Cointegration and the Dynamics of Consumption, Income, and Wealth

Consider the intertemporal budget constraint for an infinite-horizon representative agent’s consumption problem,

\[ PDV_t(C) = K_t + H_t \]

\[ = W_t \]

(1)

(2)

where \( K_t \) is the consumer’s beginning-of-period stock of physical assets, \( H_t \) is human wealth,

\[ H_t = PDV_t(Y), \]

(3)

and \( W_t \) is total wealth, human and nonhuman.

If there is a single riskless interest rate in the economy at time \( t+1, R_{t+1} \), we can define the dynamic budget constraint for total wealth as

\[ W_{t+1} = R_{t+1}[W_t - C_t]. \]

(4)

Campbell and Mankiw (?) show that if interest rates are stationary the intertemporal budget constraint implies that

\[ c_t - w_t \approx \rho(r_{t+1} - \Delta c_{t+1}) + \rho(\rho(r_{t+2} - \Delta c_{t+2}) + \rho(c_{t+2} - w_{t+2}) + \rho k) + \rho k \]

\[ = \sum_{j=1}^{\infty} \rho^j(r_{t+j} - \Delta c_{t+j}) + \rho k/(1-\rho) \]

(5)

where lower-case variables represent the logs of their upper-case equivalents and the constants \( k, \mu, \) and \( \rho \) are given by

\[ \rho = 1 - \exp(1-x) \]

(6)

\[ k = \log\rho - (1-1/\rho) \log\rho \]

(7)

\[ \mu = \sigma \log \beta \]

(8)

where \( x \) is the steady-state ratio of consumption to wealth that emerges if the interest rate is constant at \( \log R = r \).

It is hard to come up with empirical tests of the CM model, however, because human wealth \( H_t \) is unobserved and therefore total wealth \( W_t \) (and its log \( w_t \)) are unobserved. Lettau and Ludvigson (?, ?) propose the following solution to this problem. Suppose we designate the growth rate of labor income from period
to $t + 1$ as $G_{t+1}$. Then human wealth can be written

$$H_t = Y_t + Y_{t+1}/R_{t+1} + Y_{t+2}/R_{t+1}R_{t+2} \ldots$$

(9)

$$= Y_t + Y_t G_{t+1}/R_{t+1} + Y_t (G_{t+1} G_{t+2})/(R_{t+1} R_{t+2}) + \ldots$$

(10)

$$= Y_t [1 + G_{t+1}/R_{t+1} + (G_{t+1}/R_{t+1})(G_{t+2}/R_{t+2}) + \ldots]$$

(11)

$$H_{t+1} = Y_{t+1} [1 + G_{t+2}/R_{t+2} + (G_{t+2}/R_{t+2})(G_{t+3}/R_{t+3}) + \ldots]$$

(12)

$$= Y_t G_{t+1} [G_{t+1}/R_{t+1} + (G_{t+1}/R_{t+1})(G_{t+2}/R_{t+2}) + \ldots] R_{t+1}/G_{t+1}$$

(13)

$$= R_{t+1} Y_t [1 + G_{t+1}/R_{t+1} + \ldots - 1]$$

(14)

$$H_{t+1} = R_{t+1} [H_t - Y_t].$$

(15)

But note that the form of (15) is identical to the form of (4). As a result, the same steps Campbell and Mankiw (1989) used to derive (5) can be applied to generate

$$y_t - h_t = \sum_{j=1}^{\infty} \rho^j (r_{t+j} - \Delta y_{t+j}) + \rho k/(1 - \rho),$$

(16)

which can be rewritten

$$h_t = y_t - \sum_{j=1}^{\infty} \rho^j (r_{t+j} - \Delta y_{t+j}) + \rho k/(1 - \rho)$$

(17)

$$= y_t - \sum_{j=1}^{\infty} \rho^j r_{t+j}^y + \rho k/(1 - \rho).$$

(18)

where for notational simplification define the net discount rate for income as $r_{t+j}^y = r_t - \Delta y_t$.

Next, LL assume that the income growth process is stationary, so that $\lim_{j \to \infty} E_t[\Delta y_{t+j}] = g$, in which case (18) can be written

$$h_t = \kappa + y_t + z_t$$

(19)

where $z_t$ is a stationary variable. Note, however, that the stationarity of $\Delta \log y_t$ is a very strong assumption; in particular, it rules out changes in the trend of productivity growth, which we know have been quite important over the postwar period; in the US, labor productivity growth averaged about 3 percent a year from 1947 to 1973, then dropped to only about 0.9 percent a year from 1973 to 1995. Since 1995 labor productivity growth has been about 2 percent a year. So the assumption that the labor income growth process is stationary is a very questionable assumption when actually applied to US postwar data.

Note that all of these results have been derived under the assumption of perfect foresight about future interest rates and growth factors for income. If either or both of these factors is stochastic, everything is more complex.

To proceed, let’s suppose that consumers behave as though their forecasts at time $t$ reflect perfect certainty. In this case the human wealth accumulation
A mixture of the return on the portion of wealth invested in physical assets identical. Hence, the ‘return’ on human wealth is simply its growth rate, adjusted for any changes in the expected discounted growth rate of future income between periods.

LL define a new rate-of-return concept corresponding to the ‘rate of return on human wealth’ that they effectively define as

$$R_{t+1}^h = \left( \frac{H_{t+1} - Y_{t+1}}{H_t - Y_t} \right). \quad (20)$$

We can work out what this will be determined by:

$$H_{t+1} - Y_{t+1} = Y_{t+1}E_{t+1}\left[ R_{t+2}^y + R_{t+2}^yR_{t+3}^y + \ldots \right]$$

$$H_t - Y_t = Y_tE_t\left[ R_{t+1}^y + R_{t+1}^yR_{t+2}^y + \ldots \right] \quad (21)$$

$$\left( \frac{H_{t+1} - Y_{t+1}}{H_t - Y_t} \right) = \frac{G_{t+1}}{E_{t+1}\left[ G_{t+1}/R_{t+1}^y \right] + 1} \quad (22)$$

To understand this expression better, consider the case where $G$ and $R$ are constant over time. Then we obtain

$$R_{t+1}^h = G, \quad (26)$$

because $Y_{t+1} = GY_t$ and the remaining terms on the RHS of (21) and (22) are identical. Hence, the ‘return’ on human wealth is simply its growth rate, adjusted for any changes in the expected discounted growth rate of future income between periods $t$ and $t+1$. This latter component also captures the ‘human wealth effect.’

The next step is to define a rate of return on total wealth, which will be a mixture of the return on the portion of wealth invested in physical assets $A_t + Y_t - C_t$ and the portion invested in human wealth $H_t - Y_t$. If we define the portion of available period-t resources that is not consumed as savings $S_t$ we have

$$W_{t+1} = \bar{R}_{t+1}S_t = R_{t+1}(A_t + Y_t - C_t) + R_{t+1}^h (H_t - Y_t) \quad (27)$$

$$\approx S_t^h \quad \approx S_t^h \quad (28)$$
LL now define the portfolio share of savings $\omega_t = (A_t + Y_t - C_t)/S_t$ invested in physical assets, and the portfolio share of human assets as $(1 - \omega_t)$, allowing them to rewrite (28) as

$$W_{t+1} = S_t(R_{t+1}\omega_t + R_{t+1}^h(1 - \omega_t)). \quad (29)$$

Finally, LL make a critical assumption: that $\omega_t$ is a stationary variable (that is, the ratio of physical wealth to human wealth is neither rising nor falling over time). Under this assumption they derive the equation

$$c_t - \omega a_t - (1 - \omega)h_t = E_t \sum_{i=1}^{\infty} \rho^i \left\{ \left[ \omega r_{t+i} + (1 - \omega)r_{t+i}^h \right] - \Delta c_{t+i} \right\} \quad (30)$$

upon which all of their subsequent analysis is based, on the assumption that the term on the RHS is stationary.

Unfortunately, however, the theory provides no reason for $\omega_t$ to be a stationary variable. To see this consider simple PIH version of the optimal consumption model with $\sigma = 0$. In that case the model reduces to the classic Hall (?) framework in which the level of consumption follows a random walk. Since $C$ is proportional to total wealth $W$, it is apparent that the level of total wealth must also follow a random walk. But we know that income (and therefore human wealth) trends upward at rate $G$ in this model. If $H$ is rising and $W$ is constant, it must be the case that $\omega_t$ has been falling.