You are expected to answer all parts of all questions. If you cannot solve part of a question, do not give up. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.
Part I. Long Question.

Buffer Stock Saving and Balance Sheets.

The Great Recession was particularly severe in economies that experienced a larger run-up in household debt prior to the crisis.

The International Monetary Fund (2012), Mian, Rao, and Sufi (2011), and Dynan (2012) have pointed out, respectively for countries, states within the U.S., and for individual households, that those who ran up bigger debts in the period leading up to the Great Recession experienced bigger consumption drops when the Recession hit. (The figure above shows some of the IMF’s evidence). This question asks you to interpret this pattern using a modified version of the tractable buffer stock model of saving, TractableBufferStock.

1. In the modified model, rather than having their income go to zero when they become unemployed (call the period in which they become unemployed period 0), jobless persons instead receive an unemployment benefit proportional to the labor income $\ell_0 W_0$ they would have earned if they had remained employed in period 0. Assume that these benefits are financed by some new source of revenues that does not affect the employed consumer’s budget constraint; foreigners, for example.
a) Call the benefit $N_0 = \eta \ell_0 W_0$, where $0 < \eta < 1$ means that the consumer’s unemployment/retirement benefit is positive but less than the income they earned when employed. If in any period $t$ the consumer ends the period ‘in debt’ $A_t < 0$ assume that a lender who does not receive interest payments of at least $-rA_t$ in period $t + 1$ can seize any amount of the consumer’s income less than or equal to jobless benefits $N_0$ (whether the consumer is employed or unemployed). Explain why even an infinitely risk-averse private lender will be willing to lend an employed consumer any amount less than $H_{-1} \equiv N_0/r$.

**Answer:**

The PDV of the maximum amount the lender can seize is

\[
\mathbb{P}(N_0) = N_0 + \frac{N_0}{R} + \frac{N_0}{R^2} + ... \\
= N_0 (1 + \frac{1}{R} + \frac{1}{R^2} + ...)
\]

\[
= N_0 \left( \frac{R}{R-1} \right)
\]

\[
= N_0 \left( \frac{r}{R} \right),
\]

and this amount is perfectly secure so even an infinitely risk-averse lender will be willing to lend up to this amount. But if the consumer ends period $-1$ with some debt such that $A_{-1} > N_0/r$ then the dynamic budget constraint says that the consumer will arrive in period $0$ with $B_0 = A_{-1} R > R(N_0/r)$ which means that they will owe less than the maximum amount that the lender has the right to seize.

Another way to see this is to realize that the lender will want to guarantee that

\[
rA_t \leq N_0 \quad (1) \\
A_t \leq N_0 / r \quad (2)
\]

if $A_t$ is the amount with which an employed consumer ends period $t = -1$ because this condition guarantees that the consumer can pay the interest that is owed on the loan in perpetuity.

b) Explain the role of the ‘return impatience’ condition $(R\beta)^{1/\rho}/R < 1$ in guaranteeing that the solution to the unemployed consumer’s problem makes sense, in that a consumer who has ended employment with

\[
A_{-1} > -H_{-1} \equiv -N_0/r
\]

will have strictly positive consumption throughout their unemployed life.

**Answer:**

Total consumption in the first period of unemployment will be

\[
C_0 = (B_0 + \mathbb{P}(N_0)) \kappa, \quad (4)
\]

where $(B_0 + \mathbb{P}(N_0)) > 0$ is guaranteed by $A_{-1} > -H_{-1}$.
An approximate form of the marginal propensity to consume is
\[ \kappa \approx r - \rho^{-1}(r - \vartheta) \] (5)
and the role of the Return Impatience Condition (RIC) \((R\beta)^{1/\rho}/R < 1\) is that it guarantees that this is also a positive finite number. Thus, \(C_0\) is positive and finite. Thereafter, consumption grows by factor \((R\beta)^{1/\rho}\) forever, so consumption always remains a positive finite number.

c) Explain why the existence of this unemployment insurance system is equivalent (in its implications for the path of consumption) to a system in which newly unemployed consumers receive a lump sum payment of \(N_0(R/r)\) upon entering their first period of unemployment.

Answer:

In a perfect foresight model with well functioning capital markets, all that matters for the household’s level of consumption is the intertemporal budget constraint. The IBC is the same for a consumer who will receive a stream of payments with perfect certainty, or for a consumer who receives instead the PDV of that stream of payments.

For fuller treatment, see PerfForesightCRRA.

d) Call the equivalent lump sum \(\zeta W\), and suppose that for an employed consumer, labor income grows by factor \(\Gamma\) from year to year. Defining lower-case variables as the upper-case versions divided by \(\ell W\), explain why \(h_{-1} = \zeta = \eta \Gamma/r\).

Answer:

\[
\frac{h_{-1}}{\ell_{-1}} = \frac{H_{-1}}{\ell_{-1}W_{-1}} \quad (6)
\]
\[
= \frac{(N_0/r)/\ell_{-1}W_{-1}}{\ell_{-1}W_{-1}} \quad (7)
\]
\[
= \frac{(\eta \ell_0 W_0/r)/\ell_{-1}W_{-1}}{\ell_{-1}W_{-1}} \quad (8)
\]
\[
= \eta \Gamma/r. \quad (9)
\]

e) Explain why the effect of the introduction of such a system is simply to shift the consumption function in the phase diagram to the left by the amount \(h = \eta/r\). Draw an example of such a leftward shift that is large enough so that the target ratio of bank balances \(b\) is negative (so that in steady state the consumer will be in debt).

Answer:
shows such a shift, which occurs because the availability of the future benefits relaxes the ‘natural borrowing constraint’ of the consumer (the maximum amount that the consumer would voluntarily choose to borrow). The natural borrowing constraint causes the consumption function to intersect the horizontal axis at \(-h\) because a consumer with \(b > -h\) can guarantee that his consumption will be positive forever, but a consumer with \(b \leq -h\) cannot guarantee positive future consumption.

f) Explain why an expansion of unemployment benefits corresponds to a relaxation of a ‘natural borrowing constraint.’

Answer:
The ‘natural borrowing constraint’ defines an upper bound on the amount that the consumer will ever wish to borrow (subject to the constraint that the consumer must satisfy his intertemporal budget constraint with certainty). The consumer is impatient (by assumption) and so if the benefits available to be borrowed against rise, the consumer desires to take advantage of the increased borrowing capacity that lenders know now exists because of the increased resources the consumer will have in retirement. The expansion of unemployment benefits therefore gives the consumer a greater ‘natural’ borrowing capacity.

2. Using the model, for each of the experiments below, show how the phase diagram changes, and show the path of the personal saving rate of debtors leading up to, during, and after the experiment.

Experiments:

a) Suddenly and without warning, consumers become more optimistic about the probability becoming unemployed: They believe there has been a permanent
improvement in the functioning of the labor market so that the probability of unemployment \( \bar{U} \) will be lower forever. This period of optimism lasts for six years, and then suddenly reverses itself (unemployment expectations revert to their previous value).

**Answer:**

This change reduces the intensity of the precautionary saving motive. The effect of such a reduction is to twist the consumption function counterclockwise, and to reduce the target level of wealth. Because the slope of the \( \Delta m = 0 \) locus is determined by the growth rate and the growth rate changes in order to keep human wealth constant, the target does not decline as much as it would have because of the upward rotation of the \( \Delta m = 0 \) locus. But for the parameter values considered in class, the result of a reduction in unemployment risk is a decrease in target wealth and saving.

The saving rate drops sharply upon the reduction in unemployment risk, and gradually rises as the consumer’s wealth gets drawn down.
b) Suddenly and without warning, consumers become more optimistic about future income growth: They believe there has been a permanent improvement in $\Gamma$. This period of optimism lasts for six years, and then suddenly reverses itself (growth expectations revert to their previous value).

*Answer:*

The increase in the growth rate effectively makes the consumers more impatient, causing a counterclockwise rotation of the consumption function in a manner similar to that of the decrease in unemployment risk.

The path of the saving rate is given in red in the figure.

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c) Suddenly and without warning, the government announces a permanent increase in the generosity of the unemployment insurance system (specifically: $\eta$ goes from $\underline{\eta}$ before period $t$ to $\bar{\eta} > \eta$ in period $t$). Everyone believes this
change is permanent. For six years, the change in the program persists. Then, without warning, the government reduces the generosity of the unemployment system back to its original level ($\eta$ goes back down to $\eta$).

**Answer:**

The expansion in UI generosity shifts the phase diagram further to the left, as in the figure you were asked to draw for the introduction of the UI system above.

The path of the saving rate is similar to the path for the previous experiment.
Part II. Short Question.

**Friedman (1957)’s Permanent Income Hypothesis With Noisy $c$.** Answer the following questions under the assumption that Friedman’s Permanent Income Hypothesis $c_i = p_i$ is true. Koijen, van Nieuwerburgh, and Vestman (2013) present household spending data from two different sources (“survey” – designated below by an “s” superscript) and “imputed” (designated below by an “m” superscript) which can be assumed to have independent measurement errors $\epsilon_s^i \perp \epsilon_m^i$:

\[
\begin{align*}
  c_s^i & = c_i + \epsilon_s^i \\
  c_m^i & = c_i + \epsilon_m^i
\end{align*}
\]

1. Suppose that the errors from the imputed measure of spending were $\epsilon_m^i = 0 \ \forall \ i$. That is, the imputation manages to capture the true level of spending perfectly. On the other hand, the errors from the survey have variance $\sigma^2_s$. Explain what coefficients you would expect to get from the following two regressions:

\[
\begin{align*}
  c_s^i & = \alpha_0 + \alpha_1 c_m^i + u_i \\
  c_m^i & = \gamma_0 + \gamma_1 c_s^i + v_i
\end{align*}
\]

if $\sigma^2_s = \sigma^2_s$.  

*Answer:*
If $\mathbf{c}_i = \mathbf{c}_i$ and $\mathbf{c}_i = \mathbf{c}_i + \epsilon_i$ then the “true” version of (12) is

$$\mathbf{c}_i = 0 + 1 \times \epsilon_i + u_i$$

(14)

where $u_i$ is an iid variable. So the estimated coefficient should be $\alpha_1 = 1$.

Now rewrite (13) as

$$\mathbf{c}_i = \gamma_0 + \gamma_1(\mathbf{c}_i + \epsilon_i) + v_i$$

(15)

The Friedman errors-in-variables formula tells us that if we estimate (13) we will obtain

$$\gamma_1 = \left( \frac{\sigma^2_c}{\sigma^2_c + \sigma^2_{\epsilon_i}} \right)$$

(16)

but if $\sigma^2_c = \sigma^2_{\epsilon_i}$ this reduces to $\gamma_1 = 1/2$.

2. The authors compare their results to those from a similar Danish study that also has data from similar “survey” and “imputed” sources. They find that when they run a regression like (13) their $\gamma_1$ coefficient is lower than the corresponding coefficient estimated in Danish data. What hypothesis about the relative variances of $\epsilon_i$ might explain these results? What other hypothesis might explain these results even if the variances of the $\epsilon_i$ variables are the same across Denmark and Sweden? How would you expect the results

Answer:

A natural hypothesis is that the Swedish survey data have larger errors than the Danish ones. This would bias downward the estimate of $\gamma_1$ from the Swedish versus the Danish data. The authors present some evidence that the Swedish survey data are surprisingly inaccurate (though they do not have similar metrics for the Danish survey data so whether the Swedish data are actually worse than the Danish data cannot be answered using their results.)

Another natural hypothesis is that maybe the variance in $\mathbf{c}$ is different in Denmark and Sweden. If, for example, the Danes have much larger differences in permanent income than do the Swedes (perhaps the welfare state is more generous in Sweden, pushing everyone’s permanent income closer together), then even for the same size of survey measurement error, the relative size of that survey error compared to the differences in permanent income would be larger in Sweden.

3. Separately, the authors present persuasive evidence that suggests that their estimate of $\mathbf{c}_i$ is substantially better than the estimate of $\mathbf{c}_i$ from the Danish data (that is, their value of $\sigma^2_{\epsilon_m}$ is smaller than the $\sigma^2_{\epsilon_m}$ in the Danish data). If this is true, what differences might you expect in the coefficient on $\alpha_1$ if the authors were to estimate that equation on their data?

Answer:
The same formula used before yields
\[ \alpha_1 = \left( \frac{\sigma_c^2}{\sigma_c^2 + \sigma_m^2} \right) \]  \hspace{1cm} (17)

and since \( \sigma_c^2 \) is by assumption the same in the two datasets, the implication of lower measurement error in the imputed data is that \( \alpha_1 \) should be larger.
References


