Final Exam
180.604
Spring, 2012

You are expected to answer all parts of all questions. If you cannot solve part of a question, do not give up. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.
Part I. Short Discussion Questions.

1. **Risk Aversion of the Value Function.** Consider an infinite-horizon consumption optimization problem under uncertainty for a consumer solving

\[ v(m_t) = \max_{c_t} u(c_t) + \beta \mathbb{E}_t[v(m_{t+1})] \]  \hspace{1cm} (1)

where the utility function is \( u(\bullet) = (1 - \rho)^{-1} \bullet^{1-\rho} \).

a) Show that relative risk aversion of the value function is

\[ \left( \frac{-v''(m_t)c_t}{v'(m_t)} \right) = \rho c'(m_t) \]  \hspace{1cm} (2)

b) Explain why this definition suggests that consumers with lower market resource ratios \( m_t \) can be expected to be willing to spend more of their budgets on actuarially fair insurance. Explain the intuition carefully.
c) Discuss consequences of this point for an individual’s portfolio choice between risky and safe assets after big negative shock to wealth. Then discuss the implications of this result if there is a big negative shock to aggregate wealth that hits all consumers the same way. In particular, discuss whether, in a typical asset-pricing model, the general equilibrium outcome when everybody’s preferences change in the same way is likely to cause a move in asset prices that will amplify the original shock or whether the asset-pricing effect is more likely to dampen the original shock.
2. **Lingering Uncertainty and Slow Recoveries.** Reinhart and Rogoff (2009) present evidence that the recovery from recessions caused by financial crises tends to be slower than recoveries from other kinds of recessions. One idea about why this might be is that uncertainty remains high long after the acute phase of the financial crisis is over. Discuss whether Reinhart and Rogoff (2009)’s results could be explained using a tractable buffer stock model by a permanent increase in $\bar{\sigma}$, without any further assumptions about the structure of the economy and without any consideration of pricing of risky assets, capital market imperfections, costs of adjustment to investment or the capital stock, or other modifications to the production side of the economy.
Part II. Medium Length Questions.

1. Dynamics of Investment in Response to a Temporary ITC in the $\varphi$ Model.

Answer the following questions using an Abel (1981)-Hayashi (1982) $\varphi$ model of investment.

a) Leading up to date $t$, the economy is in steady state. At date $t$, the government unexpectedly introduces a permanent increase in the investment tax credit, $\zeta \uparrow$. Show the effects on a phase diagram and show dynamics of investment, capital, share prices, and $\varphi$ following the tax change.

b) Leading up to date $t$, the economy is in steady state. At date $t$, the government unexpectedly introduces a temporary increase in the investment tax credit, $\zeta \uparrow$. The low ITC will last for two years, and then the ITC will revert back to its normal level. Show the effects on a phase diagram and show dynamics of investment, capital, share prices, and $\varphi$, and the capital stock under two scenarios: (1) costs of adjustment for the capital stock, $\omega$, are high; (2) costs of adjustment are low.

c) Leading up to date $t$, the economy is in steady state. At date $t$, the government unexpectedly announces that in two years (that is, in year $t + 2$), there will be a permanent increase in the investment tax credit, $\zeta \uparrow$. Show and explain the effects on a phase diagram and show dynamics of investment, capital, share prices, and $\varphi$, and the capital stock under two scenarios: (1) costs of adjustment for the capital stock, $\omega$, are high; (2) costs of adjustment are low.

d) Explain why the logic of the examples you just went through helps understand why, whenever a member of Congress introduces a bill to increase the investment tax credit, that bill is always ‘retroactive.’ That is, if the ITC change ever passes, it will apply to investments made during the period between the introduction of the bill and its passage into law.

2. Saving and Growth Redux. Consider a Ramsey/Cass-Koopmans growth model with labor-augmenting technological progress at rate $g$ and suppose there is no population growth so that we can normalize the labor force to be $L_t = 1 \forall t$. Under standard assumptions (like Cobb-Douglas production) and standard notation (such as a capital share of $\alpha$), such a model can be normalized by the level of labor productivity, and if we assume for simplicity that there is no depreciation, the normalized problem becomes

$$\max \int_0^\infty \left( \frac{c_t^{1-\rho}}{1-\rho} \right) e^{-\vartheta t} dt$$  \hspace{1cm} (3)$$

s.t.
\[ \dot{k}_t = y_t - c_t - gk_t \]
\[ y_t = k_t^\alpha \]

where \( \dot{\vartheta} = \vartheta - g(1 - \rho) \) with time preference \( \vartheta \) and RRA \( \rho \). In such a model, it can be shown that the steady-state \( k \) is

\[ \tilde{k} = \left( \frac{\alpha}{\rho g + \vartheta} \right)^{\frac{1}{1-\alpha}} \]

and the aggregate saving rate is defined as

\[ s = \frac{y - c}{y} = 1 - c/y. \]

a) Show that the steady-state saving rate is

\[ \tilde{s} = g\tilde{k}^{1-a}, \]

and determine whether the steady-state saving rate increases or decreases when the growth rate increases.
b) In economic terms, explain the various considerations that are at work, and which might be likely to be strongest.

c) Using a phase diagram, analyze the effects of an unexpected permanent increase in $g$. Next, make a graph showing the path of the aggregate saving rate over time after the economy switches into the fast-growth regime. *Explain the reason both graphs look the way they do in intuitive terms.* Discuss what determines whether the saving rate rises or falls in the instant when consumers learn that growth has increased.
3. Beliefs, Preferences, and Choices.

Malmendier and Nagel (2011) present evidence that the experience of living through the Great Depression had a lasting, perhaps lifelong, effect on the behavior of persons who were young at the time. Specifically, they show that “Depression Babies” (DB’s, henceforth), interpreted (say) to include anyone between the ages of 5 and 25 during the Depression, tend to be less willing to allocate their portfolios to “risky” assets than people born at other times (non-DB’s, henceforth).

This question asks you to think about the implications of their research for the consequences of the Great Recession.

a) Write down the Merton (1969)-Samuelson (1969) formula derived in CRRA-RateRisk that determines the portfolio share of risky assets that will be chosen by an optimizing consumer with CRRA utility and no other risk. (Use \( c \) for the proportion of the portfolio invested in the risky asset, \( \rho \) for the coefficient of relative risk aversion, \( \phi \) for the magnitude of the risky return premium, and \( \sigma_r^2 \) for the expected variance of risky return.) If you cannot remember the formula exactly, make a guess about its form).
b) Suppose that surveys of households can reveal people’s expectations about the rate of return on the stock market (interpreted here as “the risky asset”). But suppose it is impossible to directly measure an individual’s risk aversion or perception of the market’s riskiness. Does the theory suggest that there is any way to use portfolio choices to distinguish between the following two ideas: (1) DB’s avoid risky assets because they have high risk aversion; (2) DB’s avoid risky assets because they perceive the stock market to be “riskier” than other (non-DB) people perceive it to be.
c) Now suppose that individuals’ relative risk aversion coefficients could be estimated in some other way. For example, suppose that everyone who smokes has risk aversion of 2, while all nonsmokers have risk aversion of 5. Describe how information like whether someone is a smoker or not might be useful in distinguishing the two hypotheses above (still assuming that individuals’ expected returns can be discovered by asking them survey questions).
Part III. Long Question.

Rational Inattention and Consumption Dynamics.

Reis (2006) considers the problem of a consumer who faces costs of obtaining or processing the information necessary to decide how much to consume. In the case of a continuous-time consumer with CARA utility $u(c) = -(1/\alpha)e^{-\alpha c}$ who faces an information acquisition cost $K$, Reis shows that it will be optimal to adjust consumption only at fixed intervals of length $d$. If the consumer’s time preference rate is equal to the interest rate, consumption will remain constant between these adjustment dates. Designating the level of consumption as a function of wealth $o$, the information cost as $K$, and the variance of shocks to permanent income as $\sigma^2$, Reis shows that at dates of adjustment

$$c(o; K) = c(o; 0) - \left( \frac{rK}{e^{rd} - 1} \right) - \left( \frac{\alpha r \sigma^2}{4} \right) (e^{rd} - 1)$$

(6)

and that the length of the intervals of inattentiveness (that is, the intervals during which consumption does not adjust to new information) is

$$d = \left( \frac{1}{r} \right) \log \left( 1 + \sqrt{\frac{4K}{\alpha \sigma^2}} \right)$$

(7)

1. Equation (6) says that consumption for the inattentive consumer is lower than for the consumer with zero costs of obtaining information. Provide an interpretation for each of the two reasons consumption is lower, with a discussion of why the term takes the form it does. Hint: $K$ is a real monetary expenditure that the consumer must pay in each period.
2. Give an intuitive explanation for the sign of the effects of $r$, $K$, $\alpha$, and $\sigma$ in (7).
3. Now consider an entire economy populated by inattentive consumers of this kind. Reis shows that if consumers’ decision dates are randomly distributed in the population, and the maximum length of an inattentiveness interval is $D$, then

$$
E_{t-D}[C_{t+1} - C_t] = \text{constant.}
$$

(8)

Provide an intuition for this equation, and explain how it relates to the Hall (1978) model. Contrast this result with the predictions of a model of sticky expectations or habit formation.
4. Reis shows that even a small cost of obtaining information can produce long intervals of optimal inattention. What is the key intuition for why the cost of remaining inattentive is likely to be small if consumption is set optimally during the brief instants of attention?
5. Suppose there are two kinds of macroeconomic events: “newsworthy” events (like the end of a hyperinflation, or the start of the 1991 Gulf war) and “boring” events, which do not attract much coverage in the news media. Suppose that the cost of observing “newsworthy” events is zero because there is pervasive news media coverage of those events, but the cost of observing “boring” events is \( K \) as in Reis’s model. Discuss how the predictions of the models of intattentive consumers would differ from the predictions of the habit formation model with respect to the reaction of consumption to “newsworthy” versus “boring” events.
References


