t. Then at time t+1 the position of the economy will be given by the adjacent dot in the direction of the steady-state. Because the steady states

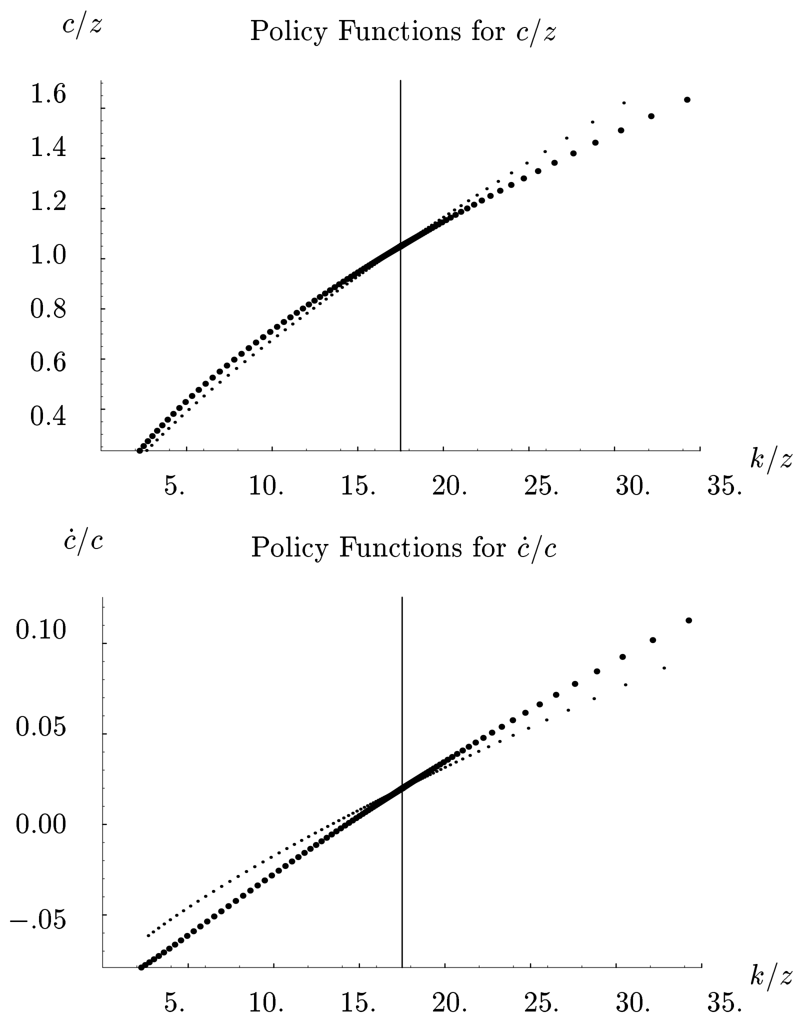

Figure 6: Optimal Policy Functions for $c/z$ and $\dot{c}/c$

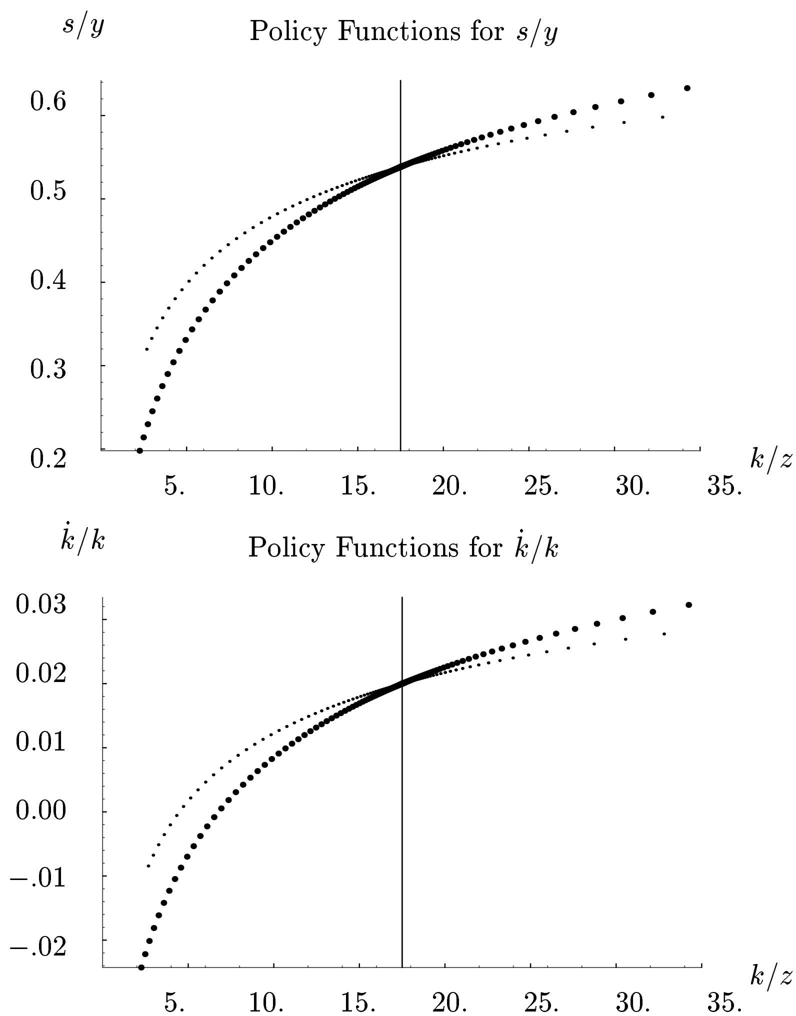


Figure 7: Optimal Policy Functions for $s/y$ and $\dot{k}/k$

of the inward- and outward-looking economies are the same, the point of intersection of the two curves represents the steady-state position of the economy. As the economy approaches the steady-state, the dots get closer to each other, indicating an aymptotic approach to the steady state.

Despite their differences, the qualitative responses of the inward and outward looking economies are similar in many respects. An economy with an initial $k/z$ ratio below its steady state will have initially low consumption and consumption growth. It will also have an initial growth rate of capital (and thus output) which is below the steady state level. Indeed, for sufficiently low initial $k/z$, both consumption growth and output growth can be negative along the initial portion of the optimal path: the intuition is that a person with an initially very high reference stock will consume at an unsustainably high level (but below his reference stock) for some period of time while he allows his reference stock to decline.

# 4 Comparison of Dynamics in Outward- and Inward-looking Economies

We now take up the question of how the dynamic response of the economy to a shock differs in the inward and outward-looking cases. As we showed above, the steady states for the two models are the same. Further, the dynamic equations of the models for the evolution of $c/z$ and $c/k$ are the same, because the relationship between $c$, $z$, and $k$ merely reflects the accumulation equations for the state variables $z$ and $k$.

Figure 8 shows the dynamics of the two models in the time dimension, for two economies that start with the same initial $k/z$ ratio, which is below its steady state.[11] The outward-looking economy starts with a lower level of consumption (seen in top figure) and a higher level

[11] The parameter values used in these time series simulations, as well as those presented below, are $\sigma = 2$, $\gamma = .5$, $\rho = .2$, $\theta = \delta = .05$, and $A$ set so that the steady state growth rate is 2 percent. For Figure 8, the initial ratio of capital to reference stock was set 10 percent below its steady state level.

of consumption growth than does the inward-looking economy. The outward-looking economy moves a bit more rapidly toward the steady state than does the inward-looking economy: In terms of the growth rate of output, the inward-looking economy moves halfway from its initial position to its steady state after 5.45 years, while the outward-looking economy does so after 4.9 years.

Why do the inward and outward looking cases differ? One way to see the intuition for this is to think about the curvature of the utility function as viewed from the persective of a decision maker. In the outward-looking case, the utility function is standard. That is, the curvature from the household's point of view is just $\sigma$: the household thinks "If I raise my consumption, $z$ will be constant, and the increase in utility is just based on $\sigma$ in my utilty function." On the other hand, in the inward- looking case, the decision maker sees that increasing consumption will not raise utility by as much as would be implied by the curvature of the utility function, because raising $c$ will also raise $z$. Thus, from the inward-looking person, the (net) increase in utility that comes from an increase in consumption is smaller – that is, the utility function is effectively more curved. So the inward-looking person acts as if he is more risk averse.

The differential behavior of the inward- and outward-looking economies raises an interesting possiblity. As argued above, utility for people with outward-looking preferences would be higher if everyone in the economy behaved as if they had inward looking preferences. That is, there is a negative externality present in the case of outward-looking preferences that depresses utility. Nonetheless, in the face of some shocks, the economy with outward looking utility will grow faster. Thus higher growth can be associated with lower utility.

## 4.1 Effects of Changing $\rho$

The parameter $\rho$ governs the speed with which reference stock adjusts. The case of $\rho = \infty$ has a natural interpretation in the case of outward-looking utility: it means that you care about how

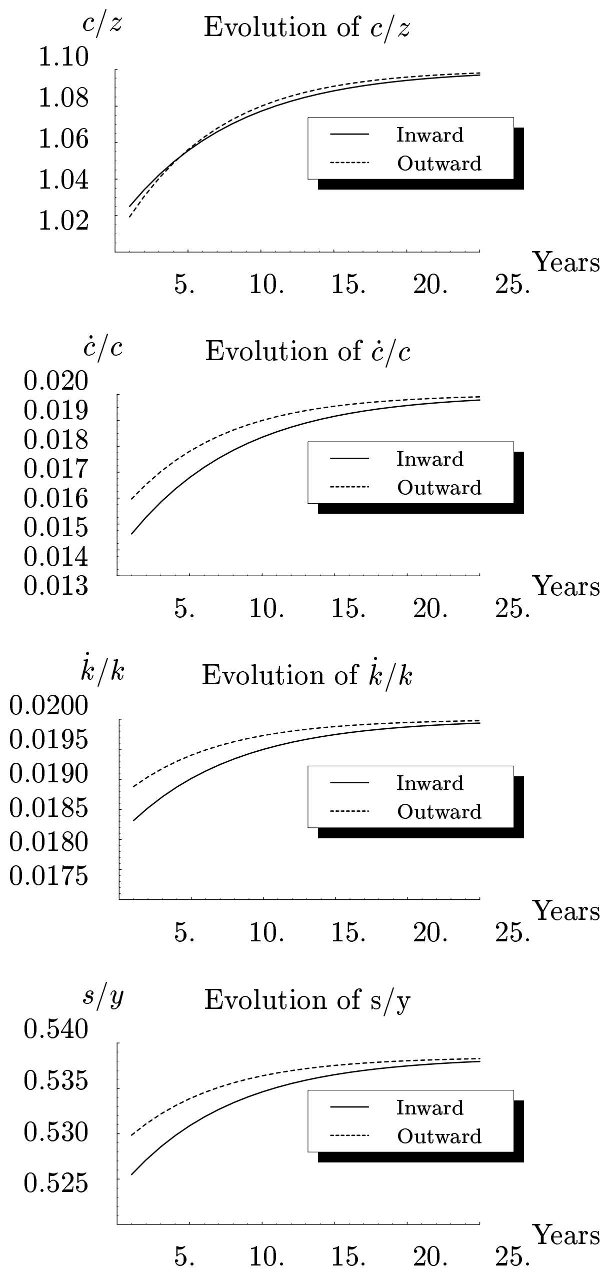


Figure 8: Dynamics Following A Negative Shock to $k/z$

your consumption compares to the contemporaneous consumption of your neighbors, rather than to a lagged average of their past consumption.[12] In the case of inward-looking utility, setting $\rho = \infty$ is equivalent to looking at a standard utility function (without any temporal dependence) in which the coefficient of relative risk aversion has been decreased.[13] For $\rho = 0$, the reference stock never changes, and so both models effectively collapse to the standard endogenous growth model of Rebelo (1991).[14]

Figure 9 shows the response of income growth to a 10 percent negative shock to the capital stock in the inward- and outward- looking cases for different values of $\rho$ in between these two extremes. For both types of economy, increasing $\rho$ both reduces the initial drop in income growth and increases the speed with which the growth rate returns to its steady state level. Hence a higher value of $\rho$ implies that the economy will return to its steady-state quicker.

The degree to which the transition paths in the inward- and outward-looking cases differs depends on $\rho$ in in an interesting fashion. Consider two otherwise-similar economies, both receiving a 10 percent shock to capital. As Figure 9 shows, for all values of $\rho$, growth in the outward-looking case is higher than in the inward- looking case. The cumulative discrepancy in their growth rates (and thus in their levels of income, once they have returned to steady state) is shown in Figure 10. The figure shows that the lower is the value of $\rho$ the larger is the

[12] Abel (1990) describes this distinction between the model where households care about the current consumption of their neighbors and the model in which they care about the lagged consumption of their neighbors as the difference between "keeping up with the Joneses" and "catching up with the Joneses."

[13] Specifically, when $\rho = \infty$, it will be the case that $z = c$, and so the utility function can be rewritten as:

$$U(c_t, z_t) = \frac{\left({c_t}^{1-\gamma}\right)^{1-\sigma}}{1-\sigma} = \frac{{c_t}^{1-\sigma-\gamma+\gamma\sigma}}{1-\sigma} \tag{41}$$

If $\sigma > 1$ (as we assume), the exponent on $c$ is a constant greater than $1 - \sigma$, which would produce behavior indistinguishable from a model with a smaller coefficient of relative risk aversion. A smaller coefficient of risk aversion, however, corresponds to a greater intertemporal elasticity of substitution. The intuition for the increase in the intertemporal elasticity is that even if your consumption drops a lot, eventually you will "get used to" the lower level of consumption and the adjustment to the new level of consumption will be less painful than it would have been had you not had a reference stock which also adjusts.

[14] Note that as long as $\rho > 0$, the steady state growth rate of output is $\frac{A-\delta-\theta}{\gamma(1-\sigma)+sigma}$, while if $\rho = 0$, the steady state growth rate is $\frac{A-\delta-\theta}{sigma}$. At first this might seem problematic. In Appendix A.4 we show that it is not.

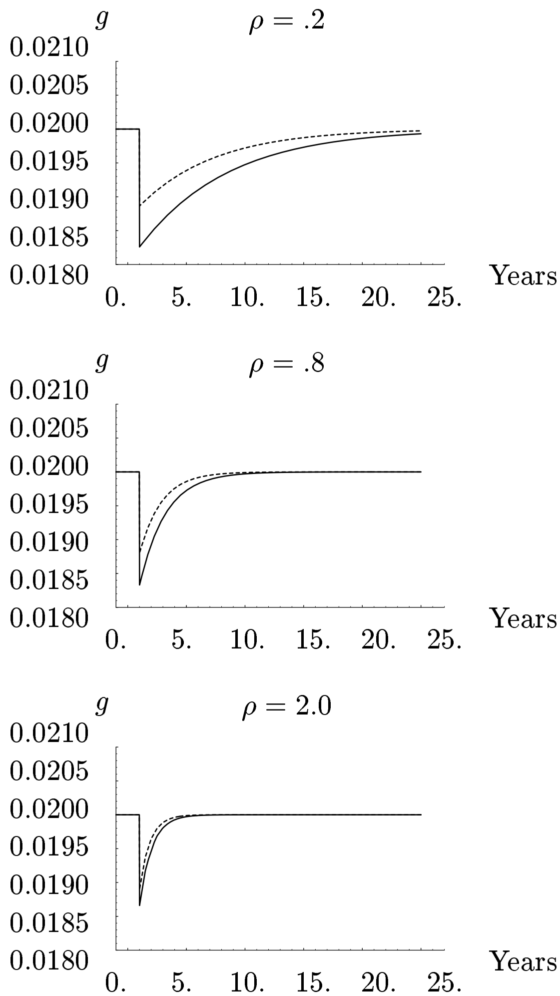


Figure 9: Effect of $\rho$ on Time Series Response to Capital Shock

cumulative effect on growth of an initial shock to capital. Further, the lower is $\rho$, the larger is the cumulative difference between the inward- and outward looking cases.

## 5 Conclusion

Our goal in this paper has been to explore two different versions of "comparison utility" in the context of a growth model. The insight underlying this class of utility functions is that an individual's utility from consumption depends both on the absolute level of consumption (as in traditional utility functions) and on how the level of consumption compares to some "reference stock."

Introducing comparison utility into the endogenous growth model leads to a rich set of dynamics. As in the model of Lucas (1988), introducing a second state variable (in our case the reference stock), leads to transitional dynamics if the economy starts with the state variables in a ratio other than that which holds in steady state. In the model that we present, the transitional dynamics are quite intuitive: an economy that starts off in the lucky state of having a low reference stock relative to its capital stock will spread out its good fortune by saving a lot and growing quickly, thus lengthening the time during which the level of consumption will be high relative to the reference stock.

Although using an endogenous growth model greatly simplifies the analysis, there are clearly aspects of reality that are left out in this sort of model. An example of how the assumption of an endogenous growth production function affects the results of the model is in the analysis of a country which starts off with an inferior technology, and is suddenly exposed to a better world technology (see, for example, the model of Parente and Prescott (1994)). In the standard endogenous growth model, such a country will experience a period in which it grows faster than the rest of the world, before finally settling down to the world growth rate. Whether the *level* of output per capita in the country will greater, less than, or equal

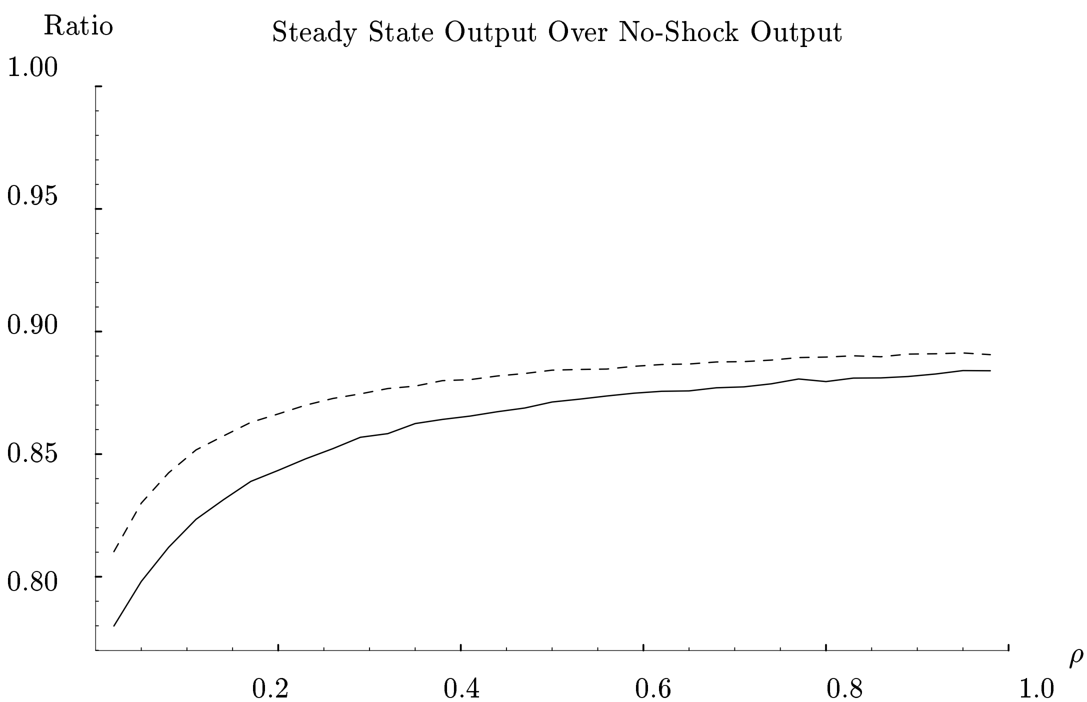


Figure 10: Effect of $\rho$ on Steady-State Output

to that of the other countries will depend on how the countries stood relative to each other at the time that the technology shock took place. Because in the endogenous growth model it is growth rates that are equalized in steady state, there is no firm prediction (if one does not know initial conditions) about relative levels of income in similar countries. In a more traditional growth model, by contrast, countries with the same preferences and technologies will eventually also have the same steady state levels of output. But along the transition path, many of the same phenomena that appear in the endogenous growth version of the model will be present. A country which is exposed to a superior world technology, and which starts out at a level of output below the world level, will experience rapid growth and high saving for the same reasons that apply in the endogenous growth model. Such a country will even temporarily grow to a level of output beyond the rest of the world before eventually settling into the world steady state level. (See Ryder and Heal (1973) for a thorough treatment of these dynamics).

The two versions of the model that we have considered differ only in how the reference stock is determined. In the outward-looking case, the reference stock is determined by the consumption of others. We consider an economy composed of identical, atomistic households who take the consumption of their neighbors, and thus the evolution of their reference stock, as given. We were also able to show how these dynamics compared in the case of a outward-looking economy and in the case of individuals who take into account the externality effect of today's consumption on the future reference stock. The outward-looking economy responds more severely to shocks to productivity or the capital stock, and returns to steady state more rapidly than does the inward-looking economy.

## Appendices

### A.1 Proof that the second steady state violates the transversality conditions.

The values of $c/z$ and $\dot{c}/c$ in the second steady state are:

$$\left(\frac{\dot{c}}{c}\right) = -\left(\frac{\rho(1-\gamma)+\theta}{\sigma(1-\gamma)}\right) \qquad \left(\frac{c}{z}\right) = 1 + \frac{-1}{\rho}\left(\frac{\rho(1-\gamma)+\theta}{\sigma(1-\gamma)}\right). \tag{42}$$

We show that this steady state violates condition (27). This condition can be rewritten in the limit as

$$\frac{\dot{\lambda}}{\lambda} + \frac{\dot{z}}{z} - \theta < 0. \tag{43}$$

We begin by showing that the in the second steady state the constraint on capital is not binding. Ignoring capital, the individual's problem is to maximize (4) subject to (6). The current value Hamiltonian is

$$H = U(c,z) + \lambda\rho(c-z) \tag{44}$$

The necessary conditions are:

$$\frac{\partial H}{\partial c} = U_c + \rho\lambda = 0 \tag{45}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial z} + \theta\lambda = (\rho+\theta)\lambda - U_z \tag{46}$$

Differentiating (45) with respect to time, and manipulating (45) and (46), yields the same values for $c/z$ and $\dot{c}/c$ as are given in (42). Thus since steady state two is the same as the solution to the individual's problem in a case where there is no capital, it is clear that the capital constraint is not binding in this steady state. Thus the costate variable on capital, $\psi$, will be zero in this steady state. From this fact and equations (21) and (22):

$$\frac{\dot{\lambda}}{\lambda} = \rho + \theta + \frac{\rho U_z}{U_c}. \tag{47}$$

Noting that

$$\frac{\dot{z}}{z} = \rho\left(\frac{c}{z} - 1\right) \tag{48}$$

and

$$\frac{U_z}{U_c} = -\gamma\frac{c}{z} \tag{49}$$

and substituting into (43) gives

$$\frac{\dot{\lambda}}{\lambda} + \frac{\dot{z}}{z} - \theta = (1-\gamma)\rho\frac{c}{z} > 0 \tag{50}$$

which violates the transversality condition.

### A.2 Constraints on the parameters such that the first steady state does not violate the transversality conditions.

In the first steady state, $c/z$ and $k/z$ are constant, thus

$$\frac{\dot{k}}{k} = \frac{\dot{c}}{c} = \frac{\dot{z}}{z} \quad , \tag{51}$$

which (combined with the observation that the constraints on both state variables are binding) in turn implies that

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\psi}}{\psi} \quad . \tag{52}$$

Given these equalities, equations (11) and (13) are equivalent in the limit. Expressing the transversality condition in terms of growth rates:

$$-\theta + \frac{\dot{k}}{k} + \frac{\dot{\psi}}{\psi} < 0. \tag{53}$$

Substituting the steady-state growth rates gives

$$\frac{A - \theta - \delta}{\gamma(1-\sigma) + \sigma} + \delta - A < 0. \tag{54}$$

This is the constraint on the parameters which allows for the first steady state to not violate transversality. This is satisfied if $A > \delta$, that is, if the economy is capable of at least maintaining its capital stock.

**A.3 Comparison of the Stable Arms for the Outward- and Inward-Looking Economies**.

We begin by analyzing how the stable arms in the two economies compare in $c/z, \dot{c}/c$ space. We then show what the position of these two-dimensional stable arms implies for initial levels of $c/z$ for a given level of $k/z$.

We begin by showing how the *acceleration* of consumption (that is, $\dot{\left[\frac{\dot{c}}{c}\right]}$) compares in the two economies for a particular set of points: those which are on the stable arm of the outward-looking economy. Define the function that gives the value of $\dot{c}/c$ along the stable arm of the outward-looking economy as a function of $c/z$ as $\Omega(c/z)$. From (17)

$$\frac{\dot{c}}{c} = \Omega(\frac{c}{z}) = \frac{1}{\sigma}(\rho\gamma(\sigma-1)(\frac{c}{z}-1) + (A-\theta-\delta)) \tag{55}$$

Differentiating this expression with respect to time and rearranging, the rate of acceleration of consumption for the outward-looking economy is

$$\dot{\left[\frac{\dot{c}}{c}\right]} = \frac{c}{z}\left[\frac{\rho\gamma(\sigma-1)}{\sigma}\left(\Omega(\frac{c}{z}) - \rho(\frac{c}{z}-1)\right)\right] \tag{56}$$

Rewriting the expression for consumption acceleration in the inward-looking economy (equation 37) to incorporate (55):

$$\begin{aligned}
\dot{\left[\tfrac{\dot{c}}{c}\right]} &= \sigma(\Omega(\tfrac{c}{z}))^2 + \Omega(\tfrac{c}{z})\left[2\gamma\rho(1-\sigma)(\tfrac{c}{z}-1) + 2\theta + \rho + \delta - A\right] \\
&- \rho^2\gamma(\gamma(1-\sigma)) + 1)\tfrac{c}{z}^2 + \tfrac{\rho\gamma}{\sigma}\tfrac{c}{z}\left(\rho\gamma(1-\sigma)(2\sigma-1) + \theta + \rho - \sigma(2\theta+\delta-A)\right) \\
&+ \tfrac{1}{\sigma}\left((\rho+\theta)(\theta+\delta-A) + \rho\gamma(1-\sigma)(\rho(\gamma(1-\sigma)+1) - 2\theta - 2\rho - \delta + A)\right)
\end{aligned} \tag{57}$$

Dividing (57) by (56) gives the ratio of the magnitudes of consumption acceleration for any point along the stable arm of the outward-looking economy:

$$\frac{\left[\frac{\dot{\dot{c}}}{c}\right]_{inward-looking}}{\left[\frac{\dot{\dot{c}}}{c}\right]_{outward-looking}} = \frac{\sigma}{\sigma - 1} \tag{58}$$

Figure 3 shows the configuration of the stable arms in $c/z$, $\dot{c}/c$ space.

The stable arm for the outward-looking economy slopes upward. Further, when $c/z$ is less than its steady state level, $\left[\frac{\dot{\dot{c}}}{c}\right]$ is positive, so by (58), consumption acceleration will be larger for an inward-looking economy starting at a point on the outward-looking stable arm than it will be for the outward-looking economy starting at the same point. Thus a inward-looking economy starting at any point on or above the stable arm of the outward-looking economy would be carried away from the steady state (see point A). Thus when $c/z$ is less than its steady state level, the inward-looking economy's stable arm must lie below that of the outward-looking economy. Similarly, when $c/z$ is above the steady state level, the inward-looking economy's stable arm is above that for the outward-looking economy.

We now show that for any value of $k/z$ below the steady state level, an inward-looking economy will start off with a higher level of $c/z$ than will the outward-looking economy. We will proceed by supposing that the $c/z$ in the inward-looking economy does *not* exceed that in the outward-looking economy, and showing that this leads to an infeasible consumption path for the outward- looking economy.

Consider the case where the two economies start off with the same level of $c/z$. From the phase diagram, it is clear that the outward-looking economy will have a higher value of consumption growth, and thus will have a more rapidly growing level of $c/z$. Since the level of $c/z$ in the inward-looking economy can never surpass that in outward-looking economy (because for equal levels of $c/z$ the the outward-looking economy will always have higher consumption growth), consumption growth will always be higher in the outward-looking economy. This, in turn, implies that for every point in time beyond the initial instant, consumption in the outward-looking economy will be higher than in the inward-looking economy. Under the assumption that the inward-looking economy was on its stable arm, the outward-looking economy would have to end up consuming at an infeasible level.

Hence, for $k/z$ below its steady state ratio, the inward-looking economy must begin at a higher level of $c/z$ than the outward- looking economy. A symmetric agrument shows that if $k/z$ is above its steady state ratio, then the inward looking economy will begin with a lower level of $c/z$ than the outward-looking economy.

**A.4 Growth Rate as $\rho$ Approaches Zero**

In the text we noted that if $\rho = 0$ both of our models boil down to the simple Rebelo model of endogenous growth. In the Rebelo (1991) model the growth rate of output is $\frac{A-\delta-\theta}{\sigma}$. As long is $\rho > 0$, however, the steady-state growth rate of output for both of our models is given by equation (19): $\frac{A-\delta-\theta}{\gamma(1-\sigma)+\sigma}$. At first it seems surprising that there should be a discontinuity in the steady state as we move between zero and a small positive value of $\rho$. The explanation for this phenomenon is that, even though two such economies with slightly different values of $\rho$ will have very different steady states, the time-paths of any of the variables of the model, starting from any initial values of the state variables, will be similar. Figure 11 shows the time paths of output growth for a series of economies which start at the same value of $k/z$, for different values of $\rho$. [15] As $\rho$ falls toward zero, two things happen: the initial growth rate approaches the growth rate of the Rebelo model, and the speed of adjustment of the economy decelerates. The limit of this process as $\rho \to 0$ is an economy where the growth rate approaches the Rebelo growth rate and the speed of adjustment away from the Rebelo growth rate approaches zero.

[15] The initial starting value of $k/z$ is the steady-state value that prevails when $\rho = .4$.

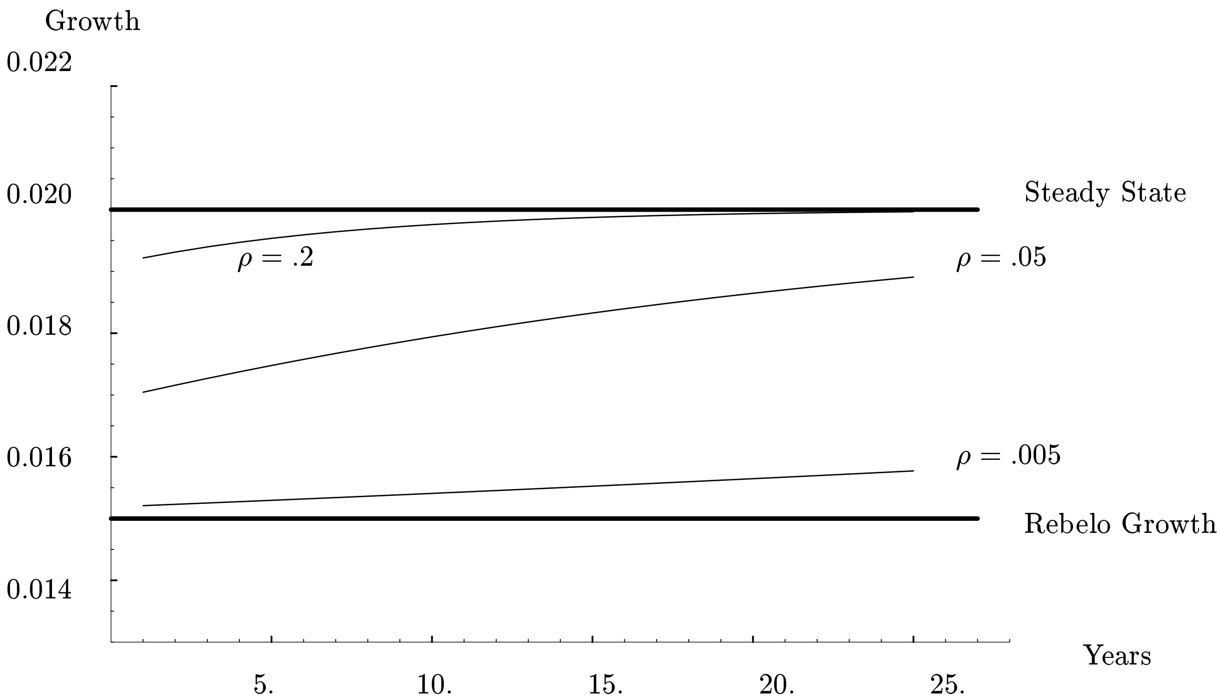


Figure 11: Growth Rates for Different Values of $\rho$

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