Solving Consumption Models with Multiplicative Habits

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Abstract

This paper provides derivations necessary for solving an optimal consumption problem with multiplicative habits and a CRRA 'outer' utility function, either for a microeconomic problem with both labor income risk and rate-of-return risk, or for a macroeconomic representative agent model.

Keywords: habit formation, relative consumption, economic growth, representative agent

1   Introduction

The last few years have seen a renewal of interest in the old[1] idea that habits may play a key role in consumption behavior. The resurgence of interest in habits has been provoked by the emergence of empirical findings that are difficult to explain using the traditional model in which utility is time-separable.[2]

The early modern theoretical models of habit formation[3] tended to take the 'subtractive' form in which utility is derived from the *difference* between current consumption and the habit stock,

$$u(c, h) \;\; = \;\; v(c - h). \tag{1}$$

where the habit stock h was usually set equal to the level of consumption in the previous period,

$$h_t \;\; = \;\; \alpha c_{t-1}, \tag{2}$$

and the 'outer' utility function $v(x)$ usually took the quadratic form. Unfortunately, quadratic utility has a host of implausible implications,[4] and has consequently largely been abandoned in the rest of the consumption literature, principally in favor of the Constant Relative Risk Aversion (CRRA) form of utility which has much more attractive properties.[5]

Some papers (notably Constantinides (1990), Dynan (forthcoming), and Campbell and Cochrane (1999)) have used CRRA utility for the 'outer' utility function, but CRRA utility in combination with the subtractive formulation of equation (1) has several theoretical problems, the gravest of which is that for microeconomically plausible parameterizations of consumption variation the accumulation equation (2) can easily lead to a zero or negative argument to the function v, generating infinite negative utility. Campbell and

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[1]Adam Smith (1776) spoke of the change over time in 'customary' consumption levels; Alfred Marshall (1898) explicitly argues (see, e.g., pages 86–91 or pages 110–111) that habits are important in consumption behavior; Pigou (1903) provides a more formal treatment; Duesenberry (1949) provides a somewhat less ancient, and recently more famous, treatment.

[2]Habits have recently been proposed in three distinct domains of macroeconomic theory. Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) have argued that habits may explain the equity premium puzzle; Carroll and Weil (1994) and Carroll, Overland, and Weil (2000) have proposed that habits may be able to explain why high growth apparently causes saving to rise; and Fuhrer (forthcoming) and Fuhrer and Klein (1998) have argued that habits may be necessary to explain the 'excess smoothness' of aggregate consumption at high frequencies. Several papers (notably van de Stadt, Kapteyn, and van de Geer (1985), Dynan (forthcoming), and Carroll and Weil (1994)) have also proposed habits as an explanation for microeconomic results.

[3]By a 'modern' treatment I mean a treatment in an explicit dynamic optimizing model.

[4]Among them, the absence of a precautionary saving motive, the existence of a 'bliss point' beyond which additional consumption reduces utility, and increasing absolute risk aversion.

[5]Recently, Alessie and Lusardi (1997) have shown how to solve the subtractive model when v is of the Constant Absolute Risk Aversion (CARA) form, thus allowing for the first time the analytical examination of the interaction between precautionary saving motives and habit formation. However, CARA utility also has some theoretically unattractive features, notably that it does not rule out negative consumption and that it implies that poor people and rich people reduce their consumption by exactly the same dollar amount in reaction to a given risk. See Kimball (1990) or Carroll and Kimball (1996) for further discussion of the unattractive properties of CARA utility.

Cochrane (1999) deal with this problem by replacing equation (2) with a highly nonlinear (and nonintuitive) function that causes habits to drop simultaneously with drops in consumption when consumption gets too close to the habit stock.

Partly in response to this and other theoretical problems with the subtractive model, the recent literature[6] seems to be trending toward the use of what might be termed the 'multiplicative' form of habits introduced by Abel (1990) and Galí (1994),

$$u(c, h) = v(c/h^{\gamma}),$$

with the habit stock a general adaptive process of the form

$$h_t = h_{t-1} + \lambda(c_{t-1} - h_{t-1}).$$

In this formulation, if consumption is always positive then h will always be positive and so a CRRA utility function can be used for v without danger of introducing negative infinite utility. Furthermore, the model's two parameters are easy to interpret: γ indexes the importance of habits, in the sense that if $\gamma = 0$ the model collapses to the standard CRRA model in which consumers only care about the level of consumption (habits are irrelevant) while if $\gamma = 1$ consumers care only about how their current consumption compares to habits and do not care at all about the level of consumption. λ indexes the speed with which habits 'catch up' to consumption; if $\lambda = 0$ the model again collapses to the CRRA model because habits are simply a constant multiplicative factor in the utility function, while if $\lambda = 1$ habits in the current period collapse to the previous period's level of consumption.

Despite its appeal, a general theoretical analysis of the multiplicative model of habits similar to the treatments for the subtractive model given in Muellbauer (1988), Constantinides (1990), Deaton (1992), and Alessie and Lusardi (1997) does not appear to have been published.[7] This paper fills that gap. Section II of the paper presents the formal model and the first order conditions that can be used to solve the model numerically, Section III explores the steady-state characteristics of the nonstochastic version of the model and derives some analytical results, and Section IV presents difference equations that characterize the evolution of the model toward the steady-state.

2 The Problem

The consumer's goal is to

$$\max \quad E_t \left[\sum_{s=t}^{T} \beta^{s-t} u(c_s, h_s) \right]$$

where β is the constant time preference factor and all variables are as usually defined.

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[6]Examples include Carroll, Overland, and Weil ((1997, 2000)), Abel (1999), Fuhrer (forthcoming), and Fuhrer and Klein (1998).

[7]Abel (1990, 1999) provides asset pricing formulas; Carroll, Overland, and Weil (1997) sketch the continuous-time perfect foresight solution; and Fuhrer (forthcoming) provides the Euler equation in the form of an infinite series and finds numerical solutions for a perfect-foresight discrete-time version with no growth, but no source known to the author provides either a general-purpose derivation under uncertainty or provides the general analytical version of the discrete-time steady-state conditions.

Assume that the utility function is given by

$$u(c,h) = \frac{(c/h^\gamma)^{1-\rho}}{1-\rho} \tag{3}$$

which implies that the derivatives of the utility function with respect to its arguments are

$$
\begin{aligned}
u^c &= (ch^{-\gamma})^{-\rho}h^{-\gamma} \tag{4}\\
u^h &= -\gamma(ch^{-\gamma})^{-\rho}ch^{-\gamma-1} \tag{5}\\
&= -\gamma u^c(c/h). \tag{6}
\end{aligned}
$$

Bellman's equation for this problem is

$$v_t(x_t, h_t) =$$
$$\max_{\{c_t, w_t\}} \quad u(c_t, h_t) \quad + \beta E_t[v_{t+1}(\tilde{x}_{t+1}, h_{t+1})] \tag{7}$$

such that

$$
\begin{aligned}
R_{t+1} &= (1-w_t)R + w_t R_{e,t+1} \tag{8}\\
x_{t+1} &= R_{t+1}[x_t - c_t] + y_{t+1} \tag{9}\\
h_{t+1} &= h_t + \lambda(c_t - h_t) \tag{10}
\end{aligned}
$$

where R is the constant gross riskfree interest factor (equal to 1 plus the riskfree interest rate), $R_{e,t+1}$ is the (ex-ante stochastic) return on the risky asset (e is mnemonic for 'equities'); w_t is the portfolio **w**eight given to the risky asset in period t; R_{t+1} is the portfolio-weighted rate of return between the end of period t and the beginning of period $t+1$; y_t is labor income in period t; x_t is 'cash-on-hand,' the total amount of resources available to be spent in period t; and the notational convention for the treatment of uncertainty is that in any expression whose expectation is being taken, a $\sim$ is put over any variable whose value is uncertain as of the date at which the expectation is taken. Thus, x_{t+1} warrants a $\sim$ in equation (7) because it is inside an $E_t[]$ expression, but does not warrant a $\sim$ in equation (9) because no expectation is being taken.

2.1 Optimality Conditions

2.1.1 First Order Conditions

The first order condition for this problem with respect to c_t is (dropping arguments for brevity and denoting the derivative of f with respect to x at time t as f_t^x):

$$
\begin{aligned}
0 &= u_t^c + \beta E_t\left(\lambda v_{t+1}^h - \tilde{R}_{t+1}v_{t+1}^x\right) \tag{11}\\
u_t^c &= \beta E_t[\tilde{R}_{t+1}v_{t+1}^x - \lambda v_{t+1}^h], \tag{12}
\end{aligned}
$$

and the FOC with respect to w_t gives:

$$
\begin{aligned}
0 &= E_t\left[v_{t+1}^x \frac{\partial \tilde{x}_{t+1}}{\partial w_t}\right]\\
&= E_t\left[\left(\tilde{R}_{e,t+1} - R\right)[x_t - c_t]v_{t+1}^x\right]. \tag{13}
\end{aligned}
$$

2.1.2 Envelope Conditions

The Envelope theorem on the variable x_t says:

$$v_t^x = \frac{\partial v_t}{\partial x_t} + \overbrace{\frac{\partial v_t}{\partial c_t}\frac{\partial c_t}{\partial x_t}}^{=0}$$

$$v_t^x = \beta E_t[\tilde{R}_{t+1}v_{t+1}^x]. \tag{14}$$

Substituting this into the FOC equation (12) gives

$$v_t^x = u_t^c + \beta E_t[\lambda v_{t+1}^h]. \tag{15}$$

Noting that $\partial h_{t+1}/\partial h_t = (1-\lambda)$, the Envelope theorem on the variable h_t says:

$$v_t^h = \frac{\partial v_t}{\partial h_t} + \overbrace{\frac{\partial v_t}{\partial c_t}\frac{\partial c_t}{\partial h_t}}^{=0}$$

$$= u_t^h + \beta E_t[v_{t+1}^h \frac{\partial h_{t+1}}{\partial h_t}]$$

$$= u_t^h + (1-\lambda)\beta E_t[v_{t+1}^h]. \tag{16}$$

2.2 Numerical Solution

As with the standard time-separable model, no analytical solutions to this model appear to exist for general forms of uncertainty. Numerical solution proceeds as follows.

The derivative of $u(c,h)$ with respect to c can be substituted into equation (12) to yield

$$(c_t h_t^{-\gamma})^{-\rho}h_t^{-\gamma} = \beta E_t\left(\tilde{R}_{t+1}v_{t+1}^x - v_{t+1}^h\lambda\right) \tag{17}$$

$$c_t^{-\rho} = h_t^{\gamma-\gamma\rho}\beta E_t\left(\tilde{R}_{t+1}v_{t+1}^x - v_{t+1}^h\lambda\right) \tag{18}$$

$$c_t = h_t^{\gamma(1-1/\rho)}\beta^{-1/\rho}E_t\left(\tilde{R}_{t+1}v_{t+1}^x - v_{t+1}^h\lambda\right)^{-1/\rho}. \tag{19}$$

Given the existence of the marginal value functions in the next period v_{t+1}^x and v_{t+1}^h, equations (19) and (13) can be jointly solved numerically for optimal c_t and w_t at some set of grid points in (x,h) space, and approximate policy functions can be constructed using any of several methods (see Judd (1998) for a catalog of options). The approximated marginal value functions can be constructed on the same (x,h) grid by substituting the optimal values of c_t and w_t into the envelope relations (14) and (16). With these marginal value functions in hand, it is then possible to solve for optimal policy in period $t-1$ and so on to any earlier period by backward recursion.

Thus, to solve the finite-lifetime version of the model, simply note that in the final period of life T the future marginal utilities are equal to zero so that

$$v_T^c = u^c(c_t, h_t)$$
$$v_T^h = u^h(c_t, h_t),$$

and backward recursion provides policy functions for all previous periods of life. An infinite-horizon solution to the model can be defined as the finite-horizon solution as the horizon approaches infinity.

3   The Steady-State

The discussion of numerical solution methods was suited to the use of the model to describe a microeconomic problem like that examined by Dynan (forthcoming) or van de Stadt, Kapteyn, and van de Geer (1985). Models with habits have also recently been applied in macroeconomic problems where the representative agent's steady-state infinite-horizon solution is relevant. It turns out that it is possible to solve analytically for the steady-state of the perfect-foresight version of the model, as follows.

Roll equation (16) forward one period to get

$$v_{t+1}^h = u_{t+1}^h + (1-\lambda)\beta[v_{t+2}^h], \tag{20}$$

which can be substituted into (15) to yield

$$u_t^c = v_t^x - \lambda\beta[u_{t+1}^h + (1-\lambda)\beta v_{t+2}^h]. \tag{21}$$

Now equation (15) can also be rolled forward one period and solved for $\beta[v_{t+2}^h]$

$$\beta[v_{t+2}^h] = (\frac{1}{\lambda})\left[v_{t+1}^x - u_{t+1}^c\right]$$

which can be substituted into equation (21)

$$u_t^c = v_t^x - \lambda\beta\left[u_{t+1}^h + \left(\frac{1-\lambda}{\lambda}\right)\left(v_{t+1}^x - u_{t+1}^c\right)\right] \tag{22}$$

$$= v_t^x - (1-\lambda)\beta[v_{t+1}^x] - \beta\left[\lambda u_{t+1}^h - (1-\lambda)u_{t+1}^c\right] \tag{23}$$

$$= v_t^x - \frac{(1-\lambda)}{R}v_t^x - \beta\left[\lambda u_{t+1}^h - (1-\lambda)u_{t+1}^c\right] \tag{24}$$

$$= (\frac{R-(1-\lambda)}{R})v_t^x - \beta\left[\lambda u_{t+1}^h - (1-\lambda)u_{t+1}^c\right] \tag{25}$$

which can be rolled forward one period and solved for v_{t+1}^x

$$v_{t+1}^x = \left(\frac{R}{R-(1-\lambda)}\right)\left[\beta\left[\lambda u_{t+2}^h - (1-\lambda)u_{t+2}^c\right] + u_{t+1}^c\right]. \tag{26}$$

Finally, from equations (25) and (14) we have

$$u_t^c = [R\beta v_{t+1}^x](\frac{R-(1-\lambda)}{R}) - \beta\left[\lambda u_{t+1}^h - (1-\lambda)u_{t+1}^c\right] \tag{27}$$

$$= \left[R\beta\left(\beta\left[\lambda u_{t+2}^h - (1-\lambda)u_{t+2}^c\right] + u_{t+1}^c\right)\right] - \beta\left[\lambda u_{t+1}^h - (1-\lambda)u_{t+1}^c\right], \tag{28}$$

which is the Euler equation for this problem. An alternative form is

$$u_t^c - \beta[Ru_{t+1}^c] = \beta\left[R\beta\left(\lambda u_{t+2}^h - (1-\lambda)u_{t+2}^c\right) - \left(\lambda u_{t+1}^h - (1-\lambda)u_{t+1}^c\right)\right]. \tag{29}$$

Note that if $\lambda = 0$ so that the reference level for 'habits' never changes, or if $\gamma = 0$ so that habits should not matter, the problem simplifies, as it should, to $u_t^c = [R\beta u_{t+1}^c]$ which is the Euler equation for the standard time-separable problem without habits.

Now let us assume that there is a perfect-foresight solution to the model in which the growth rate of consumption and the habit stock are both equal to a constant σ, so that the ratio of consumption to habits is constant at $\frac{c_t}{h_t} = \chi$ which implies that $h_t = c_t/\chi$. Substituting this formula for h into the equations for the derivatives of c and h gives

$$
\begin{aligned}
u_t^c &= c_t^{-\rho}(c_t/\chi)^{\gamma(\rho-1)} \\
&= c_t^{\rho\gamma-\gamma-\rho}\chi^{\gamma(1-\rho)} \\
u_t^h &= -\gamma u_t^c \chi.
\end{aligned}
$$

Note that this implies that we can rewrite

$$
\lambda u_t^h - (1-\lambda)u_t^c = u_t^c(-\gamma\lambda\chi - (1-\lambda)) \tag{30}
$$

Defining $\kappa = \beta(-\gamma\lambda\chi - (1-\lambda))$, rolling equation (30) forward one and two periods and substituting into equation (28) gives

$$
\begin{aligned}
u_t^c &= R\beta\left(u_{t+2}^c\kappa + u_{t+1}^c\right) - u_{t+1}^c\kappa \tag{31} \\
&= R\beta\left(u_{t+2}^c\kappa\right) + u_{t+1}^c(R\beta - \kappa) \tag{32} \\
c_t^{\rho\gamma-\gamma-\rho} &= R\beta\left(c_{t+2}^{\rho\gamma-\gamma-\rho}\kappa\right) + c_{t+1}^{\rho\gamma-\gamma-\rho}(R\beta - \kappa). \tag{33}
\end{aligned}
$$

Now if consumption is growing at rate σ each period, then $c_{t+1} = \sigma c_t$ and $c_{t+2} = \sigma^2 c_t$. Substituting these expressions into equation (33) and dividing both sides by $c_t^{\rho\gamma-\gamma-\rho}$ gives:

$$
1 = R\beta\left((\sigma^2)^{\rho\gamma-\gamma-\rho}\kappa\right) - \sigma^{\rho\gamma-\gamma-\rho}(R\beta - \kappa), \tag{34}
$$

or defining $\eta = \sigma^{\rho\gamma-\gamma-\rho}$ this becomes a quadratic equation in η:

$$
0 = 1 - \eta(R\beta - \kappa) - R\beta\eta^2\kappa \tag{35}
$$

which has the two solutions

$$
\eta = \begin{cases} \frac{1}{R\beta}, \\ -\frac{1}{\kappa}, \end{cases} \tag{36}
$$

yielding the two possible solutions for steady-state growth

$$
\sigma = \begin{cases} (\beta R)^{1/(\rho+\gamma(1-\rho))} \\ (\beta[\gamma\chi\lambda + (1-\lambda)])^{1/(\rho+\gamma(1-\rho))}. \end{cases} \tag{37}
$$

The first of these potential solutions reduces to $\sigma = (\beta R)^{1/\rho}$ if $\gamma = 0$, which matches the usual formula for consumption growth in the time-separable case. By contrast, the second solution does not reduce to the optimal time-separable solution when $\gamma = 0$ and so cannot be an optimum.[8]

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[8]Another way to see that the second solution cannot be optimal is to note that the implied growth rate is independent of interest rates.

We can also solve for the steady-state value of χ, the ratio of consumption to habits. Expand the accumulation equation for h:

$$
\begin{aligned}
h_{t+1} &= \lambda c_t + (1-\lambda)h_t \\
&= \lambda c_t + (1-\lambda)(\lambda c_{t-1} + (1-\lambda)h_{t-1}) \\
&= \lambda c_t(1 + (1-\lambda)c_{t-1}/c_t + (1-\lambda)^2 c_{t-2}/c_t \ldots) \\
&= \lambda c_t \frac{1}{1 - \sigma^{-1}(1-\lambda)}, \\
c_t/(\sigma h_t) &= (1/\lambda)\left[1 - \sigma^{-1}(1-\lambda)\right] \\
\chi &= (1/\lambda)\left[\sigma - (1-\lambda)\right]
\end{aligned}
$$

It is also possible to solve for the level of consumption in a version of the model where labor income is growing by a constant factor G from period to period and the gross interest factor R is constant (both of these conditions will hold in the steady-state of a standard neoclassical growth model). If consumption grows at rate σ every period, then the present discounted value of consumption is[9]

$$
\begin{aligned}
PDV_t(c) &= c_t(1 + \sigma/R + (\sigma/R)^2 + \ldots) \\
&= c_t \frac{1}{1 - \sigma/R}.
\end{aligned}
$$

Assuming $G < R$, the present discounted value of labor income is

$$
\begin{aligned}
PDV_t(y) &= y_t(1 + G/R + \ldots) \\
&= \frac{y_t}{1 - G/R}.
\end{aligned}
$$

Equating the present discounted value of consumption with the PDV of resources, we have

$$
\begin{aligned}
c_t \frac{1}{1 - \sigma/R} &= \frac{y_t}{1 - G/R} + x_t \\
c_t &= (1 - \sigma/R)\left[\frac{y_t}{1 - G/R} + x_t\right],
\end{aligned}
$$

or, substituting the solution for σ from above,

$$
c_t = (1 - R^{-1}(R\beta)^{1/(\rho+\gamma(1-\rho))})\left[\frac{y_t}{1 - G/R} + x_t\right].
$$

4 Dynamics of the Perfect Foresight Model

Analysis of growth models often proceeds by linearizing the model around the steady-state. For the usual neoclassical model this involves linearizing the aggregate budget

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[9]In order for this derivation to be valid, it is necessary to have $\sigma < R$.

constraint and the difference equation for consumption. We derive here the difference equations for σ and χ under the assumption that the real interest rate is constant. This is the correct procedure in an endogenous growth model with a fixed rate of return to capital; the extension to the neoclassical production function would add a third equation to the system derived here, describing the evolution of the gross interest factor as derived from the standard neoclassical production function.

The key step in obtaining the steady-state approximations is to find the difference equations that govern the evolution of χ and σ. Begin by defining $\sigma_t = c_t/c_{t-1}$ and $\chi_t = c_t/h_t$, and note that

$$c_t/h_t \;=\; \frac{\sigma_t c_{t-1}}{h_t} \tag{38}$$

$$=\; \sigma_t \frac{c_{t-1}}{h_{t-1}}\frac{h_{t-1}}{h_t} \tag{39}$$

$$=\; \sigma_t \chi_{t-1}\frac{h_{t-1}}{(1-\lambda)h_{t-1}+\lambda c_{t-1}} \tag{40}$$

$$\chi_t \;=\; \sigma_t \chi_{t-1}\frac{1}{(1-\lambda)+\lambda\chi_{t-1}}. \tag{41}$$

Substituting in for u_t^c and u_t^h in the Euler equation gives:

$$c_t^{-\rho}h_t^{\gamma(\rho-1)} \;=\; \beta\left[c_{t+1}^{-\rho}h_{t+1}^{\gamma(\rho-1)}\left(R+(1-\lambda)+\gamma\lambda\chi_{t+1}\right)-R\beta c_{t+2}^{-\rho}h_{t+2}^{\gamma(\rho-1)}\left((1-\lambda)+\gamma\lambda(\chi_{t+2})\right)\right.$$

$$1=\left[\frac{c_{t+1}}{c_t}\right]^{-\rho}\left(\frac{h_{t+1}}{h_t}\right)^{\gamma(\rho-1)}\beta\left[R+((1-\lambda)+\gamma\lambda\chi_{t+1})\right.$$
$$\left.-R\beta\left[\frac{c_{t+2}}{c_{t+1}}\right]^{-\rho}\left(\frac{h_{t+2}}{h_{t+1}}\right)^{\gamma(\rho-1)}\left((1-\lambda)+\gamma\lambda\chi_{t+2}\right)\right] \tag{42}$$

and use the fact that $(h_{t+1}/h_t)=[(1-\lambda)+\lambda\chi_t]$ (see equations (39)-(41)) to obtain

$$1=\sigma_{t+1}^{-\rho}((1-\lambda)+\lambda\chi_t)^{\gamma(\rho-1)}\beta\left[R+((1-\lambda)+\gamma\lambda\chi_{t+1})\right.$$
$$\left.-R\beta\sigma_{t+2}^{-\rho}\left((1-\lambda)+\lambda\chi_{t+1}\right)^{\gamma(\rho-1)}\left((1-\lambda)+\gamma\lambda\chi_{t+2}\right)\right] \tag{43}$$

$$\sigma_{t+1}^{\rho}((1-\lambda)+\lambda\chi_t)^{\gamma(1-\rho)}/\beta-R-((1-\lambda)+\gamma\lambda\chi_{t+1})=$$
$$-R\beta\sigma_{t+2}^{-\rho}\left((1-\lambda)+\lambda\chi_{t+1}\right)^{\gamma(\rho-1)}\left((1-\lambda)+\gamma\lambda\chi_{t+2}\right))$$

$$\sigma_{t+2}^{-\rho}\;=\;\frac{R+((1-\lambda)+\gamma\lambda\chi_{t+1})-\sigma_{t+1}^{\rho}((1-\lambda)+\lambda\chi_t)^{\gamma(1-\rho)}/\beta}{R\beta\left((1-\lambda)+\gamma\lambda\chi_{t+2}\right)\left((1-\lambda)+\lambda\chi_{t+1}\right)^{\gamma(\rho-1)}} \tag{44}$$

$$\sigma_{t+2}\;=\;\left[\frac{R+((1-\lambda)+\gamma\lambda\chi_{t+1})-\sigma_{t+1}^{\rho}((1-\lambda)+\lambda\chi_t)^{\gamma(1-\rho)}/\beta}{R\beta\left((1-\lambda)+\gamma\lambda\chi_{t+2}\right)\left((1-\lambda)+\lambda\chi_{t+1}\right)^{\gamma(\rho-1)}}\right]^{-1/\rho}. \tag{45}$$

Equations (41) and (45) are difference equations for χ and σ which can be linearized or log-linearized around the steady-state values derived above to allow analysis of the near-steady-state behavior of the model.

5   Conclusions

This paper provides the derivations necessary to solve a problem with multiplicative habits and a CRRA outer utility function, either for a microeconomic problem with both labor income risk and rate-of-return risk, or for a perfect-foresight macroeconomic representative agent model. These solutions should be useful for researchers who want to further explore the properties of multiplicative habit formation models.

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