You are expected to answer all parts of all questions. If you cannot solve part of a question, do not give up. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.
Part I

Productivity Growth and Dynamic Inefficiency in the OLG Model.

Consider a Diamond (1965) OLG economy like the one in the handout OLGModel, assuming logarithmic utility and a Cobb-Douglas aggregate production function,

\[ Y_t = F(K_t, P_t L_t) \]  

(1)

where \( P_t \) is a measure of labor productivity that grows by

\[ P_{t+1} = G P_t \]  

(2)

from period to period. Assume that population growth is zero (\( \Xi = 1 \); for convenience normalize the population at \( L_\tau = 1 \ \forall \ \tau \)), and assume that productivity growth has occurred at the rate \( g = G - 1 \) forever.

One unit of the quantity \( PL \) is called an ‘efficiency unit’ of labor: It reflects a unit of labor input to the production process.

1. Assume that \( F(K, PL) \) is a Constant Returns to Scale function, and show how to rewrite the capital accumulation equation

\[ K_{t+1} = A_{1,t} \]  

(3)

in per-efficiency-unit terms as

\[ k_{t+1} = a_{1,t}/G \]  

(4)
2. Show that under these assumptions, the process for aggregate $k$ dynamics is

$$k_{t+1} = \left( \frac{(1 - \epsilon) \beta}{G_{t+1}(1 + \beta)} \right) k_t^\epsilon$$

(5)
3. Derive the steady-state level of $k_t$ that the economy achieves if the rate of productivity growth is constant at $G_t = G \forall t$. 
Now suppose that the economy had been growing at this constant rate $G$ since the beginning of time, but all of a sudden at the beginning of period $t$ everybody learns that henceforth and forever more, productivity will grow at a faster rate than before, $\hat{G} > G$.

4. Define the new steady-state as $\tilde{k}$. Will this be larger or smaller than the original steady state $\bar{k}$? *Explain your answer.*
5. Next, use a diagram to show how the $k_{t+1}(k_t)$ curve changes when the new growth rate takes effect, and show the dynamic adjustment process for the capital stock toward its new steady-state, assuming that the economy was at its original steady state leading up to period $t$. 
6. Define an index of aggregate consumption per efficiency unit of labor in period $t$ as $\chi_t = c_{1,t} + c_{2,t}/G$, and derive a formula for the sustainable level of $\chi$ associated with a given level of $k$. 
7. Derive the conditions under which a marginal increase in the productivity growth rate $g$ will result in an increase in the steady-state level of $\chi$, and explain in words why this result holds. (You can leave the term $\partial \bar{k}/\partial g$ unevaluated in your answer, using only what we know about this term from above).
Part II

Dynamic Inefficiency and the Great Recession.

Consider a Diamond (1965) OLG economy like the one in the handout OLGModel, with no population growth and no depreciation, but where technological progress causes wage rates to rise by a factor $G$ from one young generation to the next.

1. On a diagram showing the relationship between the sustainable level of consumption per capita $\chi_t = c_{1,t} + c_{2,t}/G$ and the level of capital per capita $k$, indicate the level of $k$ beyond which the economy is in a state of dynamic inefficiency. Explain why the terminology makes sense: That is, explain why points in the region of dynamic inefficiency are “inefficient” in some “dynamic” sense. Is the economy also inefficient in a static (one-period) sense?
2. Consider the following quote from Brad deLong’s web page in 2011:

There is ... an overwhelming case for borrow-and-spend right now. Why? Because the thirty-year Treasury inflation-indexed security rate at 1.62% per year is lower than the expected long-run growth rate of the real economy right now of close to 3% per year. ... If the economy ever gets itself into a situation in which risk-adjusted long-run interest rates are lower than the risk-adjusted expected long-run growth rate of the economy, it is dynamically inefficient—and government should borrow and spend and keep borrowing and spending until at least it drives long-term interest rates up to and above the risk-adjusted expected long-run growth rate.

Discuss the conditions under which deLong’s argument is a proper application of the theory of dynamic inefficiency in the Diamond (1965) model. Specifically, address these questions:

a) Does the interest rate on riskfree long-term Treasury bonds correspond to the interest rate that is used in the theory? Why or why not?
b) Is deLong right in comparing the real interest rate to the overall growth rate of the economy, rather than (say) per-capita income growth? Defend your answer intuitively and analytically.
Interest rates are endogenous to saving decisions in the OLG model. For the rest of this question, assume instead that interest rates are determined by the global capital market, and can therefore be taken as exogenous to the U.S. economy.

3. The 30 year real interest rate that deLong quotes is very low compared to previous historical experience. Suppose that the period of low interest rates turns out to apply only for one 30-year generation: The returns \( R_{t+1} \) earned between period \( t \) and \( t + 1 \) will be low. Furthermore, suppose that wage growth continues to be \( G \) in every period. Call the generation that is old when the shock hits generation \( t - 1 \) (so that the generation that is young in the shock period is generation \( t \) and so on). Use the generational accounting framework outlined in class to figure out which generation or generations are hurt by the crisis, and why.
4. Now consider a different crisis, in which there is no effect on interest rates, but the pattern of income growth is affected. $G_t = 1 < G$ where $G$ is the normal rate of growth. But between $t$ and $t + 1$, the growth rate is $G^2$ so that the level of income of the generation that is young at $t + 1$ is the same as was expected before the crisis (this may be realistic: after the Great Depression ended, wage growth was fast enough in the subsequent 3 decades to restore U.S. wage rates to their pre-1929 trend). Describe the effects of this crisis on the generational accounts of the various generations.
5. Assuming that the lessons learned from the previous exercises are correct, briefly describe implications about how the recent crisis, which affected both wage growth and interest rates, might differently affect different generations.
References