Final Exam
180.604
Spring, 2011
Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, do not give up. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.
Part I. Short Discussion Questions.

1. **Capital Market Imperfections and Slow Recoveries.** Reinhart and Rogoff (2009) present evidence that the recovery from recessions caused by financial crises tends to be slower than recoveries from other kinds of recessions. Use the model of capital market imperfections presented in class to discuss some reasons why this might be so (under the assumption that ‘other kinds of recessions’ mostly are caused by temporary factors that do not impede the efficient functioning of capital markets).

   **Answer:**

   There is no single ‘correct’ answer to this question; good answers included those that explained why the kinds of factors captured in that model (like a breakdown in the ability of financial intermediaries to assess the likelihood of default) might be long-lasting rather than transitory.

2. **Effects of a Positive Productivity Shock in an RBC Model.**

   The rate of productivity growth in the United States during the two-year period from late 2008 to late 2010 was much higher than historical norms, though most forecasters by late 2010 were expecting that the rate of productivity growth would subside to more normal levels in 2011 and thereafter. Describe the effects that such a ‘positive productivity shock’ should have had on the total amount of labor hours during this period in a Real Business Cycle model of the type advocated by Prescott (1986). Does this match well with the actual experience of 2008-2010?

   **Answer:**

   In the Prescott model, a transitory positive productivity shock causes a boost to wages which induces people to work harder in order to take advantage of the temporarily high wage rate. Thus, the Prescott model would have implied robust real wage growth accompanied by a boom in labor hours during the 2008-2010 period. To put it mildly, this is not what happened.
3. **Stimulative Effects of Transitory Versus Permanent Tax Cuts.** Consider a buffer stock model like the one presented in class but with a permanent ‘payroll’ wage tax $\tau$ so that the employed consumer’s income is wage income times the untaxed proportion $\tau$. This model relates the ratio of consumption to permanent income, $c = C/(\tau W)$, to the ratio of market resources to permanent income $m = M/(\tau W)$. Use this model to rank the relative magnitudes of the effects on consumption you would expect from each of the following policies: (1) A transfer of size $X$ targeted to people with large values of $m$ (so the person receives income of $\tau W + X$); (2) a permanent tax cut targeted to people with large values of $m$; (3) a transitory transfer (as described above) targeted to people with small values of $m$; (4) a permanent tax cut targeted to people with small values of $m$. (Your answer will be graded not just on whether you rank the four options correctly, but also on how well you explain why you obtain the answer you do. A diagram or two may be helpful in explaining your reasoning).

*Answer:*

The model implies that the marginal propensity to consume out of permanent changes in income will be fairly high (close to one) for both rich and poor households. However, it implies a much higher MPC out of transitory shocks for poor than for rich households; this is an implication of the concavity of the consumption function.
Part II. Medium Length Questions.

1. Effects of a ‘Global Saving Glut’ (Bernanke (2005)) on Housing.

Consider a small open economy in which firms and individuals can borrow and lend money risklessly according to the global interest factor $R$.

All goods are produced in perfectly competitive markets with no costs of adjustment, except for one: Housing. Housing is built by real estate developers, but each developer faces costs of adjustment. Suppose all houses are rented from the developers (nobody owns their own home). The rental market is perfectly competitive, with a long-run production function

$$ y_t = h_t F(H_t) $$

where lower case variables indicate the value of an atomistic individual developer’s stock of housing, while upper-case variables refer to the economy-wide housing stock. (Assume there is a mass one of real estate developers). Assume that the aggregate function takes the form $F(H) = H^\alpha$ for some $\alpha < 1$.

a) Suppose interest rates have been constant in this economy for a long time and the housing market has settled down to its long-run equilibrium at date $t$. Explain why (in the absence of any taxes) that equilibrium will be

$$(r + \delta) = F'(\hat{H})$$

Answer:

This model is just a pure relabeling of the $\phi$ model, where the variable $H$ has been substituted for $K$. The equation in question is the equilibrium point of the $\phi$ model where $\hat{K} = \hat{H}$ and taxes are zero.

b) Now suppose that as of date $t$ a ‘global saving glut’ arrives on the world economy, permanently driving world interest rates down to $r$ which is below the original $r$. Draw the appropriate phase diagram both for the individual firm and for the housing industry as a whole. Then draw diagrams showing how the path of housing prices for this economy over time.

Answer:

Results are depicted in the figure.

So, the instant interest rates drop, house prices jump up, but then they gradually decline over time back to their original level.

At the level of an individual perfectly competitive firm, the initial equilibrium is one in which the $\dot{\lambda} = 0$ and $\dot{h} = 0$ loci are on top of each other at $\lambda = 1$. The drop in interest rates causes $\lambda$ to shift vertically upward so that it is a horizontal line above the $\dot{h} = 0$ locus; whatever the firm’s initial level of $h$, it will now be expanding $h$. However, as the economy-wide amount of the housing stock $H$
expands, the $\dot{\lambda} = 0$ locus shifts down gradually over time, eventually settling back down at the $\lambda = 1$ level.

c) Ben Bernanke argued in 2005 that low world interest rates did in fact reflect a global saving glut. If he believed that, and also thought that the model sketched above was a good description of how the housing market works, what would he have predicted for the path of house prices over time?

Answer:

He would have believed that the decline in interest rates would produce a temporary housing price boom and that after that housing prices would gradually fall back down to their equilibrium value over time. ($\lambda$ here is the equivalent of the price of a house, so the upward shift in the $\lambda$ saddle path means prices jump up then gradually readjust downward.)

d) In years following Bernanke’s ‘global savings glut’ speech, the financial sector in the U.S. and elsewhere lent vast amounts of money to home buyers, based on a financial analysis that assumed that ‘housing prices never fall.’ Can you think of any variant of a model like the one here which might justify an assumption of permanently higher prices (you are permitted to assume interest rates remain low forever, if necessary).

Answer:

The only idea discussed in class that could be invoked to tell such a story was the one that assumed that the firms have some monopoly power; in that case we learned that share prices do not necessarily reflect the marginal value of an additional unit of capital. The most obvious thought in this context is to point out that the supply of land in desirable locations is fixed, so the prices of houses built on those desirable spots could be permanently higher than the prices determined just by the cost of building (which is what the $\varphi$ model says is the ultimate equilibrium price of a unit of capital).

e) In response to the collapse of house prices over the past couple of years, the Congress passed some temporary tax credits (the ‘new homebuyer’s tax credit’ being the most prominent) with the goal of propping up house prices. What does economic theory suggest about the short- and long-term consequences of temporary tax credits of this kind.

Answer:

The analysis of such a credit is identical to the analysis of a temporary investment tax credit in $\varphi$Model.

2. Infrastructure and Growth.

Standard growth models ignore the role of a country’s public ‘infrastructure’ (roads, bridges, sewers, other public facilities) in determining a country’s level
of income per capita. Yet looking across countries, it seems clear that countries
with well-designed and well-maintained infrastructure are more prosperous than
countries with crumbling, deficient, or nonexistent infrastructure.
Suppose we can capture the long-run effect of government infrastructure expendi-
tures $e$ in the per-capita production function:
\begin{equation}
    f(k, e) = k^\alpha e^\eta
\end{equation}
where a country with more infrastructure spending has a higher value of $e$. (As-
sume the population and the level of productivity are normalized at 1, and $\eta < 1$).
Suppose government infrastructure expenditures translate one-for-one into produc-
tive efficiency $e$, and assume that the government must satisfy a balanced budget
criterion by the use of lump-sum taxes of amount $\tau$:
\begin{equation}
    e = \tau.
\end{equation}
For simplicity, suppose that the capital stock is exogenously fixed at $k = \bar{k}$ and does
not depreciate but cannot be augmented by extra saving (there is an endowment
of capital).
a) Calculate the level of taxes that maximizes per-capita after-tax income
$f(k, e) - \tau$ and explain intuitively the reasons for the effects that the
parameters have on the optimal choice of government expenditures.
Answer:
\begin{equation}
    \max_e \bar{k}^\alpha e^\eta - e
\end{equation}
has FOC with respect to $e$ of
\begin{equation}
    \bar{k}^\alpha \eta e^{\eta-1} = 1
\end{equation}
\begin{equation}
    e = (\bar{k}^\alpha \eta)^{1/(1-\eta)}
\end{equation}
which says that expenditures/taxes will be higher when 1) the capital
stock is higher (because there is more productivity to “enhance” by
government expenditures; 2) when the coefficient on capital is higher ($\alpha$ is larger), for the same reasons; 3) $\eta$ is larger, because the larger is
$\eta$ the smaller is the rate at which government efficiency improvements
have diminishing marginal productivity effects.
b) Now suppose this economy suffers from corruption. Specifically, some of
the tax revenues that are raised do not get spent on efficient government
expenditures but instead are wasted. Again using $e$ for the amount of efficient
expenditures, and again imposing the balanced budget constraint, the new
level of after-tax income is
\begin{equation}
    f(\bar{k}, e) - \tau
\end{equation}
where $\chi > 1$ measures the degree of corruption. Thus, taxes paid $\tau$ exceed expenditures $e$ (the extra taxes represent waste and corruption). Now calculate the level of $e$ that maximizes after-tax per capita output. Is it higher or lower than in the honest economy (where $\chi = 1$)? Why? Is there a cost to the economy beyond the fact that the tax burden is higher by amount $\chi$? Why?

Answer:
The FOC are
\[
\bar{k}^\alpha \eta e^{\eta - 1} = \chi \tag{9}
\]
\[
e = \left(\frac{\bar{k}^\alpha \eta}{\chi}\right)^{1/(1-\eta)} \tag{10}
\]
and since $\chi > 1$ this is clearly a smaller number than the $e$ that was optimal for the honest economy. Notice that after-tax income is lower for two reasons: 1) with a lower $e$ the economy produces less output; 2) with a higher $\chi$ the effective tax rate is higher. So pretax income is less while taxes are higher.

c) Hall and Jones (1999) find that, looking across countries in the world, only a very small proportion of the differences in output per capita are explained by differences in private capital, natural resources, or other private factors of production. Discuss how this finding might be related to the modeling choice above to assume a fixed level of capital $\bar{k}$. Speculate on whether permitting private capital accumulation would be likely to reinforce, undermine, or leave unchanged the results from the baseline model. If the theory sketched above about the effectiveness of infrastructure investment were true, discuss what effect it would have had for Hall and Jones to include some measure of public capital (as well as private capital) in their model.

Answer:
The Hall and Jones (1999) finding suggests that private capital accumulation is not one of the main influences that make some countries rich and others poor, so an intensive and complex study of optimal intertemporal allocation decisions may not yield much fundamental insight about the process of economic growth.

Permitting private capital accumulation would likely reinforce but not change the logic outlined above; in the more efficient economy, the incentives to save (returns on capital) would be higher, and therefore it is likely that there would be more saving.

If the theory were true, we should expect to see measures of infrastructure per capita having a high degree of explanatory power for measures of income per capita.

Part III. Long Question.
An Entrepreneur’s Problem.
Consider a firm that wants to maximize the present discounted value of profits after subtracting off costs of investment.

\[ k_t \] - Firm’s capital stock at the beginning of period \( t \)
\[ f(k) \] - The firm’s total output (depends only on \( k \))
\[ \tau \] - Tax rate on corporate earnings
\[ \tau' = 1 - \tau \] - Portion of earnings untaxed
\[ \pi_t = f(k_t)\tau' \] - After tax revenues
\[ i_t \] - Investment in period \( t \)
\[ j(i, k) \] - Adjustment costs associated with investment \( i \) given capital \( k \)
\[ \zeta \] - Investment tax credit (ITC)
\[ P = \zeta' = 1 - \zeta \] - Cost of 1 unit of (price-1) investment after ITC
\[ \xi_t = (i_t + j_t)\zeta' \] - After-tax expenditures (purchases plus adjustment costs) on investment
\[ \beta = 1/R \] - Discount factor for future profits (inverse of interest factor)

The firm’s goal is to pick the sequence of values of \( i_t \) that solves:

\[
e(k_t) = \max_{\{i_t\}^\infty} \sum_{n=0}^{\infty} \beta^n (\pi_{t+n} - \xi_{t+n})
\] (11)

subject to the transition equation for capital,

\[
k_{t+1} = (k_t + i_t)\gamma
\] (12)

where \( \gamma = (1 - \delta) \) is the amount of capital left after one period of depreciation at rate \( \delta \).\(^1\) \( e_t \) is the value of the profit-maximizing firm: If capital markets are efficient this is the equity value that the firm would command if somebody wanted to buy it.

\(^1\)There are some small differences between the formulation of the model here and in qModel. Here, investment costs are paid at the time of investment and the depreciation factor applies to \( (k_t + i_t) \) rather than just \( k_t \). These changes simplify the computational solution without changing any key results.
1. Show that the Bellman equation for the firm can be derived as follows:

\[ e_t(k_t) = \max_{\{i_t\}} \pi_t - \xi_t + \beta e_{t+1} ((k_t + i_t) \gamma) \]  \hspace{1cm} (13)

\textbf{Answer:}

\[ e_t(k_t) = \max_{\{i_t\}} \sum_{n=0}^{\infty} \beta^n (\pi_{t+n} - \xi_{t+n}) \]

\[ = \max_{\{i_t\}} \pi_t - \xi_t + \beta \left[ \max_{\{i_{t+1}\}} \sum_{n=0}^{\infty} \beta^n (\pi_{t+1+n} - \xi_{t+1+n}) \right] \]

\[ = \max_{\{i_t\}} \pi_t - \xi_t + \beta e_{t+1} ((k_t + i_t) \gamma) \]
Define $j_t^i$ as the derivative of adjustment costs with respect to the level of investment.

2. Show that the first order condition for optimal investment implies

$$\tau \beta e_{t+1}^k(k_{t+1})$$

and provide a verbal interpretation of the condition.

**Answer:**

$$0 = -\tau \beta e_{t+1}^k(k_{t+1})$$

$$\tau \beta e_{t+1}^k(k_{t+1})$$

In words: The marginal after-tax cost of an additional unit of investment (the LHS) should be equal to the discounted marginal value of the resulting extra capital (the RHS).

3. Now use the Envelope theorem to derive the Euler equation for investment

$$\tau \beta \left[ e_{t+1}^k + (1 + j_t^i - j_t^{i+1}) \right]$$

**Answer:**

The Envelope theorem says

$$e_t^k(k_t) = \tau \beta f_k(k_t) - \tau \beta e_{t+1}^k(k_{t+1}) - \tau \beta e_{t+1}^k(k_{t+1})$$

$$\beta e_{t+1}^k(k_{t+1})$$

So the corresponding $t+1$ equation can be substituted into (14) to obtain

$$\tau \beta \left[ e_{t+1}^k + (1 + j_t^i - j_t^{i+1}) \right]$$

which is the Euler equation for investment.

Now suppose that a steady state exists in which the capital stock is at its optimal level and is not adjusting, so costs of adjustment are zero: $j_t = j_{t+1} = j_t^i = j_t^{i+1} = j_t^k = j_{t+1}^k = 0$.

4. Use the investment Euler equation to show that the steady state level of the capital stock $k$ satisfies the equation below, and provide an intuitive interpretation of the equation.

$$\tau \beta \left( \tau + \tau f^k(k) \right)$$

**Answer:**
If $j_t^i = j^i_{t+1} = j^k_{t+1}$ then (17) reduces to

$$P = R^{-1} \beta [\mathcal{H}(k) + P] \quad (21)$$

$$PR = \tau(P + \mathcal{H}(k)) \quad (22)$$

so that the capital stock is equal to the value that causes its after-tax marginal product to match the interest factor, after compensating for depreciation.

Another way to analyze this problem is in terms of the marginal value of capital, \( \lambda_t \equiv e^f_k(k_t) \).

5. Show that in the vicinity of the steady state, assuming that adjustment costs are approximately zero, the equation for \( \lambda \) will be

$$\lambda_t = \frac{\mathcal{H}(k_t) - Pj^k_t + \Delta \lambda_{t+1}}{(1 - \beta \tau)} \quad (23)$$

and use this equation along with the transition equation for capital to draw a phase diagram in \((k, \lambda)\) space for this model. Be sure to explain why the \( \Delta \lambda_{t+1} = 0 \) locus is downward sloping in the vicinity of the steady state.

**Answer:**

Rewrite (18) as

$$\lambda_t = \frac{\mathcal{H}(k_t) - Pj^k_t + \beta \tau (\lambda_t + \lambda_{t+1} - \lambda_t)}{(1 - \beta \tau)} \quad (24)$$

$$= \frac{\mathcal{H}(k_t) - Pj^k_t + \beta \tau (\lambda_t + \Delta \lambda_{t+1})}{(1 - \beta \tau)} \quad (25)$$

$$(1 - \beta \tau)\lambda_t = \mathcal{H}(k_t) - Pj^k_t + \Delta \lambda_{t+1} \quad (26)$$

$$\lambda_t = \frac{\mathcal{H}(k_t) - Pj^k_t + \Delta \lambda_{t+1}}{(1 - \beta \tau)} \quad (27)$$

and the phase diagram is constructed using the \( \Delta \lambda_{t+1} = 0 \) locus. In the vicinity of the steady state, we can assume \( j^k_t \approx 0 \) in which case the \( \Delta \lambda_{t+1} = 0 \) locus becomes

$$\lambda_t = \frac{\mathcal{H}(k_t)}{(1 - \beta \tau)} \quad (28)$$

which implies (since \( f^k(k_t) \) is downward sloping in \( k_t \)) that the \( \Delta \lambda_t = 0 \) locus (that is, the \( \lambda_t(k_t) \) function that corresponds to \( \Delta \lambda_t = 0 \)) is downward sloping.

The phase diagram is depicted in figure 2.

The steady state of the model will be the point at which \( k_{t+1} = k_t = \hat{k} \), implying from (12) a steady-state investment rate of

$$\hat{k} = (\hat{k} + \hat{i}) \tau \quad (29)$$

$$\hat{i} = (1 - \tau)\hat{k} / \tau = (\delta / \tau)\hat{k} \quad (30)$$
and solving (22) for $\mathcal{F}^k(\hat{k})$

$$
\left( \frac{\mathcal{P}(1 - \beta\tau)}{\beta} \right) = \mathcal{F}^k(\hat{k})
$$

(31)

which can be substituted into (28) to obtain the steady-state value of $\lambda$:

$$
\hat{\lambda} = \left( \frac{\mathcal{R}\mathcal{P}}{1 - \beta} \right).
$$

(32)

We now wish to modify the problem in two ways. First, we have been assuming that the firm has only physical capital, and no financial assets. Second, we have been assuming that the people running the firm only care about the level of profits; suppose instead we want to assume that they must live off the dividends of the firm, and thus they are maximizing the discounted sum of utility from dividends $u(c_t)$ rather than just the level of discounted profits. (Note that we designate dividends by $c_t$; dividends were not explicitly chosen in the $\varphi$-model version of the problem, because the Modigliani-Miller theorem says that the firm’s value is unaffected by its dividend policy).

We call the maximizer running this firm the ‘entrepreneur.’ The entrepreneur’s level of monetary assets $m_t$ evolves according to

$$
m_{t+1} = \pi_{t+1} + (m_t - \xi_t - c_t) R.
$$

(33)

That is, next period the firm’s money is next period’s profits plus the return factor on the money at the beginning of this period, minus this period’s investment and associated adjustment costs, minus dividends paid out (which, having been paid out, are no longer part of the firm’s money).

6. Show that the entrepreneur’s Bellman equation can now be written

$$
v_t(k_t, m_t) = \max_{\{i, c_t\}} \left( u(c_t) + \beta v_{t+1}(k_{t+1}, m_{t+1}) \right)
$$

Answer:

Value is simply the discounted sum of utility from future dividends:

$$
v_t(k_t, m_t) = \max_{\{i, c_t\}} \sum_{n=0}^{\infty} \beta^n u(c_{t+n})
$$

$$
= \max_{\{i, c_t\}} \left( u(c_t) + \beta \sum_{n=0}^{\infty} \beta^n u(c_{t+1+n}) \right)
$$

$$
= \max_{\{i, c_t\}} \left( u(c_t) + \beta v_{t+1}(k_{t+1}, m_{t+1}) \right).
$$

Assume that $f$ and $j$ do not depend directly on $m_t$. That is, their partial derivatives with respect to $m_t$ are zero.
7. Use the first order condition with respect to dividends and the Envelope theorem with respect to money to show that the Euler equation for dividends is
\[ u'(c_t) = R \beta u'(c_{t+1}). \] (34)

Answer:

FOC wrt \( c_t \):
\[ u'(c_t) = R \beta v_{t+1}^m \] (35)

Envelope wrt \( m_t \):
\[ v_{t+1}^m = R \beta v_{t+1}^m \] (36)

and combining the FOC with the Envelope theorem we get the usual
\[ v_{t+1}^m = R \beta v_{t+1}^m \]
\[ = u'(c_{t+1}) \]
\[ = R \beta u'(c_{t+1}) \]
\[ = u'(c_{t+1}) \]

where the last line follows because we have assumed \( R \beta = 1 \).

8. Next explain why the value function can be rewritten as
\[ v_t(k_t, m_t) = \max_{\{i_t, m_{t+1}\}} u((\pi_{t+1} - m_{t+1})/R + m_t - \xi_t) + \beta v_{t+1}(k_{t+1}, m_{t+1}) \] (37)

Answer:

This holds because maximizing with respect to \( m_{t+1} \) (subject to the accumulation equation) is equivalent to maximizing with respect to the components of \( m_{t+1} \).
9. Now show that for the version in (37) the FOC with respect to $i_t$ is

$$u'(c_t)(P(1 + j^i_t) - \mathcal{F}_{t+1}^{k} \gamma/R) = \gamma \beta v_{t+1}^k$$ (38)

**Answer:**

This holds because the derivative of the RHS of (37) with respect to $i_t$ is

$$u'(c_t) \left( \frac{\partial \mathcal{F}_{t+1} \partial k_{t+1}}{\partial i_t} \right) / R - \frac{\partial P_{j_t}}{\partial i_t} + \beta \left( \frac{\partial k_{t+1}}{\partial i_t} \right) v_{t+1}^k(k_{t+1}, m_{t+1})$$ (39)

(remember that $m_{t+1}$ is a control variable and thus its derivative with respect to investment is zero) so the FOC translates to

$$u'(c_t)(\mathcal{F}_{t+1}^{k} \gamma/R - \mathcal{P}_{j_t}) + \beta \gamma v_{t+1}^k = 0$$ (40)

which reduces to (38).

10. Now use the envelope theorem with respect to $k_t$ to show that

$$v_t^k = u'(c_t)(\mathcal{F}_{t+1}^{k} \gamma/R - \mathcal{P}_{j_t}) + \beta \gamma v_{t+1}^k$$ (41)

**Answer:**

This can be seen by directly taking the derivative of the RHS of (37) with respect to $k_t$:

$$u'(c_t) \left( \frac{\partial \mathcal{F}_{t+1} \partial k_{t+1}}{\partial k_t} \right) / R - \frac{\partial \mathcal{P}_{j_t}}{\partial k_t} + \beta \left( \frac{\partial k_{t+1}}{\partial k_t} \right) v_{t+1}^k$$ (42)

and noting that the Envelope theorem tells us the derivatives with respect to the controls $m_{t+1}$ and $i_t$ are zero while $\partial k_{t+1}/\partial k_t = \gamma$.

11. Next show how to combine (38) and (41) to derive the Euler equation for investment

$$P(1 + j^i_t) = \gamma \beta \left[ \mathcal{F}_t^{k} (k_{t+1}) + P(1 + j^i_{t+1} - j^k_{t+1}) \right].$$ (43)

**Answer:**

To see this, start with the Envelope theorem,

$$v_t^k = u'(c_t)(\mathcal{F}_{t+1}^{k} \gamma/R - \mathcal{P}_{j_t}) + \beta \gamma v_{t+1}^k \text{ from (38)}$$ (44)

$$= u'(c_t)(\mathcal{F}_{t+1}^{k} \gamma/R - \mathcal{P}_{j_t}) + u'(c_t)(P(1 + j^i_t) - \mathcal{F}_{t+1}^{k} \gamma/R)$$ (45)

$$= u'(c_t)P(1 + j^i_t - j^k_t)$$ (46)

which means that we can rewrite (38) substituting the rolled-forward version of (46)

$$u'(c_t)(P(1 + j^i_t) - \mathcal{F}_{t+1}^{k} \gamma/R) = \gamma \beta v_{t+1}^k$$
\[ \beta u'(c_{t+1})\mathbb{P}(1 + j_{t+1}^i - j_{t+1}^k) \]
\[ \mathbb{P}(1 + j_{t}^i) = \beta [\mathbb{R}^k(k_{t+1}) + \mathbb{P}(1 + j_{t+1}^i - j_{t+1}^k)] \]

where the last line follows because with \( R\beta = 1 \) we know that \( c_{t+1} = c_t \)

implying \( u'(c_{t+1}) = u'(c_t) \).

12. Comment on the fact that the Euler equation for investment for the firm being run by a utility-maximizing manager, (43), is identical to the Euler equation for the profit maximizing manager, (17), to discuss whether it matters, in this model, whether managers maximize profits or utility. Similarly comment on whether there would be any evidence from consumption dynamics that the consumer was running a business with costly capital adjustment.

**Answer:**

Since behavior (for either a firm manager or a consumer) is determined by Euler equations, and the Euler equations for both consumption and investment are identical in this model to the Euler equations for the standard models, there is no observable consequence for investment of the fact that the firm is being run by a utility maximizer, and there is no observable consequence for consumption of the fact that the consumer owns a business enterprise with costly capital adjustment.

13. Now consider a firm of this kind that happens to have arrived in period \( t \) with positive monetary assets \( m_t > 0 \) and with capital equal to the steady-state target value \( k_t = \hat{k} \).

Suppose that an executive steals all the firm’s monetary assets and disappears.

Use the investment and consumption Euler equations to show the consequences for monetary assets, capital, dividends, and investment subsequently.

**Answer:**

The consequences for the firm are depicted in figure 3.

Dividends follow a random walk. Thus, there is a one-time downward adjustment to the level of dividends to reflect the stolen money. Thereafter dividends are constant, as are monetary assets (which are constant at zero forever).

The theft of the money has no effect on investment or the capital stock, because the firm’s investment decisions are made on the basis of whether they are profitable and the theft of the money has no effect on the profitability of investments.

14. Now consider another kind of shock: The firm’s main building gets hit by a meteor, destroying some of the firm’s capital stock. Again show dynamics of monetary assets, capital, consumption, and investment.

**Answer:**
The results are depicted in figure 4.

Again, because dividends follow a random walk, what the firm’s managers do is to assess the effect of the meteor shock on the firm’s total value and they adjust the level of dividends downward immediately to the sustainable new level of dividends. Thereafter there is no change in the level of dividends.

Investment is more complicated. The firm’s capital stock is obviously reduced below its steady-state value by the meteor, so there must be a period of high investment expenditures to bring capital back toward its steady state. However, the firm started out with monetary assets of zero. Therefore the high initial investment expenditures will be paid for by borrowing, driving the firm’s monetary assets to a permanent negative value (the firm goes into debt to pay for its rebuilding). Gradually over time the capital stock is rebuilt back to its target level, and investment expenditures return to zero (or the level consistent with replacing depreciated capital).
Figure 1 Decrease in $R$: phase diagrams with saddle paths (dashed-black and continuous-red lines respectively pre and post the $R$ change) and impulse response functions
Figure 2 Phase Diagram

Figure 3 Negative shock to $m_t$
Figure 4  Negative shock to $k_t$
References


