Final Exam
180.604
Spring, 2000
Answers

This exam consists of two parts. The first part contains two long questions, each of which has multiple subparts. The second part is a set of three shorter discussion-type questions. READ THE QUESTIONS CAREFULLY BEFORE STARTING TO ANSWER THEM. Finally, when I ask you to explain something I mean it! You will not get full credit if you only provide math and no explanation. Please write your answers on the exam itself; if you run out of room, use the back of the preceding page.

Part I.
Question A
Consider the following Ramsey/Cass-Koopmans growth model with labor-augmenting technological progress at rate $g$:

$$\max \int_0^{\infty} u(C_t)e^{-\theta t} dt \quad (1)$$

subject to:

$$\dot{K}_t = Y_t - C_t - \delta K_t \quad (2)$$

$$Y_t = K_t^\alpha (Z_t L_t)^{1-\alpha}$$

$$\dot{Z}_t = g$$

$$u(C) = \frac{C^{1-\rho}}{1-\rho}$$

and suppose there is no population growth so that we can normalize the labor force to be $L_t = 1 \forall t$. Finally, normalize the initial level of productivity to $Z_0 = 1$.

a. Rewrite the problem in per-efficiency-unit terms, designating small-letter variables as equal to the capital-letter variable divided by labor productivity, e.g. $c_t = C_t/Z_t$, and show that in such an economy, the steady-state will be given by the point where $\dot{K}/K = \dot{C}/C = \dot{Y}/Y = g$. (Hint: Begin by rewriting the utility function in terms of consumption per efficiency unit and the growth in productivity since time 0.) Use the first order condition for consumption per efficiency unit in the rewritten problem to show that the steady-state normalized capital stock will be given by

$$k = \left[ \frac{\rho g + \theta + \delta}{\alpha} \right]^{\frac{\alpha}{1-\alpha}} \quad (3)$$
Answer:
The utility function is rewritten as
\[
\frac{C_t^{1-\rho}}{1-\rho} = \frac{(c_tZ_t)^{1-\rho}}{1-\rho} \tag{4}
\]
\[
= \frac{c_t^{1-\rho}Z_t^{1-\rho}}{1-\rho} \tag{5}
\]
\[
= \frac{c_t^{1-\rho}(Z_0e^{gt})^{1-\rho}}{1-\rho} \tag{6}
\]
\[
= \frac{c_t^{1-\rho}e^{\theta(1-\rho)t}}{1-\rho} \tag{7}
\]

Rewrite \( \dot{K} \) as
\[
\frac{dK}{dt} = \frac{d(k_tZ_t)}{dt} \tag{8}
\]
\[
= \frac{dk_t}{dt}Z_t + k_t \frac{dZ_t}{dt} \tag{9}
\]
\[
= (\dot{k}_t + gk_t)Z_t \tag{10}
\]
so that equation (2) becomes
\[
(\dot{k}_t + gk_t)Z_t = Y_t - C_t - \delta K_t \tag{11}
\]
and the optimization problem can be rewritten as
\[
\max \int_0^\infty \left( \frac{c_t^{1-\rho}}{1-\rho} \right) e^{-\hat{\theta}t} dt \tag{12}
\]
\[
\text{s.t.} \quad \dot{k}_t = y_t - c_t - (\delta + g)k_t \tag{13}
\]
\[
y_t = k_t^a \tag{14}
\]
where \( \hat{\theta} = \theta - g(1-\rho) \).

The steady-state of this model will occur at the point where \( \dot{k}_t = \dot{c}_t = 0 \). But since \( k_t = K_t/Z_t \) and \( c_t = C_t/Z_t \), \( \dot{k} \) and \( \dot{c} \) can be zero only if \( K \) and \( C \) are growing at the same rate as \( Z \). Thus we know that in the steady-state,
\[
\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = g. \quad (16)
\]

The consumption Euler equation tells us that
\[
\frac{\dot{c}}{c} = \rho^{-1}(f'(k) - (\delta + g) - \theta) \quad (17)
\]
\[
= \rho^{-1}(f'(k) - (\delta + g) - \theta + g(1 - \rho)) \quad (18)
\]
\[
= \rho^{-1}(f'(k) - \delta - \theta - \rho g) \quad (19)
\]

but we know that in steady-state, \(\dot{c}/c = 0\), implying
\[
f'(k) = \theta + \delta + \rho g \quad (20)
\]
\[
\alpha k^{\alpha - 1} = \theta + \delta + \rho g \quad (21)
\]
\[
k = \left(\frac{\theta + \delta + \rho g}{\alpha}\right)^{1/(\alpha - 1)} \quad (22)
\]
\[
= \left(\frac{\alpha}{\theta + \delta + \rho g}\right)^{1/(1 - \alpha)} \quad (23)
\]

b. Briefly discuss the effect of labor productivity growth on the absolute marginal product of capital in this model, and discuss the implications of this finding for the return to saving (the effective interest rate). (Hint: ‘absolute’ means don’t normalize by \(Z\)).

Answer:

The MPK in this model is
\[
F_k(K, L) = \alpha K^{\alpha - 1} Z^{1 - \alpha}. \quad (24)
\]

Thus, if labor productivity \(Z\) were constant, the marginal product of capital would be constant. Since \(Z\) is assumed to grow over time (and \(1 - \alpha > 0\)), the marginal product of capital grows over time. Thus, a given unit of capital becomes more valuable (in absolute terms) as time passes.

Since a given amount of saving is more valuable when there is labor productivity growth, and one might expect saving to rise as a result of its rising rewards.

c. Using a phase diagram, analyze the dynamic effects of increasing the degree of impatience \(\theta\) for an economy which starts off in period 0 in the steady-state
corresponding to the old, lower \( \theta \). Then make a graph showing the path of the aggregate saving rate over time. Explain both graphs in intuitive terms; be sure, in particular, to explain why the saving rate returns to its steady-state over time. Show and discuss how the graphs would change with a higher or lower coefficient of relative risk aversion.

Answer:

Increasing \( \theta \) reduces the steady-state capital stock, because the denominator of (23) is increased. This makes sense, because it says that making people more impatient implies that saving and the equilibrium capital stock will be lower. This means that the \( \dot{c}/c = 0 \) locus shifts left; the effect is shown in the attached figure (unfortunately, for some reason the dots in the phase diagrams are printing as \(<<\) symbols instead; ignore this problem).

The dynamics are as follows: At the instant when the representative agent becomes more impatient, there is a sharp increase in consumption (corresponding to a sharp decline in saving in the second figure; see below). Over time, the lower saving rate means that the capital stock per capita falls, and the interest rate rises. The increasing interest rate induces consumers to increase their saving rates, and the economy eventually returns to a new steady-state with a lower capital stock and a higher interest rate.

As for the effects of \( \rho \), recall that a low value of \( \rho \) means a high intertemporal elasticity of substitution, i.e. a great willingness to tolerate large differences in the level of consumption over time. Thus the saddle path is much more steeply sloped for \( \rho \) low than for \( \rho \) high. A more steeply sloped saddle path means that the initial jump up in consumption is much greater if \( \rho \) is low, meaning that the initial plunge in the saving rate is greater if \( \rho \) is low. However, the steady-state level of the saving rate is the same in both models (since the value of \( k^{SS} \) is the same). Thus the smaller drop in the saving rate initially in the model with high \( \rho \) is made up for by a slower return to the steady-state saving rate over time.

The reason the saving rate gradually rises after the initial plunge is that the interest rate is rising because the capital stock per capita is falling, and a higher interest rate encourages consumers to save more.

The path of saving over time is shown in figure 2. At time 0, the instant when \( \theta \) rises, there is a sharp drop in the saving rate. The drop is sharper when the coefficient of relative risk aversion is low (the
$k = S_{\text{low}}$, $r = q_{\text{low}}$

$\kappa - \omega$

$\kappa = 0$, $\theta_{\text{low}}$

$\kappa = 0$, $\theta_{\text{high}}$

Saddle Path, $\rho_{\text{low}}$

Saddle Path, $\rho_{\text{high}}$

Figure 1:
Figure 2:

solid line) than when it is high (the gray line). However, because the steady-state aggregate capital stock is the same in both models, the equilibrium saving rate must also be the same. Thus, both curves asymptote to the same level of saving, but the speed with which the economy approaches the steady-state is greater in the model with a lower value of $\rho$.

d. Now analyze the effects of an unexpected permanent increase in $g$, the rate of labor-augmenting technological progress. Again make a phase diagram, and compare the effects of $g$ on the $\dot{c} = 0$ and $\dot{k} = 0$ loci with the effects of $\theta$, and explain the reason for any similarities or differences.

Make a graph showing the path of the aggregate saving rate over time after the economy switches into the fast-growth regime. Explain the reason both graphs look the way they do in intuitive terms. Discuss what determines whether the saving rate rises or falls in the instant when consumers learn that growth has increased. (Hint: Whether the new steady-state saving rate is higher or lower depends on parameter values, as shown in last year’s exam. You can make whatever
assumption you wish about whether the new steady-state saving rate is higher or lower, but given your assumption about the new steady-state you can draw the path of saving over time.)

Answer:
The effects of $g$ on the $\dot{c} = 0$ locus are qualitatively the same as an increase in $\theta$: the locus moves left because the steady-state level of the capital stock is lower. The reason that $\theta$ and $g$ have similar effects is that increasing either of them makes consumers want to spend more now; higher $\theta$ directly makes them more impatient, and higher $g$ means that they will be richer in the future than they are now, and because they are forward-looking this extra future income means they can ‘afford’ to spend more now (this is essentially the human wealth effect).

Unlike $\theta$, $g$ also affects the $\dot{k} = 0$ locus. This is because $k$ is capital per efficiency unit of labor, and if efficiency units of labor start growing
faster, then capital must also grow faster in order to maintain capital-per-efficiency-unit constant.

A key insight for understanding the behavior of the saving rate is that the increase in $g$ also implies that the rate of return on capital rises (as noted in part b), because the capital will be more productive than previously expected because it will be combined with more effective labor in the future. The higher interest rate, in turn, gives the representative agent an incentive to save more than at the old interest rates. Whether the equilibrium saving rate rises or falls therefore depends on whether the human wealth effect is larger or smaller than the intertemporal substitution effect. If $\rho$ is very low (so that the IES is very high), it is possible that the saving rate will actually rise when $g$ rises. However, as shown on last year’s exam, the steady-state relationship between growth and saving is negative for most plausible parameter values.

Whether the new steady-state gross saving rate is higher or lower than the initial saving rate (right after the change in $g$) depends on param-
eter values (see last year’s final exam for the results). However, several things are unambiguous: 1) since the new equilibrium $k$ is lower, the new equilibrium interest rate is higher than before the increase in $g$; 2) the final steady-state saving rate is higher than the saving rate experienced immediately after the increase in $g$, which follows simply from the fact that rising interest rate over time causes a rising saving rate; 3) the variation in saving rates over time is less for $\rho$ large than for $\rho$ small, because for $\rho$ large consumers are less willing to substitute consumption over time.

**Question B.**

Mankiw (2000). Consider an economy in which there are two kinds of consumers: “Keynesian” consumers who are assumed simply to set their consumption equal to their current labor income and who own no capital, and “Ramsey” consumers who solve a traditional perfect foresight dynamic optimizing problem to determine their consumption. Suppose there is no population growth or productivity growth in this economy. The aggregate production function is Cobb-Douglas in labor and capital, $K^\alpha L^{1-\alpha}$ and both capital and labor markets are perfect implying that interest rates and wages are given by

\begin{align*}
    r &= \alpha K^{\alpha-1} L^{1-\alpha} \\
    &= \alpha k^{\alpha-1} \\
    &= f'(k) \\
    w &= (1 - \alpha) K^\alpha L^{-\alpha} \\
    &= (1 - \alpha) k^\alpha
\end{align*}

where as usual we define capital per capita as $k = K/L$. Assume CRRA utility $u(c) = c^{1-\rho}/(1 - \rho)$ for all consumers. Indicate per capita variables for the two classes of consumers by a subscript, i.e. consumption per Ramsey consumer is $c_r$, total labor supplied by Ramsey consumers is $L_r$, and so forth.

Thus the maximization problem for the Ramsey consumers is

\[
\max \int_t^\infty u(c_r) e^{-\theta t} dt
\]

\[
\dot{k}_r = (f'(k) - \delta) k_r + w L_r - c_r
\]

where $\delta$ is the depreciation rate and $\theta$ is the time preference rate.

a. Show that the steady-state capital stock in this model is the same as in an economy entirely populated by ‘Ramsey’ consumers. Discuss the plausibility of this result if the proportion of the population that is Ramsey is very small.

*Answer:*
The first order condition from the optimization problem yields a standard consumption Euler equation:

\[
\frac{\dot{c}_r}{c_r} = \rho^{-1}(f'(k) - \delta - \theta)
\]  

and the solution to this equation will clearly come at the point where \(f'(k) = \delta + \theta\). Since this equation does not depend on the proportion of the population that is Ramsey, the steady-state capital stock is the same regardless of what proportion of the population consists of Ramsey consumers.

A famous paper by Uzawa (1968) first made the point that if an economy contains multiple infinite-horizon consumers with different time preference rates, then in steady-state the entire capital stock will belong to the most-patient agent. The result for this problem can be interpreted as an example of an Uzawa economy in which the Keynesian consumers are simply interpreted as more impatient than the Ramsey consumers.

Of course, this result leans very heavily on the assumption that the Ramsey consumers are infinitely lived. If there is only one Ramsey consumer and everyone else is Keynesian, it might take millions of years for the capital stock to reach the Ramsey predicted level.

I can’t resist quoting an alternative answer provided on one of the exams: “If the Ramsey population is very small, then they would receive large profits from their capital. This formulation is implausible as we all know that if there were a small number of owners of capital exploiting the working class, eventually there would be a revolution and everyone would share the capital and be very happy. The capitalists should really try stock options to prevent such needless bloodshed.” I gave the author full credit, on the grounds that he is merely repeating the point I made about the “rule of law.”

Now assume that the population is divided half-and-half between Ramsey and Keynesian consumers: \(L_r = L_k = (1/2)\).

b. Calculate three measures of inequality for this economy: the fraction of total labor income going to the ‘poor’ (the Keynesian consumers), their fraction of aggregate wealth, and their fraction of aggregate income. (Assume that capital’s share in GDP is \(\alpha = 1/3\).) How does the ranking of these three measures of inequality compare with the ranking in real economies?
Since labor is supplied inelastically by both poor and rich, they earn exactly the same labor income so the ratio $w_L k / (w_L r + w_L k) = 1/2$.

In steady-state, the Ramsey consumers own the entire capital stock and the Keynesian consumers own no capital, so the ratio of wealth of the poor to aggregate wealth is zero.

The total income of the Keynesian consumers is their labor income. The total income of the Ramsey consumers is their labor income plus the entire amount of aggregate capital income.

$$\frac{y_k}{y_k + y_r} = \frac{wL_k}{w(L_k + L_r) + \alpha k^\alpha}$$  \hspace{1cm} (33)

$$= \frac{(1 - \alpha)k^\alpha L_k}{(1 - \alpha)(L_k + L_r) + \alpha k^\alpha}$$  \hspace{1cm} (34)

$$= \frac{(1 - \alpha)L_k}{(1 - \alpha)(L_k + L_r) + \alpha}$$  \hspace{1cm} (35)

$$= \frac{(1 - \alpha)/2}{1 - \alpha + \alpha}$$  \hspace{1cm} (36)

$$= \frac{(1 - \alpha)}{2}$$  \hspace{1cm} (37)

$$= \frac{2/3}{2}$$  \hspace{1cm} (38)

$$= \frac{1}{3}$$  \hspace{1cm} (39)

Thus, the greatest inequality is in wealth, followed by total income, with the least inequality in labor income. This is the same ranking as prevails in real economies.

c. Discuss whether a model like this is consistent with the evidence that aggregate consumption exhibits excess sensitivity.

*Answer:*

This is just a version of the Campbell-Mankiw model, which was originally proposed precisely in order to explain excess sensitivity. Thus the model is certainly consistent with the evidence on excess sensitivity.

d. Consider an economy like this one which has reached the steady-state level of the capital stock. Suppose that the preferences of the Keynesian consumers are identical to the preferences of the Ramsey consumers and the only reason the
Keynesian consumers have always set \( c_k = y_k \) is because they did not have access to financial markets so that they were physically unable to save. Discuss what would happen in this economy if the Keynesian consumers were suddenly granted access to financial markets. Discuss in particular the dynamics of inequality.

Answer:
Rather than blindly setting \( c_k = y_k \), the Keynesian consumers now will obey the optimality conditions for consumption,

\[
\frac{\dot{c}_k}{c_k} = \rho^{-1}(f'(k) - \delta - \theta)
\]

which is the same as the optimality condition for the Ramsey consumers. But we assumed that the economy had reached the steady-state capital stock, so that \( f'(k) = \delta + \theta \). Thus the Keynesian consumers will have optimal consumption growth of zero, and so they will choose to continue setting consumption equal to income even though they are now unconstrained! Thus, the inequality that came about during the period when the Keynesian consumers were constrained will be perpetuated forever after, because steady-state interest rates must be at exactly the right level to make everybody want to spend exactly their income.

Part II. Short Questions.

a. Steve Ballmer, the Chief Executive Officer of Microsoft, was recently asked by the Washington Post whether he believed that Microsoft shareholders might benefit from splitting Microsoft up into multiple “Baby Bill” companies each of which would own a copy of the operating system and Microsoft Office programs. Ballmer replied:

“To split up the company and allow two or more competitors to go head-to-head selling similar products would drive the price of the software so low that neither company could make a profit,” Steve Ballmer said.

“I was an economics major,” Ballmer said. “I learned enough of economics to know that if you have two guys selling the same thing and the marginal cost is zero, the price point on that is a well-known economic fact. And yet you have people say that it will increase shareholder value. I would vehemently dispute any notion there would be enhanced shareholder value on breakup.”
Using $q$ theory, discuss whether this argument is consistent with Microsoft’s claim in court that it is not a monopoly.

**Answer:**
Abel (1981) and Hayashi (1981) showed that marginal $q$ and average $q$ are the same only if there is perfect competition at the level of individual firms. Since Ballmer admits that the marginal cost is zero and the marginal price of Windows is positive, he is implicitly admitting that the market’s high value for Microsoft’s stock (its high average $q$) reflects monopoly power. His argument that there is only one equilibrium price (zero) when there is competition is a further admission that Microsoft has a lot of monopoly power.

Assuming that Ballmer is right about the consequences of a splitup, and that the Hayashi/Abel neoclassical $q$ model is also right, does Microsoft’s current extremely high value for Tobin’s $q$ imply that the company should be engaging in large amounts of investment to expand its current lines of business? (Hint: Most sales of Windows are to computer manufacturers who install the program on computers that they sell).

**Answer:**
As we discussed, one circumstance in which marginal $q$ will be very different from average $q$ is when the firm has monopoly power. In this case the firm’s ongoing operations may be highly profitable (due to its monopoly), but it may have no ability to expand those profits by increasing production, because marginal revenue will be less than marginal cost.

Think of it this way: If Microsoft decided to double the number of copies of Windows that it wanted to sell, how much would it have to cut the price? Since most copies of Windows are sold on new computers, Microsoft would need to cut the price of Windows enough to double sales of new computers. It seems very likely that Microsoft would have to create a huge subsidy to computer manufacturers to make the price of computers fall far enough to double sales. This amounts to charging a negative price for Windows (say, -$300). Clearly charging a negative price for its main product would not increase Microsoft’s profits!

Thus, if both Ballmer and Hayashi/Abel are right, the high value of $q$ reflects purely monopoly profits, and marginal $q$ is probably quite low, and so the theory does not imply that Microsoft should be doing a lot of investment spending to expand its current lines of business, i.e. to sell more copies of Windows or Office.
(iii) In fact, Microsoft does engage in very large amounts of “investment” spend-
ing, in the form of research and development of new programs (and buying
existing companies, sometimes companies whose products compete with Mi-
crosoft’s and sometimes companies who offer products that Microsoft does
not offer). Comment about what this high level of investment spending sug-
gests about Microsoft’s views about its ability to obtain monopolies in the
future.

*Answer:*

High levels of investment today must be justified by high expected
revenues in the future. If Microsoft is engaged in developing new
software which in the future will have a zero marginal cost but high
marginal price, it must by definition be making those investments in
the hopes of obtaining more monopoly power in the future (perhaps
in new markets) than it has now, or perhaps in hopes of maintaining
its current monopoly power.

(iv) Discuss what the q model would suggest about why it might be a rational
decision for Microsoft to give away Internet Explorer for free, given that Mi-
crosoft has spent hundreds of millions of dollars in developing the program.

*Answer:*

Once again, this is a rational decision if by giving away IE for
free, Microsoft either preserves existing monopoly power or creates
new monopoly power in the future. Giving away IE for free can
be considered as an ‘investment’ which purchases future monopoly
power.

To be fair to Microsoft, the software industry is probably a particu-
larly poor place to try to apply q theory, because since the marginal
cost of production is close to zero, the whole industry is about mo-
nopolistic competition, which q theory doesn’t handle very well.

b. Consider a small open economy (whose gross interest rate $R$ is fixed by world
capital markets) with a population of consumers who face random transitory
shocks to their incomes. The consumers solve the maximization problem:
\[ V(X_t) = \max E_t \left[ \sum_{s=t}^{\infty} ((1 - p)\beta)^{s-t} u(C_s) \right] \] (41)

\[ \text{s.t.} \] (42)

\[ X_{t+1} = R[X_t - C_t] + P_{t+1}\epsilon_{t+1} \] (43)

\[ P_{t+1} = GP_t \] (44)

\[ E_t[\epsilon_{t+1}] = 1 \quad \forall \ t \] (45)

\[ u(C_t) = \frac{C^{1-\rho}}{1-\rho} \] (46)

where all variables are as usually defined and \((1 - p)\) is the (constant) probability that the consumer will live another period (death occurs with probability \(p\)). Thus consumers discount future utility for two reasons: One is that they are impatient as traditionally defined \((\beta)\); the other is that they know that they only care about their future utility if they are alive in the future period. Assume that the transitory shock to income \(\epsilon_t\) is a white noise random variable which can occasionally take on a value of \(\epsilon_t = 0\), but never falls below zero. All consumers have the same level of permanent income \(P_t\) in a given period. When a consumer dies, he is replaced by a new ‘heir’ who is born in the same period and receives the dying consumer’s wealth as a bequest.

(i) What is the ‘impatience condition’ such that the consumption rules in this economy will be well-defined?

\textit{Answer:}

This is just a standard buffer-stock model of consumption, with the sole modification that the effective time discount factor is now \((1 - p)\beta\) rather than the usual \(\beta\). Thus the ‘impatience condition’ is that

\[ ((1 - p)\beta R)^{1/\rho} < G \] (47)

(ii) Discuss what the growth rates of aggregate labor income, aggregate wealth, and aggregate consumption will be in the steady-state in this economy, and use the second-order approximation to the household-level consumption Euler equation to discuss how these steady-states can be reconciled with individual optimization (the ‘second-order approximation’ is the equation that involves the variance of consumption growth).

\textit{Answer:}

Aggregate labor income, consumption, and wealth will all grow at rate \(G\) in steady-state. We know this because the impatience
condition prevents the $X_t/P_t$ ratio from rising too high, and precautionary saving motives prevent the ratio from falling too low. If $X_t/P_t$ can’t rise or fall forever, then in the long run $X_t$ must grow at the same rate as $P_t$. If $X_t$ and $P_t$ are growing at the same rate, the aggregate budget constraint says that $C_t$ must also grow at that rate.

A few people were tripped up by the fact that some consumers in this economy die, and thought that this might mean that income or wealth or consumption did not grow at rate $G$. But when they ‘die’, they are replaced by an identical clone who has exactly the same preferences, inherits the same wealth, and has the same permanent income and income uncertainty - in other words, the new ‘heir’ is identical in every respect to the agent who died, and so from the standpoint of the model the ‘death’ has no effect on the behavior of the dynasty. (This reminds me of the student in Classics class who said on an exam that “Homer was not written by Homer but by another man with the same name.”)

Mathematically, the consumption Euler equation says that

$$1 = R\beta E_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\rho} \right], \quad (48)$$

the second-order approximation to which is

$$E_t[\Delta \log C_{t+1}] = \rho^{-1}(r - \delta - \rho) + (\rho/2)\text{var}(\Delta \log C_{t+1}). \quad (49)$$

The way this equation (which does not directly involve $G$) can be reconciled with a requirement that aggregate consumption growth must be equal to $G$ is through the variance term. If wealth is too low, the variance term will be large, boosting consumption growth. If wealth is very high, the variance term will be very low, meaning that consumption growth will be low. The only value of the variance term consistent with stable wealth is the value which guarantees that aggregate consumption grows at rate $G$.

(iii) Suppose that the probability of death suddenly falls because a new vaccine is invented. Using a diagram, discuss the short run and long run effects on aggregate consumption, aggregate wealth, and the variance of consumption growth across consumers in the population.

Answer:
A decline in $p$ is mathematically identical to a decline in impatience $\delta$ (an increase in $\beta$). The short run effect of a decline in impatience
is to cut the level of aggregate consumption (an increase in the aggregate saving rate). As wealth grows, the variance term in the second-order approximation declines, until the new equilibrium is achieved in which the higher average level of wealth holdings across consumers implies a lower average value of the variance term. The standard diagrams that we used in class to examine the effects of a decline in the time preference rate can be used without modification here.

c. Explain the principal problem with the neoclassical growth models of Solow and Cass-Koopmans which induced Paul Romer and others to develop models of ‘endogenous growth.’ Explain which parameter in the neoclassical model can be modified to make the neoclassical model come arbitrarily close to the Rebelo AK model. Explain the empirical reasons behind the traditional choice for that parameter value, and explain the kinds of assumptions that must be made to justify the alternative value of the parameter implicit in the AK model.

Answer:
The problem with the neoclassical growth model is that when one does the Solow growth accounting exercise one finds that virtually all of economic growth is due to something that is unexplained in the model: exogenous ‘technological progress.’ It is very unsatisfying to claim to have a ‘model of economic growth’ which leaves the causes of almost all economic growth unexplained!

This defect caused Romer and others to try to rethink the circumstances under which it is possible to achieve long-lasting economic growth as a consequence of economic choices. The conclusion of this line of inquiry is summarized in the Rebelo AK model: To sustain perpetual economic growth, it is necessary to have some factor of production which can be accumulated forever without any decline in its marginal product ($K$ in the Rebelo model).

In class, I showed that you can think of the $Y = AK$ model as the limit of the standard Solow growth model $Y = AK^\alpha L^{1-\alpha}$ as $\alpha$ approaches 1. The traditional choice for $\alpha$, however, is $\alpha = 0.33$. This value comes from the fact that capital’s share of aggregate income is roughly 0.33, and in a perfectly competitive economy with a Cobb-Douglas production function it can be shown that the share of national income each factor will receive will be equal to its coefficient in the aggregate production function. Because in a perfectly competitive economy each factor is paid its marginal product and with a CRS Cobb-Douglas production function the share of the factor in aggregate income is equal to...
the value of its exponent in the production function. Thus, in order to assume $\alpha = 1$ it is necessary to abandon either the perfect competition assumption or the assumption that each factor is paid its aggregate marginal product. The simplest assumption to make is that there are externalities to capital accumulation that work at the aggregate level to yield a much higher productivity of aggregate capital than the productivity an individual firm can capture by increasing its own capital. A slightly more devious approach is to redefine $K$ as ‘knowledge’ and assume that knowledge can be accumulated forever.

References
