You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.
Part I: Analytical Question

Saving Rates and Interest Rates. PerfForesightCRRA showed that in a perfect foresight infinite horizon CRRA-utility consumption model, the fact that

$$ c_t = (h_t p_t + b_t) \kappa $$

implies that

$$ c_t \approx \frac{h_t}{1 + \theta} \left[ 1 - \frac{\theta}{\gamma} \right] + (r - \rho^{-1}(r - \theta)) b_t $$

where

$$ h_t = \text{The ratio of human wealth to permanent noncapital income} $$

$$ \gamma = \text{Growth rate of permanent noncapital income} $$

$$ \theta = \rho^{-1}(r - \theta) - \gamma $$

$$ \equiv \text{‘growth impatience rate’} $$

1. Using the fact that the approximate MPC in this model is $\kappa \approx r - \rho^{-1}(r - \theta)$, derive (2) from (1)
2. Explain, in words, why the ‘return impatience condition’ \( \hat{p}_r < 0 \) needs to be imposed in order for the model to have a sensible solution. Explain, in words, why the ‘finite human wealth’ condition, \( \gamma < r \) needs to be imposed. Explain, in words, what imposing the ‘growth impatience condition’ \( \hat{p}_\gamma < 0 \) accomplishes and why it might be desirable to impose that condition in an infinite-horizon model. (Reminder: \( \hat{p}_r \equiv \rho^{-1}(r - \vartheta) - r \).)
3. Use (2) and the fact that the level of saving can be defined as total income minus total consumption:

\[ s_t \approx r a_{t-1} + p_t - c_t \]  \hspace{1cm} (4)

to show that the ratio of saving to permanent labor income \( p_t \) is approximately

\[ s_t \approx \frac{p_t}{r - \gamma} + \rho^{-1}(r - \vartheta)a_{t-1} \]  \hspace{1cm} (5)

and that the ratio of saving to total income (the ‘saving rate’) is

\[ \bar{s}_t = \left( \frac{p_t}{r - \gamma} + \rho^{-1}(r - \vartheta)a_{t-1} \right) / \left( 1 + r a_{t-1} \right) \].  \hspace{1cm} (6)
4. Now use (6) and the following calibration:

\[
\begin{align*}
    r &= 0.08 \tag{7} \\
    \vartheta &= 0.04 \tag{8} \\
    \rho &= 2 \tag{9}
\end{align*}
\]

to calculate the saving rates at the following combinations of \(a_{t-1}\) and \(\gamma\):

<table>
<thead>
<tr>
<th>(a_{t-1})</th>
<th>(\gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.025</td>
</tr>
<tr>
<td>0</td>
<td>0.030</td>
</tr>
<tr>
<td>10</td>
<td>0.025</td>
</tr>
<tr>
<td>10</td>
<td>0.030</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.025</td>
</tr>
<tr>
<td>(\infty)</td>
<td>0.030</td>
</tr>
</tbody>
</table>

and use the results to construct a graph showing the relationship between \(a_{t-1}\) and the saving rate for the range \(a_{t-1} \in [0, 10]\), showing on the same graph the asymptote of the saving rate as \(\lim_{a_{t-1} \to \infty}\).
5. Opinions about the long-term growth rate of income, $\gamma$, are widely divergent today. Some scholars (e.g., Robert Gordon) believe that productivity growth in the U.S. is likely to be slow, perhaps 1 percent, over the next 50 years, while others (e.g., Erik Brynjolfsson) are considerably more optimistic, projecting little if any slowdown from the growth rate of 2.5 percent a year that has characterized the postwar period. Furthermore, beliefs about long-run growth have become considerably more pessimistic in the period since the Great Recession.

a) Explain why these facts cast doubt upon this model as a useful or reliable guide to understanding actual saving choices. Relate this point to the argument of Summers (1981) about the magnitude of the human wealth effect in perfect foresight models.
b) Explain why the saving rate is higher in this model than in the previous one.
c) Compare the degree of sensitivity of the saving rate to the growth rate of income in this model compared to the perfect foresight model. Give the intuition for any differences.
Figure 1  Saving Rate By $a_{t-1}$ in a Buffer Stock Model

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Saving Rate By $a_{t-1}$ in a Buffer Stock Model}
\end{figure}
Part II: Discussion Questions

1. **Secular Stagnation.** In 2013, Larry Summers delivered a widely discussed lecture at which he warned that the U.S. and European economies may face a risk that the next decade will be a period of “secular stagnation” like the one that has gripped Japan over the past 25 years. He proposed that a substantial increase in government spending could be an effective way to respond to this risk.

   a) If the extra spending advocated by Summers were directly in the form of transfers to households with a high marginal propensity to consume, can the argument described above be completely understood as an assertion that the U.S. is now in a state of “dynamic inefficiency?” (Hint: Mention Ricardian Equivalence in your answer).

   b) In fact, Summers did not advocate that the extra spending should all be directed to households who would spend it. He has advocated substantial increases in government investment in infrastructure, education, and other “investment” goods. Explain under what conditions this advice is what would be called for in a dynamically inefficient economy. Explain why this would or would not be an appropriate response.
c) Elsewhere, DeLong and Summers (2012) have argued that when the economy is operating far below full employment, an increase in “aggregate demand” may be able to call forth a substantial increase in aggregate supply. This is essentially a Keynesian story. If DeLong and Summers (2012) are right, how would that relate to your answer to question (b) about dynamic inefficiency?
2. A recent literature stimulated by Bloom (2009) has argued that shocks to uncertainty are a major source of business cycle fluctuations; Carroll, Slacalek, and Sommer (2012) argue that a majority of the increase in the U.S. personal saving rate during the Great Recession reflected increases in labor income uncertainty. Explain, using both math and words, why this argument cannot be investigated using a model in which a representative consumer with quadratic utility makes optimal consumption choices.
3. **Fisherian Separation** (Fisher (1930)). Explain what is meant by the “Fisherian separation” proposition, then explain how relaxing each of the following assumptions in the Fisher model might undermine the proposition:

   a) Liquidity constraints
   b) Uncertainty
   c) Failure to optimize intertemporally
   d) Time inconsistent preferences
References


