The Time Series Behavior of Consumption in the Buffer Stock Model

1 Theory

Consider a consumer solving the maximization problem

\[ V_t(X_t, P_t) = \max_{\{C_t\}} u(C_t) + E_t \left[ \sum_{s=t+1}^{T} \beta^{s-t} u(C_s) \right] \]  \hspace{1cm} (1) \]

s.t.

\[ S_t = X_t - C_t, \] \hspace{1cm} (2) \]
\[ W_{t+1} = R S_t, \] \hspace{1cm} (3) \]
\[ X_{t+1} = W_{t+1} + Y_{t+1}, \] \hspace{1cm} (4) \]
\[ Y_{t+1} = P_{t+1} \epsilon_{t+1}, \] \hspace{1cm} (5) \]
\[ P_{t+1} = P_t G_{t+1} N_{t+1}. \] \hspace{1cm} (6) \]

where the variable \( X_{t+1} \) indicates the level of the consumer’s ‘cash-on-hand,’ the sum of current income and current wealth \( W_{t+1} \), which equals the proceeds from investment at gross interest rate \( R = (1 + r) \) of the unconsumed resources (‘savings’ \( S_t \)) at the end of the previous period. The income process is subject to i.i.d. transitory shocks (\( \epsilon_{t+1} \)) and permanent shocks (\( N_{t+1} \)) both of which have mean values of one, \( E_t[\tilde{\epsilon}_{t+1}] = E_t[\tilde{N}_{t+1}] = 1 \). As usual, the recursive nature of the problem allows us to rewrite the problem as:

\[ V_t(X_t, P_t) = \max_{\{C_t\}} u(C_t) + \beta E_t \left[ V_{t+1}(\tilde{X}_{t+1}, \tilde{P}_{t+1}) \right]. \] \hspace{1cm} (7) \]

This problem is essentially identical to problems that have been analyzed in a number of papers on ‘buffer-stock saving’ beginning with Deaton (1991) and Carroll (1992). As written, the problem has two state variables, the level of permanent income \( P_t \) and the level of cash-on-hand \( X_t \). One of the convenient features of this problem is that if utility is of the Constant Relative Risk Aversion form \( u(C) = C^{1-\rho} / (1 - \rho) \) it is possible to normalize all variables by the level of permanent income \( P_t \) and thereby to effectively reduce the number of

\[ 1 \]The notational convention is that stochastic variables have a \( \sim \) over them when their expectation is being taken, but not otherwise, on the grounds that equations where the expectation is being taken are equations where the time period from which the equation is being viewed is well-specified. Hence we write \( P_{t+1} = G_{t+1} N_{t+1} P_t \) but if we need the period-\( t \) expectation we would write \( E_t[\tilde{P}_{t+1}] = P_t E_t[\tilde{G}_{t+1} \tilde{N}_{t+1}]. \]
state variables in the problem to one. Specifically, defining lower-case \(x_t = X_t/P_t\), \(c_t = C_t/P_t\), and so on, consider the problem in the second-to-last period of life,

\[
V_{T-1}(X_{T-1}, P_{T-1}) = \left(1 \over 1 - \rho\right) \max_{c_{T-1}} C_{T-1}^{1-\rho} + E_{T-1}[\beta \bar{X}_{T}^{1-\rho}]
\]

\[
= \left(1 \over 1 - \rho\right) \max_{c_{T-1}} (P_{T-1} c_{T-1})^{1-\rho} + E_{T-1}[\beta (P_{T} \bar{X}_{T})^{1-\rho}]
\]

\[
= \left(1 \over 1 - \rho\right) P_{T-1}^{1-\rho} \left( \max_{c_{T-1}} c_{T-1}^{1-\rho} + E_{T-1}[\beta (\bar{G}_{T} \bar{N}_{T} \bar{X}_{T})^{1-\rho}] \right)
\]

where we define \(v_T(x_T) = x_T^{1-\rho}/(1 - \rho)\) and

\[
v_t(x_t) = \max_{c_t} u(c_t) + \beta E_t[(\bar{G}_{t+1} \bar{N}_{t+1})^{1-\rho} v_{t+1}(\bar{x}_{t+1})] \tag{8}
\]

s.t.

\[
s_t = x_t - c_t, \tag{9}
\]

\[
x_{t+1} = \left( R \over \bar{G}_{t+1} \bar{N}_{t+1} \right) s_t + \epsilon_{t+1} \tag{10}
\]

Note that if we assume that \(E_t[\bar{G}_{t+1}] = G \forall t\) equation (8) has only a single state variable, \(x_t\), and recursion on this equation yields a ‘normalized’ value function for any period prior to \(T-1\). The full value function \(V_t(X_t, P_t)\) is recovered simply from \(V_t(X_t, P_t) = P_t^{1-\rho} v_t(X_t/P_t)\). The first order condition is

\[
c_t^{-\rho} = R \beta E_t[(\bar{G}_{t+1} \bar{N}_{t+1})^{-\rho} \bar{c}_{t+1}^{-\rho}] \tag{11}
\]

and Carroll (2004) shows that the problem defines a contraction mapping if the ‘impatience condition’ originally derived by Deaton (1991)

\[
R \beta E_t[(\bar{G}_{t+1} \bar{N}_{t+1})^{-\rho}] < 1 \tag{12}
\]

holds, so that as the horizon recedes the consumption function \(c_t(x_t)\) approaches an invariant function \(c(x)\) which we define as the infinite-horizon solution to the problem.

### 2 The Perfect Foresight Case

It is well known that the solution to the unconstrained perfect certainty version of this model where \(G_t = G, N_t = \epsilon_t = 1 \forall t\) is

\[
C_t = \left[1 - R^{-1} (R \beta)^{1/\rho}\right] (H_t + W_t), \tag{13}
\]

where \(W_t\) is physical wealth and \(H_t\) is human wealth,

\[
H_t = \frac{P_t}{1 - G/R}. \tag{14}
\]
which means that since \( X_t = P_t + W_t \rightarrow x_t = 1 + w_t \) the consumption function in ratio form can be written as

\[
c(x_t) = [1 - R^{-1}(R\beta)^{1/\rho}](h_t + w_t) \quad (15)
\]

\[
= [1 - R^{-1}(R\beta)^{1/\rho}](h_t - 1 + w_t) \quad (16)
\]

\[
= [1 - R^{-1}(R\beta)^{1/\rho}](h_t - 1 + x_t). \quad (17)
\]

### 2.1 The MPC Out Of Transitory Income

Thus, if we give a consumer an extra dollar of wealth \( W_t \) the marginal propensity to consume out of that wealth will be

\[
c'(x) = [1 - R^{-1}(R\beta)^{1/\rho}]. \quad (18)
\]

For example, if we choose \( R = 1, \beta = 0.96, \rho = 2 \) we obtain \( c'(x) = (1 - (1 - 0.04)^{1/2}) \approx (1 - (1 - (1/2)(0.04))) = .02 \forall x \). More generally, for any plausible parameter values, this model implies a very low MPC out of transitory shocks to income.

### 2.2 The MPC Out Of Permanent Income

The natural definition of the MPC out of permanent income in this model is

\[
\frac{dC_t}{dP_t} = \frac{1 - (R\beta)^{1/\rho}/R}{1 - G/R} \quad (19)
\]

which is equal to one if \((R\beta)^{1/\rho} = G\). Note the relationship of this condition to the ‘impatience’ condition which says that consumers are impatient if

\[
(R\beta)^{1/\rho} < G, \quad (20)
\]

because if this condition holds they will desire to spend more than their current incomes. Thus, if consumers are exactly on the border between patience and impatience, the MPC out of permanent income is equal to one. If consumers are impatient, the MPC out of permanent income is greater than one.

### 3 The Buffer Stock Model

#### 3.1 The MPC Out Of Transitory Income

One of the reasons models with precautionary saving have become popular is that, as shown in Kimball (1990) and Carroll and Kimball (1996), precautionary saving boosts the MPC out of transitory income. The amount by which precautionary motives boost the MPC cannot be determined analytically, but numerical solution of the model finds that the MPC out of transitory shocks can be potentially as high as 0.40 for reasonable parameterizations of the model (Carroll 1997).
Table I
Steady-State Results for Alternative Parameter Values

<table>
<thead>
<tr>
<th>Parameter Value</th>
<th>Average Growth Rate of Aggregate Consumption</th>
<th>Average Growth Rate of Household Permanent Income</th>
<th>Average Growth Rate of Household Consumption</th>
<th>Aggregate Personal Saving Rate</th>
<th>Average MPC out of Wealth</th>
<th>Average Net Wealth</th>
<th>Target Net Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>g = .00</td>
<td>0.00</td>
<td>-0.005</td>
<td>-0.005</td>
<td>0.002</td>
<td>0.16</td>
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<td>0.62</td>
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<tr>
<td>g = .02†</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.007</td>
<td>0.33</td>
<td>0.35</td>
<td>0.32</td>
</tr>
<tr>
<td>g = .04</td>
<td>0.04</td>
<td>0.035</td>
<td>0.035</td>
<td>0.011</td>
<td>0.42</td>
<td>0.28</td>
<td>0.25</td>
</tr>
<tr>
<td>δ = .00</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
<td>0.15</td>
<td>0.66</td>
<td>0.61</td>
</tr>
<tr>
<td>δ = .04†</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.007</td>
<td>0.33</td>
<td>0.35</td>
<td>0.32</td>
</tr>
<tr>
<td>δ = .10</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.005</td>
<td>0.46</td>
<td>0.25</td>
<td>0.23</td>
</tr>
<tr>
<td>r = .00†</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.007</td>
<td>0.33</td>
<td>0.35</td>
<td>0.32</td>
</tr>
<tr>
<td>r = .02</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.009</td>
<td>0.26</td>
<td>0.45</td>
<td>0.42</td>
</tr>
<tr>
<td>r = .04</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.014</td>
<td>0.17</td>
<td>0.65</td>
<td>0.61</td>
</tr>
<tr>
<td>ρ = 1</td>
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<td>0.015</td>
<td>0.015</td>
<td>0.003</td>
<td>0.49</td>
<td>0.14</td>
<td>0.11</td>
</tr>
<tr>
<td>ρ = 2†</td>
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<td>0.015</td>
<td>0.015</td>
<td>0.007</td>
<td>0.33</td>
<td>0.35</td>
<td>0.32</td>
</tr>
<tr>
<td>ρ = 5</td>
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<td>0.015</td>
<td>0.015</td>
<td>0.024</td>
<td>0.14</td>
<td>1.13</td>
<td>1.08</td>
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<tr>
<td>σ_{ln N} = .05</td>
<td>0.02</td>
<td>0.019</td>
<td>0.019</td>
<td>0.006</td>
<td>0.38</td>
<td>0.30</td>
<td>0.28</td>
</tr>
<tr>
<td>σ_{ln N} = .10†</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.007</td>
<td>0.33</td>
<td>0.35</td>
<td>0.32</td>
</tr>
<tr>
<td>σ_{ln N} = .15</td>
<td>0.02</td>
<td>0.007</td>
<td>0.007</td>
<td>0.011</td>
<td>0.22</td>
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<tr>
<td>σ_{ln Z} = .05</td>
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<td>0.015</td>
<td>0.015</td>
<td>0.006</td>
<td>0.33</td>
<td>0.32</td>
<td>0.30</td>
</tr>
<tr>
<td>σ_{ln Z} = .10†</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.007</td>
<td>0.33</td>
<td>0.35</td>
<td>0.32</td>
</tr>
<tr>
<td>σ_{ln Z} = .15</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.008</td>
<td>0.32</td>
<td>0.39</td>
<td>0.35</td>
</tr>
<tr>
<td>p = .001</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.004</td>
<td>0.41</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>p = .005†</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.007</td>
<td>0.33</td>
<td>0.35</td>
<td>0.32</td>
</tr>
<tr>
<td>p = .010</td>
<td>0.02</td>
<td>0.015</td>
<td>0.015</td>
<td>0.010</td>
<td>0.30</td>
<td>0.46</td>
<td>0.43</td>
</tr>
</tbody>
</table>

† Designates the base value of the parameter.
3.2 The MPC out of Permanent Income

Because the level of consumption can be rewritten as \( C_t = c(x_t)P_t \) for some invariant function \( c(x) \), the only way the elasticity of consumption with respect to permanent income \( P_t \) in this model can be different from one is if there is a correlation between \( P_t \) and \( x_t \). But of course such a correlation does exist: Equations (6) and (10) indicate that both \( P_t \) and \( x_t \) are influenced by the realization of the stochastic shock to permanent income \( N_t \). Furthermore, both will reflect residual effects of the previous shocks to permanent income, \( N_{t-1}, N_{t-2}, \ldots \). It is these effects of the permanent shocks on the cash-on-hand to permanent-income ratio that will be the key to understanding the results below.

Another important insight is that if the distribution of \( x_t \) is ergodic (which to my knowledge remains an important unproven conjecture, but which both intuition and simulations suggest is true), then eventually the infinite-horizon marginal propensity to consume out of any permanent shock to labor income must be one because ergodicity of \( x_t \) means that the expectation as of time \( t \) of \( x_s \) as \( s \to \infty \) is the same for any current realization of \( N_t \) (and indeed for any past sequence of realizations of \( N_{t-1}, N_{t-2}, \ldots \)), implying that as \( s \to \infty \) the time-\( t \) expectation of \( c(x_s)P_s \) depends only on the level of \( P_t \).

But as noted above, the ‘marginal propensity to consume’ out of a shock has traditionally been defined as the immediate effect, not the total eventual effect, and so we now turn to the question of how consumption is affected in period \( t \) by the contemporaneous realization of the shock to permanent income \( N_t \).

Assuming \( G_t = G \ \forall \ t \), the natural definition of the marginal propensity to consume out of permanent shocks to labor income is the derivative of \( C_{t+1} \) with respect to \( N_{t+1} \), given an initial level of savings \( S_t = s_tP_t \) carried over from the previous period,

\[
\frac{dC_{t+1}}{dN_{t+1}} = \frac{dP_{t+1}c(x_{t+1})}{dN_{t+1}}
\]

\[
= \frac{d}{dN_{t+1}} \left[ GP_tN_{t+1}c\left( \frac{R}{GN_{t+1}}s_t + \epsilon_{t+1} \right) \right].
\]

This equation reveals a minor conceptual difficulty: the effect of \( N_{t+1} \) on \( C_{t+1} \) depends not only on the value of \( s_t \) but also on the realization of \( \epsilon_{t+1} \), and so in principle there are two ‘state variables’ (other than the scaling variable \( P_t \)) which determine the marginal propensity to consume out of permanent income. However, since \( \epsilon_{t+1} \) is an i.i.d. random variable, it is easy and intuitive to calculate
the expectation of (22) as
\[
E_t \left[ \frac{d}{dN_{t+1}} GP_t N_{t+1} c_{t+1} \right] = GP_t E_t \left[ \tilde{N}_{t+1} \frac{dc(\tilde{x}_{t+1})}{d\tilde{N}_{t+1}} + c(\tilde{x}_{t+1}) \right]
\]
(23)
\[
= GP_t E_t \left[ \tilde{N}_{t+1} c'(\tilde{x}_{t+1}) \frac{d}{d\tilde{N}_{t+1}} \left( \frac{R}{G \tilde{N}_{t+1}} s_t + \tilde{c}_{t+1} \right) + c(\tilde{x}_{t+1}) \right]
\]
(24)
\[
= GP_t E_t \left[ c(\tilde{x}_{t+1}) - c'(\tilde{x}_{t+1}) \frac{R}{G \tilde{N}_{t+1}} s_t \right].
\]
(25)

Now the marginal propensity out of a shock to permanent income in period \( t + 1 \) is plausibly defined as the \textit{proportion} of the change in \( C \) to the change in \( P \) caused by the shock. Since the effect of the value of \( N_{t+1} \) on \( P_{t+1} \) is given by \( GP_t N_{t+1} \), it is plausible to define the expected MPC out of permanent shocks as
\[
\chi(s_t) = E_t \left[ c(\tilde{x}_{t+1}) - c'(\tilde{x}_{t+1}) \frac{R}{G \tilde{N}_{t+1}} s_t \right].
\]
(26)
\( \chi(s_t) \) is the expression that I will henceforth refer to as the (expected) marginal propensity to consume out of shocks to permanent labor income.

Finally, it is revealing to relate (26) to the perfect-foresight case,
\[
C_{t+1} = \left[ 1 - R^{-1}(R\beta)^{1/\rho} \right] \left( \frac{G N_{t+1} P_t}{1 - G/R} + W_{t+1} \right)
\]
(27)
\[
c_{t+1} = \left[ 1 - R^{-1}(R\beta)^{1/\rho} \right] \left( \frac{1}{1 - G/R} + w_{t+1} \right)
\]
(28)
\[
= \left[ 1 - R^{-1}(R\beta)^{1/\rho} \right] \left( \frac{1}{1 - G/R} + R \frac{1}{G \tilde{N}_{t+1}} s_t \right)
\]
(29)
so that
\[
c_{t+1} - c_{t+1}' \frac{R}{G \tilde{N}_{t+1}} s_t = \left[ 1 - R^{-1}(R\beta)^{1/\rho} \right] \left( \frac{1}{1 - G/R} + \frac{R}{G \tilde{N}_{t+1}} s_t \right)
\]
(30)
\[
- \left[ 1 - R^{-1}(R\beta)^{1/\rho} \right] \frac{R}{G \tilde{N}_{t+1}} s_t
\]
(31)
\[
= \frac{1 - R^{-1}(R\beta)^{1/\rho}}{1 - G/R},
\]
(32)
so in the perfect foresight case this formula generates the same result as derived earlier in equation (20) (as it should).

Equation (26) by itself gives us some interesting results. The definition of precautionary saving is the amount by which consumption is reduced when uncertainty is introduced. Thus we know that the \( c_{t+1} \) term in equation (26) will be
lower than in the certainty case. Furthermore, Carroll and Kimball prove that precautionary saving increases the value of $c'(x_{t+1})$. Hence this equation tells us that precautionary saving unambiguously reduces $\chi$ - and, furthermore, the amount by which the MPC out of permanent shocks is reduced is directly related to the amount by which the MPC out of transitory shocks is increased.

3.3 The Deaton Case (No Transitory Shocks)

Note first how this expression maps into Deaton’s finding that for consumers who begin with zero savings the marginal propensity to consume out of $P_{t+1}$ is one. Such consumers have $s_t = 0$ and therefore the second term on the RHS in equation (26) drops out. Deaton also assumed that there were no transitory shocks to income, so that $\epsilon_{t+1} \equiv 1$. Finally, his consumers were sufficiently impatient so that their consumption at $c(x)$ was equal to one at $x = 1$. Hence the marginal propensity to consume out of permanent income was given by the expression $\chi(0) = E_t[c(1)] = 1$.

To really understand Deaton’s result, it is necessary to recall why it must be that $c(1) = 1$ Consider the first order condition for the unconstrained optimization problem,

$$c(x_t)^{-\rho} = R\beta E_t[(G\tilde{N}_{t+1})^{-\rho}c(x_{t+1})^{-\rho}].$$

The consumer will be constrained at $c_t = x_t = 1$ iff the marginal utility of consuming 1 is greater than the marginal utility of saving $s_t = 0$, i.e. if

$$1 > R\beta E_t[(G\tilde{N}_{t+1})^{-\rho}c((R/G\tilde{N}_{t+1}) * 0 + 1)^{-\rho}]$$

where the second line follows from the first because with $s_t = 0$ we know that $x_{t+1} = \epsilon_{t+1} = 1 = x_t$. But notice that equation (33) is identical to the impatience condition which we have already imposed, equation (12). Thus in imposing the impatience condition we guarantee Deaton’s result that a consumer with zero savings who experiences only permanent shocks will remain at zero savings forever. Zero savings is an absorbing state.

What Deaton was unable to prove, but conjectured must be true, was that a liquidity-constrained consumer who starts with positive savings will always eventually run down that wealth to reach the absorbing state of zero. Consider the accumulation equation for savings:

$$s_{t+1} = x_{t+1} - c_{t+1} = (R/GN_{t+1})s_t + 1 - c(1 + (R/GN_{t+1})s_t).$$

---

2 The following is intended as a loose intuitive argument rather than a rigorous derivation; in particular it mixes logic from the constrained and unconstrained optimization problem. See Deaton (1991) for the rigorous version.

3 Substitute (10) into (9) and roll forward one period.
Carroll and Kimball (2001) show that the marginal propensity to consume out of transitory income in a problem with liquidity constraints is always greater than the MPC in the unconstrained case. We also know, from combining Kimball (1990) and Carroll and Kimball (1996), that the MPC in the unconstrained case with labor income risk is greater than the MPC without labor income risk. As noted above, the solution to this problem in the unconstrained case with no labor income uncertainty is of the form

\[ c(x_t) = c(h - 1 + x_t) \]

where

\[
\begin{align*}
\xi &= (1 - R^{-1}(R\beta)^{1/\rho}), \\
h &= \frac{1}{1 - G/R}.
\end{align*}
\]

Since we know that \( c(1) = 1 \) in the Deaton model, we know from the chain of MPC inequalities described above that

\[
c(1 + (R/GN_{t+1})s_t) > c(1) + \xi(R/GN_{t+1})s_t = 1 + \xi(R/GN_{t+1})s_t.
\]

Substituting in equation (36),

\[
s_{t+1} < (R/GN_{t+1})s_t - \xi(R/GN_{t+1})s_t < (R/GN_{t+1})s_t(1 - \xi)
\]

From this we have (substituting (37) into (39))

\[
s_{t+1} < (R/GN_{t+1})s_t - \xi(R/GN_{t+1})s_t = (R/GN_{t+1})s_t R^{-1}(R\beta)^{1/\rho} = [(R\beta)^{1/\rho}/G\tilde{N}_{t+1}]s_t
\]

implying

\[
E_t[s_{t+1}] < E_t[(R\beta)^{1/\rho}/G\tilde{N}_{t+1}]s_t.
\]

But note that if \( \tilde{N}_{t+1} \) is lognormally distributed then the impatience condition implies that the expression in brackets on the RHS of equation (40) is less than one, implying

\[
E_t[s_{t+1}] < s_t.
\]

Thus, at any positive level of savings \( s_t > 0 \), savings are expected to fall toward zero. Note that this condition does not guarantee that savings ever reach zero in finite time, because in principle it is possible (though arbitrarily improbable) to draw an arbitrarily long sequence of low draws of \( N_t \). On the other hand, equation (41) does rule out the possibility that Deaton raised (but doubted) that some positive level of savings \( w \) could exist such that if \( s_t > w \) the consumption rule might be such as to never allow savings to fall below \( w \), thus preventing the consumer from ever reaching the absorbing state of \( s_t = 0 \). Hence, in Deaton’s model, savings falls toward zero, and when it reaches zero, the MPC out of permanent shocks is one ever after.
3.4 The General Case (Transitory and Permanent Shocks)

In the real world households experience both transitory and permanent shocks to their incomes. The natural supposition might be that since nothing can be done to insulate consumption in the long run against the permanent shocks, the presence or absence of transitory shocks should not affect the MPC out of permanent shocks. This section shows otherwise.

Consider the behavior of consumption around the ‘target’ level of savings $s^*$ defined as the level of savings such that $E_t[\tilde{s}_{t+1}] = s^*$

$$s_{t+1} = (R/GN_{t+1})s_t + \epsilon_{t+1} - c((R/GN_{t+1})s_t + \epsilon_{t+1})$$  \hspace{1cm} (42)

$$E_t[\tilde{s}_{t+1}] = E_t[(R/G\tilde{N}_{t+1})s_t + 1 - E_t[c((R/G\tilde{N}_{t+1})s_t + \tilde{\epsilon}_{t+1})]$$

$$s^* = s^*E_t[(R/G\tilde{N}_{t+1})] + 1 - E_t[c((R/G\tilde{N}_{t+1})s^* + \tilde{\epsilon}_{t+1})]$$

$$E_t[c((R/G\tilde{N}_{t+1})s^* + \tilde{\epsilon}_{t+1})] = 1 + s^*(E_t[(R/G\tilde{N}_{t+1})] - 1).$$  \hspace{1cm} (43)

Now recall that Carroll and Kimball\cite{Carroll and Kimball 1996} have shown that the marginal propensity to consume under uncertainty is strictly greater than the MPC in the corresponding perfect certainty model, and therefore we know that $c'(x_{t+1}) > \underline{c}$ where as above $\underline{c} = 1 - R^{-1}(R\beta)^{1/\rho}$. Using these facts in the formula for $\chi(s^*)$ gives

$$\chi(s^*) = E_t \left[ c(\tilde{x}_{t+1}) - c'(\tilde{x}_{t+1}) \frac{R}{GN_{t+1}} s_t \mid s_t = s^* \right]$$  \hspace{1cm} (44)

$$< 1 + (E_t[R/G\tilde{N}_{t+1}] - 1)s^* - \underline{c}E_t[R/G\tilde{N}_{t+1}]s^*$$  \hspace{1cm} (45)

$$= 1 + (E_t[R/G\tilde{N}_{t+1}](1 - \underline{c}) - 1)s^*$$  \hspace{1cm} (46)

$$= 1 + (E_t[R/G\tilde{N}_{t+1}](R^{-1}(R\beta)^{1/\rho} - 1)s^*$$  \hspace{1cm} (47)

$$= 1 + (E_t[(R\beta)^{1/\rho}/G\tilde{N}_{t+1}] - 1)s^*$$  \hspace{1cm} (48)

But as noted above, if $\tilde{N}_{t+1}$ is lognormally distributed, the impatience condition \cite{Impatience Condition} implies that $E_t[(R\beta)^{1/\rho}/G\tilde{N}_{t+1}] < 1$ and thus that $\chi(s^*) < 1$ if $s^* > 0$. Hence, at the target level of savings the marginal propensity to consume out of permanent income shocks is strictly less than one.

We can also say something about how $\chi(s_t)$ varies with the level of savings. Its derivative with respect to $s_t$ is given by

$$\left( \frac{d}{ds_t} \right) \chi(s_t) = E_t \left[ c'(\tilde{x}_{t+1})(R/G\tilde{N}_{t+1}) - c'[\tilde{x}_{t+1}](R/G\tilde{N}_{t+1}) - c''[\tilde{x}_{t+1}](R/G\tilde{N}_{t+1})^2 s_t \right]$$

$$= E_t[-c''[\tilde{x}_{t+1}](R/G\tilde{N}_{t+1})^2 s_t].$$  \hspace{1cm} (49)

\cite{Carroll 2004} proves that such a target will exist if consumers satisfy the impatience condition.
But Carroll and Kimball (1996) prove that for problems in the class considered here the consumption function is strictly concave, $c''[x] < 0$, and since $(R/G\tilde{N}_t+1)^2$ is certainly positive, equation (49) implies that the marginal propensity to consume out of permanent shocks is increasing in the level of savings.

These results appear to be the most that can be said analytically about the characteristics of $\chi(s_t)$. To obtain quantitative results it is necessary, as usual with problems of this type, to turn to simulations.

4 Simulation Results

Some of the model’s parameters can be estimated empirically. In particular, Carroll (1992) finds that household-level data from the Panel Study of Income Dynamics are reasonably well characterized by the assumption that the permanent shock $N_t$ is lognormally distributed with standard deviation $\sigma_N = .10$, while the process for transitory income has two parts: With probability $p$, income is zero, and with probability $(1 - p)$ the transitory shock $\epsilon_t$ is equal to $1/(1 - p)$ times the value of a shock drawn from a lognormal distribution with standard deviation $\sigma_\epsilon = .10$ and mean value one, so that $E_t[\tilde{\epsilon}_{t+1}] = 1$ as assumed above. Income growth at the household level is between roughly $G = 1.02$ and $G = 1.03$.

This leaves three parameters to be chosen, $\beta$, $R$, and $\rho$. Although interest rates are of course measurable, it is not clear which asset category in the real world to assume corresponds to the model’s $w$, so in practice the choice of interest rate is unconstrained within a reasonable range (say, $R = 1.00$ (which corresponds to the average real after-tax return on a perfectly riskless asset like 3-month T-bills) to $R = 1.03$ (which corresponds roughly to average returns on corporate debt)).

Many of the above results depended on the impatience condition (12). The most interesting way to organize the simulation results is therefore with respect to how close or far the set of parameter values comes to satisfying the impatience condition. Under the assumption that $N$ is lognormally distributed, the impatience condition can be rewritten as

$$R\beta E_t[(G\tilde{N}_{t+1})^{-\rho}] = R\beta G^{-\rho} \exp(\rho \sigma_N^2/2 + \rho^2 \sigma_N^2/2)$$

$$\approx \left(\frac{R\beta}{1 + \rho g}\right) (1 + \rho \sigma_N^2/2 + \rho^2 \sigma_N^2/2)$$

$$= R\beta \left(\frac{1 + 0.01 * (\rho/2 + \rho^2/2)}{1 + \rho0.02}\right)$$

where the last line holds if $g = 0.02$ and $\sigma_N^2 = 0.01$. If we assume that the coefficient of relative risk aversion $\rho = 3$, a common value in the literature, the
Table 1: Simulation Results

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Mean $s$</th>
<th>Mean $c'(x)$</th>
<th>Mean $\chi(s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.14</td>
<td>0.08</td>
<td>0.91</td>
</tr>
<tr>
<td>0.98</td>
<td>0.77</td>
<td>0.18</td>
<td>0.87</td>
</tr>
<tr>
<td>0.96</td>
<td>0.64</td>
<td>0.24</td>
<td>0.85</td>
</tr>
<tr>
<td>0.94</td>
<td>0.59</td>
<td>0.28</td>
<td>0.84</td>
</tr>
</tbody>
</table>

The solution method is the standard one of backwards iteration from the last period of life. In all cases reported the iteration was continued for 200 periods. Simulation mean values reported in the table are the means across a population of 500 agents at the end of 100 periods of simulation using the converged consumption rule. Results did not change if more agents or more periods were used.

The reader may be surprised that the mean value of $w$ is only 1.14 when $\beta = 1.00$, given that $\beta = 1.00$ is very close to the exact point at which the impatience condition begins to hold. There appear to be two main reasons for this. First, as noted in the preceding footnote, the consumption rule used is not actually the infinite-horizon converged rule, but rather the rule that obtains 200 periods from the end of life; while the difference between 200 periods and an infinite number of periods may seem small, it may be important in this knife-edge case. Second, the numerical solution method naturally makes use of a discrete approximation to the lognormal distributions for $\tilde{N}$ and $\tilde{\epsilon}$. While these approximations are very accurate, they are not perfectly accurate, and in particular the approximation method misses the extremely low probability events of extremely negative shocks to $\tilde{N}$, thereby slightly understating the precautionary saving motive.
would tend to reduce $\chi(s_t)$; on the other hand, we know that the marginal propensity to consume declines with the level of wealth, which would tend to increase $\chi(s_t)$. Apparently these two effects are of roughly offsetting magnitude across different values of $\beta$.

5 The Reaction of Consumption to News About Future Income

5.1 The Perfect Foresight Model

In the lecture on the time series behavior of consumption in the Hall-Flavin model, we defined human wealth $H_t$ as the present discounted value of future income, and concluded that consumption was equal to

$$C_t = \left( \frac{r}{R} \right) (H_t + W_t).$$

Consider, for example, how consumption would change upon receipt of news that the expected growth rate of income has changed from $g = 0.02$ to $g = 0.03$ if the interest rate is $R = 1.04$, assuming current income is $Y_t$:

$$\Delta C_{t+1} = \Delta \left( \frac{r}{R} \right) \left( \frac{Y_t}{0.04 - 0.03} - \frac{Y_t}{0.04 - 0.02} \right)$$

$$= \Delta \left( \frac{r}{R} \right) (100Y_t - 50Y_t)$$

$$\approx \Delta 0.04(50Y_t)$$

so consumption will increase upon receipt of this news by an amount equal to two years’ worth of income.

This formula applies for any time series process for income $Y_t$. Thus when news about future income arrives in this model, all that matters is the magnitude of the change in the expected present discounted value of future income.

5.2 The Buffer Stock Model

Carroll [Carroll (1997)] demonstrates that the buffer-stock model can imply that the reaction of consumption to changes in $H_t$ can be virtually nonexistent if the changes are anticipated to be in the distant future. Figure 1 shows the results graphically.

The figure confirms that the MPC out of changes in expected income that mostly lie in the distant future can be close to zero in the buffer-stock model.

The figure does not answer the question of how much one would expect consumption to change in reaction to news of a positive transitory shock to income that is expected to occur in the immediate future. Consider, for example, the case where a buffer-stock consumer learns in period $t$ that there will be an ‘extra’
positive shock to income of expected size \( v \) next period. Furthermore, assume for simplicity that we are considering a model with no permanent shocks. The corresponding accumulation equation will be

\[
\hat{x}_{t+1} = \left( \frac{R}{G} \right) (x_t - c_t) + \epsilon_{t+1} + v\mu_{t+1} + \epsilon_{t+1} + v\mu_{t+1} (60)
\]

where \( E_t[\mu_{t+1}] = 1 \).

If consumption today did not react at all, a first order Taylor expansion says that the expected change in the level of tomorrow’s consumption can be approximated by

\[
E_t[c(\hat{x}_{t+1} + v\mu_{t+1})] \approx E_t[c(\hat{x}_{t+1}) + c'(\hat{x}_{t+1})v\mu_{t+1}] (61)
\]

\[
= E_t[c(\hat{x}_{t+1})](1 + vE_t[c'(\hat{x}_{t+1})]/E_t[c(\hat{x}_{t+1})]) (62)
\]

\[
= E_t[c(\hat{x}_{t+1})](1 + \gamma). (63)
\]

On the other hand, suppose the consumer increased consumption today by \( (G/R)v \), the full expected present discounted value of the future increase in resources. Then defining \( c_0^t \) as the level that consumption would have taken in the absence of the shock we would have

\[
\hat{x}_{t+1} = (R/G)(x_t - c_0^t) - (G/R)v + \epsilon_{t+1} + v\mu_{t+1} (64)
\]

and obviously \( E_t[\hat{x}_{t+1}] = E_t[\tilde{x}_{t+1}] \). However, although the mean level of cash-on-hand would be the same as if there were no shocks \( v \), the variance of cash-on-hand

Figure III

Figure 1: Reaction to News of an Increase in \( g \) in CEQ and Buffer-Stock Models
would be increased because there are now two shocks, $\mu_{t+1}$ and $\epsilon_{t+1}$ that govern the variation of $x_{t+1}$ around its mean, rather than just $\epsilon_{t+1}$ as before. Hence the variance of $c_{t+1}$ would be higher than before. In other words, spending in anticipation of the higher resources means the consumer enters the period with lower resources and thus must endure a higher degree of uncertainty about the level of future consumption.

Now consider a final possible reaction to the news: Consumption today increases proportionally by the amount $\omega < v(G/R)$ such that the expected proportional increase in current and future consumption is the same,

$$\log c_t^0 + \omega = E_t[\log c_{t+1}^0 + \omega]$$

(65)

where $c_{t+1}^0$ is the level of consumption that would have prevailed in the absence of the $v$ shock.

Now consider the second-order logarithmic approximation to the consumption Euler equation,

$$E_t[\log c_{t+1} - \log c_t] \approx \rho^{-1}(r - \delta) + \left(\frac{\rho}{2}\right) E_t[(\log c_{t+1} - \log c_t)^2],$$

$$\approx \rho^{-1}(r - \delta) + \left(\frac{\rho}{2}\right) \left\{ (E_t[(\log c_{t+1} - \log c_t)])^2 + \text{var}(\log c_{t+1} - \log c_t) \right\}$$

(66)

Now notice that if we define $\hat{c}_t$ such that $\log \hat{c}_t = \log c_t^0 + \omega$ and $\hat{c}_{t+1}$ as the $c$ that results from behaving optimally in period $t+1$ given that you have consumed $\hat{c}_t$ in period $t$, we have that

$$E_t[\log \hat{c}_{t+1} - \log \hat{c}_t] = E_t[\log c_{t+1}^0 - \log c_t^0].$$

(67)

Note that this also implies that $(E_t[(\log \hat{c}_{t+1} - \log \hat{c}_t)])^2 = (E_t[(\log c_{t+1}^0 - \log c_t^0)])^2$. However, there is a problem in making (66) hold for $\hat{c}_t$ and $\hat{c}_{t+1}$: By consuming more in the first period, the policy under consideration implies that the consumer arrives in the second period with fewer resources, so that the variance term in (66) will be higher. However, for an arbitrarily small value for $v$, the increase in the variance term is arbitrarily small. Thus the policy of increasing $c_t$ by $\omega$ comes very close to satisfying the first order conditions.

What can we conclude from this? Basically, that the reaction of consumption in the buffer-stock model to information about immediate and very small future increases in transitory income should be very close to the reaction in the perfect foresight model: Consumption increases immediately by the full amount of its eventual adjustment.

Combining this result with the result from Carroll (1997), we can summarize the reaction of consumption to news about future income as follows. If the information is about a very large, and very distant, increase in future income, consumption will change by only a tiny fraction of the response that would obtain in the perfect foresight model. If the information is about a very small and very near increase in future income, consumption reacts in almost the
same way as it would to a current shock with PDV equal to the PDV of the expected future change in income.

To quantify these results, it is necessary to turn to simulation methods.

6 Excess Smoothness and Excess Sensitivity in a Buffer-Stock Economy

Ludvigson and Michaelides (2001) attempt to examine whether buffer-stock saving can explain the excess smoothness and excess sensitivity of consumption documented by Campbell and Deaton (1989).

Recall our discussion of excess smoothness from the earlier lecture on the time series behavior of aggregate consumption. We showed that if the level of income follows a random walk,

\[ \Delta \log Y_t = \alpha_0 + \epsilon_t \]

then consumption too should follow a random walk,

\[ \Delta \log C_t = \beta_0 + \epsilon_t \]

because the change in income is expected to be permanent and therefore consumption should change by the full amount of the change in income. This implies that the standard deviation of changes in consumption should equal the standard deviation of changes in income,

\[ \sigma_{\Delta C} = \sigma_{\Delta Y}. \]

We also showed that if income growth rates followed an AR(1) process,

\[ \Delta y_t = \alpha_0 + \alpha_1 \Delta y_{t-1} + \epsilon_t, \]

then when one receives the information that income today is higher than expected, the expected eventual total change in income is given by \( 1/(1-\alpha_1) \) and so

\[ \sigma_{\Delta C} = \left( \frac{1}{1-\alpha_1} \right) \sigma_{\Delta Y}. \]

LM estimate time series processes for quarterly and annual wage and salary income in the US, and find that the annual data are well represented by a random walk, while the quarterly data are well captured by an AR(1) with coefficient estimate \( \alpha_1 = .26 \).

The first column of Table 1 from LM presents statistics on the ratio of the standard deviation of consumption growth to the standard deviation of income.
growth for US annual and quarterly data. The table shows that the standard deviation of annual nondurable consumption growth is only 61 percent as large as the standard deviation of wage and salary income growth (recall that if income is a random walk and consumption is determined by the PIH, this statistic should be 1.00). For quarterly data the ratio of standard deviations is 0.68 (recall that if income follows an AR(1) with coefficient 0.22, the PIH model implies that the standard deviation of consumption growth should be roughly \(1/(1-.22) = 1.26\) times as large as the standard deviation of income growth).

The second column of Table 1 presents the coefficient on lagged consumption in a regression of the form

\[
\Delta \log C_{t+1} = \gamma_0 + \gamma_1 \Delta \log Y_t + \epsilon_{t+1}. \tag{73}
\]

Thus, this is a measure of whether there is ‘excess sensitivity’ of consumption growth to lagged income growth. LM find evidence of such excess sensitivity at both the quarterly and the annual frequency.

LM’s goal was to see whether these features of the data can be reproduced by a buffer-stock model. Specifically, they solve a model like the one described above, but where all households experience the same (stochastic) shocks to income growth \(G_t\). Thus, they consider two possibilities for \(G_t\): either \(\log G_t = \alpha_0 + \epsilon_t\) (the annual random walk) or \(\log G_t = \alpha_0 + 0.26 \log G_{t-1} + \epsilon_t\) (the quarterly AR(1) process).

Upon obtaining the solution, LM simulate an economy composed of buffer-stock consumers, and report the results in their Table 2. They find that the buffer-stock model implies a smoothness ratio of 0.99 when aggregate income follows a random walk, virtually unchanged from the implication of the PIH model.

How can we reconcile this result with the earlier finding that the MPC out of permanent shocks is substantially less than one? The answer is that the statistic \(\chi\) only indicated the immediate response of consumption to a shock to permanent income in the first period. Of course eventually consumption must respond fully to the permanent shock, and the statistic that LM calculate is really a measure of the total eventual response rather than the first-period response. Hence their ‘smoothness ratio’ of essentially 1 is not inconsistent with \(\chi = 0.8\).

When aggregate income follows an AR(1) process, LM find that the smoothness ratio for the buffer-stock model is in the range 1.06-1.09 - much smaller than the ratio 1.26 implied by the CEQ PIH model. The reason for this is essentially as follows. Consider a consumer who receives the good news that there has been a positive shock to permanent income. In the PIH model, that consumer would instantly increase his consumption by a factor of 1.26 times as large as the income shock. In a buffer-stock model, however, the consumer may not have enough current wealth to be able to increase his consumption that much. Thus consumption reacts less to the shock than it would in the CEQ PIH model.

Of course, the quarterly data yield a smoothness ratio of only 0.68. Thus
the buffer-stock model closes about a third of the gap between the data and the theory.

The other test that LM examine is of the model’s predictions for excess sensitivity. In principle, a buffer-stock model can explain excess sensitivity: Consumption may not react fully today to news about future income because the consumer may not have a large enough buffer stock of assets to allow consumption to respond fully. Thus part of the response may be delayed until the income is actually in hand.

LM report results on the implications of their buffer-stock model for the smoothness ratio in the second columns of the results in table 2. When aggregate income follows a random walk, the model generates very little excess sensitivity. This is intuitive: when income follows a random walk, you have the full effect of the shock on income immediately, and there is no difficulty about being unable to spend today on the basis of expected further increases in future income, because you do not expect such increases. The more interesting case is when income growth is positively serially correlated. Results are shown in the second panel of the table: The excess sensitivity coefficient is now on the order of 0.06. While this is larger than the zero implied by the PIH theory, excess sensitivity is still notably less than is found in the aggregate data, where excess sensitivity was on the order of 0.15.

After producing these results with a standard version of the buffer-stock model, LM consider a simple modification of the model. They propose that consumers cannot distinguish any of the three kinds of shocks they observe from each other. In other words, all they know in a given period is how much their income changed, and not whether that change was due to an idiosyncratic transitory shock, an idiosyncratic permanent shock, or an aggregate shock (which has whatever time series properties the aggregate shocks are assume to have).

The reason that increasing consumers’ confusion might change the time series characteristics of aggregate consumption is that consumers will systematically misperceive the aggregate shocks as having the average time series characteristics of their income shocks as a whole. Since a substantial portion of household-level shocks are transitory, consumers will react to an aggregate shock as though it too were largely transitory. Thus consumption will react sluggishly to aggregate shocks, generating both excess smoothness and excess sensitivity.

Results for this model are presented in Table 3. The annual model still implies a smoothness ratio of nearly one, so the confusion hypothesis does not help much there. Furthermore, the excess sensitivity parameter, while larger than before, is still considerably below the empirical estimate.

Results are better for the quarterly model. The excess smoothness parameter is now substantially less than one (although still greater than the empirical estimate of 0.68), and the excess sensitivity parameter is actually larger than the empirical counterpart.

The final conclusion of the paper is that both buffer-stock saving and incomplete information can help make the model match the empirical data better.
However, there is still a substantial gap between the even the hybrid theoretical model and the empirical data, so more work is necessary.

7 The Time Series Behavior of Consumption in a Buffer-Stock Economy

The LM tests are not the route I would have taken to explore the implications of the buffer-stock model for the dynamics of aggregate consumption. Instead, I would focus on the model’s predictions for the relationship between consumption growth and predictable income growth.

Recall the Campbell-Mankiw finding that when an equation of the form

\[ \Delta \log C_{t+1} = \alpha_0 + \alpha_1 E_t[\Delta Y_{t+1}] + \epsilon_{t+1} \] (74)

is estimated, the result is generally a value for the coefficient \( \alpha_1 \) of around 0.5. The CEQ PIH model implies that consumption reacts immediately to any information received, so the coefficient on predictable income growth should be zero.

One of the most distinctive predictions of the buffer-stock model, however, is that on average the growth rate of consumption must be equal to the average permanent growth rate of labor income. In the notation above, if \( g = E_t[\log G] \),

\[ \Delta \log C_{t+1} = g. \] (75)

Now consider an economy which switches (at low frequency) between two growth regimes, \( G^H = 1.03 \) and \( G^L = 1.01 \), and suppose that the transition probabilities are small, \( \Pr(S_{t+1} = H - S_t = L) = 0.05 \), \( \Pr(S_{t+1} = L - S_t = H) = 0.05 \). Then roughly speaking, we could expect consumption growth to be about 3 percent while the economy is in the fast-income-growth regime and 1 percent while the economy is in the slow-income-growth regime (if the economy is composed mostly of buffer-stock consumers).

Thus if we were to estimate a Campbell-Mankiw equation

\[ \Delta \log C_{t+1} = \alpha_0 + \alpha_1 E_t[\Delta Y_{t+1}] + \epsilon_{t+1} \] (76)

using an indicator of the aggregate state as our instrument for \( E_t[\Delta Y_{t+1}] \) we should expect to estimate a coefficient around \( \alpha_1 = 1 \) (or at least not much less than one).

However, the logic discussed above indicated that if we learn something today about an immediate, one-off increase in income that is happening soon, consumption reacts to it immediately and almost as much as in the CEQ PIH model. Thus if we were to estimate the C-M equation using instruments that were highly correlated with transitory predictable movements in income, we would expect to estimate a coefficient of \( \alpha_1 \) close to zero.

Devising a procedure to test the differing implications of the model for low and high frequency predictable growth in income would make an excellent paper.
Relative Smoothness and Excess Sensitivity: U.S. Aggregate Data

<table>
<thead>
<tr>
<th>Relative Smoothness</th>
<th>Excess Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Annual Data</td>
<td></td>
</tr>
<tr>
<td>$\Delta C_t / C_{t-1}$</td>
<td>.48</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
</tr>
<tr>
<td>$\Delta C_{t}^{ND} / C_{t}^{ND}$</td>
<td>.61</td>
</tr>
<tr>
<td></td>
<td>(.06)</td>
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<td>$\Delta C_{t}^{S} / C_{t-1}^{S}$</td>
<td>.43</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
</tr>
<tr>
<td>B. Quarterly Data</td>
<td></td>
</tr>
<tr>
<td>$\Delta C_t / C_{t-1}$</td>
<td>.47</td>
</tr>
<tr>
<td></td>
<td>(.04)</td>
</tr>
<tr>
<td>$\Delta C_{t}^{ND} / C_{t}^{ND}$</td>
<td>.68</td>
</tr>
<tr>
<td></td>
<td>(.05)</td>
</tr>
<tr>
<td>$\Delta C_{t}^{S} / C_{t-1}^{S}$</td>
<td>.46</td>
</tr>
<tr>
<td></td>
<td>(.03)</td>
</tr>
</tbody>
</table>

Notes: The sample period begins with the first quarter of 1947 and ends with the fourth quarter of 1997. The column labeled “Relative Smoothness” reports the ratio of the standard deviation of the aggregate consumption growth measure in the row, to the standard deviation of aggregate labor income growth. In parentheses are the standard errors for this ratio, computed by GMM. The column labeled “Excess sensitivity” reports the OLS coefficient of consumption growth on lagged labor income growth. OLS standard errors are in parentheses. The consumption growth measure $\Delta C_t / C_{t-1}$ is growth in real, per capita nondurables and services expenditure less shoes and clothing. $\Delta C_{t}^{ND} / C_{t}^{ND}$ is nondurables expenditure growth less shoes and clothing and $\Delta C_{t}^{S} / C_{t-1}^{S}$ is growth in services expenditure. Labor income is compiled from the NIPA components as wages and salaries plus other labor income minus personal contributions for social insurance minus taxes. Taxes are defined as the fraction of wage and salary income in total income, times personal tax and non tax payments. This measure is also per capita and is deflated by the PCE chain-type price deflator.

Figure 2: Table 1: US NIPA Data
<table>
<thead>
<tr>
<th>Risk Preference</th>
<th>Low Variance</th>
<th>High Variance</th>
<th>High Relative Variance</th>
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<tbody>
<tr>
<td>A. Annual Model, I.I.D. Income growth</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>$\rho = 1$</td>
<td>0.99 0.004 (0.004)</td>
<td>0.99 0.000 (0.009)</td>
<td>0.98 0.000 (0.008)</td>
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<tr>
<td>$\rho = 2$</td>
<td>0.98 0.006 (0.004)</td>
<td>0.98 0.009 (0.010)</td>
<td>0.97 0.012 (0.009)</td>
</tr>
<tr>
<td>PIH</td>
<td>1.00 0.000 (0.004)</td>
<td>1.00 0.000 (0.010)</td>
<td>1.00 0.000 (0.009)</td>
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<tr>
<td>B. Quarterly Model, AR(1) Income growth</td>
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<td></td>
</tr>
<tr>
<td>$\rho = 1$</td>
<td>1.06 0.063 (0.017)</td>
<td>1.06 0.059 (0.019)</td>
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<tr>
<td>$\rho = 2$</td>
<td>1.09 0.055 (0.018)</td>
<td>1.08 0.041 (0.022)</td>
<td>1.07 0.035 (0.024)</td>
</tr>
<tr>
<td>PIH</td>
<td>1.26 0.000 (0.018)</td>
<td>1.26 0.000 (0.022)</td>
<td>1.26 0.000 (0.024)</td>
</tr>
</tbody>
</table>

Notes: The first number in each cell is the mean smoothness ratio (the ratio of the standard deviation of consumption growth to the standard deviation of income growth) over 100 simulations; the second number is the mean excess sensitivity coefficient (the OLS coefficient estimate from a regression of consumption growth on lagged income growth). The standard deviation across simulations for each parameter is given in parentheses. The column labeled “Risk Preference” indicates whether these statistics are reported for a buffer stock model with coefficient of relative risk aversion, $\rho$, or for a representative agent PIH model, where the representative agent receives the aggregate income process. The column labeled “Low Variance” reports these statistics for cases where $\sigma_u = .07, \sigma_n = .05$; the column labeled “High Variance” reports these statistics for cases where $\sigma_u = 0.1, \sigma_n = .08$. The column labeled “High Relative Variance” reports these statistics for cases where the ratio of the standard deviation of the transitory to permanent shock, $\sigma_u/\sigma_n$, is double that of the high variance case. The AR parameter, $\phi$, is set equal to 0.23.

Figure 3: Table 2: Standard Buffer Stock Model
<table>
<thead>
<tr>
<th>Risk Preference</th>
<th>Low Variance</th>
<th>High Variance</th>
<th>High Relative Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Annual Model, I.I.D. Income growth</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho = 1 )</td>
<td>0.96 (0.005)</td>
<td>0.92 (0.011)</td>
<td>0.84 (0.016)</td>
</tr>
<tr>
<td>( \rho = 2 )</td>
<td>0.92 (0.008)</td>
<td>0.85 (0.016)</td>
<td>0.77 (0.022)</td>
</tr>
<tr>
<td>PIH</td>
<td>1.00 0.00</td>
<td>1.00 0.00</td>
<td>1.00 0.00</td>
</tr>
</tbody>
</table>

| B. Quarterly Model, AR(1) Income growth |
| \( \rho = 1 \) | 0.93 (0.014) | 0.91 (0.019) | 0.88 (0.017) |
| \( \rho = 2 \) | 0.91 (0.016) | 0.84 (0.019) | 0.77 (0.017) |
| PIH | 1.26 0.00 | 1.26 0.00 | 1.26 0.00 |

Notes: The first number in each cell is the mean smoothness ratio (the ratio of the standard deviation of consumption growth to the standard deviation of income growth) over 100 simulations; the second number is the mean excess sensitivity coefficient (the OLS coefficient estimate from a regression of consumption growth on lagged income growth). The standard deviation across simulations for each parameter is given in parentheses. The column labeled “Risk Preference” indicates whether these statistics are reported for a buffer stock model with coefficient of relative risk aversion, \( \rho \), or for a representative agent PIH model, where the representative agent receives the aggregate income process. The column labeled “Low Variance” reports these statistics for cases where \( \sigma_u = .07, \sigma_n = .05 \); the column labeled “High Variance” reports these statistics for cases where \( \sigma_u = 0.1, \sigma_n = .08 \). The column labeled “High Relative Variance” reports these statistics for cases where the ratio of the standard deviation of the transitory to permanent shock, \( \sigma_u/\sigma_n \), is double that of the high variance case. The AR parameter, \( \phi \), is set equal to 0.23.

Figure 4: Table 3: Buffer Stock Model with Incomplete Information
References


