Problem Set 2

180.616b

1. This problem asks you to solve the two period consumption problem in the presence of uncertainty in *both* the rate of return $\mathcal{R}$ *and* the level of future labor income. Furthermore, you should allow the returns on the two kinds of uncertainty to be correlated with each other.

   A simple method for approximating two correlated but jointly lognormally distributed variables is described in `Equiprobable`. The method approximates the continuous realizations of the stochastic variables into a set of equiprobable bins, with n distinct discrete realizations in each dimension.

   Formally, therefore, your task is to numerically solve the following optimization problem at time $T-1$:

$$\mathrm{v}_{T-1}(m_{T-1}) \;=\; \max_{\mathrm{c}_{T-1}} \left\{ \frac{c_{T-1}^{1-\rho}}{1-\rho} + \mathfrak{v}_{T-1}(m_{T-1} - c_{T-1}) \right\} \tag{1}$$

   where

$$\mathfrak{v}_{T-1}(a_{T-1}) \;=\; \beta_T \left(\frac{1}{n^2}\right) \sum_{i=1}^{n} \sum_{j=1}^{n} \Gamma_T^{1-\rho} \frac{(\mathcal{R}_{i,j} a_{T-1} + \Theta_i)^{1-\rho}}{1-\rho}, \tag{2}$$

   where $(\mathcal{R}_{i,j}, \Theta_i)$ are grid points selected using the method of Equiprobable discretization. You may solve the problem using either `Mathematica` or `Matlab`.

   Your solution should have the following characteristics:

   a) Calibrate the rate of return to have a standard deviation of 0.2, and a mean return of $\mathbb{E}[\mathcal{R}] = 1.04$

   b) In your code, call the variable that governs the correlation between the two kinds of shocks `CorrIncRet`. You should show how your results depend on that coefficient (for example, by showing how the consumption rules differ for various different choices of `CorrIncRet`).

   The core `Mathematica` and `Matlab` code that solves the simpler version of this problem with only a single stochastic variable is called `DoAll.m` in the `SolvingMicroDSOP` code archive (the detailed description is in Section "The Approximate Consumption and Value Functions" in the lecture notes.)

   The programming rules will be similar to those in your 604 class. We will create a 616 folder in the Department common drive; and please upload your solutions (codes, graphs, results, explanations, etc.) and your evaluations to your folder there.

   Full urls for the two resources mentioned above are:

   - `http://econ.jhu.edu/people/ccarroll/SolvingMicroDSOPs.zip`
   - `http://econ.jhu.edu/people/ccarroll/public/lecturenotes/consumption/Equiprobable.zip`

1 Appendix – Detailed Description of Mathematica Archive

The `Mathematica` folder contains several files, and the core among them is called `DoAll.nb`. It proceeds in the following manner:

a) at the beginning it clears everything in the memory, and set the working directory as the place where the `.nb` file is located.

b) `setup_everything.m` loads three .m files. $\ll$ means to ask `Mathematica` kernel to read in a file, evaluate each expression in it, and return the last one.

c) `discreteApprox.m` invokes the discrete approximation function and for $\sigma = 0.1$ plots Figure 1 to show how it works intuitively. Notice that in this example, $\sigma = 0.1$, while in the actual `setup_params.m`, σ is chosen to be 1.

d) With these the three functions at time $T-1$ are ready; and the next is to plot them as in Figure 2-4.

2 Assignment

Table 1 Assignment For PS #02

Person	Language	Evaluated by
Garcia Molina, Daniel	Matlab or Mathematica	Xin, Yi
Hong, Minho	Matlab or Mathematica	Wang, Mingjian
Li, Delong	Matlab or Mathematica	Verrier, Jeanne
Ma, Chang	Matlab or Mathematica	Garcia Molina, Daniel
Nguyen, Tu H	Matlab or Mathematica	Qayyum, Shaiza
Oey, Terrance	Matlab or Mathematica	Hong, Minho
Qayyum, Shaiza	Matlab or Mathematica	Li, Delong
Tigges, David J	Matlab or Mathematica	Ma, Chang
Verrier, Jeanne	Matlab or Mathematica	Nguyen, Tu
Wang, Mingjian	Matlab or Mathematica	Oey, Terrance
Xin, Yi	Matlab or Mathematica	Tigges, David

3 FAQ

a) **Q:** When I look at the Mathematica code for `functions.m`, I notice a Θ`Loop` term that I don't see defined anywhere else. I see that it is used three times in the $\mathfrak{v}$ equation, but I am still confused by how it is used.

A: $\mathfrak{v}$ is a function with the argument being the end of period asset a_t:

$$\mathfrak{v}_{T-1}(a_{T-1}) = \beta\left(\frac{1}{n^2}\right)\sum_{i=1}^{n}\sum_{j=1}^{n}\Gamma_T^{1-\rho}\frac{(\mathcal{R}_{T,j}a_{T-1}+\Theta_i)^{1-\rho}}{1-\rho} \quad (3)$$

$$= \beta\Gamma_T^{1-\rho}\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\frac{1}{n^2}\right)\mathrm{v}_T(\underbrace{\mathcal{R}_{T,j}a_{T-1}+\Theta_i}_{m_T}) \quad (4)$$

$$(5)$$

Implementing this in `Mathamatica` will call the `Sum` function: if you search the help file for `Sum`, you will see that to compute $\sum_{i=1}^{i=i_{max}} f(i)$, the corresponding code is `Sum[f,` $\{i, i_{max}\}$`]`. Here i is a local variable, with the value iterating from 1 to i_{max}; this is different from other parameters whose value need to be defined beforehand. So in our example the code works as follows: (1) for Θ`Loop` = 1, calculate the corresponding m_T, $\mathrm{v}_T(m_T)$, and finally the term $\frac{1}{n}\mathrm{v}_T(m_T)$ which is what $\Theta\text{Prob}[[\Theta\text{Loop}]]v[mtp1, \text{PeriodsToEnd} - 1]$ means; (2) for Θ`Loop` = 2 until Θ`Loop` = `Length`[Θ`Vals`], do the same thing; (3) the `Sum` function will add up those terms; (4) outside the `Sum` function, the summation will be multiplied by $\beta\Gamma_T^{1-\rho}$. This completes the definition of the $\mathfrak{v}_{T-1}(a_{T-1})$ function.

To wrap up, the key is to understand the meaning of `Sum` in Mathematica. Here Θ`Loop` is just a name for an iterator; if you wish, you can rename Θ`Loop` as i generically.

b) **Q:** I was hoping you could clarify what `cInterpFunc =` {♯&} means (it is located in `setup_lastperiod.m`). I tried piecing together what ♯ and & separately, but I don't see how they work together here.

A: This is a special trick in Mathematica; and the reference is by searching for `Function` help file. In a nutshell, you can define a 45 degree line as `f[x_]:=x;`, and an alternative definition is `f =` ♯&. You can check that the two definitions are identical, for example by plotting their shapes. With either definition of the 45 degree line, `cInterpFunc =` $\{f\}$ means designating the 45 degree line as the first element in the `cInterpFunc` which is a `List`. The current version ignores the need to give the 45 degree line function a name, and merges the two steps into one.

References