Saving and Growth in the Ramsey/Cass-Koopmans Model

In the neoclassical growth model with labor-augmenting technological progress at rate $\gamma$, utility function $u(c) = c^{1-\rho}/(1-\rho)$, time preference rate $\vartheta$ and depreciation rate $\delta$ the steady-state will be at the point where the growth rate of consumption is equal to the growth rate of labor-augmenting technological progress, $\gamma$,

$$\frac{\dot{C}}{C} = \rho^{-1}(f'(k) - \delta - \vartheta)$$

which implies that

$$f'(k) = \rho \gamma + \vartheta + \delta$$
$$\alpha k^{\alpha - 1} = \rho \gamma + \vartheta + \delta$$
$$k = \left[ \frac{\rho \gamma + \vartheta + \delta}{\alpha} \right]^{\frac{1}{1-\alpha}}.$$  

The aggregate gross saving rate is defined as

$$s = \frac{y - c}{y}$$
$$= \frac{k^{\alpha} - c}{k^{\alpha}}$$
$$= 1 - c/k^{\alpha}.$$  

In steady-state by definition

$$\dot{k} = 0$$

but from the capital accumulation equation we know that

$$\dot{k} = k^{\alpha} - (\delta + \gamma)k - c$$

so in steady-state

$$c = k^{\alpha} - (\delta + \gamma)k.$$ 

This can be substituted into (8) to obtain

$$s = 1 - \frac{k^{\alpha} - (\delta + \gamma)k}{k^{\alpha}}$$
$$= (\delta + \gamma)k^{1-\alpha}$$

and the expression for the steady-state level of capital per capita can be substituted in to yield

$$s = (\delta + \gamma) \left( \frac{\rho \gamma + \vartheta + \delta}{\alpha} \right)^{-1}$$
$$= \left( \frac{\alpha(\gamma + \delta)}{\delta + \vartheta + \rho \gamma} \right)$$
The derivative of this expression with respect to $\gamma$ is

$$\frac{ds}{d\gamma} = \frac{\alpha(\delta + \vartheta + \rho \gamma) - \alpha(\gamma + \delta)}{(\delta + \vartheta + \rho \gamma)^2} \quad (16)$$

$$= \frac{\alpha(\delta(1 - \rho) + \vartheta)}{(\delta + \vartheta + \rho \gamma)^2} \quad (17)$$

This will be positive if its numerator is positive, i.e. if

$$\rho\delta < \vartheta + \delta \quad (18)$$

$$\rho < 1 + \vartheta/\delta. \quad (19)$$

A typical assumption is $\vartheta = .04$ and $\delta = .08$, implying that the steady-state relationship between saving and growth in the neoclassical model is positive only if the coefficient of relative risk aversion $\rho$ is less than 1.5. Typically we assume values of $\rho$ in the range from 2 to 5, so the model leads us to expect a negative relationship between saving and growth.