Stock Market Valuation

1 The Level of the Price

The traditional theory of the stock market states that the ‘rational’ price of a share of stock is the present discounted value of the stream of dividends that will be paid to the person owning the share:

\[ P_t = D_t + \frac{D_{t+1}}{R_{t+1}} + \frac{D_{t+2}}{R_{t+1}R_{t+2}} \ldots \]  
\[ = D_t + \frac{G_{t+1}D_t}{R_{t+1}} + \frac{G_{t+1}G_{t+2}D_t}{R_{t+1}R_{t+2}} \ldots \]  
\[ = D_t[1 + (G_{t+1}/R_{t+1}) + (G_{t+1}G_{t+2}/R_{t+1}R_{t+2}) \ldots \]  

If the growth rate of dividends from period to period is constant at \( G \) and the interest rate is constant at \( R \) this becomes

\[ P_t = D_t[1 + (G/R) + (G/R)^2 + (G/R)^3 + \ldots \]  
\[ = D_t \sum_{i=0}^{\infty} (G/R)^i \]  
\[ = D_t \frac{1}{1 - (G/R)}. \]

But if we assume that \( g = G - 1 \) and \( r = R - 1 \) are ‘small’ we know that \( G/R \approx 1 + g - r \) and this expression becomes

\[ \frac{P_t}{D_t} \approx \frac{1}{r - g}. \]

This is known as the “Gordon formula.” The tricky thing in applying the formula is to know what to assume for \( R \) and \( G \). The interest rate \( R \) should be the interest rate ‘appropriate’ for discounting risky quantities. The usual assumption is that \( R = R_f + r_p \) where \( R_f \) is the rate of return on perfectly safe (riskfree) assets and \( r_p \) is the rate-of-return premium that people demand as compensation for the risk inherent in future dividends.

2 The Random Walk

2.1 The Law of Iterated Expectations

Suppose that a security price at time \( t \), \( P_t \), can be written as the rational expectation of some ‘fundamental value’ \( V^* \) conditional on information available at time \( t \) (the usual example of the ‘fundamental value’ in question is the present discounted value of
dividends). Then we have

\[ P_t = \mathbb{E}_t[V^*]. \]  
(8)

(9)

The same formula holds in period \( t + 1 \):

\[ P_{t+1} = \mathbb{E}_{t+1}[V^*]. \]  
(10)

(11)

Then the expectation of the change in the price over the next period is

\[ \mathbb{E}_t[P_{t+1} - P_t] = \mathbb{E}_t[\mathbb{E}_{t+1}[V^*] - \mathbb{E}_t[V^*]] \]  
(12)

\[ = \mathbb{E}_t[V^*] - \mathbb{E}_t[V^*] \]  
(13)

\[ = 0 \]  
(14)

because any information known at time \( t \) must be known at time \( t + 1 \) and so the only thing that should cause a change in prices should be the arrival of new information that was not known at time \( t \).