Government Accounting and the Government Budget Constraint

### Definitions

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
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<tbody>
<tr>
<td>G&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Government spending on everything except transfers and interest</td>
</tr>
<tr>
<td>T&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Net taxes and other revenues (taxes minus transfers)</td>
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<tr>
<td>B&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Government bonds (debt) at the beginning of period t</td>
</tr>
<tr>
<td>r</td>
<td>Interest rate on government bonds, paid in period t</td>
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<tr>
<td>R</td>
<td>(1+r)</td>
</tr>
<tr>
<td>PDEF&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Government primary deficit = (G&lt;sub&gt;t&lt;/sub&gt; - T&lt;sub&gt;t&lt;/sub&gt;)</td>
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<tr>
<td>DEF&lt;sub&gt;t&lt;/sub&gt;</td>
<td>Government total deficit = (G&lt;sub&gt;t&lt;/sub&gt; + rB&lt;sub&gt;t&lt;/sub&gt; - T&lt;sub&gt;t&lt;/sub&gt;)</td>
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### Variables

The government budget deficit is financed by selling government bonds. The government must sell enough bonds so that the proceeds will pay for the current spending it cannot pay for with tax revenue (this fact is called the ‘government budget constraint,’ which just says that the government must obtain the money it spends either by taxation or by borrowing). Thus the stock of government bonds accumulates according to:

\[
B_{t+1} - B_t = \text{DEF}_t \\
= (G_t + rB_t) - T_t \\
B_{t+1} = G_t + RB_t - T_t \\
B_{t+1} + (T_t - G_t) = RB_t \\
B_t = B_{t+1}/R + (T_t - G_t)/R \\
\]

but

\[
B_{t+1} = B_{t+2}/R + (T_{t+1} - G_{t+1})/R \\
\]

so

\[
B_t = |B_{t+2}/R + (T_{t+1} - G_{t+1})/R|/R + (T_t - G_t)/R \\
= (T_t - G_t)/R + [T_{t+1} - G_{t+1})/R]/R + ... \\
- B_t = PDEF_t/R + [PDEF_{t+1}/R]/R + ... \\
which means that \\
\[
RB_t = PDV_t(T) - PDV_t(G) \\
\]

But the expression on the right just represents the present discounted value of future primary surpluses (a primary surplus is the negative of a primary deficit). What this equation says is that future primary surpluses must be large enough to pay off the current stock of debt. That is, the government can’t default on its bonds.

Note that there is no constraint on what the government can do in any particular single period. It can make G<sub>t</sub> as large as it wants, or run a deficit as large as it wants, in any single period. This means that the government can effectively make future generations pay for the spending of current generations.
Defining Generational Accounts

Consider a very simple model of government taxes and transfers. In this model, consumers have only two “stages” of life, which we will call “young” and “old.” While they are “young” consumers work and earn money, some of which they save to finance their spending when they are “old” and retired. For convenience, assume for that all generations are the same size. Formally, define:

\[ \tau_{y,t} \] - Per capita taxes minus transfers for generation young in period t
\[ \tau_{o,t} \] - Per capita taxes minus transfers for generation old in period t

Generally, young people pay more in taxes than they receive in transfers, so \( \tau_y \) is a positive number, while old people receive more in transfers (such as Social Security benefits) than they pay in taxes, so \( \tau_o \) is a negative number.

The “Generational Account” for the generation that is young in period t is defined as the present discounted value of the taxes they will pay in their lifetime, minus the present discounted value of the transfers they will receive from the government:

\[ GA_t = \tau_{y,t} + \tau_{o,t+1}/R \]

Now we will show that it is possible to rewrite the government’s budget constraint in terms of generational accounts. Call total net revenues received by the government \( T_t \). Obviously total net revenues at a point in time equal the sum of the net revenues from the young and net revenues from the old:

\[ T_t = \tau_{y,t} + \tau_{o,t} \]

The present discounted value of net revenues over the future can be rewritten as:

\[
P DV(T_t) = T_t + \frac{T_{t+1}}{R} + \frac{T_{t+2}}{RR} + ... \]
\[= \tau_{y,t} + \frac{\tau_{y,t+1}}{R} + \frac{\tau_{y,t+2}}{RR} + ... \]
\[+ \tau_{o,t} + \frac{\tau_{o,t+1}}{R} + \frac{\tau_{o,t+2}}{RR} \]
\[= \tau_{o,t} + \left[ \tau_{y,t} + \frac{\tau_{o,t+1}}{R} \right] + \left[ \tau_{y,t+1} + \frac{\tau_{o,t+2}}{R} \right]/R + ... \]
\[= \tau_{o,t} + GA_t + \frac{GA_{t+1}}{R} + \frac{GA_{t+2}}{RR} + ... \]

What does this exercise in algebra teach us? Basically, that the government can treat any single generation as well or as badly as it likes, if it makes up for what it does by treating other generations differently. In particular, the government has the ability to confer a large benefit on, say, a generation that is voting today, and to pay for that benefit by offsetting unfavorable changes in the generational accounts of future generations.
Generational Accounting Analysis of PAYG Social Security

Suppose that before period t there had been no Social Security system and no government programs of any kind, and then in period t the government introduces a Pay As You Go (PAYG) Social Security system which it says will remain the same size (per capita) forever. PAYG means that the Social Security taxes from the young in period t are used to pay the Social Security benefits received by the old in the same period.

In our notation from the Generational Accounting framework, call the net tax paid by the young generation at time t $\tau_{y,t}$ and the net tax paid by the old $\tau_{o,t}$. Then the PAYG Social Security system is characterized by a single figure, $\tau^*$, the size of the taxes:

$$\tau^* = \tau_{y,t} = -\tau_{o,t} = \tau_{y,t+1}... \text{ and so on.}$$

The equality $\tau_{y,t} = -\tau_{o,t}$ just expresses the fact that taxes paid by the young are equal to benefits received by the old.

First let’s figure out the effects of the Social Security system on the Generational Accounts of different generations. Consider first the generation that was young in the period before the Social Security system was introduced. This generation paid no taxes into the system, but receives the same benefits as all the future generations who will pay into the system. The generational account for this generation is therefore:

$$GA_{t-1} = \tau_{y,t-1} + \tau_{o,t} / R = -\tau^*/R$$

Since generational accounts represent net taxes, this generation’s negative generational account means that it is better off by the amount $\tau^*/R$.

Now consider the following generations. Their generational accounts will be given by:

$$GA_t = \tau_{y,t} + \tau_{o,t+1}/R$$  \hspace{1cm} (11)

$$= \tau^* (1 - 1/R)$$  \hspace{1cm} (12)

$$= \tau^* (R/R - 1/R)$$  \hspace{1cm} (13)

$$= \tau^* (R - 1)/R$$  \hspace{1cm} (14)

$$= \tau^* (r/R)$$  \hspace{1cm} (15)

Obviously, as long as the interest rate is positive, this generation is worse off, because $\tau^*, r, \text{ and } R$ are all positive numbers. The generational account will be identical for all future generations, so all future generations are worse off. Thus, the unfunded benefits received by the first generation are essentially paid for by making all future generations permanently worse off. Why are the young generations made worse off by introducing the Social Security system? Because it takes away from them an amount $\tau^*$ when young and returns only the same $\tau^*$ when old. If that money had been invested (rather than handed to the elderly), it would have earned interest $r \tau^*$. So the young generations are worse off by an amount equal to the present discounted value of the lost interest on their Social Security contributions.
Generational Accounting Analysis of Funded Social Security

Suppose the government introduces a “Fully Funded” Social Security system which it says will remain the same size (per capita) forever. “Fully Funded” means that the government takes the Social Security taxes paid by workers and puts the money aside (saves it) on behalf of the contributor; old people get benefits only based on how much they paid into the system while working plus the accumulated interest earnings on those contributions.

In our notation from the Generational Accounting framework, again call the net tax paid by the young generation at time $t$ $\tau_{y,t}$ and the net tax paid by the old $\tau_{o,t}$. Because the system is funded, now the benefits received by the elderly include interest, so:

$$\tau_{o,t+1} = - R \tau_{y,t}$$

First let’s figure out the effects of the Social Security system on the Generational Accounts of different generations. Consider first the generation that was young in the period before the Social Security system was introduced. This generation paid no taxes into the system, and will receive no benefits, so their generational account is zero.

Now consider the following generations. Their generational accounts will be given by:

$$GA_t = \tau_{y,t} + \tau_{o,t+1} / R = \tau_{y,t} - \tau_{y,t} = 0$$

So their generational account is zero too. In fact, the GA’s of all generations are left unchanged by the funded Social Security system.

Why does this result differ from the unfunded system? The crucial difference is that in the unfunded system, no money is actually saved, and therefore no interest is earned; in the funded system, the government sets aside the tax revenues and saves them, and so it does earn interest.