The Romer (1986) Model of Growth

Romer (1986) relaunched the growth literature with a paper that presented a model of increasing returns in which there was a stable positive equilibrium growth rate that resulted from endogenous accumulation of knowledge. This was an important break with the existing literature, in which technological progress had largely been treated as completely exogenous.[1]

In Romer's model, firm j's production function is of the form

$$y_{t,j} = A_t \mathrm{F}(k_{t,j}, \ell_{t,j}) \tag{1}$$

where aggregate output-augmenting technological progress is captured by A_t. Its capital accumulates without depreciation,

$$\dot{k}_{t,j} = i_{t,j}. \tag{2}$$

Firms and individuals are distributed along the unit interval with a total mass of 1, as in `Aggregation` (and, importantly, there is no population growth). Thus, aggregate investment is, e.g.,

$$I_t = \int_0^1 i_{t,j} dj. \tag{3}$$

Romer assumes that the aggregate stock of knowledge in the economy is proportional to the cumulative sum of past aggregate investment

$$\Xi_t = \int_{-\infty}^t I_v dv \tag{4}$$

which, not coincidentally, is identical to the size of the aggregate capital stock,

$$K_t = \int_{-\infty}^t I_v dv. \tag{5}$$

Romer makes the crucial assumption that the effect of the stock of knowledge determines productivity via

$$A_t = \Xi_t^\eta \tag{6}$$

where $\eta < 1$. Thus, suppressing the t subscript, the firm-level Cobb-Douglas production function can be written

$$y_j = k_j^\alpha \ell_j^{1-\alpha} \Xi^\eta \tag{7}$$

which is CRS at the firm level in (k, ℓ) holding aggregate knowledge Ξ fixed.

Aggregate output is

$$Y = K^\alpha L^{1-\alpha} \Xi^\eta \tag{8}$$

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[1]See also the prescient paper by Arrow (1962); Afred Marshall articulated similar ideas in the late 19th century.

Dividing by the size of the labor force L (or, equivalently, normalizing to $L = 1$), we have

$$y = k^\alpha \Xi^\eta. \tag{9}$$

Now assume that households maximize a typical CRRA utility function, but each household ignores the trivial effect its own investment decision has on aggregate knowledge. Thus from the individual firm/consumer's perspective, the marginal product of capital is $\alpha k_{t,i}^{\alpha-1} \ell_{t,i}^{1-\alpha} \Xi_t^\eta$. If we normalize the model by assuming that the aggregate quantity of labor adds upt to $L_t = 1$, we can set up and solve the Hamiltonian to obtain

$$\dot{c}_{t,i}/c_{t,i} = \rho^{-1}(\alpha k_{t,i}^{\alpha-1} \Xi_t^\eta - \vartheta). \tag{10}$$

But if all households are identical and $\Xi_t = K_t$, this means that aggregate consumption per capita evolves according to

$$\begin{aligned} \dot{c}_t/c_t &= \rho^{-1}(\alpha k_t^{\alpha-1} \Xi_t^\eta - \vartheta) \\ &= \rho^{-1}(\alpha k_t^{\alpha+\eta-1} - \vartheta). \end{aligned} \tag{11}$$

A balanced growth path can occur in this economy if $\alpha + \eta = 1$, in which case

$$\dot{c}_t/c_t = \rho^{-1}(\alpha - \vartheta) \tag{12}$$

so there is constant growth forever at a rate that depends on the degree of impatience and capital's share in output.

Note finally that the steady-state growth rate that would be chosen by the social planner is

$$\dot{c}_t/c_t = \rho^{-1}(\alpha + \eta - \vartheta), \tag{13}$$

because the social planner would take into account the fact that the externalities imply that there are higher returns to capital accumulation at the social level than at the individual level. Thus, this model implies that capital accumulation should be subsidized if the social planner wants to induce the private economy to move toward the social optimum.

References

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