

Requiem for the Representative Consumer? Aggregate Implications of Microeconomic Consumption Behavior

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1 Introduction

Macroeconomists pursuing microfoundations for aggregate consumption have generally adopted one of two approaches: either to model microeconomic consumption behavior carefully and then to aggregate, or to thoroughly understand the behavior of a ‘representative consumer’ in general equilibrium, then to introduce microeconomic risk and heterogeneity. The broad conclusion from the ‘bottom up’ approach has been that precautionary saving and microeconomic heterogeneity can profoundly change behavior (Stephen P. Zeldes (1989); Angus S. Deaton (1991); Christopher D. Carroll (1992)). The broad conclusion from the ‘top down’ approach has been that precautionary saving is of little importance in determining the aggregate capital stock (S. Rao Ayigari (1994); Per Krusell and Anthony A. Smith (1998)), leading some economists to conclude that heterogeneity is unimportant for macroeconomic purposes. This paper shows that while general equilibrium effects do imply that the aggregate magnitude of precautionary saving is modest, nevertheless when a model with uninsurable idiosyncratic risk is modified so that it can match key micro facts, it produces behavior which is very different in important respects from the representative agent economy. This leads to the conclusion that for many purposes, the representative consumer model should be abandoned in favor of a model which matches key microeconomic facts.¹

¹See Kirman (1992) for a broader critique of the representative agent model.

2 On the Concavity of the Consumption Function

Unfortunately, the theoretical conditions under which an economy composed of many individuals will behave exactly as though it contains a single representative agent ('exact aggregation holds') are very stringent. The most problematic requirement is that consumers can completely insure their income against idiosyncratic shocks. In reality, household-level income data that include information on the existing sources of insurance (such as unemployment insurance, government transfers, and support from family and friends) show large fluctuations in post-tax, post-transfer idiosyncratic income, and there is now a large literature showing that consumption responds strongly to uninsured income shocks (a few examples are work by John H. Cochrane (1991), Orazio P. Attanasio and Stephen J. Davis (1996), Jonathan McCarthy (1996), and Tullio Jappelli and Luigi Pistaferri (1999).)

Uninsurable risk prevents aggregation because risk causes the consumption policy function to become nonlinear (it becomes strictly concave, even in the absence of liquidity constraints (Carroll and Miles S. Kimball (1996))). Figure 1 presents an example, drawn from the model specified below. The ratio of consumption C to permanent labor income wL , $c = C/wL$, is a concave function of the ratio of total current resources (nonhuman wealth plus current income) X to permanent labor income, $x = X/wL$, for a microeconomic consumer for whom interest rates, wages, and labor supply are

fixed at their steady-state levels. This nonlinearity implies that the distribution of wealth will affect the level of aggregate consumption, the average marginal propensity to consume (MPC), and other aggregate statistics.

Despite the global nonlinearity of $c[x]$, it is relatively smooth, and is almost linear at large values of x . If aggregate wealth were distributed relatively tightly around some large value of x , aggregate behavior would closely resemble the behavior of a representative consumer with wealth equal to the mean of the distribution. Conversely, if wealth is very unequally distributed, the grounds for hoping for any ‘approximate aggregation’ result are much weaker. This figure therefore indicates that the structure of the wealth distribution is of key importance for understanding macroeconomic behavior.

Consider what the figure implies about a statistic which is critical to the analysis of fiscal and monetary policies: the aggregate marginal propensity to consume. Concavity implies that the MPC is much higher at low wealth than at high wealth. If there are many consumers with little wealth we would expect an aggregate MPC much higher than implied by the representative agent model; if most consumers had large amounts of wealth, we would expect the representative consumer model to perform well. Alternatively, we can reason in reverse: we can measure the average MPC, and if it turns out to be much larger than implied by the representative agent model, we

can conclude that many consumers are holding levels of wealth that are in the steeply sloping region of the consumption function.²

3 The Micro Facts

The top panel of table 1 presents information on the distribution of wealth across US households. The data show that the ratio of wealth to labor income for households in the top third of the wealth distribution is enormously higher than the ratio in the bottom two thirds of the distribution, whether the measure of wealth is total net worth or liquid assets. (The same qualitative pattern holds true of the ratio of wealth to total income, and at all ages.)

Representative agent models are typically calibrated to match an aggregate wealth/income ratio like the one in the first column of the table. The table shows that the typical household's wealth is much smaller than the wealth of such a representative agent. Judging from figure 1, this would lead us to expect that the behavior of the median household may not resemble the behavior of a representative agent with a wealth-to-income ratio similar to the aggregate ratio.

Empirical evidence bears out this prediction. Below, we show that the annual MPC predicted by a standard representative agent model is about 0.04. Many empirical

²Of course, a high average MPC might be explained by models other than the rational, time-consistent optimization model employed here; see David Laibson (1997) for an alternative.

analyses performed with household datasets in the 1950s and 1960s found an annual MPC in the range of 0.2 to 0.4.³ A more recent literature, starting with Robert E. Hall and Fredrick S. Mishkin (1982) and with contributions by Annamaria Lusardi (1996), Jonathan McCarthy (1996), Nicholas S. Souleles (1999), and Jonathan Parker (1999), and others has found annual MPC's typically in the range of 0.2 to 0.5.

4 Four Models

Consider a standard model where a representative agent maximizes the discounted sum of expected future utility $E_t [\sum_{s=t}^{\infty} \beta^{s-t} C_s^{1-\rho} / (1-\rho)]$ subject to an aggregate capital accumulation constraint:

$$K_{t+1} = (1 - \delta)(X_t - C_t), \quad (1)$$

$$X_{t+1} = K_{t+1} + \theta_{t+1} K_{t+1}^\alpha L_{t+1}^{1-\alpha}, \quad (2)$$

where K_{t+1} is capital at the start of period $t + 1$, equal to undepreciated savings from period t , and X_t is total resources available for consumption in period t , the sum of capital and current income $\theta_t K_t^\alpha L_t^{1-\alpha}$; θ is an aggregate productivity shock. We consider first a version of the model where there are no aggregate shocks and the aggregate labor supply are normalized to one ($\{\theta_t, L_t\} = \{1, 1\} \forall t$).

The first row of the bottom panel of Table 1 presents the statistics of interest in

³See Thomas Mayer (1972) or Milton A. Friedman (1963) for summaries of the early evidence.

this model under conventional parametric choices and considering the model period as a quarter.⁴ The ratio of the steady-state capital stock to steady-state labor income is 3.906, and the MPC is 0.04 at an annual rate.⁵

Today, the standard version of this model is one with aggregate shocks but no uninsurable idiosyncratic shocks. Following Krusell and Smith, consider a version of the model where there are two aggregate states: a ‘good’ state where the aggregate productivity parameter is $\theta = 1.01$ and a ‘bad’ state where the aggregate productivity parameter is $\theta = 0.99$, and the model is parameterized so that the economy spends half its time on average in each state, and the average duration of expansions and contractions is identical and equal to 8 quarters. Furthermore, to capture the cyclical variability in the unemployment rate, assume that the aggregate labor supply is $L = .96$ in the good state and $L = .90$ in the bad state. The second row of the bottom panel of Table 1 presents the key results. The effect of the aggregate uncertainty on the aggregate capital/income ratio (the precautionary saving effect) is modest: the average value of the K/wL ratio rises by only about 0.6 percent. The reason the precautionary effect is so modest is obvious from figure 1: the representative agent has a very large amount of wealth, and therefore spends essentially all of its time in a region where the

⁴Specifically, mostly following Per Krusell and Anthony A. Smith, we assume $\rho = 3$, $\alpha = .36$, $\delta = 0.025$, $\beta = 0.99$. Under these parameter values, the model substantially underpredicts the empirical K/wL ratio, but this problem could be rectified by assuming a higher β .

⁵Details of the calculation can be found in the appendix. Here and henceforth, ‘annual rate’ MPCs are defined to be four times the quarterly MPC.

consumption function is very flat.

The greatest contribution of Krusell and Smith (1998) is to show how to solve for the dynamic behavior of a model where households are subject to uninsurable idiosyncratic risk as well as aggregate risk. Using their methodology, we now solve a version of the model where fluctuations in aggregate labor supply reflect fluctuations in employment of individual households. Krusell and Smith assume that unemployment spells represent periods when a household's labor income is zero. Here, for greater realism, we assume the existence of an unemployment insurance system that replaces half of permanent wage income. The third row of the bottom panel of table 1 presents the results. The first important conclusion is that, as Krusell and Smith found, adding idiosyncratic risk makes little difference to the magnitude of the aggregate capital/labor income ratio, which rises by only a little over 1 percent when the idiosyncratic risk is added. The remaining columns show why idiosyncratic risk has so little effect: the distribution of wealth is fairly tightly centered around the steady-state average level of wealth. Returning to figure 1, again the essential reason aggregate precautionary saving is modest is that even after the introduction of idiosyncratic shocks, the vast majority of consumers have high levels of wealth fairly close to the level that was held by the representative agent in the model without idiosyncratic shocks. This high-mean, low variance wealth distribution generates an attractive 'approximate aggregation' re-

sult: behavior of the economy is very similar in essentially all respects to behavior in the representative agent model. Thus, the approximate aggregation result depends critically on the model's *failure* to capture either of the key microeconomic facts cited above: the extreme skewness of the wealth distribution and the (consequent) high average value of the marginal propensity to consume.

Fortunately, a final simple modification makes the model capable of generating both skewness in the wealth distribution and a high MPC: we relax the assumption that all consumers have identical tastes. Specifically, suppose that there are two classes of consumers, a 'patient' group with quarterly time preference factor of 0.99 and an 'impatient' group with a time preference factor of $\beta = 0.975$ for an annual rate of 10 percent.⁶ Suppose further that the impatient consumers compose 2/3 of the population.

Before presenting the results, a brief theoretical digression. Long ago, Hirofumi Uzawa (1968) showed that in a nonstochastic economy populated by infinitely-lived agents with different time preference rates, eventually the entire capital stock will be owned by the agent with the lowest time preference rate, because at any aggregate interest rate higher than his time preference rate the most patient agent will accumulate wealth indefinitely. The reverse logic shows that any agent who is less patient will run down his wealth indefinitely, so the patient agent eventually owns all the capital.

⁶Marco Cagetti (1999) estimates time preference rates even lower than 0.975 for many consumers.

As shown in the next-to-last row of table 1, the wealth distribution is now highly skewed, in a manner roughly similar to the data,⁷ and the average annual MPC is almost 0.2. Note that aggregate precautionary saving is *lower* in this model than in the model where all consumers have identical tastes, because the patient agents whose behavior determines the size of the aggregate capital stock now hold much more wealth than the typical agent held before, and are much farther out to the right in figure 1 where the consumption function is nearly linear. The last row shows that under the alternative assumption of log utility ($\rho = 1$), the wealth distribution becomes even more skewed and the MPC is nearly 0.5.⁸

A final point. Many economists are uncomfortable explaining the inequality of the wealth distribution by assuming that consumers have differing tastes. But similar results can be obtained by assuming identical tastes but differing expectations about income growth.⁹ Perhaps the most attractive interpretation is one in which consumers labelled as ‘impatient’ here are thought of as young consumers in the ‘buffer-stock’ saving phase of their life cycle because they anticipate an age profile of rapid income

⁷Because the net worth of the median household is mostly housing equity, which may be illiquid and difficult to use for high-frequency consumption smoothing, it is not clear whether the right goal is to match net worth or liquid assets.

⁸Krusell and Smith also show that adding heterogeneous preferences results in a much more realistic distribution of wealth, and a higher correlation between aggregate consumption and income.

⁹Mark Huggett (1996) argues that much of the inequality of the wealth distribution is attributable to differences in expectations about income growth between working life and retirement. Vincenzo Quadrini and José-Víctor Ríos-Rull (1997) examine various other mechanisms for matching the wealth distribution.

growth through roughly age 50, while the model's 'patient' consumers represent consumers in the latter phase of the life cycle or in retirement who expect slow or no income growth.¹⁰ The crucial requirement for many purposes is likely to be simply that the model have multiple classes of households, some with little wealth and a high MPC and some with substantial wealth and a low MPC - qualitatively, a structure similar to that of Hall and Mishkin (1982) and of John Y. Campbell and N. Gregory Mankiw (1989), though with important differences caused by the stochastic environment.

5 Conclusions

Constructing secure microfoundations for macroeconomic models has long been a central goal of macroeconomists. An apparent message from several recent papers (especially Ayigari (1994)) that have introduced idiosyncratic risk into representative agent economies has been that microeconomic heterogeneity may not matter much for macroeconomic outcomes. This paper argues that the models which produce this 'approximate aggregation' result do not really have solid microfoundations, in the sense that they do not match the key micro facts of a skewed wealth distribution and a high MPC.¹¹ When the model is modified in ways that help it to capture these micro

¹⁰See Carroll (1997), Pierre-Olivier Gourinchas and Jonathan Parker (1999), or Cagetti (1999) for just such an interpretation of life cycle patterns of saving; see Gourinchas (1999) for an ambitious attempt to solve a general equilibrium model of this type with a full specification of life cycle behavior.

¹¹By 'approximate aggregation' I mean that a representative agent model is a good approximation in all important macroeconomic dimensions. Nothing in this paper undermines Krusell and Smith's finding that the evolution of the economy is well captured by an AR(1), which they call a 'quasi-aggregation' result, but which does *not* imply that aggregate data can be rationalized by a

facts, the behavior of the resulting aggregate economy differs from the behavior of the representative agent economy in ways that may be very important for understanding aggregate fluctuations and analyzing the effects of economic policies, though perhaps not for analyzing the long-run questions typically addressed in growth models.

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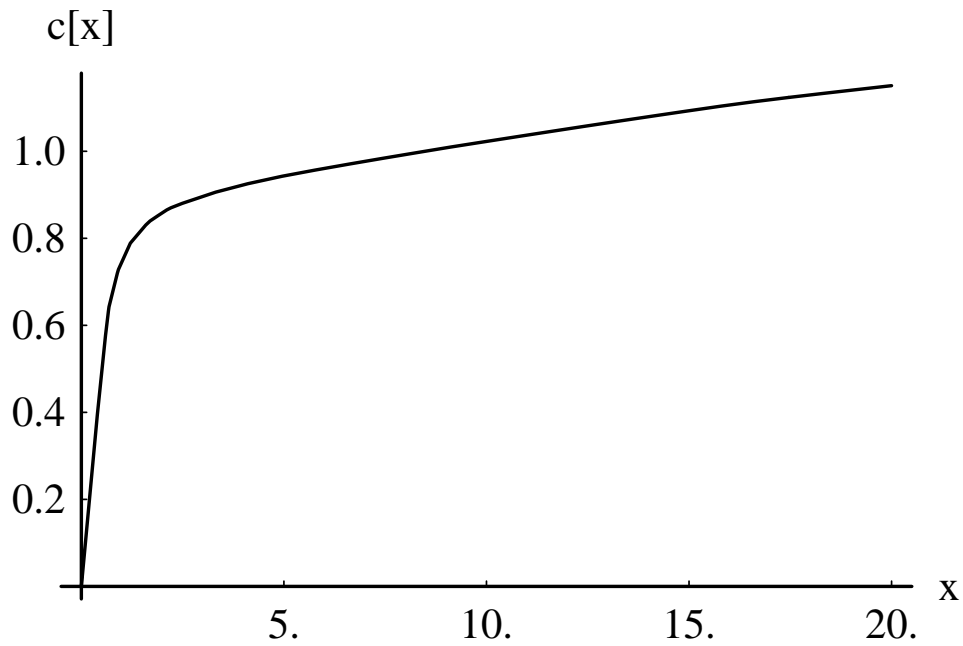
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Table 1: SCF Data and Model Results

Source	Agg K/wL	K/wL By K Percentile		Agg MPC
		0-66	67-100	
<i>Empirical Data</i>				
K = Net Worth	6.2	1.2	10.8	-
K = Liquid Assets	1.5	0.1	2.9	-
<i>Models</i>				
RepAgent	3.906	-	-	0.040
RepAgent+AShocks	3.929	-	-	0.043
AShocks+IShocks	3.984	3.62	4.71	0.050
AShocks+IShocks+Hetero				
Baseline prefs	3.916	0.35	11.09	0.197
$\rho=1.00$	3.909	0.14	11.56	0.468

Notes: The first column is the ratio of total aggregate wealth to total aggregate annual labor income. The second column reports, for the consumers in the bottom 2/3 of the wealth distribution, the ratio of their total aggregate wealth to their total aggregate annual labor income; the third column reports the corresponding statistics for the consumers in the top third of the wealth distribution. Empirical data are from the 1995 Survey of Consumer Finances; similar results hold for earlier surveys. The four models are described in the text. RA = Representative Agent; AShocks = aggregate shocks; IShocks = idiosyncratic shocks; Hetero indicates the model with preference heterogeneity. Further details of the data and theory can be found in the technical appendix to the paper, available at <http://www.econ.jhu.edu/people/ccarroll/requiem.html>.

Figure 1: The Concave Consumption Function



Note: The figure shows $c[x]$ for the third model described in the text, for an unemployed consumer during the ‘good’ aggregate state, where both c and x are normalized by permanent quarterly wage and salary income. (For comparison, the numbers in Table 1 are normalized by annual rather than quarterly income.)

Appendix to “Requiem for the Representative Consumer?”

Following Krusell and Smith (1998), we assume an aggregate production function of the Cobb-Douglas form, $\bar{Y} = \theta_t \bar{K}_t^\alpha \bar{L}_t^{1-\alpha}$, where we are denoting aggregate variables by an overbar. It is convenient to rewrite the model in terms of a Cobb-Douglas aggregate of capital and an adjusted labor stock \bar{P}_t (where P is mnemonic for Productive labor) as follows (this is essentially just a normalization):

$$\begin{aligned}\bar{Y}_t &= \theta_t \bar{K}_t^\alpha \bar{L}_t^{1-\alpha} \\ &= \bar{K}_t^\alpha \bar{P}_t^{1-\alpha} \\ \bar{P}_t &= \theta_t^{1/(1-\alpha)} \bar{L}_t\end{aligned}$$

Because the aggregate production function is CRS in (\bar{K}, \bar{P}) and we assume perfectly competitive labor and capital markets we can write:

$$\bar{Y} = r(\bar{K}, \bar{P})\bar{K} + w(\bar{K}, \bar{P})\bar{P}.$$

Defining $G_{t+1} = \bar{P}_{t+1}/\bar{P}_t$ (essentially the growth rate in labor efficiency), the representative agent’s problem in this economy is¹²

$$V(\bar{X}_t, \bar{P}_t) = \max_{\{\bar{C}_t\}} u(\bar{C}_t) + \beta E_t[V(\tilde{\bar{X}}_{t+1}, \tilde{\bar{P}}_{t+1})] \quad (\text{A.1})$$

such that

$$\bar{K}_{t+1} = (1 - \delta)(\bar{X}_t - \bar{C}_t), \quad (\text{A.2})$$

$$\bar{X}_{t+1} = \bar{K}_{t+1} + \bar{Y}_{t+1}, \quad (\text{A.3})$$

$$\bar{Y}_{t+1} = r(\bar{K}_{t+1}, \bar{P}_{t+1})\bar{K}_{t+1} + w(\bar{K}_{t+1}, \bar{P}_{t+1})\bar{P}_{t+1}, \quad (\text{A.4})$$

$$\bar{P}_{t+1} = G_{t+1}\bar{P}_t. \quad (\text{A.5})$$

¹²Variables inside an expectations operator whose value is uncertain as of the date at which the expectation is being taken have a \sim over them.

It turns out that it is possible and convenient to normalize everything by \bar{P}_t . Define lower-case variables as the normalized version of the upper case variables, e.g. $\bar{y}_t = \bar{Y}_t/\bar{P}_t$, and note that

$$\begin{aligned}
\bar{Y}_t &= \bar{K}_t^\alpha \bar{P}_t^{(1-\alpha)} \\
\bar{y}_t &= \bar{k}_t^\alpha \\
r(\bar{K}_t, \bar{P}_t) &= \alpha(\bar{K}_t/\bar{P}_t)^{\alpha-1} \\
&= \alpha \bar{k}_t^{\alpha-1} \\
w(\bar{K}_t, \bar{P}_t) &= (1-\alpha)(\bar{K}_t/\bar{P}_t)^\alpha \\
&= (1-\alpha)\bar{k}_t^\alpha.
\end{aligned}$$

Now consider the problem

$$v(\bar{x}_t, \bar{P}_t) = \max_{\{\bar{c}_t\}} u(\bar{c}_t) + \beta E_t[\tilde{G}_{t+1}^{1-\rho} v(\tilde{x}_{t+1}, \tilde{P}_{t+1})] \quad (\text{A.6})$$

such that

$$\bar{k}_{t+1} = [(1-\delta)/G_{t+1}](\bar{x}_t - \bar{c}_t), \quad (\text{A.7})$$

$$\bar{y}_{t+1} = \alpha \bar{k}_{t+1}^{\alpha-1} \bar{k}_{t+1} + (1-\alpha)\bar{k}_{t+1}^\alpha, \quad (\text{A.8})$$

$$\bar{x}_{t+1} = \bar{k}_{t+1} + \bar{y}_{t+1} \quad (\text{A.9})$$

$$= \bar{k}_{t+1}(1 + \alpha \bar{k}_{t+1}^{\alpha-1}) + (1-\alpha)\bar{k}_{t+1}^\alpha \quad (\text{A.10})$$

$$= [(1-\delta)/G_{t+1}](\bar{x}_t - \bar{c}_t)(1 + \alpha \bar{k}_{t+1}^{\alpha-1}) + (1-\alpha)\bar{k}_{t+1}^\alpha \quad (\text{A.11})$$

By considering the solution to this problem back from some hypothesized last period of the economy's existence, it is easy to show that $V(\bar{X}_t, \bar{P}_t) = \bar{P}_t^{1-\rho} [v(\bar{x}_t, \bar{P}_t)]$. Thus solving the above problem for $\bar{c}[\bar{x}_t, \bar{P}_t]$ yields the solution for $\bar{C}[\bar{X}_t, \bar{P}_t] = \bar{c}[\bar{x}_t, \bar{P}_t]\bar{P}_t$.

Denoting the derivative of v with respect to x as v^x , the first order conditions for the normalized problem are

$$\begin{aligned} 0 &= u(\bar{c}_t) + \beta E_t G_{t+1}^{1-\rho} v^x(\tilde{x}_{t+1}, \tilde{P}_{t+1}) \frac{\partial \tilde{x}_{t+1}}{\partial \bar{c}_t} \\ u(\bar{c}_t) &= \beta E_t G_{t+1}^{-\rho} v^x(\tilde{x}_{t+1}, \tilde{P}_{t+1}) (1-\delta)(1 + \alpha \bar{k}_{t+1}^{\alpha-1}) \\ 1 &= \beta E_t [G_{t+1}^{-\rho} (\bar{c}_{t+1}/\bar{c}_t)^{-\rho} (1-\delta)(1 + \alpha \bar{k}_{t+1}^{\alpha-1})]. \end{aligned}$$

where the leap from between the last two lines comes from applying the envelope theorem to derive $v^x(x_{t+1}, P_{t+1}) = u(c_{t+1})$.

Now consider the steady-state of a version of the model where there are no productivity shocks of any kind so that $\bar{c}_{t+1} = \bar{c}_t$ and $G_{t+1} = 1 \forall t$. Denoting the steady-state capital stock by \bar{k} with no time subscript, in the steady-state the first order condition reduces to

$$\begin{aligned} 1 &= \beta(1-\delta)(1 + \alpha \bar{k}^{\alpha-1}) \\ \bar{k} &= \frac{1}{\alpha} (\beta(1-\delta) - 1)^{1/(\alpha-1)} \\ &= \frac{\alpha\beta(1-\delta)^{1/(1-\alpha)}}{1 - \beta(1-\delta)} \end{aligned}$$

For the baseline parameter values considered in the paper, $\{\alpha, \beta, \delta\} = \{0.36, 0.99, 0.025\}$, this formula implies that $\bar{k} \approx 36.516$.¹³ For comparability with empirical data, table 1 in the text reports the ratio of the steady-state capital stock to steady-state labor income,

$$\frac{\bar{k}}{(1-\alpha)\bar{k}^\alpha} = \bar{k}^{1-\alpha}/(1-\alpha)$$

¹³Krusell and Smith report a mean value for their definition of capital of 11.54. Their definition differs from the one here in that they do not normalize by labor input. Since they assume an average value of labor input equal to 0.3271 (personal communication from Per Krusell), the appropriate comparison is of 36.516 to $11.54/0.3271=35.28$. The minor discrepancy is caused by the fact that we assume depreciation occurs between periods, while Krusell and Smith assume depreciation within the period.

yielding $K/wL = 15.625$ as the ratio of capital to quarterly labor income. Because a year's labor income is equal to four times a quarter's, this yields the annualized figure in the table of 3.906.

Turning to the version of the problem with idiosyncratic heterogeneity, denote the consumer's employment status in period t by the variable e_t . Krusell and Smith assume a two-point distribution for e_t : either the consumer is unemployed, in which case $e_t = 0$ and the consumer earns no wage income, or the consumer is employed and $e_t = 1$. We assume that periods of unemployment correspond to a value of $e_t = 0.5$, to capture the existence of unemployment insurance and other transfers to the unemployed. Furthermore, we choose a value of e_t in the employed state so that the average value of e_t in the population is always equal to one. For example, in the 'bad' state where the unemployment rate is 10 percent, we set the value of e_t in the employed state to $e_t = (1 - 0.10 \cdot 0.5) / (1 - 0.9) = 1.055$, implying that $p^u e_t + p^e e_t = 1$ where p^u and p^e denote the proportions of the population who are unemployed and employed.

As noted in the text, the aggregate state transition process is chosen so that the expected duration of expansions and contractions is eight quarters. Denoting the aggregate good state by g and the bad state by b , the overall state transition matrix for an individual (obtained directly from Krusell and Smith; see their paper for further calibration information) is shown in Table A.2.

Table A.2: Transition Probabilities

		Today's state			
		(g,1)	(b,1)	(g,0)	(b,0)
Tomorrow's state	(g,1)	0.850694	0.122917	0.583333	0.093750
	(b,1)	0.115885	0.836111	0.031250	0.350000
	(g,0)	0.024306	0.002083	0.291667	0.031250
	(b,0)	0.009115	0.038889	0.093750	0.525000

(g = good times, b = bad times, 1 = employed, 0 = unemployed)

Thus, the consumer's idiosyncratic income in period t is given by the interest on the consumer's holdings of capital plus the consumer's labor income,

$$y_t = r(\bar{k}_t, \bar{P}_t)k_t + w(\bar{k}_t, \bar{P}_t) \quad (A.12)$$

$$= \alpha \bar{k}^{\alpha-1} k_t + (1 - \alpha) \bar{k}_t^\alpha \quad (A.13)$$

where the variables remain in lower case to indicate that we are still normalizing by the aggregate level of labor productivity \bar{P}_t and the variable k_t does not have a bar over it because it represents the individual consumer's personal holdings of capital. Because the expectation of \bar{P}_t across consumers is equal to one, the aggregated value of equation (A.13) is equal to the formula for aggregate income, equation (A.8).

Following Krusell and Smith define Γ_t as the measure (distribution) of consumers over holdings of x and employment status at time t , and denote the law of motion for Γ as H so that $\Gamma_{t+1} = H(\Gamma, \bar{P}_t, \bar{P}_{t+1})$. Imposing household-level liquidity constraint $c_t \leq x_t$, the individual consumer's problem (imposing liquidity constraints) is to solve

$$v(x_t, \epsilon_t; \bar{P}_t, \Gamma_t) = \max_{\{c_t\}} u(c_t) + \beta E_t[G_{t+1}^{1-\rho} v(\tilde{x}_{t+1}, \tilde{\epsilon}_{t+1}; \tilde{\bar{P}}_{t+1}, \tilde{\Gamma}_{t+1})] \quad (A.14)$$

such that

$$c_t \leq x_t \quad (A.15)$$

$$k_{t+1} = [(1 - \delta)/G_{t+1}](x_t - c_t), \quad (A.16)$$

$$y_{t+1} = \alpha \bar{k}_{t+1}^{\alpha-1} k_{t+1} + (1 - \alpha) \bar{k}_{t+1}^\alpha \quad (A.17)$$

$$x_{t+1} = k_{t+1} + y_{t+1} \quad (A.18)$$

$$= k_{t+1}(1 + \alpha \bar{k}_{t+1}^{\alpha-1}) + (1 - \alpha) \bar{k}_{t+1}^\alpha \quad (A.19)$$

$$= [(1 - \delta)/G_{t+1}](x_t - c_t)(1 + \alpha \bar{k}_{t+1}^{\alpha-1}) + (1 - \alpha) \bar{k}_{t+1}^\alpha$$

$$\Gamma_{t+1} = H(\Gamma_t, \bar{P}_t, \bar{P}_{t+1}) \quad (A.20)$$

The reason the consumer needs to know the law of motion for Γ is that the consumer needs to know the future values of interest rates and wages, and those depend on the

evolution of the aggregate capital stock, which in turn in principle depends upon the entire distribution of wealth.

Denoting the four possible aggregate states by gg , gb , bb , and bg , where the first letter indicates last period's state (good or bad) and the second letter denotes the current period's state, consider the following simple rule of thumb for evolution of the capital stock:

$$\bar{k}_{t+1} = \begin{cases} a_0 + a_1 \bar{k}_t & \text{if AggState} = gg, \\ b_0 + b_1 \bar{k}_t & \text{if AggState} = gb, \\ c_0 + c_1 \bar{k}_t & \text{if AggState} = bb, \\ d_0 + d_1 \bar{k}_t & \text{if AggState} = bg \end{cases} \quad (\text{A.21})$$

Now suppose that consumers solve the idiosyncratic optimization problem outlined above under some reasonable assumption about the values of $a_0 \dots d_1$, and consider simulating an economy populated by consumers who share this common assumption about these values.¹⁴ If the actual evolution of the capital stock is captured well by the AR(1) approximation, Krusell and Smith call the solution an 'approximate' equilibrium. The extent to which the equilibrium differs from the exactly correct solution will depend on how well the AR(1) process fits the data.

Following Krusell and Smith, our solution algorithm is as follows. 1) Begin with an assumption that the law of motion in all four aggregate states is the same, and is given by

$$\bar{k}_{t+1} = \bar{k} + .98(\bar{k}_t - \bar{k}) \quad (\text{A.22})$$

where \bar{k} with no time subscript corresponds to the steady-state solution for the non-stochastic model described above. 2) Solve for the optimal individual decision rules

¹⁴We solve using backward iteration from a final period in which the decision rule is assumed to correspond to the decision of a partial equilibrium agent who assumes that wages and interest rates are forever fixed at their steady-state values, because there is a standard linear analytical decision rule for this problem.

given this assumption. 3) Simulate the behavior of an economy populated by 400 consumers using those decision rules for 10,000 periods, discarding the first 1000 periods to allow the system to reach steady-state. 4) Using OLS, estimate the set of equations (A.21) on the data generated by the simulations. We then endow the consumers with new expectations about the evolution of \bar{k} that correspond to the estimated coefficients in the simulated data, solve for the optimal decision rules given those expectations, and repeat the process until expectations correspond closely to the actual time series process. Results do not change when the number of consumers or the number of periods in the simulation are increased.

Like Krusell and Smith, we find that the state-dependent AR(1) process does a spectacularly good job in fitting the simulated data: the R^2 's are typically above 0.999. All of the code is written in *Mathematica*. Solving the most complicated model (with idiosyncratic and aggregate risk and heterogeneous preferences) takes about 24 hours on a 333 Mhz Pentium II-class laptop computer.