Final Exam
Consumption Dynamics: Theory and Evidence
Spring, 2004
Answers

This exam consists of two parts. The first part is a long analytical question. The second part is a set of short discussion questions. Each part of the exam counts equally. If you cannot solve part of the long analytical question, do not give up. The question is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.
Part I. Long Analytical Question.

Consider a consumer solving a standard consumption problem

$$\max E_t \left[ \sum_{s=t}^{\infty} \beta^{t-s} u(\tilde{c}_s) \right]$$  \hspace{1cm} (1)

subject to the constraint

$$a_{t+1} = R[a_t + y_t - c_t]$$  \hspace{1cm} (2)

where $a_t$ is the consumer’s current spendable assets, $y_t$ is current labor income, $R$ is the constant gross interest factor and $\beta = 1/(1 + \theta)$ is the time preference factor where $\theta$ is the time preference rate. Suppose the consumer has quadratic utility $u(c) = - (\gamma/2)(c - c^*)^2$ where $c^*$ is the ‘bliss point’ level of consumption.

1. Assume that $\beta = 1/R$ and derive Hall’s random walk proposition for consumption,

$$\Delta c_{t+1} = \epsilon_{t+1},$$  \hspace{1cm} (3)

where $\epsilon_{t+1}$ is a white noise variable. Explain why Flavin’s finding that the regression equation

$$\Delta c_{t+1} = \alpha_0 + \alpha_1 \Delta y_t.$$  \hspace{1cm} (4)

yields a coefficient $\alpha_1$ that is positive and statistically significant is a rejection of Hall’s model.

**Answer:**

The first order condition is the standard

$$u'(c_t) = R\beta E_t[u'(\tilde{c}_{t+1})]$$  \hspace{1cm} (5)

$$-\gamma c_t = E_t[-\gamma \tilde{c}_{t+1}]$$  \hspace{1cm} (6)

$$E_t[\tilde{c}_{t+1}] = c_t$$  \hspace{1cm} (7)

$$c_{t+1} = c_t + \epsilon_{t+1}$$  \hspace{1cm} (8)

$$\Delta c_{t+1} = \epsilon_{t+1}$$  \hspace{1cm} (9)

where $\epsilon_{t+1}$ is an expectational variable, which means that it is uncorrelated with any variable whose value was known at time $t$.

Flavin’s finding that lagged income growth is related to current consumption growth violates the random walk proposition because the value of income growth last period was known when last period’s consumption was decided. Any information contained in last period’s income growth should therefore have been incorporated into the level of consumption last period and should not be related to future changes in the level of consumption.
2. Now assume that the process for labor income is

\[ p_t = p_{t-1} + \phi_t \quad (10) \]
\[ y_t = p_t + \gamma_t, \quad (11) \]

where \( \gamma_t \) is a white noise variable representing a transitory shock to labor income and \( \phi_{t+1} \) is a white noise variable representing a shock to permanent labor income, \( E_t[\gamma_{t+n}] = E_t[\phi_{t+n}] = 0 \ \forall \ n > 0 \). Show that the formula for level of consumption is

\[ c_t = p_t + \left( \frac{R}{R-1} \right) (a_t + \gamma_t). \quad (12) \]

Answer:

The IBC says that the expected PDV of consumption must equal the expected PDV of total wealth, human and nonhuman:

\[ E_t \left[ \sum_{s=t}^{\infty} R^{t-s} c_s \right] = a_t + E_t \left[ \sum_{s=t}^{\infty} R^{t-s} y_s \right] \]
\[ E_t \left[ \sum_{s=t}^{\infty} R^{t-s} c_t \right] = a_t + \gamma_t + E_t \left[ \sum_{s=t}^{\infty} R^{t-s} p_s \right] \]
\[ \left( \frac{1}{1 - R^{-1}} \right) c_t = a_t + \gamma_t + p_t \left( \frac{1}{1 - R^{-1}} \right) \]
\[ c_t = \left( R/R - 1/1 \right) (a_t + \gamma_t) + p_t \]
\[ c_t = \left( \frac{R}{R-1} \right) (a_t + \gamma_t) + p_t. \]

3. Now consider a consumer who in periods \( t - n \) for \( n > 0 \) has experienced \( \gamma_{t-n} = \phi_{t-n} = 0 \); that is, this consumer has experienced no shocks to income in the (recent) past. Suppose this period the consumer draws \( \gamma_t = 1 \) and \( \phi_t = 0 \).

Draw carefully-labelled diagrams showing the time paths of past, current, and expected consumption and past, current and expected wealth from the vantage point of period \( t \) after the shock has become known (that is, show on one graph \( c_{t-2}, c_{t-1}, c_t, E_t[c_{t+1}], E_t[c_{t+2}], \ldots \) and on the other graph show the equivalent for \( a_t \)).

Answer:
Figure 1: Path of $c$ and $a$ after a shock $\gamma$
Now consider a large economy populated by \( N \) consumers solving this optimization problem, all of whom have the same time preference factor and other parameters. Define aggregate variables by the upper-case; for example, aggregate consumption is

\[
C_t = \sum_{i=1}^{N} c_{i,t}
\]

where there are \( N \) consumers in the economy and \( c_{i,t} \) is consumption of consumer \( i \) in period \( t \).

4. Show that aggregate versions of equation (3) and (12) hold, and explain what special feature(s) of the model allow this result to be obtained.

Answer:

\[
C_t = \sum_{i=1}^{N} c_{i,t} = \sum_{i=1}^{N} [(r/R) (a_{i,t} + \gamma_{i,t}) + p_{i,t}] = (r/R)(A_{t} + \Gamma_{t}) + P_{t}.
\]

The crucial feature of the model that allows us to aggregate analytically is the linearity of the consumption rule in \( p, w, \) and \( \gamma \).

Now suppose that not every consumer updates his expectations in every period. Instead, assume that expectations are ‘sticky’: each consumer updates his expectations with probability \( \lambda \) in each period. In periods when expectations are not updated, the consumer continues to consume the same amount as he consumed in the last period when expectations were updated.

For example, consider a consumer who happens to have updated his expectations in period \( t \). Designating \( c_{i,t}^r \) as the level of consumption that a rational consumer with completely up-to-date expectations would choose, this consumer’s actual consumption will equal the rational level,

\[
c_{i,t} = c_{i,t}^r.
\]

If this consumer does not update his expectations in period \( t + 1 \), he will continue to consume \( c_{i,t+1} = c_{i,t} = c_{i,t}^r \). He continues consuming this same amount until he updates his expectations again, which occurs randomly with probability \( \lambda \).
5. Explain why this set of assumptions leads to the following representation for the level of aggregate consumption,

\[ C_t = \lambda C_t^r + \lambda(1 - \lambda)C_{t-1}^r + \lambda(1 - \lambda)^2 C_{t-2}^r + \ldots \]  

(18)

where \( C_t^r \) is the level of aggregate consumption that would have occurred if all consumers in the economy were rational. Hint: Define the set of consumers who update their expectations in period \( t \) as \( \Lambda_t \), for whom average consumption is

\[ \bar{c}^A_t = \frac{1}{(\lambda N)} \sum_{\Lambda_t} c_{i,t}^r. \]  

(19)

where if the economy is large the number of consumers who update their expectations every period will be essentially exactly \( \lambda N \), and define an analogous expression for \( \bar{c}_t^r \), the average per-capita level of consumption that would prevail if all consumers were rational.

[Space for your answer is on the next page]

**Answer:**

We are assuming that the probability of adjusting one’s expectations is independent of the level of income, wealth, or consumption; therefore the average levels of wealth and income among those consumers who adjust their expectations this period should equal the average levels of wealth and income in the economy as a whole. The level of consumption per capita that would prevail if all consumers were rational is

\[ \bar{c}_t^r = \frac{1}{N} \sum_{i=1}^{N} c_{i,t}^r. \]  

(20)

But since the set of consumers who updated was randomly selected from the population, the average level of consumption-per-capita for the updaters must be equal to the average level of rational-consumption-per-capita for the population as a whole,

\[ \bar{c}_t^A = \bar{c}_t^r \]  

(21)

Hence total aggregate consumption by the updaters will be \( \lambda N \bar{c}_t^A = \lambda (Nc_t^r) = \lambda C_t^r \).

Now consider the period-\( t \) non-updaters. Among these consumers, a fraction \( (1 - \lambda) \) updated in the last period. The same chain of logic we went through for the period-\( t \) updaters will have applied to the period-\( t - 1 \) updaters in the previous period. Therefore among
these consumers the period-\(t\) average level of per-capita consumption will be \(c_t^r\). How many such consumers will there be? Well, some fraction \(\lambda\) of last period’s updaters will have updated again this period (remember that the probability of updating for any consumer is independent of the time since last update), so the expected number of such consumers (didn’t update this period, did update last period) is \((1 - \lambda)\lambda N\) and their total consumption should be 

\[(1 - \lambda)\lambda Nc_t^r = \lambda(1 - \lambda)c_t^r.\]

Similar logic leads to the rest of the equation.

6. Show that if equation (18) holds, then the process for aggregate consumption can be rewritten as

\[\Delta C_{t+1} = \lambda \Delta C_{t+1}^r + (1 - \lambda) \Delta C_t\]  

(22)

and explain why \(\lambda \Delta C_{t+1}^r\) should be very close to a white noise variable.

Answer:

\[C_t = \lambda \left[ C_t^r + (1 - \lambda)C_{t-1}^r + (1 - \lambda)^2 C_{t-2}^r + \ldots \right]\]  

(23)

\[C_{t+1} = \lambda \left[ C_{t+1}^r + (1 - \lambda)C_t^r + (1 - \lambda)^2 C_{t-1}^r + \ldots \right]\]  

(24)

\[C_{t+1} = \lambda \left[ C_{t+1}^r \right] + (1 - \lambda) \lambda \left( C_t^r + (1 - \lambda)C_{t-1}^r + \ldots \right)\]  

(25)

\[C_{t+1} = \lambda \left[ C_{t+1}^r \right] + (1 - \lambda)C_t\]  

(26)

\[\Delta C_{t+1} = \lambda \Delta C_{t+1}^r + (1 - \lambda) \Delta C_t\]  

(27)

The short answer for why \(\lambda \Delta C_{t+1}^r\) should be close to a random walk is that it should be close to the change in consumption of a rational consumer between periods \(t\) and \(t + 1\), and the change in consumption of a rational consumer is white noise. However, the exactly correct answer is as follows.

Theory tells us that if aggregate consumption had been chosen fully rationally in period \(t\) then \(\Delta C_{t+1}^r\) would be white noise; specifically, we know that

\[\eta_{t+1} = (r/R)[R(P_t + \Gamma_t + A_t - C_t^r) + \Gamma_{t+1}] - C_t^r\]  

(28)

\[= (r/R)\Gamma_{t+1}\]  

(29)

for some white noise \(\Gamma_{t+1}\) which implies \(\eta_{t+1}\) is also white noise. The only difference between the RHS of this expression and the actual value of \(\Delta C_{t+1}^r\) is the presence of the \(r\) superscript on \(C_t\) in square brackets. Thus we know that

\[\Delta C_{t+1}^r = \eta_{t+1} + (r/R)[R(C_t^r - C_t)]\]  

(30)

\[= \eta_{t+1} + r(C_t^r - C_t)\]  

(31)
So equation (27) can be rewritten as
\[
\Delta C_{t+1} = (1 - \lambda)\Delta C_t + r(C^r_t - C_t) + \eta_{t+1}
\] (32)
where $\eta_{t+1}$ is a white noise variable.

However, (26) implies
\[
C_t = \lambda C^r_t + (1 - \lambda)C_{t-1}
\] (33)
\[
C^r_t = \frac{C_t - (1 - \lambda)C_{t-1}}{\lambda}
\] (34)
\[
C^r_t - C_t = \frac{(1 - \lambda)C_t - (1 - \lambda)C_{t-1}}{\lambda}
\] (35)
\[
= \left(\frac{1 - \lambda}{\lambda}\right)\Delta C_{t-1}
\] (36)
which can be substituted into (32) to yield
\[
\Delta C_{t+1} = (1 - \lambda)\Delta C_t + r\left(\frac{1 - \lambda}{\lambda}\right)\Delta C_t + \eta_{t+1}
\] (38)
\[
= \left[(1 - \lambda) + r\left(\frac{1 - \lambda}{\lambda}\right)\right]\Delta C_t + \eta_{t+1}
\] (39)
\[
\approx (1 - \lambda)\Delta C_t + \eta_{t+1}
\] (40)
where the approximation holds so long as $r(1 - \lambda)/\lambda$ is small relative to $(1 - \lambda)$.

7. Show how aggregate consumption and wealth would be expected to respond in this economy to a transitory positive shock to aggregate labor income like the shock considered above in part 2. In particular, discuss whether the MPC out of the shock in the first period is higher than, the same as, or lower than in the fully rational model and why, and explain the consequences of this for the ultimate effect of the shock on the steady-state level of wealth. Compare this with the ultimate effect on wealth in the fully rational model.

Answer:
The MPC out of the shock is lower than in the fully rational model; in fact, only fraction $\lambda$ of consumers react in the current period, and those consumers who do react change their consumption by the same fraction as the fully-rational ones would, so the MPC in the first period is clearly $\lambda(r/R) < (r/R)$.

Consider the case of a positive shock, as before. In the first period consumption rises only by $\lambda r$, rather than the full amount corresponding to the permanent income associated with the new level of
wealth. Therefore aggregate wealth in period $t + 1$ will be greater than it would have been in the fully-rational model. Similarly for all subsequent periods. Thus, in contrast with the fully-rational model, the sluggish adjustment of consumption to the shock means that the shock has a permanent effect on the level of aggregate wealth, and therefore on the level of aggregate consumption.

8. Is the ‘sticky expectations’ model consistent with the evidence on the excess smoothness of aggregate consumption? What empirical strategy could be employed to estimate $\lambda$ using aggregate data?

Answer:
Yes, the sticky expectations model says that consumption growth today can be statistically related to any variable that is related to lagged consumption growth. In particular, if lagged consumption growth is related to lagged income growth (as it certainly will be), then there should be a statistically significant effect of lagged income growth on current consumption growth if expectations are sticky.

If the model derived here could be taken literally, it would suggest estimating an equation of the form

$$\Delta C_t = \alpha_0 + \alpha_1 \Delta C_{t-1} + u_t$$

and interpreting the coefficient $\alpha_1$ as a measure of $(1 - \lambda)$.

However, if there is potential measurement error in $C_{t-1}$ the coefficient estimate obtained from estimating (41) would be biased toward zero for standard errors-in-variables reasons (just as regressing consumption on actual income yields a downward-biased estimate of the response of consumption to permanent income), which means that the estimate of $\lambda$ would be biased toward 1 (i.e. a model in which everyone adjusts all the time). Thus, direct estimation of (41) might not be a reliable way to estimate $\lambda$. 
Figure 2: Path of $c$ and $a$ after a shock $\gamma$ with $\lambda$ updating
Part II. Short Discussion Questions.

1. Discuss the effects of a decrease in the time preference rate on the path of consumption growth over time and the target level of wealth in a buffer stock model of saving. (If $\beta = 1/(1 + \theta)$ then $\theta$ is the time preference rate and $\beta$ is the time preference factor.)

Answer:

The effect of a decrease in the time preference rate is identical to the effect of an increase in interest rates. The level of consumption drops. In the short run, the growth rate of consumption is faster. In the long run the growth rate of consumption returns to its steady-state rate, which is the same as the growth rate of permanent income. The target level of assets is higher, as can be seen from the fact that the formula for the variance of consumption growth implies a lower variance for a higher interest rate, and a lower variance can be achieved only with a higher stock of wealth.
2. In a recent Hopkins dissertation, Martin Sommer (2001) found that an equation of the form

\[ \Delta \log C_{t+1} = \alpha_0 + \alpha_1 \Delta \log C_t + \epsilon_{t+1} \]  

(42)

fits U.S. aggregate consumption data fairly well with a coefficient estimate of \( \alpha_1 \approx 0.7 \). Explain why this result is inconsistent with the usual model of consumption, and briefly discuss an alternative model of consumption mentioned in class that could explain a result like this.

Answer:
According to the Hall (1978) theory of consumption, no lagged variable (including lagged consumption growth) should have predictive power for current consumption growth. A coefficient of \( \alpha_1 = 0.7 \) implies that consumption growth is a very long way from a random walk.

A handout on habit formation indicated that models with habits can imply serial correlation in consumption growth of precisely this kind, where the coefficient on lagged consumption growth indicates the strength of the habit formation motive.
3. Discuss the short- and long-run consequences of an increase in income growth for the capital-to-labor-income ratio in a Ramsey model of economic growth, and explain why it is not possible to estimate the marginal propensity to consume out of wealth by doing a cross-country regression of consumption/labor income ratios on captial/labor income ratios, according to the theory.

Answer:

This was discussed extensively in the handout RamseyCointegration.
References
