1 Theory

Assume that each infinitely-lived consumer in the economy is endowed with one unit of labor which is supplied inelastically in an aggregate labor market, and that the individual consumer’s labor income is subject to multiplicative idiosyncratic transitory shocks $\epsilon_{i,t+1}$ such that $E_t[\epsilon_{i,t+1}] = 1$. Idiosyncratic ‘permanent labor income’ is defined as the level of labor income that would obtain for individual $i$ if the transitory shock to labor income took on its mean value of one. Permanent labor income $P_{i,t}$ is also subject to lognormally distributed shocks $N_{i,t+1}$ (again such that $E_t[N_{i,t+1}] = 1$) in each period; the marginal propensity to consume out of permanent income will be determined by examining the response of consumption to these shocks.\(^2\)

Denoting by $H_{i,t}$ the habit stock inherited from past consumption behavior, and designating total resources available for consumption (the sum of undepreciated past capital, current capital income, and current labor income) as $X_{i,t}$, the consumer’s optimization problem for a finite horizon beginning at the current period $t$ and ending at period $T$ (and dropping the $i$ subscripts to reduce clutter) is given by

$$
V_t(X_t, P_t, H_t) = \max_{\{C_s\}_t^T} u(C_t, H_t) + E_t \left[ \sum_{s=t+1}^{T} \beta^{s-t} u(C_s, H_s) \right]
$$

such that

\begin{align*}
K_t &= X_t - C_t, \\
H_{t+1} &= H_t + \lambda(C_t - H_t), \\
X_{t+1} &= (R - \delta)K_t + Y_{t+1}, \\
Y_{t+1} &= P_{t+1}\epsilon_{t+1}, \\
P_{t+1} &= GP_tN_{t+1},
\end{align*}

where $K_t$ is the proportion of total available resources from the beginning of the

\(^1\)The notational convention will be that stochastic variables have a $\sim$ over them when their expectation is being taken, but not otherwise, on the grounds that equations where the expectation is being taken are the only kinds of equations where the time period from which the equation is being viewed is well-specified. Hence we write the transitory shock to labor income in period as $\epsilon_{i,t+1}$ but its expectation as $E_t[\epsilon_{i,t+1}]$.

\(^2\)Note that the assumption that $N$ is lognormally distributed and that $E[N] = 1$ imply that $\log N \sim N(-\sigma_N^2/2, \sigma_N^2)$, i.e. $N$ is lognormally distributed and $\log N$ has variance $\sigma_N^2$ and mean $-\sigma_N^2/2$. 

1
period $X_t$ that have not been consumed at the end of the period. Habits as of
the beginning of next period $H_{t+1}$ will have moved from where they started out in
this period ($H_t$) toward this period’s consumption. Next period’s total resources
$X_{t+1}$ will equal the gross return $R = (1 + r)$ on this period’s end-of-period capital,
minus depreciation $\delta$, plus labor income $Y_{t+1}$. Labor income will equal permanent
labor income $P_{t+1}$ multiplied by the transitory shock $\epsilon_{t+1}$, and permanent labor
income next period will equal permanent labor income this period increased by
the gross productivity growth factor $G = (1 + g)$ and multiplied by the stochastic
shock to permanent income $N_{t+1}$. As usual, the recursive nature of the problem
allows us to rewrite the maximand as:

$$V_t(X_t, P_t, H_t) = \max_{\{c_t\}} u(C_t, H_t) + \beta E_t \left[ V_{t+1}(\tilde{X}_{t+1}, \tilde{P}_{t+1}, H_{t+1}) \right]. \quad (7)$$

Following C-O-W we assume that the utility function is of the form

$$u(C_t, H_t) = \frac{(C_t/H_t^{\gamma})^{1-\rho}}{1-\rho}. \quad (8)$$

The model as written has three continuously-valued state variables, $X_t$, $H_t$, and $P_t$. It turns out that the model can be recast by dividing all stock and flow
variables by the level of permanent income, and it then becomes a problem in
two state variables and permanent income becomes essentially a multiplicative
scaling term. Specifically, define $x_t = X_t/P_t$, $c_t = C_t/P_t$, and so on (where the
subscript $t$ on a state variable indicates that the value of the variable is known at
the beginning of the period. The accumulation equation for $x_t$ is

$$X_{t+1} = R [X_t - C_t] + Y_{t+1} \quad (9)$$

$$x_{t+1} P_{t+1} = R P_t [x_t - c_t] + P_{t+1} \epsilon_{t+1} \quad (10)$$

$$x_{t+1} = (R/G N_{t+1}) [x_t - c_t] + \epsilon_{t+1} \quad (11)$$

and the equation for $h_{t+1}$ is

$$H_{t+1} = H_t + \lambda (C_t - H_t) \quad (12)$$

$$h_{t+1} P_{t+1} = h_t P_t + \lambda (c_t P_t - h_t P_t) \quad (13)$$

$$h_{t+1} G N_{t+1} P_t = P_t (h_t + \lambda (c_t - h_t)) \quad (14)$$

$$h_{t+1} = \frac{(h_t + \lambda (c_t - h_t))}{G N_{t+1}} \quad (15)$$

In analyzing problems of this kind it is often useful to define the value of the
program at two points within a period: before and after the control variables have
been chosen. To do so however requires that we give names to the state variables
at the end of the period that are distinct from their names at the beginning. At
the end of the period the state variables for this problem are capital $k_t = x_t - c_t$
and $\overline{h}_t = h_t + \lambda (c_t - h_t)$, and the end-of-period value function is

$$\Omega_t(k_t, \overline{h}_t) = E_t \left[ (G \tilde{N}_{t+1})^{(1-\rho)(1-\gamma)} \tilde{v}_{t+1} (((R/G \tilde{N}_{t+1}) k_t + \tilde{\epsilon}_{t+1}, \overline{h}_t/G \tilde{N}_{t+1}) \right]. \quad (16)$$
and, denoting the derivative of a function $f$ with respect to an argument $z$ as $f^z$ we have

$$\Omega_t^c(k_t, \overline{h}_t) = E_t \left[ (G\tilde{N}_{t+1})^{\rho \gamma - \rho - \gamma} Rv_{t+1} \right]$$

$$\Omega_t^h(k_t, \overline{h}_t) = E_t \left[ (G\tilde{N}_{t+1})^{\rho \gamma - \rho - \gamma} v_{t+1}^h \right].$$

(17)

Using this definition, the maximization problem becomes simply

$$v_t(x_t, h_t) = \max \{c_t, h_t\} \ u(c_t, h_t) + \beta \Omega_t(x_t - c_t, h_t + \lambda (c_t - h_t)),$$

(19)

and we define the infinite-horizon solution as the limit of the finite horizon solution as the horizon $T$ approaches infinity.\(^\text{3}\)

### 1.1 Optimality Conditions

#### 1.1.1 The First Order Condition for $c_t$

The first order condition for this problem with respect to $c_t$ is:

$$0 = P_t^{(1-\gamma)(1-\rho)} \left\{ u^c(c_t, h_t) + \beta E_t \left( (G\tilde{N}_{t+1})^{(1-\gamma-\rho)} \left[ v_{t+1}^x \frac{-R}{G\tilde{N}_{t+1}} + v_{t+1}^h \frac{\lambda}{G\tilde{N}_{t+1}} \right] \right) \right\}$$

$$u^c(c_t, h_t) = \beta E_t \left[ (G\tilde{N}_{t+1})^{(\rho \gamma - \rho - \gamma)} (Rv_{t+1}^x - \lambda v_{t+1}^h) \right]$$

(20)

$$(c_t h_t^{-\gamma}) - \rho h_t^{-\gamma} = \beta E_t \left[ (G\tilde{N}_{t+1})^{(\rho \gamma - \rho - \gamma)} (Rv_{t+1}^x - \lambda v_{t+1}^h) \right]$$

(21)

$$c_t^{-\rho} h_t^{\gamma (\rho - 1)} = \beta E_t \left[ (G\tilde{N}_{t+1})^{(\rho \gamma - \rho - \gamma)} (Rv_{t+1}^x - \lambda v_{t+1}^h) \right]$$

(22)

$$c_t^{-\rho} = h_t^{(1 - \rho)} \beta E_t \left[ (G\tilde{N}_{t+1})^{(\rho \gamma - \rho - \gamma)} (Rv_{t+1}^x - \lambda v_{t+1}^h) \right]$$

(23)

$$c_t = h_t^{(1 - 1/\rho)} \left\{ \beta E_t \left[ (G\tilde{N}_{t+1})^{(\rho \gamma - \rho - \gamma)} (Rv_{t+1}^x - \lambda v_{t+1}^h) \right] \right\}^{-1/\rho}$$

(24)

$$c_t = h_t^{(1 - 1/\rho)} \left\{ \beta \left[ \Omega_t^c - \lambda \Omega_t^h \right] \right\}^{-1/\rho}$$

(25)

Since we solve the problem backwards from the end of life, at any point $t$ we should have in hand the functions $v_{t+1}^x$ and $v_{t+1}^h$, so in principle we can solve equation (25) to find $c_t$ for a grid of possible values of $(x_t, h_t)$, yielding values for $v_t(x_t, h_t), v_t^x(x_t, h_t),$ and $v_t^h(x_t, h_t)$ and so on recursively until the consumption rules have converged.

\(^3\)It is necessary to impose some restrictions on parameter values to ensure convergence; see below for a discussion of the necessary assumption.
1.1.2 Applying the Envelope Theorem

The envelope theorem on the variable $x_t$ says:

\[
v^x_t = \frac{\partial v_t}{\partial x_t} + \frac{\partial v_t}{\partial c_t} \frac{\partial c_t}{\partial x_t}
\]

\[
v^x_t = \beta E_t \left[ (G_{t+1})^{\rho - \gamma - \rho} R v^x_{t+1} \right]
\]

\[
v^x_t = \beta \Omega^x_t
\]

Thus, equation (25) above can be rewritten as

\[
u^c_t = v^x_t - \beta E_t \left[ (G\tilde{N}_{t+1})^{\rho - \gamma - \rho} \lambda v^h_{t+1} \right]
\]

\[
v^x_t = u^c_t + \beta E_t \left[ (G\tilde{N}_{t+1})^{\rho - \gamma - \rho} \lambda v^h_{t+1} \right]
\]

\[
v^x_t = u^c_t + \lambda \beta \Omega^h_t
\]

Noting that $\partial h_{t+1}/\partial h_t = (1 - \lambda)/GN_{t+1}$, the envelope theorem on the variable $h_t$ says:

\[
v^h_t = \frac{\partial v_t}{\partial h_t} + \frac{\partial v_t}{\partial c_t} \frac{\partial c_t}{\partial h_t}
\]

\[
v^h_t = u^h_t + \beta E_t \left[ (G\tilde{N}_{t+1})^{(1-\rho)(1-\gamma)} \lambda v^h_{t+1} \frac{\partial h_{t+1}}{\partial h_t} \right]
\]

\[
v^h_t = u^h_t + (1 - \lambda) \beta E_t \left[ (G\tilde{N}_{t+1})^{\gamma - \gamma - \rho} \lambda v^h_{t+1} \right]
\]

\[
v^h_t = u^h_t + (1 - \lambda) \beta \Omega^h_t
\]

Using the first order conditions (30) and (31), the problem is solved recursively using numerical methods from the last period of life until successive decision rules have converged, and we denote the converged consumption rule $c[x_t, h_t]$. The methods in Carroll (2001) can be extended to this case to prove that the successive rules will converge if

\[
R \beta E_t [(G_{t+1} N_{t+1})^{-\hat{\rho}}] < 1.
\]

where $\hat{\rho} = \rho + \gamma(1 - \rho)$ and this condition corresponds to a requirement that consumers are sufficiently impatient that as wealth approaches infinity eventually a point arrives at which they would wish to consume more than their current labor and capital income.\(^4\)

\(^4\)This is the counterpart to the impatience condition derived in Deaton (1991) for the model with liquidity constraints and in Carroll (2001) in the unconstrained case.
2 Parameterization

We begin by parameterizing the baseline version of the model with intertemporally separable utility, then consider parameterizing the habit formation version of the model.

We choose fairly conventional assumptions for the time preference rate $\beta = .97$ and net interest rate $R - \delta = 1.03$, where the time period of the model is interpreted as a year. In the absence of Japanese data I use an estimate by Cohen, Hassett, and Kennedy (1995) that the aggregate depreciation rate in the U.S. is $\delta = .06$ (this may seem low, but recall that since we are examining the overall aggregate saving rate the appropriate definition of capital is not just machinery but also structures, both residential and non-residential). We assume a coefficient of relative risk aversion of $\rho = 3$. Finally, Carroll (1992) and several other studies find that the household labor income process in the United States is relatively well-characterized by the assumption that $N$ is lognormally distributed with standard deviation $\sigma_N = 0.10$; that household income is occasionally close to zero (principally during unemployment spells); and that when $\epsilon$ is not close to zero it is distributed lognormally with $\sigma_\epsilon^2 = 0.10$; in the absence of corresponding empirical results for Japan we use these numbers. The unemployment rate in Japan has typically ranged from 2-4 percent over most of the postwar period. Accordingly I set the probability of an unemployment spell in which $Y = 0$ at $p = 0.02$ or two percent.\(^5\)

Fortunately, recent work by Fuhrer (2000) provides some evidence about how to parameterize the habit formation component of the model. Fuhrer (2000) estimates a perfect foresight version of the C-O-W model on quarterly U.S. data and obtains a fairly tight parameter estimate of $\gamma = 0.8$ for the parameter that indexes the importance of habits in utility. In the absence of specific data for Japan, we follow Fuhrer and set $\gamma = 0.8$.

One implication of habit formation is that the short-run value of the coefficient of relative risk aversion differs from the long-run value (or, equivalently, the intertemporal elasticity of substitution is smaller over the short-term than over the long-term). In fact, it is precisely this difference between short-run and long-run elasticities that helps habit formation models resolve the equity premium puzzle without implying absurdly risk-averse behavior in other dimensions (say, occupational choice); the difference comes from the fact that one has time to ‘get used to’ low-frequency fluctuations, in the sense that the habit stock has time to adjust (and thereby diminish the impact on utility).

For the purposes of this paper, I will parameterize the baseline habit formation model in such a way as to keep the infinite-horizon coefficient of relative risk aversion

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\(^5\)I chose a number less than the observed unemployment rate because the spells in this model are assumed to last for a full year and that there are no unemployment benefits or other social insurance mechanisms. Since both of these assumptions are probably too strong, I compensate by choosing a low value of $p$. 

aversion equal to the plausible value of three specified in the baseline model with intertemporally separable utility. As C-O-W show, the infinite-horizon coefficient of relative risk aversion when utility is of the form (8) is equal to \((\rho + \gamma(1 - \rho))\).

To make this quantity equal to 3 given that \(\gamma = 0.8\) as estimated by Fuhrer, it is necessary to assume an instantaneous coefficient of relative risk aversion of \(\rho = 11\).

The final parametric choice is for the value \(\lambda\) of the speed at which habits catch up to consumption. I use as a baseline the value used in C-O-W for most of their analysis, \(\lambda = 0.3\). This value implies that the half-life with which habits move toward consumption is only two years (because \((1 - .3)^2 = .49 \approx 1/2\)).

### 3 Saving and Growth Redux

As a preliminary experiment with the model, we need to reexamine the question that motivated the original work in C-O-W: is the apparent fact that growth causes saving consistent with the baseline model without habits, and if not does adding habits help?

Before attempting to answer this question we must specify the stochastic process governing growth in this economy. Roughly speaking, the postwar history of growth in Japan falls into three periods. The first of these was the period of astonishingly rapid growth from the late 1940s to the early 1970s, which far exceeded the growth experience of any other country in any previous era of human history. The second was the period beginning in the early 1970s and ending in the late 1980s when Japan still managed to achieve an impressive growth rate but not one that was radically higher than the growth achieved in other industrialized countries. The third era is of course the decade of the 1990s when Japan’s growth has been dismal. In none of these three cases can a plausible case be made that the change in ‘growth regime’ was widely anticipated in advance. The simplest model that seems to capture this pattern of experience is a first-order Markov transition process for the underlying level of the growth process. For simplicity of analysis, I will assume a two-state rather than a three-state Markov process that switches between \(G^L = 1.01\) during the Low growth state and \(G^H = 1.04\) during the High growth state with transition matrix where the transition probabilities are chosen so that the expected duration of the high and low growth periods is 30 years, which roughly corresponds to the length of time Japan stayed in the high-growth regime (late 40’s to mid 70’s). In concord with this baseline transition matrix, our simulation results will track an economy over a 90 year period that consists of a 30-year-long period of slow growth followed by a 30-year-long period of fast growth followed by a switch back to the low-growth state for the final 30 periods.
Having specified the actual growth process, we now turn to the question of how consumers perceive that growth process. A truly realistic model of households’ perceptions about the income process would probably be one in which households engage in Bayesian updating with respect to their beliefs about which state the economy is in and about the values in the transition matrices. We present two sets of results, each of which corresponds to one of the extreme possibilities: either consumers always have perfect knowledge of the current state of the economy and the transition probabilities (consumers are ‘smart’) or they attach equal probability at all times to the probability that the economy is in each of the two states (consumers are ‘dumb’) and thus they always forecast growth to be equal to the mean of growth in the low state and growth in the high state.

A last detail is how we specify the initial state of our economy at the beginning of the simulations. The usual procedure is to simply simulate the model for a long ‘presample’ period and then to begin monitoring it at the beginning of the period that is defined as the ‘sample’ period. Our procedure yields similar results but is more efficient: in the first period of ‘life’ of our economy we endow all of the consumers in our model with a stock of cash-on-hand and a habit stock equal to the ‘target’ values that are implied by their consumption rules and their expectations. Specifically, the target value \( x^* \) for \( x_t \) is defined as the value of \( x \) such that \( E_t[\tilde{x}_{t+1}] = x_t \), and similarly \( h^* \) is the value of \( h_t \) such that \( E_t[\tilde{h}_{t+1}] = h_t \). Actually, it is necessary to define \( x^* \) and \( h^* \) jointly, because the value of each variable affects the value of the other. Thus

\[
\begin{align*}
x_{t+1} & = \left( R/G_{t+1}N_{t+1} \right) (x_t - c_t) + \epsilon_{t+1} \\
E_t[\tilde{x}_{t+1}] & = E_t[R/\tilde{G}_{t+1}] \exp[\sigma_N^2](x_t - c_t) + 1 \\
x^* & = E_t[R/\tilde{G}_{t+1}] \exp[\sigma_N^2](x^* - c[x^*, h^*]) + 1 \quad (33)
\end{align*}
\]

where we use the fact that if \( \log N \sim N(-\sigma_N^2/2, \sigma_N^2) \) then \( E[(1/N)] = \exp[\sigma_N^2] \).

We also can write

\[
\begin{align*}
h_{t+1} & = \frac{h_t + \lambda(c_t - h_t)}{G_{t+1}N_{t+1}} \\
E_t[\tilde{h}_{t+1}] & = E_t \left[ \frac{(1 - \lambda)h_t + \lambda c_t}{\tilde{G}_{t+1}N_{t+1}} \right] \\
h^* & = ((1 - \lambda)h^* + \lambda c[x^*, h^*]) \exp[\sigma_N^2]E_t[1/\tilde{G}_{t+1}] \quad (34)
\end{align*}
\]

and the target values of \( x^* \) and \( h^* \) are those values that satisfy equations (33) and (34) simultaneously.

### 3.1 Dumb Consumers

As will be evident shortly, the actual average amount of wealth held by consumers in this economy tends to be fairly close to the target stocks defined by equations
(33) and (34). This is convenient because it allows us to use the expressions for the targets to gain insight about the behavior of the aggregates.

For example, consider a consumer in the baseline model (without habits) who happens to hold exactly what he perceives to be his target stock of wealth in period $t$, $x_t = x^*$. But recall that the 'target' is wealth is defined with respect to the expectation about the growth rate of permanent income $G$, which dumb consumers perceive to be given by $\Pr(G_{t+1} = G_H) = \Pr(G_{t+1} = G_L) = .5$. Thus, when the economy is in the 'high' state, the actual mathematical expectation of $E_t[1/G_{t+1}|\text{Truth}] < E_t[1/G_{t+1}|\text{Dumb}]$. It follows that $E_t[\tilde{x}_{t+1}|\text{Truth}] < E_t[\tilde{x}_{t+1}|\text{Dumb}] = x^*$. We will call this story henceforth the 'G shrinks $x$' effect.

What can we conclude about the saving rate on the basis of the ‘$G$ shrinks $x$’ effect? We know from Carroll and Kimball (1996) that the consumption function for this problem is strictly concave with a slope greater than the slope in the perfect certainty case, and so we know that reducing wealth will reduce consumption by more than the reduction in interest earnings. Hence the saving rate for this consumer next period will be higher than the saving rate in the current period. In other words, faster-than-expected growth in permanent income shrinks $x_t$ which boosts precautionary saving. The converse logic holds when the economy is in the slow-growth regime. Hence for the economy with dumb consumers we should expect to see the saving rate rise when the economy switches to the high-growth regime and fall in the slow-growth regime.
Figure 1 presents the results when we initialize a population of 5000 agents with the target value of $x^*$ in period 1 and then simulate the 90 year sequence described above. The first substantive conclusion is thus that the ‘$G$ shrinks $x$’ effect is virtually negligible: While the saving rate is indeed higher in the fast-growth than in the slow-growth regime, the difference in saving rates is so small that it is difficult to detect.

Now consider the theory for what we should expect to see in the habit-forming economy. The ‘$G$ shrinks $x$’ effect continues to hold in this model, so we can continue to expect a modest boost to the saving rate from this effect. However, there is another effect in the model with habits. Recall that habits evolve according to

$$ h_{t+1} = \frac{h_t + \lambda (c_t - h_t)}{GN_{t+1}} $$

(35)

Note that exactly the same logic holds with respect to the habits-to-permanent-income ratio $h_t$ in this equation as held for the wealth-to-permanent-income ratio in the ‘$G$ shrinks $x$’ effect: faster growth will shrink the habits ratio. Furthermore, the effect of a lower level of $h_t$ on $c_t$ is similar to that of lower $x_t$: lower habits ‘drag down’ consumption and thereby boost the saving rate. We will call this the ‘$G$ shrinks $h$’ effect.

The simulation results for the economy with dumb habit-forming consumers (where again all 5000 consumers are initialized in period 1 with $\{x_1, h_1\} = \{x^*, h^*\}$)
are depicted in figure 2. In contrast to the results for the dumb non-habit-formers, in Figure 2 the increase in the saving rate is huge: the gross saving rate goes from around 16 percent to around 28 percent. Thus the ‘$G$ shrinks $h$’ effect is apparently quantitatively much more important than the ‘$G$ shrinks $x$’ effect, and as a result the saving rate increases sharply in the wake of the increase in growth.

### 3.2 Smart Consumers

Before examining the results that obtain when consumers are ‘smart’ in the sense that they correctly perceive the aggregate income process, another theoretical excursion will be helpful. The standard perfect-foresight permanent income model of consumption without uncertainty implies that the level of consumption is given by

$$C_t = (1 - R^{-1}(R\beta)^\text{IES})(K_t + \frac{P_t}{1 - G/R})$$  \hspace{1cm} (36)$$

$$c_t \approx (r - \rho^{-1}(r - \theta))\left[k_t + \frac{1}{r - g}\right]$$  \hspace{1cm} (37)$$

where IES stands for the intertemporal elasticity of substitution \(1/\rho\), \(\theta = 1/\beta - 1\), and the approximation holds for ‘small’ \(\{r, g, \theta\}\). If we set the aggregate value of \(k = 4\) and assume a value of \(r = .03\) and \(g = .01\) it is clear that the elasticity of consumption with respect to the growth rate of income will be colossal. This enormous ‘human wealth effect’ is a well-known problem with the perfect foresight/certainty equivalent life cycle model of consumption, emphasized, for example, by Tobin (1967), Carroll and Summers (1991), and Deaton (1992);\(^6\) Viard (1993) has shown that a similar problem afflicts models of aggregate consumption that rely on the permanent income hypothesis.

There was no human wealth effect of switching from $G^L$ to $G^H$ and back in the simulations for dumb consumers, because the definition of ‘dumb’ was that their expectations of $G$ did not change when the actual aggregate state changed. However, the human wealth effect will clearly exist for smart consumers who do understand the aggregate growth process. What is not so clear \textit{a priori} is whether the human wealth effect will be large or small: Carroll (1997) has shown that the introduction of uncertainty can reduce the human wealth effect from enormous to negligible if consumers are assumed to be sufficiently impatient, so we must turn to simulation results to see whether the effect is large or small in this particular context.

Figure 3 shows the time path of the aggregate gross saving rate for smart consumers without habits. The transition to the fast growth regime which occurs

\(^6\)Although some authors, notably Horioka (1997, 1997), have argued that movements in the Japanese saving rate can be explained by demographic trends, recent work by Deaton (1997, 2000) finds little support for the proposition that demographic differences can explain cross-country saving differentials, a finding confirmed by Rodrik (1999).
in period 30 results in a dramatic plunge in the saving rate, indicating that the human wealth effect is quite strong: when consumers learn that they are going to have vastly higher incomes in the future, they go on a spending spree. Over the next twenty years the saving rate gradually climbs back up somewhat, but it never reattains the level that prevailed before the economy shifted to the high-growth regime.

When the economy switches back to the low-growth regime in period 60, the aggregate saving rate instantly leaps upward, corresponding to the inverse human wealth effect: consumers suddenly learn that they will be much poorer in the future than they had previously anticipated, so their consumption plummets.

The next figure shows what happens when consumers are again smart but now have habit-forming utility. The intial plunge in the saving rate when the economy switches into the high-growth regime is much smaller than the plunge that happened in the non-habit-forming economy. Furthermore, within a few years after the switch, the gross saving rate has regained and subsequently exceeds its rate in the low-growth regime.

The logic behind the differences here is somewhat subtle. One’s first intuition is that habits must somehow directly reduce the human wealth effect. This turns out to be false. Carroll (2000) shows that in this model of habit formation the
The formula for consumption in the perfect-foresight version of the model is

$$C_t = (1 - R^{-1}(R \beta)^{IHIES})(K_t + \frac{P_t}{1 - G/R}).$$  \hspace{1cm} (38)

where IHIES stands for the Infinite Horizon Intertemporal Elasticity of Substitution. But remember that in parameterizing the habit formation model we chose the parameter values precisely to keep the infinite horizon coefficient of relative risk aversion in the habits model the same as in the baseline model without habits, and since the IHIES is just the inverse of the infinite horizon CRRA, IHIES = IES and therefore the level of consumption implied by equation (38) is identical to the level of consumption that would occur in the intertemporally separable model, equation (37). Thus, the human wealth effect is exactly as strong in the perfect certainty version of the habit formation model as in the perfect certainty version of the model without habits!

The only possible conclusion is that the differences in outcomes between the two models (and in particular, the ability of the model with habits to explain the positive causality from growth to saving) have to do with the effects of uncertainty on the precautionary saving motive. With a $\gamma$ of 0.8, it was necessary to assume that $\rho = 11$ in order to make the long-term intertemporal elasticity of substitution in the habit formation model match the IES in the baseline model. Thus consumers in this model have an extremely strong aversion to high-frequency fluctuations in consumption and consequently have an extremely strong precautionary saving
motive with respect to high frequency risks. As noted earlier, Carroll (1997) shows that the precautionary saving motive can vastly decrease the size of the human wealth effect for impatient consumers, essentially because such consumers are not willing to spend today on the basis of their expected mean level of future income, because of the risk of catastrophically low utility if they fail to save and they experience a bad income shock.
References


